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Resistive Forces against the Motion of a Model Car Rolling on Sand (II)

Kiyoshi TAKAHASHI*

I. Introduction

Though it is not yet made clear what is the resistance which acts on a car rolling on sand, the problem of the car resistance should be induced from that of the wheels. From the results of some experiments carried out on a single wheel, we succeeded to get an outline of the resistance problem of cars progressing on sand in general.

In these experiments described below, the vertical and horizontal components of the resistance which represent the relations between the magnitude and the direction were measured as function of the depth of sinking in sand. And by the use of that relation, the relations between the two components were determinted.

Additionally, the dependency of the above relations on the velocity of the wheel, the size of the sand particles and the packing state of sand were investigated.

On the other hand, the revolution of the wheels depends on various conditions and there exists some amount of slip between the wheel and the sand. Experiments carried out on the slip problem are also reported[1].

II. Apparatus and methods of the resistance measurement

The sand used in this experiment was sieved out into 5 species and dried fully. Reposing angle, apparent density and the mean diameter of each species are given in Table 1.

Table 1. Apparent specific gravity, angle of repose, and average diameter of sand

Sand (No.)	Apparent specific gravity	Angle of repose (°)	Average diameter of sand (cm)
1	1.55 ～ 1.60	39.	0.43
2	1.50 ～ 1.58	38.5	0.31
3	1.46 ～ 1.57	37.	0.17
4	1.40 ～ 1.55	35.5	0.07
5	1.27 ～ 1.47	29.5	0.026

A sand box (70×300×45 cm) filled withsand wasslide on horizontal rails at a constant velocity. The force needed to hold the wheel at a determined positon

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1) Repoted at the Session of the Oyo-Butsuri-Gakkai held in April and November, 1951.

with some amount of sinking in sand, which represents the resistance, was measured.

As shown in Fig. 1, the axle of the wheel O was fixed into a frame having the shape of the letter C to which the bar AB was connected.

At the approximate central portion of AB was attached the axle P parrallel to the axle of the wheel, which in turn was fixed another C-type frame. The bar CD was connected to this frame and at the center of PD the axle Q was fixed to the columes E and F.

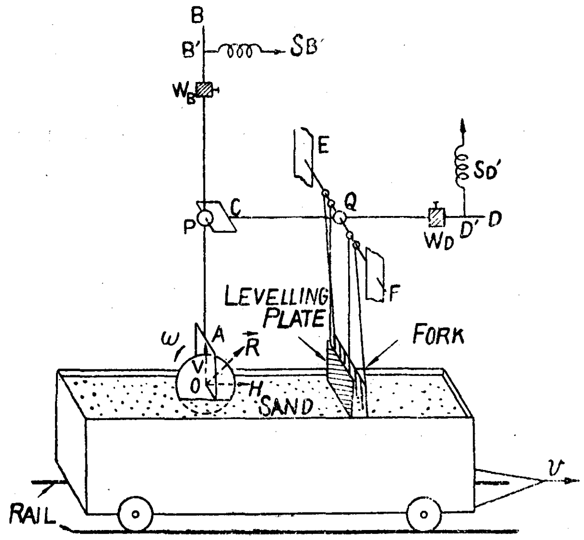

Fig. 1. Apparatus

To keep the packing state of sand constant, the fork and a levelling plate were installed in the apparatus which were to rake and flatten the surface of the sand when the box proceeded backwards. At the beginning of each step of the experiments the sand was made to be in a constant virgin state raked soft and flattened.

The velocity of the car was 1.1 cm/sec.

First of all, the weight W_B was moved so as to place the center of gravity of the part OB at the center of P and then PD was fixed horizontally. At a given depth of the wheel sunk in sand the box was moved forwords with the wheel being kept in a balanced position between the torque given by the resistance of sand and that applied by the spring balance SB' at the point B' near B. The readings of the balance was read while keeping OB vertical.

The total resistance R given by the sand on the wheel is, since the friction of the axle is negligibly small, directed normally towards the alxe.

The torque given by R at P is the same as that given by the horizontal component of R. Accordingly, the horizontal component H of R is given by the equation.

$$H = \frac{PB'}{OP} f_1$$

where f_1 is the reading of $S_{B'}$. H is therefore measurable.

For the measurement of the vertical component of R, after the coincidence of the center of gravity of the part OB with p by the sliding the weight W_B, OB and PD were fixed at right angle and the center of gravity of the whole system OB and PD is made to coincide with Q by the weight W_D. In the movement of the sand box, the torque caused by R at Q is counterbalanced by the spring balance $S_{D'}$ placed at right angle at D' near D. The reading f_2 is made on $S_{D'}$ when PD is in a horizontal position.

