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Ripple Voltage of D.C. Generator (II) Correction Formula for Non-Uniform Air-Gap of Salient Pole Machine and Saturation of Teeth*

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Abstract

In the preceeding report, a constant air-gap δ_0 under pole-arc had been chosen as the standard of air-gap variation due to rotation. In this report, method of corrections for non-uniform air-gap of salient pole machines and saturation of teeth are explained, and corrected equation of induced voltages is brought forward.

By these corrections, we must use $\alpha(x)$, which varies according to the position x, instead of the constant air-gap δ_0. Analyse $\alpha(x)$ into Fourier series; then, the formula in the preceeding report can be used only by substituting $1/\delta_0$ with α_0, as the fundamental terms of ripple voltage.

I. Introduction

In the preceeding report,[1] a constant air-gap δ_0 under pole-arc had been chosen as the standard of various air-gap due to rotation, in order to clarify the plot of theory. However, corrections of it are naturally necessary for non-uniform air-gap of actual salient pole machine, as air-gap length is not constant.

The variation of flux distribution caused by the unevenness of the rotor surface will sufficiently be regarded as proportional to the air-gap variation, when m.m.f. is all consumed in the air-gap, as mentioned in the preceeding report. But, when the teeth are saturated, corrections become necessary on consideration of m.m.f. required by the teeth.

This report is to discuss the method of these corrections, and to lead to the correction formula, in which the formula of the preceeding report is set as the fundamental terms and useful only after being multiplied by the correction coefficients.

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1) T. HORII , J. I. E. E. of Japan 72. 606 (Oct. 1952)
T. HORII , This proceedings, 3. 102 (1950)

II. Correction for Non-Uniform Air-Gap

When the average path length and width of unit tube of magnetic force produced only by N pole at point x from the original point, which is set on the neutral point in Fig. (1), are $\delta_N(x)$ (under the pole $\delta_N(x) = \delta_0 = \text{const.}$) and $b_N(x)$ respectively, and the width of it on the surface of armature is $a_N(x)$, the flux distribution by N pole for salient pole machine is[2)]

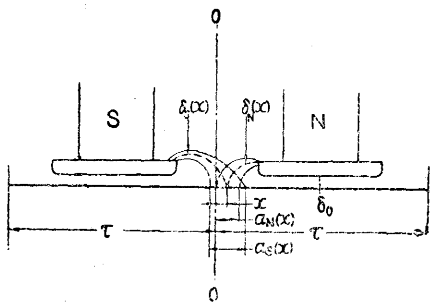

Fig. 1. Air-gap length of salient pole machine

$$B_N(x) = B(\delta_N(x)) \cdot b_N(x) / a_N(x) \quad (1)$$

where $B(\delta_N(x))$ is the flux density corresponding to the magnetic path length $\delta_N(x)$. Let the sectional area at spontaneous position along the tube be $S(\mu)$, and the magnetic reluctance of the tube be $R_N(x)$, then along the tube,

$$R_N(x) = \int_0^{\delta_N(x)} \frac{d\mu}{S(\mu)} \simeq \frac{\delta_N(x)}{b_N(x)}$$

Then, the m. m. f. needed for air-gap at x is

$$V_N(x) = K_N(x) \cdot \delta_N \cdot B_N(x) \quad (2)$$

where

$$K_N(x) \equiv 0.8 a_N(x) / b_N(x)$$

The magnetic potential difference between the pole surface and the armature surface will be constant, when the saturation of teeth is disregarded, but it depends upon x because the saturation cannot be disregarded. In the case of the *smooth rotor*, Eq. (2), defining with suffix 0, is reduced to

$$B_{N0}(x) = \frac{V_N(x)}{K_N(x) \cdot \delta_N(x)} \quad (3)$$

Similarly, the flux distribution by S pole is

$$B_{S0}(x) = \frac{V_S(x)}{K_S(x) \cdot \delta_S(x)} \quad (4)$$

The resultant flux distribution $B_0(x)$ is, by Eqs. (3) and (4),

$$B_0(x) = B_{N0}(x) + B_{S0}(x) \quad (5)$$

Put the variation of flux density $\Delta B(x)$, when magnetic path length at x is varied as much as ε due to the uneveness of the rotor. Under the conditions

2) For Example, Arnold, "Gleichstrommaschinen," Bd. I. §89

$\varepsilon \ll \delta_N(x)$, $\delta_S(x)$, from Eqs. (3), (4), and (5),

$$\begin{aligned} \Delta B(x) &= \frac{V_N(x)}{K_N(x)\cdot\{\delta_N(x)-\varepsilon\}} + \frac{V_S(x)}{K_S(x)\cdot\{\delta_S(x)-\varepsilon\}} - B_0(x) \\ &= \frac{V_N(x)\cdot\varepsilon}{K_N(x)\cdot\delta^2{}_N(x)} + \frac{V_S(x)\cdot\varepsilon}{K_S(x)\cdot\delta^2{}_S(x)} \end{aligned} \tag{6}$$

