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Resistive Forces against the Motion of a Model Car rolling on Sand (III)

(Received March 5, 1954)

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Abstract

Improvements were added to the experimental apparatus used before so as to take the simultaneous measurements of the horizontal and vertical components of the resistive force acting upon a wheel rolling on sand dried in natural state and also the depth of the sinking of the wheel in sand. The range of measurement was widened at the same time.

By the use of these new apparatus changes of the constants K and n which appear in the empirical formula $H=KV^n$, which relates the horizontal components H and the vertical component V of the resistive force acting upon a disc-shaped wheel were mesasured under different conditions of the experiment, namely the diameter and the thickness of the wheel and the packing state of the sand. The results obtained were applied to the study of the relationship between the traction resistance of a car and the position of its gravity center.

I Apparatus and Methods of Measurement

Just as it was in the previous experiments, a sand box ($75\times300\times45$cm^3) filled with sand was slid on (lower) rails instead of moving the car. The sand was raked with a fork and its surface was smoothened every time before each cycle of the measurement to give a virgin state.

The apparatus shown in Fig. 1 is suspended by a hook C and a spring balance S_V, keeping the horizontal state of the base plate by means of levels L_1, and L_2, the reading of S_V, which is the load acting upon the wheel is regulated by the movement of the movable poise Q. After that, the apparatus is removed from the hook and placed on the sand while the rollers R_1 and R_2 are mounted on the upper rails.

A part of the weight of the whole apparatus is supported by the wheel W, the amount of which is measured by S_V, and the rest is supported by R_1 and R_2.

The counter-weight G is used to cancel the frictional resistance of the latter.

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During the motion of the sand box toward the left in the figure, the screws A_1 and A_2 are turned so that the distance between the base of the apparatus and the rollers becomes appropriate to keep the base and the pulling rope horizontal; naturally the pulley P is adjusted also to come to a proper position. Thus when the motion of the wheel levels off and the base of the apparatus and the rope are horizontal, the reading on the spring balance S_H indicates the horizontal component of the resistance acting upon the wheel.

The readings on scales attached to the screws A_1 and A_2 show the depth of sinking of the wheel in the sand. The sand used in this experiment was the No. 5 of the previous report (the mean diameter 0.026 cm, angle of repose 28.5°, apparent density 1.27–1.47) and the velocity of the wheel was 4.1 cm/sec.

II Results

1) The functional relation among the horizontal compoment H, the vertical component V and the depth of sinking h in the sand,

$$H=K'h^{n'} \qquad V=K''h^{n''} \qquad H=KV^{n}$$

where K', K'', K, n', n'', n are all constant, holds for the wheel of Fig. 2 are Table 1,

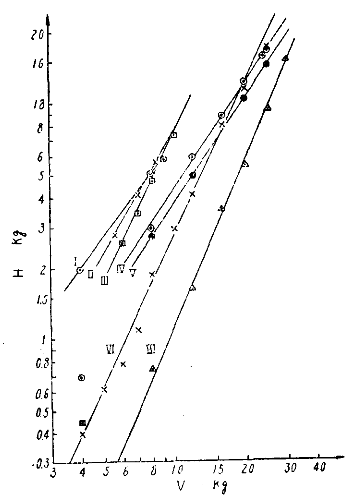

Fig. 2. Relations between H and V under some conditions shown on Table 1

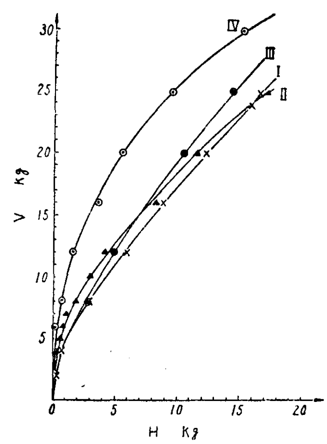

Fig. 3. Relations between H and V under some conditions shown on Table 1

Generally speaking, the resistance acting upon a wheel is considered to be composed of two parts: the one acting upon the central part of the cylindrical surface of the wheel and the other acting upon the side plane and its nearby positions.

