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Resistive Forces against the Motion of a Model Car Rolling on Sand (I)

(Recieved Oct. 10, 1952)

Kiyoshi TAKAHASHI*

Abstract

Previously the author carried out experiments on the resistive forces of sand upon a single wheel rolling on dried[1] sand Referring to those results, various kinds of experiments are described in this report which have been tried with the object of measuring the resistances experienced by a model car oonstructed specially for the purpose, with three or four wheels rolling on sand.[2]

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Discussions are added also.

I. Apparatus and Methods of Measurement

Car: For the car with three wheels, the wheelbase (the distance between the front and the rear axles) was 45 cm. and the tread (the distance between the two rear wheels) was 22.5 cm. For the car with four weels, the wheelbase and the tread were 45 cm. and 22.5 cm. respectively, just the same as those of the former.

Wheels: The wheels were made from "Rowan" wood with a diameter of 20 cm. and a thickness of 5 cm.

Three types of different shapes were adopted as shown in Fig. 1. The depth of grooves of the wheel B, and the height of sharp edge of the wheel C, are 2.5 cm.

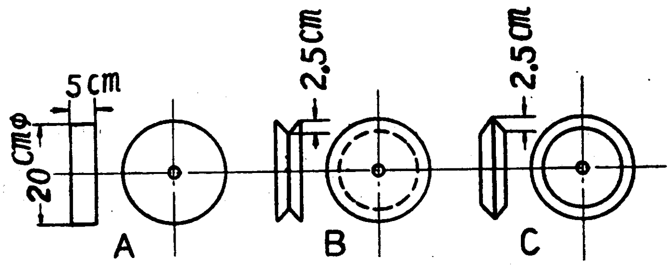

Fig 1. Shapes of Wheels

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1) Read at the meeting of the society of Applied Physics, Japan Apr. 4, 1951 and Nov. 1, 1951.

2) Read at the meeting of the society of Applied Physics, Japan, Oct. 9, 1952.

Velocity of the car: The sand was placed in a box (300×70×45,cm). The car was placed on this sand and was held at a fixed position. The sand box was driven on horizontal rails at a constant speed, thereby producing a relative velocity between the sand and the car.

Sand: The particles of sand were comparatively round shape as they were "Kawazuna" (river sand). They were sieved into 5 kinds according to their sizes and the mean diameter of 170 particles of the sand that was used in this experiment was 0.026 cm. The repose angle 39°30′ and the specific gravity was from 1.27 to 1.47. The sand was dried fully prior to the test. To acquire the constancy of the packing condition, a fork and a plate were rigged at fixed positions to rake the sand and then to level flat the surface of the sand when the sand box was moving. The measurements were made during the return cycles of the motion of the sand box.

Measurments of the resistance: Two ends of the pulling rope were fixed on the two symmetrical positions of the front axle. One end of another rope was fastened to the center of the former rope. The other end of that rope was connected to a spring balance through a pulley. The pulley was so rigged as to slide vertically in order that the ropes may be held horizontal between the front axle and the pulley.

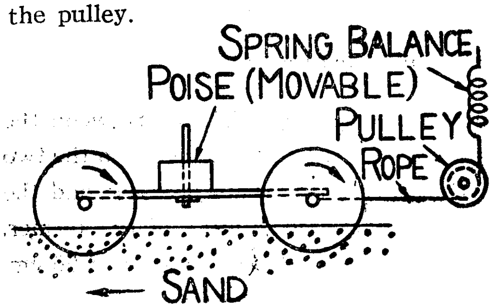

Fig. 2. Schematic Diagram of the Aparatus

II. Purpose of the Experiment

The experiments were carried out from the following five viewpoints by the measurement of the resistance experienced by the car with three or four wheels in the aforementioned manner.

1. The relation between the weight of the car and the resistance when the car is pulled at a constant velocity with the same load on each wheel on the sand of virgin state. Furthermore the influences of the wheel shape on this relation.

2. Unlike the above problem where the experiment is to be carried out on the raked virgin sand (sand in soft state),[3] it is intended to get informations in this case on the relations between the resistance and the repetition eycles of the passage of the car without refreshing the state of the sand, in order to get easily informations on the relation between the resistance and the packing state of sand.

