Title	A note on kinetic energy of fluid motion
Sub Title	
Author	鬼頭, 史城(Kito, Fumiki)
Publisher	慶應義塾大学藤原記念工学部
Publication year	1952
Jtitle	Proceedings of the Fujihara Memorial Faculty of Engineering Keio University Vol.5, No.18 (1952.) ,p.67(14)- 69(16)
JaLC DOI	
Abstract	It is pointed out that, we can deduce from Kelvin's extension of Green's theorem, when a body immersed in a water region vibrates, it's theoretical value of virtual mass is the same whether the water is flowing with a steady potential-flow or there is no flow at all.
Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00050018-0014

A Note on Kinetic Energy of Fluid Motion

(Received April 24, 1953)

Fumiki KITO*

Abstract

It is pointed out that, we can deduce from Kelvin's extension of Green's theorem, when a body immersed in a water region vibrates, it's theoretical value of virtual mass is the same whether the water is flowing with a steady potential-flow or there is no flow at all.

Let us consider a connected region (extending to infinity or not) bounded by one or more closed surfaces S_1, S_2, ········ as sketched in Fig. 1. Let barriers necessary to reduce the region to a simply connected one (σ_1, σ_2, ······) be drawn. Let two functions ϕ, Φ be given, both of which are continuous functions and are differentiable, throughout the region. Let the cyclic constants of the function ϕ be k_1, k_2, ······ . Then we have, by Kelvin's extension of Green's Theorem : —

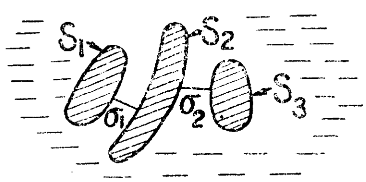

Fig. 1

$$\iint \phi \frac{\partial \Phi}{\partial n} ds + k_1 \iint \frac{\partial \Phi}{\partial n} d\sigma_1 + k_2 \iint \frac{\partial \Phi}{\partial n} d\sigma_2 + \cdots\cdots\cdots\cdots$$
$$= -\iiint \left[\frac{\partial \phi}{\partial x} \frac{\partial \Phi}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \Phi}{\partial y} + \frac{\partial \phi}{\partial z} \frac{\partial \Phi}{\partial z} \right] dx\, dy\, dz$$
$$- \iiint \phi \nabla^2 \Phi \, dx\, dy\, dz \qquad (1)$$

where the surface-integrations on the left-hand side extend, the first over the original boundaries of the region only, and the rest over the several barries σ_1, σ_2, ······ . $\partial/\partial n$ indicates the derivative with respect to inwardly drawn normal to the boundary surface of the reduced region. (Lamb, Hydrodynamics, Art. 53)

Now, let us assume that (a) $\nabla^2\Phi = 0$ throughout the region, (b) $\partial\Phi/\partial n = 0$ on the bonndary surfaces S_1, S_2, ········ (c) $k_1 = 0$, $k_2 = 0$ ······ . Then we have, no matter whether Φ is cyclic or not.

$$\iiint \left[\frac{\partial \Phi}{\partial x} \frac{\partial \phi}{\partial x} + \frac{\partial \Phi}{\partial y} \frac{\partial \phi}{\partial y} + \frac{\partial \Phi}{\partial z} \frac{\partial \phi}{\partial z} \right] dx\, dy\, dz = 0 \qquad (2)$$

In order to apply this equation to a case of fluid motion, let Φ and ϕ represent two velocity potentials such that

*) 鬼頭史城 Dr. Eng., Professor at Keio University

$$\left.\begin{aligned} V_x &= \frac{\partial \Phi}{\partial x}, \quad V_y = \frac{\partial \Phi}{\partial y}, \quad V_z = \frac{\partial \Phi}{\partial z} \\ v_x &= \frac{\partial \phi}{\partial x}, \quad v_y = \frac{\partial \phi}{\partial y}, \quad v_z = \frac{\partial \phi}{\partial z} \end{aligned}\right\} \tag{3}$$

As to the potential Φ we assume that $\nabla^2\Phi = 0$ throughout the region, and $\partial\Phi/\partial n = 0$ on the surfaces S_i. This means that Φ is the potential corresponding to a steady potential-flow of incompressible fluid around the rigid bodies S_1, S_2, ········· . ϕ may be any continuous and single valued function of (x, y, z), but it may depend on time t. For example, it may be a solution of wave-equation

$$\nabla^2\phi = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

or a potential function such that $\nabla^2\phi = 0$.