The sign of f_2 is chosen to be positive when acting towords upside and negative when acting towords downside in Fig. 1, The vertical component V of R given by the following equation,

$$V = \frac{H \cdot OP + f_2 \cdot QD'}{PQ}$$

Two wheels were used. Both were made of iron disc with diameter of 31.6 cm. The thickness were 0.83 cm and 0.63 cm. The length of OP, PB, PQ and QD are as follows:

$$OP = 66.0\,\mathrm{cm} \qquad PQ = 47.8\,\mathrm{cm}$$

$$PB' = 66.0\,\mathrm{cm} \qquad QD' = 72.7\,\mathrm{cm}$$

To measure the revolution velocity of the wheel, after OB and PD were fixed at right angle and PD horizontal, the distance S which was travelled by the sand box with velocity v in a single cycle of the wheel should be measured. Let the angular velocity of the wheel be ω,

$$\omega = \frac{2\pi}{S/v}$$

which makes possible the determination of ω.

III. Results

1. The relation between R and the depth h of the sinking of the wheel in sand, the relation between the horizontal component H and the vertical component V.

The relation of the logalithmic plots of H and against h, all of them were found to be linear as shown in Fig. 2 in the experimental range where h was below about 70% of the wheel radius.

Therefore, H and V stand in the relation with h roughly as

$$H = K_1 h^{n_1}$$

$$V = K_2 h^{n_2}$$

where K_1, K_2, n_1 are n_2 are constants differing in each experiment as given by the Table 2.

From the relation between H and h, and V and h follows that between H and V as shown in Fig. 3, a sort of the vector diagram for R with h being the parameter.

Table 2. Constants K_1,K_2,n_1 and n_2 of the equation $H=K_1h^{n_1}$, or $V=K_2h^{n_2}$ shown in Fig. 2

Curve	Thickness (cm)	Sand (No.)	Component (kg)	Depth (cm)							K_1,K_2 × 10^2	n_1,n_2
				2.5	4	5.5	7	9	11	11.7		
I ×	0.83	2	V	1.239	2.416	3.789	5.090	8.135	11.16	—	31.9	1.46
II ◎	〃	5	V	1.037	1.881	2.854	3.847	5.513	6.939	—	31.6	1.29
III ×	〃	2	H	0.303	0.730	1.434	2.233	4.284	6.596	—	3.94	2.13
IV ◎	〃	5	H	0.261	0.649	1.184	1.799	3.015	4.261	—	4.99	1.86
V ⚠	0.63	5	V	0.837	1.524	2.363	3.196	4.451	5.548	5.937	26.1	1.23
VI ⚠	〃	5	H	0.214	0.530	0.946	1.454	2.425	3.419	3.886	3.93	1.86

Since $H=K_1h^{n_1}$, $V=K_2h^{n_2}$ as dicussed above, the following relation holds for V and H. $V=KH^n$ where k and n are constants and

$$n=n_2/n_1, \quad K=(K_2/K_1)^{n_2/n_1}.$$

Furthermore as $n_1>n_2$, n_1, $n_2>0$ the H-V curve is convex upside. The H-V curve of Fig. 3 indicate that it has a tendency to become higher when the particles of the sand are large or the state of the sand becomes harder or the thickness of the wheel is large.

That tendency seems to appear also in the case where the velocity of the box is raised.

It is apparent from Fig. 2 and 3 that in the exprimental range $V>H$ and therefore R makes larger angle than 45° against the horizontal plane. Taking that angle to be θ,