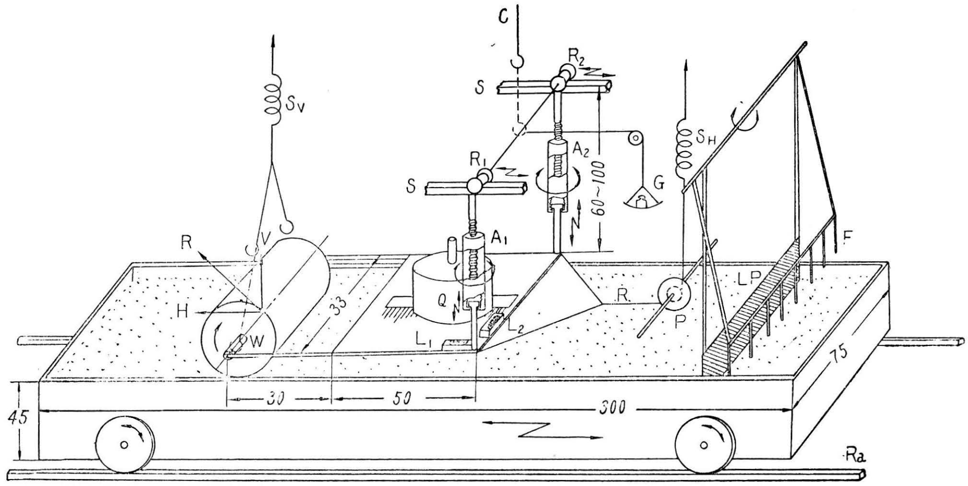

Fig. 1_{A1}. Apparatus (Unit cm.)

H.	horizontal component	S_H.	spring balance to measure *H.*	*Q.*	movable poise	*S.*	support
V.	vertical component	S_V.	spring balance to measure *V.*	R_1,& R_2.	roller	*F.*	fork
R.	resistance	L_1& L_2.	level	*R.*	rope	*LP.*	leveling plate
		W.	wheel	A_1& A_2.	adjusting screw		
		P.	pulley	*G.*	balancing weight	*Ra.*	rail

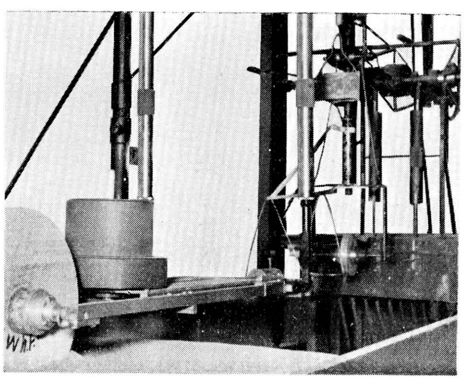

Fig 1_{A_2}. Apparatus

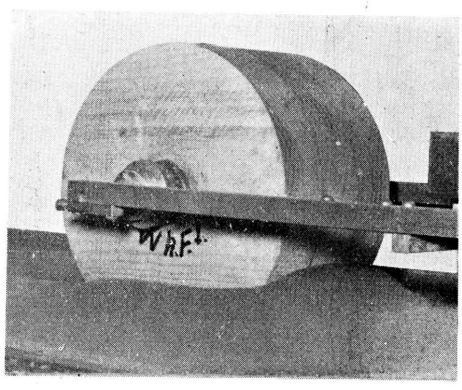

Fig 1_B. A wheel travelling on Sand

Curve		wheels (cm) Thickness	wheels (cm) Diameter	n
⊙	I	5	20	1.42
×	II	7	20	1.73
□	III	20	20	2.20
⊙	IV	5	28.5	1.48
●	V*	5	28.5	1.46
×	VI	10	28.5	2.11
◬	VII	20	28.5	2,30

Table 1. The value of n in the formula $H=KV^n$, for each curve in Fig. 2

* In the case of sand pressed under wheel, 20cm. in thickness and 28.5cm. in diameter, loaded to 30Kg

Curve		Wheels (cm) Diameter	Wheels (cm) Thickness
×	I	28.5	5
▲	II	28.5	10
●	III*	28.5	5
⊙	IV	28.5	20

Table 2. Cvrves on Fig. 3

V(kg)	H (kg) Thickness T and Diameter D of wheels: T=5cm, D=28.5cm	T=10cm, D=28.5cm	T=20cm, D=28.5cm	T=5cm, D=20cm	T=20cm, D=15cm	T=7cm, D=20cm	* T=5cm, D=28.5cm
2	0.35			0.37			
4	0.70	0.40		2.01			
5		0.62		5.1	0.91		
5.6						2.80	
6		0.80	0.21		2.61		
7		1.11			3.51	4.18	
8	2.99	1.90	0.76	5.11	4.74		2.75
8.4						5.74	
9					5.89		
10		2.97			7.44		
12	5.93	4.17	1.64				4.95
16	8.76	8.18	3.60				
20	12·19	11.55	5.54				10.44
24	15.89						
25	16.61	17.06	9.48				14.27
30			15.16				

Table 3. Measured H and V plotted on Fig. 2 and Fig. 3

* In the case of Pressed sand under 30kg. loaded wheel 20cm. in thickness and 28.5cm in diameter

In the case of a thick wheel it may be presumed that the proportion of the latter becomes small while the former contributes the main part resulting in the liner proportionality of the resistance to the thickness. It can be seen from Fig. 2 and Table 1 that though the values of n are rather different between the wheels with thickness 5cm and 10cm respectively and with the same diameter, the difference becomes very small between the wheels with thickness 10cm.