Around the neutral point, because the flux density is small, the potential difference of the teeth can be neglected. Therefore, in Eq. (6), put $V_N(x) = -V_S(x)$, and $K_N(x) \simeq K_S(x)$, then

$$\Delta B(x) = B_0(x)\cdot\varepsilon\cdot(1/\delta_N(x) + 1/\delta_S(x))$$

Under N pole, $\delta_N(x) = \delta_0 \ll \delta_S(x)$, so it is enough to be regarded as $1/\delta_N(x) \gg 1/\delta_S(x)$. Then, if we put

$$1/\delta_0(x) \equiv 1/\delta_N(x) + 1/\delta_S(x)$$

for the entire boundary of x, $\Delta B(x)$ can be represented by

$$\Delta B(x) \cong B_0(x)\cdot\varepsilon/\delta_0(x) \tag{7}$$

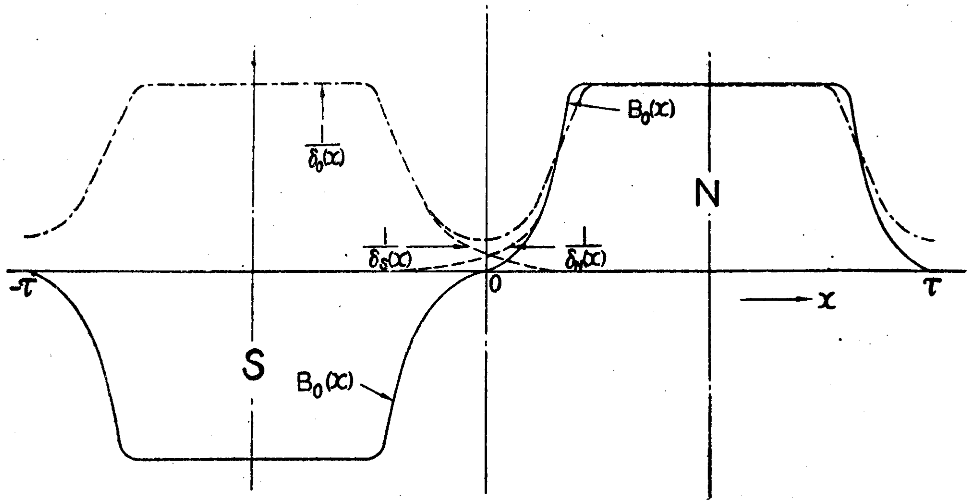

Fig. 2. Relation between $B_0(x)$ and $1/\delta_0(x)$

For low-frequency ripple, the values of $\delta_N(x)$, $\delta_S(x)$ and δ_0 must be corrected by Carter coefficients.

By use of $\delta_0(x)$ for δ_0, the formulas for non-uniform air-gap can be obtained. Then,

$$\text{Air-gap distributed m.m.f.} = \delta_0(x) \sum_k \beta_k(t)\cdot\sin k\left(\frac{\pi}{\tau}x + \gamma_k\right) \tag{8}$$

Air-gap for uneven rotor $\delta(x)$ is,

$$\delta(x) = \delta_0(x) - \sum_{m=1}^{\infty} \varepsilon_m \sin m\left(\frac{\pi}{\tau}x + \frac{v\pi}{\tau}t + \kappa_m\right) \tag{9}$$

$$B(x) = \frac{\sum_k \beta_k(t)\cdot\sin k\left(\frac{\pi}{\tau}x + \gamma_k\right)}{1-\sum \varepsilon_m/\delta_0(x)\cdot\sin m\left(\frac{\pi}{\tau}x + \frac{v\pi}{\tau}t + \kappa_m\right)} \tag{10}$$

The relation between $B_0(x)$ and $1/\delta_0(x)$ is shown in Fig. 2.