$$\theta=\tan^{-1}\frac{V}{H}=\tan^{-1}\left(\frac{K_2}{K}h^{\,n_2-n_1}\right)$$

As $n_2-n_1<0$, θ become infinite at $h=0$ and decreases monotonously with

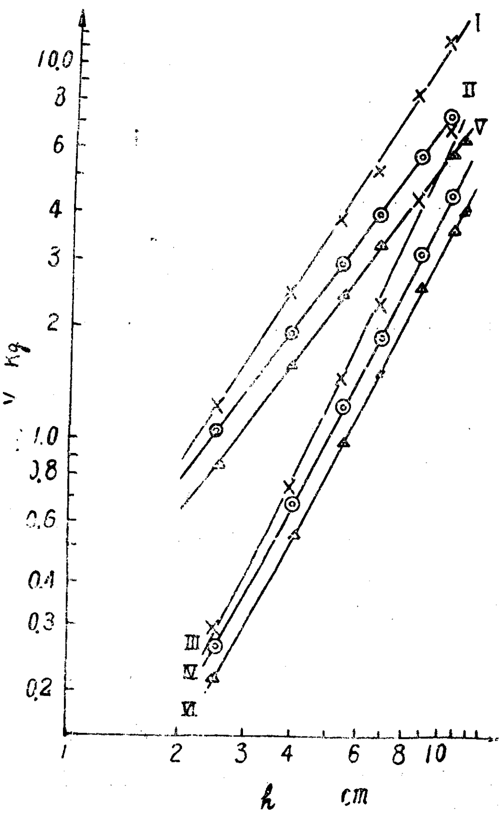

Fig. 2. Relations between Hrizontal or Vertical component H or V and depth h.

Table 3. Constants K and n of the equation $V=KH^n$

Sand (No.)	2	5	
Thickness (cm)	0.83	0.83	0.63
K	2.92	2.55	2.40
n	0.685	0.696	0.686

the increase of h

The dependency of R which is equal to

$$R=\sqrt{K_1^2 h^{2n_1}+K_2^2 h^{2n_2}}$$

and

$$\theta=\tan^{-1}\left(\frac{K_2}{K_1}h^{\,n_2-n_1}\right)$$

on h is shown in Fig. 4.

Since the relation between H and V have been clarified, we were able to explain the existence of a position for the center of gravity of a car which will give the least tractive resistance on the sand for a given load and the experimental results on the relation concerning the position.

Letting the weight of the car be W, the tractive resistance be F and the representative number of the wheel be i (for the four wheeled car $i=$ 1,2,3,4.)

$$W=\sum V_i \quad \cdots\cdots \quad (1)$$

$$F=\sum H_i \quad \cdots\cdots \quad (2)$$

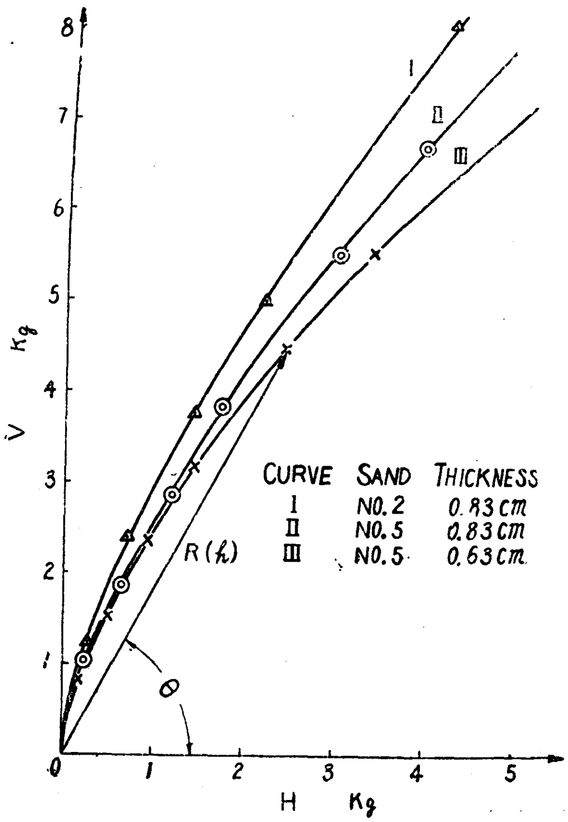

Fig.3. Vector diagram of R with parameter h

The condition for the minimum of F under the constant W becomes regarding H_i as a function of V_i,

$$\delta W=\sum\delta V_i=0$$

$$\delta F=\sum\frac{\partial H_i}{\partial V_i}\delta V_i=0$$

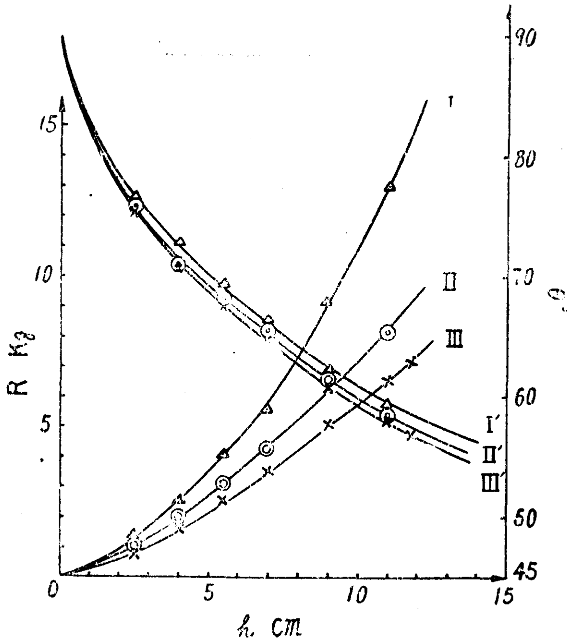

Fig. 4. Relations between adsolute value or angle respect to the horizon, of R and h