The difference between the resistances of two wheels with the same diameter and different thickness were calculated and plotted in Fig. 4 and given in Table 4. Suffixes I, II and III show that the V and H values came from the wheels of thickness 5, 10 and 20cm respectively, the diameter being 28.5cm in common.

We may regard these differences as composed purely of that part of the resistance which is proportional to the length of the central part of the wheel surface measured along the the axle, free from that part of resistance acting at and near the side planes of the wheel.

Therefore, when we plot the difference of resistance in each case against those of logarithmically shown in Fig 4, the gradient may be considered to represent the value of n at the thickness very large compared with the diameter, and moreover, to be the maximum value of n since n does not change with diameter.

This value was measured to be about 2.7.

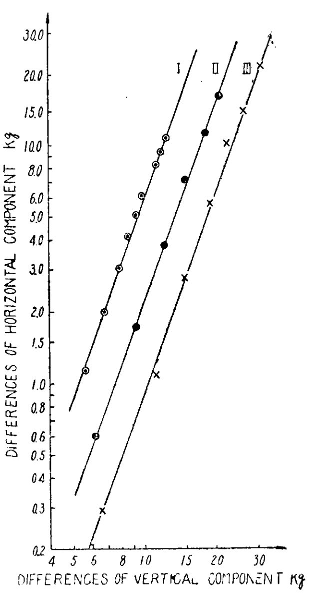

Fig 4.

Curve		Vertical	Horizontal
⊙	1	$V_{II} - V_{I}$	$H_{II} - H_{I}$
●	2	$V_{III} - V_{II}$	$H_{III} - H_{II}$
×	3	$V_{III} - V_{I}$	$H_{III} - H_{I}$

Table 4. Curves on Fig. 4

2) Fig 2 and Table 1 show that n is nearly the same for wheels with the same thickness and different diameter. This fact can be seen also from the experimental of the traction resistance of cars, if we regard the constancy of n against

the change of the packing state of sand.

3) The packing state of the sand does not change n. Since the traction resistance and the weight of W of a car equipped with the same type wheels stand in the relation $F=kW^{\nu}$ and the equation $H=KV^{n}$ holds for a single wheel, the following arguments results. For a 4 wheeled car,

$$H_i=K_iV_i^{n_i},\ \sum_{i=1}^{4} V_i=W,\quad \sum_{i=1}^{4} H_i=F,\quad F=kW^{\nu}. \tag{1}$$

where the suffix i runs from 1 to 4 representing each wheel.

If we take $V_i=W/a_i$ where $\sum_{i=1}^{4} 1/a_i=1$ then the relations

$$H_i=\frac{K_i}{a_i^{n_i}}W^{n_i}\text{ , therefore }\sum_{i=1}^{4}\frac{K_i}{a_i^{n_i}}W^{n_i}=kW^{\nu} \tag{2}$$

must hold, which means that $n_i=\nu$. Thus n never changes among eaeh wheel, even when the front and rear ones face different states of packing of the sand.

In order to verify the above conclusion, the sand in a virgin state was pressed in advance by rolling a wheel with diameter of 28.5cm and thickness of 20cm under the load of 30kg and thereupon the reistance upon a wheel was measured, resulting to the curve of Fig. 2 and Table 1 which does not show any change of n as clarified by the comparision with the curve IV.

4) Position of the gravity center of a car. Since the curves I and II in Fig. 3 intersect each other, the equation,

$$\frac{\partial H_1}{\partial V_1}=\frac{\partial H_2}{\partial V_2} \tag{3}$$

Which gives the position of the gravity center, *G. C.*, corresponding to the minimum value of the traction resistance, (Suffixes 1 and 2 indicate the front and rear wheels respectively), shows that this position may lie in front of that of *G. C.* at which each wheel carries equal load. Appropriate selection of the thickness of the wheels makes this possible. Actually these positions were observed (refer to Fig. 6, Table 5, Table 6, Curves IV and V).