3. Changes in the resistance by the displacement of the center of gravity, the weight of the car being held constant.

3) The expression "the sand in soft state or soft state of sand" means the soft packing condition of sand such as was raked by a fork against the expression "the sand in hard state or hard state of sand" which means the hard one such as was treaded on.

4. The influences of the repetitions of the passage on the relations between the position of the center of gravity and the resistance.

5. The relation of the velocity to the resistance.

The results are described in the following section.

III. Results

These results were obtained at the common speed of 1.3 cm/sec, excepting in the case 5.

1. The resistances of the car with the four wheels of the shape A, B and C respectively were found to be in the order $A<C<B$ in any case.

The resistance of the car with three wheels of the type A was larger than that of the any with 4 wheels.

Logarithmic plots of the resistance against the weight of the car became liner in the range measured as shown in Fig. 3.

The gradient n was $2>n>1$.

The resistance F and the weight W are given by

$$F=KW^n$$

where K and n are constants.

The values of these constants are calculated when unit of F and W are in gramm in Table 1 for each case.

The degree of the sinking in the sand of the wheel under the same load was found to be least in the case of the wheel A.

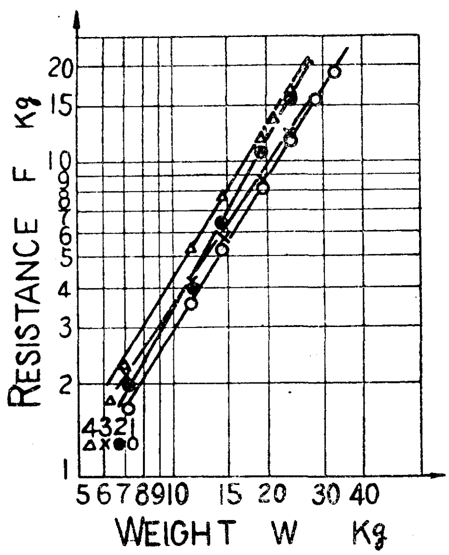

Fig.3. Logarithmic Plots of The Resistance F as Function of The Weight W for Four Curves

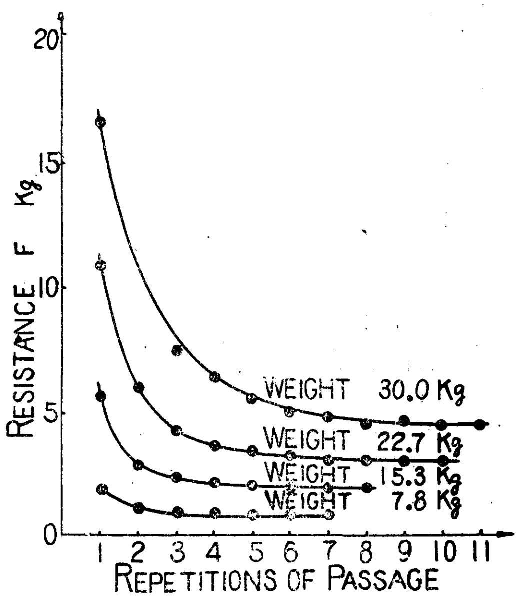

Fig. 4. Resistance versus Repetitions of Passage at Different Weight

Table. 1. The constants K and n of the formula $F=KW^n$, for each curve in Fig. 3

Curve	Shapes and Number of Wheels		K × 10³	n
1 ○	A	4	1.00	1.61
2 ●	B	4	1.72	1.82
3 ×	C	4	2.77	1.52
4 △	A	3	1.17	1.63

2. By the repetition of the passage of the car with four wheels of the type *A* the resistance decreased gradually approaching constant values which differed on each weight as shown in Fig. 4.

3. There is a position of the center of gravity where the resistance becomes minimum under a given load. For the case of the wheel *A* and the weight of 21.3kg. see Fig 5.

This phenomenon is observed clearly, especially when the sand is in a raked virgin state (confer 3). The position lies somewhat in the backward direction from the center of gravity where the same load acts on each wheel, both in the case of three and four wheels.