Introducing the Lagrange indeterminate multiplier λ,

$$\sum\left(\frac{\partial H_i}{\partial V_i}+\lambda\right)\delta V_i=0$$

$$\therefore \frac{\partial H_i}{\partial V_i}+\lambda=0$$

$$\therefore \frac{\partial H_i}{\partial V_i}=\frac{\partial H_2}{\partial V_2}=\cdots\cdots \quad \cdots\cdots(3)$$

The position of the center of gravity (load distribution) where the tractive resistance becomes minimum, is determined by (3) with the results coinciding with the ex perimental results[2].

2. Relation between the resistance and the velocity.

Table 6 seems to indicate that H decreased while V increased slightly with the velocity[3].

Table 4. R and θ at each depth h which were ploted in Fig. 4

Curve	Sand (No.)	Thickness (cm)	*h* cm						
			2.5	4.0	5.5	7.0	9.0	11.0	11.7
I ◬ R kg	2	0.83	1.28	2.52	4.05	5.56	9.19	12.96	——
I' ◬ θ°			76.6	72.8	69.3	66.3	62.2	59.3	——
II ◎ R kg	5	0.83	1.07	1.99	3.09	4.25	6.28	8.15	——
II' ◎ θ°			75.9	71.0	67.5	64.9	61.3	58.4	——
III × R kg	5	0.63	0.86	1.60	2.25	3.51	5.07	6.52	7.10
III' × θ°			75.6	70.9	68.2	65.6	61.4	58.4	56.8

Accordingly the H-V curve in Fig. 3 is supposed to move upwards when the velocity is increased.

3. On the revolution velocity of the wheel

As given before $\omega=\frac{2\pi}{s/v}$

In case of a wheel whith radius r, let ω be ω_0 when $S=2\pi r$ Calucurated ω/ω_0

2) Reported at the Session of the Oyo-Butsuri-Gakkai held in November, 1952. and April 1953

3) The change of the tractive resistance of the car by the velocity was not detected in the range from 0~30 cm/sec.

and the relation $\omega/\omega_0 \cdot h$ may be represented as Fig. 5, Table 7. If there were not any friction of the axle of the wheel, ω/ω_0 seems to approach 1 as h goes to zero as indicated by the dotted line in the Fig. 5.

Table 5. Curves in Fig. 2)

Curve			Former (cm) 12	10	8	6	4	2	0	Backward (cm) 2	4	6	8	10	12
4-Wheeled Car	2	1st kg	—	11.829	11.163	10.492	10.241	9.855	0.712	9.677	9.613	9.628	9.787	10.187	10.723
	3	2nd 〃	—	6.733	6.455	5.841	5.594	5.311	5.117	5.138	4.981	4.910	5.061	5.536	5.698
		3rd 〃	—	4.729	4.378	4.201	3.994	3.866	3.824	3.824	2.919	3.891	4.027	4.241	6.623
		4th 〃	—	3.886	3.719	4.621	3.479	3.420	3.342	3.400	3.412	3.442	3.578	3.686	3.989
	4	5th 〃	—	3.501	3.350	5.233	3.215	3.193	3.108	3.181	3.130	3.20	3.289	3.402	3.619
		6th 〃	—	3.348	3.129	3.074	3.049	3.011	2.960	3.108	3.023	3.040	3.096	3.207	3.422
		7th 〃	—	3.187	3.025	2.956	2.969	2.907	2.886	2.936	2.906	2.931	3.004	3.104	3.218
		8th 〃	—	3.133	2.955	2.848	2.925	2.858	2.801	2.764	2.852	2.862	2.861	2.972	3.113
	Final	9th 〃	—	2.998	2.908	2.930	2.829	2.842	2.756	2.793	2.808	2.781	2.799	2.888	3.055
3-Wheeled Car	1	1th 〃	—	—	—	—	14.285	13.909	13.822	13.843	13.791	13.720	13.868	—	14.406