The following is the calculation of the position of *G. C.* for the minimum value of the traction resistance. Substituting the equations

$$H_i=K_iV_i^{n_i},\ V_1+V_2=W/2 \tag{4}$$

in (1), we get, $n_1K_1V_1^{n_1-1}=n_2K_2V_2^{n_2-1}$,

or, $(n_1K_1/n_2K_2)^{1/(n_2-1)}\cdot V_1^{(n_1-n_2)/(n_2-1)}=V_2/V_1$. (5)

If the coefficients n_i and K_i are known which depend on the conditions of the wheels and the sand, the ratio of V_2/V_1 or the position of *G. C.* for the minimum traction resistance can be determined by (5) and the corresponding load W is calculated by (4). Therefore graphical plot of the relation between V_1 and W allows the determination of V_1 against a given load W at once.

Curve No.	Weight (Kg)	Wheels: Diameter (20cm), Thickness (cm): Front	Rear
I	21.3	2	5
II	21.3	2	7
III	21.3	5	5
IV	30.0	7	2
V	16.4	7	2
VI	30.2	5	5
VII	15.0	2	2

Table 5. Curves on Fig. 6

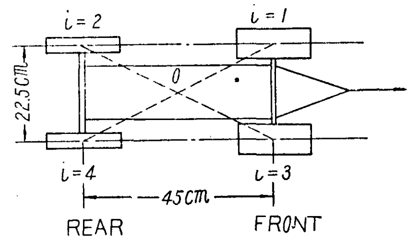

Fig. 5. Numbering for the wheels point 0 corresponds to the position of center of gravity in Fig. 6

	The Center of Gravity (cm)	Resistance F(kg): Front wheel $\{D=20cm, T=7cm\}$ Rear wheel $\{D=20cm, T=2cm\}$ $W=16.4kg$	Front wheel $\{D=20cm, T=7cm\}$ Rear wheel $\{D=20cm, T=2cm\}$ $W=30.0kg$	F. W. $\{D=20cm, T=7cm\}$ R. W. $\{D=20cm, T=2cm\}$ $W=15,0kg$	F. W. $\{D=20cm, T=5cm\}$ R. W. $\{D=20cm, T=5cm\}$ $W=30.2kg$
Front	12	8.55	19.74		
	11		19.13		
	10	8.23	19.15		
	9	8.03	18.67		
	8	7.87	18.52		18.82
	7.4			8.27	
	7	7.63	18.43		
	6.2			7.98	
	6	7.55	17.96		
	5.0	7.45	17.80	7.90	
	4.0	7.33	17.41		17.19
	3.1			7.63	
	3	7.25	17.23		
	2	7.19	17.21		
	1.2			7.35	
	1	7.08	17.32		16.72
	0	7.05	17.33	7.18	16.50
Rear	1	7.09	17.44		
	1.2			7.13	
	1.9			7.08	
	2	7.15	17.47		16.48
	2.5			7.10	
	3	8.28	17.51		16.58
	3.1			7.14	
	4	7.40	17.95		16.52
	5	7.51	18.04	7.26	
	6				16.90
	6.2			7.38	
	7	7.96			
	8				17.17
	8.7			7.86	
	12.0				19.62
	12.4			8.38	

Table 6. Relation between the tractive resistance and the position of the center of gravity of the car with four wheels

III Conclusions

a) Thickness of the wheel giving minimum traction resistance varies with load.

b) Position of the gravity center which minimizes the traction resistance may lie in front of that of the gravity center corresponding to the equal loading of the wheels.

c) Position of the gravity center for minimum traction resistance of a car with a given amount of load can be determined by the equation (5).

IV Acknowledgement

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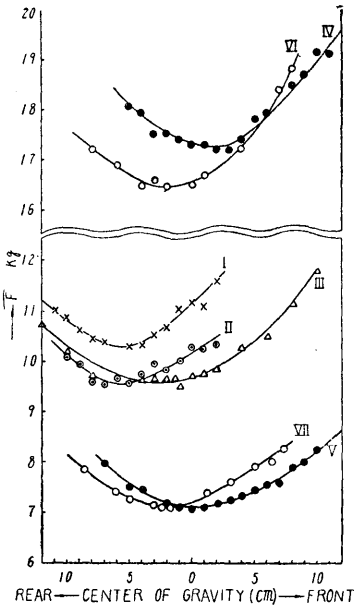

Fig. 6. Relations between tractive force F and position of center of gravity of a car with four wheels of different thicknesses shown on Table 5 and with some weight

1) Read at the meeting of the society of Applied physics, Japan Nov. 1953

2) I of this report and read at the meeting of the society of Applied physics, Japan Apr. 1953