The resistance becomes large when the center of gravity is displaced to the left or to the right side, accompanied with the appearance of a moment which tends to turn the direction of the progression of the car to that side where the center of gravity has been displased to.

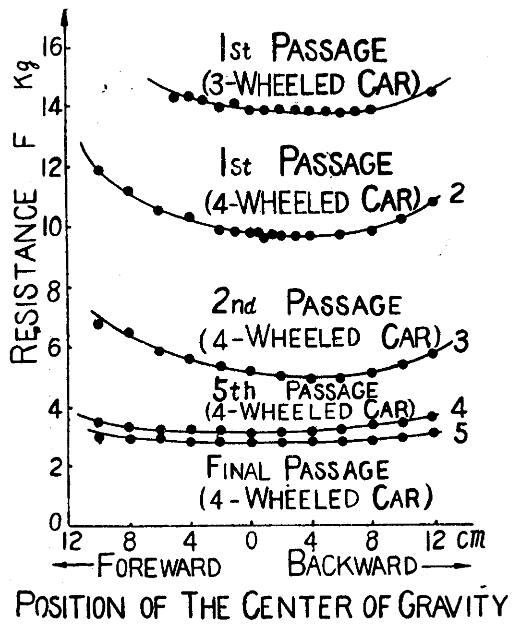

Fig. 5. The Relations Between The Resistance and The Posttion of The Center of Gravity at Each Passage

The relation of the resistance and the position of the center of gravity in the fore or backward direction under a constant weight is explained as follows.

The previous experiments[2)] of the author has revealed the facts described below.

In pulling a wheel of type *A* which is sunk in the sand to some depth and is progressing with a constant speed, rolling freely, the total resistive force is directed to the center of the axle as long as no friction is present on the axle, and the direction changes with the depth of the sinking in the sand.

The resistance is represented by a vector in Fig. 6. in the $OX-OY$ plane where OX and OY are vertical and horizontal directions respectively. The end point of the arrow vector moves on the curves auch as 1 and 2 in Fig. 6.

The curve 1 corresponds to the case where the sand has been raked and remains in a soft state, whereas the curve 2 corresponds to the case where the sand is in hard treaded state. In both cases the gradients are vertical in the zero limit of the sinking. As the depth of the sinking increased, the gradient to OX decreases monotonouslly.

As the vertical component of the resistance corresponds to the load acting on a wheel and the horizontal component corresponds to the resistance against the progression of the car, the curves 1 and 2 can be considered to represent the state met by the front and rear wheels respectively.

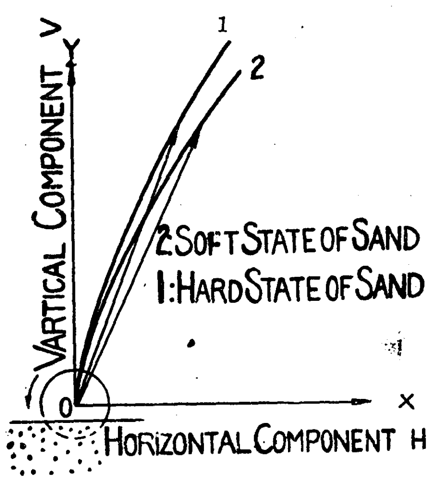

Fig. 6. Vector diagram of the resistance acting upon a single wheel rolling on sand

From this assumption the position of the center of gravity for which the resistance or the force needed for pulling becomes minimum can be calculated as follows.

This problem is an isoperimetric problem. Now, first let us consider the case of the car with four wheels, the load on each wheel being V_i, and its resistance H_i ($i=1,2,3,4.$)(See Fig.7)

As H_i is the function of V_i, put

$$H_i = \psi_i(V_i) \qquad (1)$$

Of course $\sum V_i$ equals the weight of W of the car,

$$\sum V_i = W = \text{constant} \qquad (2)$$

Conditions to minimumizs $\sum H_i$ is determined from (1) and (2).