Consider now the motion of fluid, obtainable by superposition $\phi + \Phi$ of two potentials ϕ and Φ. Kinetic energy T of fluid at any instant, contained in the region, is given by

$$T = \frac{\rho}{2}\iiint\Big[(V_x+v_x)^2 + (V_y+v_y)^2 + (V_z+v_z)^2\Big]\,dx\,dy\,dz$$

According to the above lemma (2), we have

$$T = T_0 + T_1 \tag{4}$$

where

$$\left.\begin{aligned} T_0 &= \frac{\rho}{2}\iiint\Big[V_x^2 + V_y^2 + V_z^2\Big]\,dx\,dy\,dz \\ T_1 &= \frac{\rho}{2}\iiint\Big[v_x^2 + v_y^2 + v_z^2\Big]\,dx\,dy\,dz \end{aligned}\right\} \tag{5}$$

Thus, the total kinetic energy of fluid is equal to the sum of kinetic energies of two component motions represented respectively by Φ aud ϕ.

CASE A Let us consider a closed rigid boundary D inside of which there may exist several rigid bodies B_1, B_2, ········· . (Fig. 2) Let Φ denote any potential flow through the space thus formed. Φ may be many valued. Also, ϕ be any other single valued potential of flow, for example, motion of fluid generated by small oscillations of bodies B. Then the kinetic energy of combined motion of the fluid is given by the formula (4) of superposition.

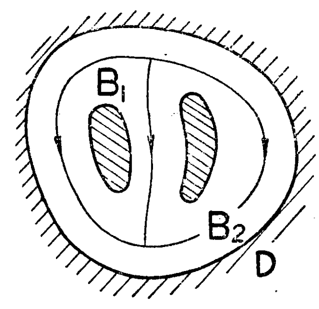

Fig. 2

CASE B Let Φ be a potential flow through several bodies B_1, B_2, ········· . (Fig. 3) Imagine a closed boundary surface D which is very far away from any one of them. Let ϕ be another motion of the fluid, for example generated by oscillations of bodies B_1, B_2, ········· , and such that at an infinite distance from the bodies we have $\phi \to 0$. In this case $\phi \frac{\partial \Phi}{\partial n} = 0$ on the boundary surface D. So that the formula (4) of superposition also holds true in this case.

CASE C Take the case of two-dimensional flow. Let there be bodies B_i infinite in number and arranged in a row spaced at equal distances. Let Φ be the potential of a potential-flow through the row of bodies B, which is a periodic function of period L. ϕ be some other motion as before. Draw a curve ab passing

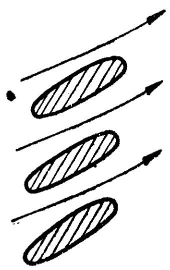

Fig. 3

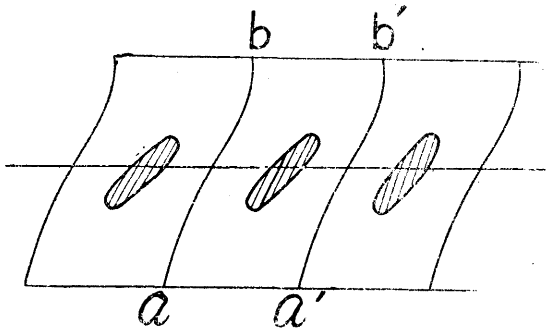

Fig. 4

through the fluid, and another cnrve $a'b'$ which is obtained by translation of ab by the distance L. Suppose that when we make the points a, b, a', b' infinitely distant from the row of bodies B_i, we have $\phi \to 0$. Taking the region enclosed by contour $abb'a'$, we find, as before, that the law of superposition (4) also holds.

As application of these inferences, suppose that there exist steady flow around the bodies B in any one of water regions, Fig. 2, 3 or 4, with or without the circulation around each body B. Let this flow be represented by a velocity potential Φ. Moreover, suppose that each body B is vibrating, and are causing vibratory motions of surrounding water. This vibration of water gives rise to so called phenomenon of "virtual mass" on the vibration of bodies B themselves. In this case, however, the total kinetic energy of water is given by (4), and consist of the sum of steady part T_0 and vibratory part T_1. Thus we see that the steady flow Φ has no effect upon the "virtual mass" of water of a body or bodies which are vibrating in water region.