Wheele A. Velocity 1.1cm/sec. Sand No. 5. Weight 21.3kg. Wheelbase and tread were 45cm and 22.5cm respectively

Table 6. H and V at several velocities

	h cm	2.5	4.0	5.5	7.0	9.0	11.0
H kg	v=1.1cm/sec	0.26	0.65	1.18	1.80	3.02	4.27
	v=8.3cm/sec	—	0.50	1.02	1.69	2.92	4.22
V kg	v=1.1cm/sec	1.04	1.88	2.85	3.85	5.51	6.94
	v=8.3cm/sec	1.04	2.01	2.98	4.04	5.75	7.25

In cases where the thickness of the wheel is small and the action of the sand on the wheel is chiefly on the side surfaces of the wheel, ω/ω_0 is larger than 1

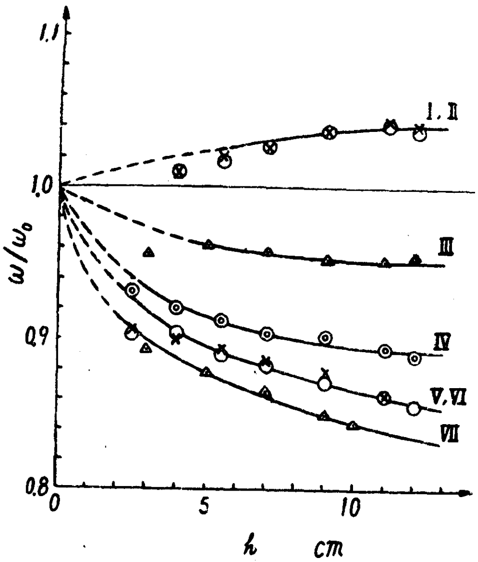

Fig. 5. Relations between angular velocity ratio ω/ω_0 and depth h

Table 7. Curves in Fig. 5

Curve	Thickness (cm)	Sand (No.)	Velocity (cm/sec)	ω/ω_0 at each depth h cm						
				2.5(cm)	4.0(cm)	5.5 (cm)	7.0 (cm)	9.0 (cm)	11.0(cm)	12.0(cm)
I ◎	0.15	5	1.1	—	1.008	1.016	1.028	1.038	1.041	1.037
II ×	0.15	5	8.1	—	1.007	1.021	1.028	1.038	1.044	1.040
III △	0.15	2	1.1	—	—	—	0.957	0.953	0.953	0.954
IV ◎	0.63	5	1.1	0.925	0.919	0.911	0.993	0.903	0.894	0.888
V ◎	0.83	5	1.1	0.903		0.889	0.881	0.872	0.864	0.856
VI ×	0.83	5	8.1	0.906	0.900	0.894	0.885	0.879	0.864	—
VII △	0.83	2	1.1	—	—	—	0.957	0.953	—	—

and the linear velocity of the perimeter of the wheel is larger than the velocity of the progression

As can be seem in Fig. 5 ω/ω_0 decreases with the increase of the thickness of the wheel and that of the size of the sand praticles.

But because ω/ω_0 does not change with v the revolution velocity becomes proportional to the velocity of the progression.

Experimental restlts described above on the revolution velocity of the tracted wheels make some disecussions possible on the slip for the case when the wheel self-drives.

It may be said that in the case of the traction, the revolution veleocity of the wheel under a constant load or a constant V becomes small when the wheel is thick. From this follows that in the case of self-drive the thicker the wheel the smaller the slip.

For the larger size of the sand particles the same argument holds.

IV. Conclusion

From the results descrived above, following conclusions can be drawn which cover the range of these experiments.

1. Dependency of the two compnents H and V of the resistance R and the angle θ between R and H on h is given by

$$H=K_1h^{n_1} \qquad V=K_2h^{n_2}$$

$$R=\sqrt{K_1^2h^{2n_1}+K_2^2h^{2n_2}}$$

$$\theta=tan^{-1}\left(\frac{K_2}{K_1}h^{n_2-n_1}\right)$$

2. The relation between H and V is expressed by the same form as the above formula,

$$V=KH^n$$

3. In the above formulas both H and V increases with the size of sand particles, the hardness of the packing state of the sand and the thickness of the wheel. There-by the rate of the increase of V is higher than that of H

4. The change of the resistance by the velocity is very small and may be neglected.

5. The angular velocity of the wheel decreases with the thickness of the wheel and the size of sand particles. It is proportional, however to the progression velocity.

Acknowledgement

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