As the front wheels ($i=1,3,$) and the rear wheels ($i=2,4,$) are under the same condition respectively, then,

$$H_i = \psi_i(V_i) \qquad (3)$$

$$\sum V_i = \frac{W}{2} = \text{Constant} \qquad (4)$$

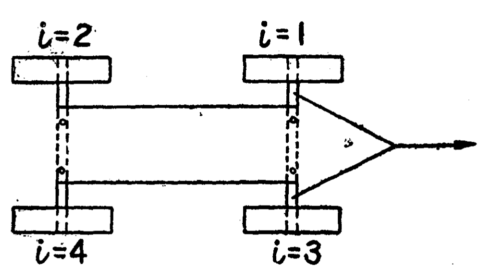

Fig. 7. Numbering for the wheels

where $i=1$, 2. From (3) and (4) the variation leads to

$$\sum \frac{\partial H_i}{\partial V_i} \delta V_i = 0 \qquad (5)$$

$$\delta\sum V_i = \sum\delta V_i = 0 \tag{6}$$

Introducting an undeterminate multipler λ of Lagrange, we get

$$\sum\left(\frac{\partial H_i}{\partial V_i} + \lambda\right)\delta V_i = 0 \tag{7}$$

i, e. $$\frac{\partial H_i}{\partial V_i} + \lambda = 0 \tag{8}$$

i, e. $$\frac{\partial H_1}{\partial V_1} = \frac{\partial H_2}{\partial V_2} \tag{9}$$

The curves 1 and 2 in the Fig. 6. show the relation between V_1, H_1, and V_2, H_2, And as long as $V_1 = V_2$,

$$\frac{\partial H_1}{\partial V_1} > \frac{\partial H_2}{\partial V_2}$$

both $\frac{\partial H_1}{\partial V_1}$ and $\frac{\partial H_2}{\partial V_2}$ increase monotonously with the increment of V_1 or V_2.

Therefore, from (9), the following condition is introduccd;

$$V_1 < V_2 \tag{10}$$

From (10), we find that the minimum resistance of the car with four wheels is obtained in the case when the center of gravity of the car lies in a slightly backwards posision from the center of the car, where every wheel will be equally loaded. In the case of the car with three wheels ($i=1, 2, 3,$) similarly as the above, We get

$$\frac{\partial H_i}{\partial V_1} = \frac{\partial H_2}{\partial V_2} = \frac{\partial H_3}{\partial V_3} \tag{11}$$

But, as every wheel is under the same condition,

$$V_1 = V_2 = V_3 \tag{12}$$

Thus to make the resistance of the car with three wheels $\sum H_i$ minimum, the load carried by each wheel should be the same.

Results shown in curve 1 in Fig. 5, however, contradict with this conclusion. This discrepancy seems to be due to the fact that the front wheel leaves influence on the sand in the passage of the rear wheels because of the short distance between the passage of the front wheel and those of the two rear wheels.

4. Results are plotted in the curves 2, 3, 4 and 5 of Fig. 5. After the constancy of the resistance is attained, change of resistance caused by the displacement of the position of the center of gravity seems to become small, and at same time the resistance is minimum when the center of gravity lies in the position where each wheel carries equal load. Changes of the resistance by the displacement to the left or right side of the center of gravity are also very small.

5. Table. 2. shows the results for the car with three wheels. Generally speaking, the resistance does not change with the velocity in the experimental range.

IV. Conclution

1. The most suitable form of the wheel seems to be in the intermediate between

Table. 2. The measured resistance at different velocity and weight

Velocity cm/sec	Resistance F Kg		
	weight 7.2kg	Weight 14.5kg	Weight 23.8kg
0.35	2.25	7.54	16.34
1.3	2.24	7.67	16.51
24	2.30	7.49	15.93

A and C, in regard to the resistance and the degree of the sinking in the sand,

2. The resistance F as a function of the weight W is described by the following formula.

$$F = KW^n$$

where K and n are constants.

3. In the case of the hard state of the sand, the resistance is far smaller than in the case of the soft state of the sand.

4. There exists a certain position of the center of gravity where the resistance becomes minimum for each experimental condition.

The resistance is affected by the displacement of the center of the gravity, more in the case of a soft state of it, and the harder the state of sand, the nearer the position of the center of the gravity approaches the position where every wheel carries equal load.

5. The change of the velocity does not affect the resistance in the range investigated in this experiment.

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