III. Correction for the Saturation of Teeth

In Fig. 3, curve M and straight lines $\delta_0(x)$ and $\delta_0(x)\pm\varepsilon$ represent magnetization curve of teeth and air-gap of path length $\delta_0(x)$ and $\delta_0(x)\pm\varepsilon$ respectively, and curves A_1, A_2 and A_3 represent magnetization curve of teeth+air-gap, when air-gap length is $\delta_0(x)$, $\delta_0(x)\pm\varepsilon$ respectively. Because the saturation of pole and armature other than teeth are low, it can be regarded that air-gap+teeth is between these two equipotential surfaces. Let their potential difference be AT_0; then, in Fig. 3, we get the points of intersection of A_1, A_2 and A_3 with the perpendicular at point AT_0. ΔB, when the air-gap is varied from $\delta_0(x)$ as much as $\pm\varepsilon$, is according to the condition $\varepsilon \ll \delta_0(x)$,

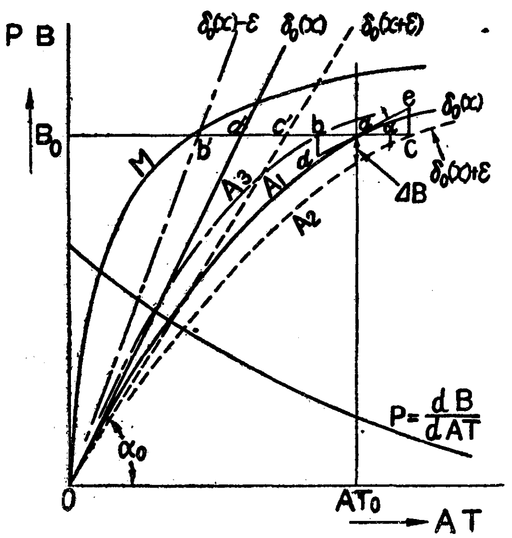

Fig. 3. Relation between ΔB and ε for saturated teeth

$$\Delta B = \left(\frac{dB}{dAT}\right)_{AT_0}\cdot\Delta AT \tag{11}$$

For the air-gap $\delta_0(x)$, B and AT are proportional as shown in the figure ; therefore

$$AT_l = k\cdot\delta_0(x)\cdot B \tag{12}$$

and, as $ab = ac = a'b' = a'c'$,

$$\Delta AT = ab = a'b' = \Delta AT_l = k\varepsilon B_0 \tag{13}$$

By substituting Eq. (13), into Eq. (11), and by using $1/\tan\alpha_0$ instead of $k\delta_0(x)$ in Eq. (12),

$$\Delta B = \left(\frac{dB}{dAT}\right)_{AT_0} k\varepsilon B_0 = \frac{\left(\frac{dB}{dAT}\right)_{AT_0}}{\left(\frac{dB}{dAT_l}\right)}\frac{\varepsilon}{\delta_0(x)}B_0 = B_0\left(\frac{\tan\alpha}{\tan\alpha_0}\right)\cdot\frac{\varepsilon}{\delta_0(x)} \tag{14}$$

Comparing Eq. (14) with Eq. (7), it will be clear that $\tan\alpha/\tan\alpha_0$ should be corrected when the teeth are saturated. This value is varied with x, as the value of $\delta_0(x)$ in Fig. 3 is varied with situation x.

IV. Simultaneous Correction for Non-Uniform Air-Gap and Saturation

Let

$$\left(\frac{\tan\alpha}{\tan\alpha_0}\right)\frac{1}{\delta_0(x)} \equiv \alpha(x) \tag{15}$$

Calculate the value of $\tan\alpha/\tan\alpha_0$ with situation x; then, $\alpha_0(x)$ curve will be obtained as shown in Fig. 4. This curve is drawn as a straight line under the pole arc, as the air-gap $\delta_0(x)=\delta_0=$const. and $\tan\alpha/\tan\alpha_0$ becomes also constant.

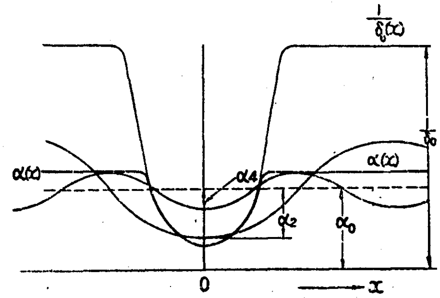

Fig. 4. $\alpha(x)$ curve and its harmonics

Outside of the pole arc, flux density of teeth is so small that it requires no correction, and $\alpha(x)$ curve resolves into $1/\delta_0(x)$, for $\tan\alpha/\tan\alpha_0 \cong 1$. Therefore $\alpha(x)$ can be represented by a straight part under the pole arc, and $1/\delta_0(x)$ curve between the point of intersection with the straight line and the neutral point.

As shown in the figure, $\alpha(x)$ curve has the same sign under N and S poles; therefore it can be developed into Fourier series with even terms only.

$$\alpha(x) = \alpha_0 + \alpha_2 \cos 2\pi x/\tau + \alpha_4 \cos 4\pi x/\tau + \cdots\cdots = \sum_{n=0}^{\infty}\alpha_{2n}\cos\frac{2n\pi}{\tau}x \tag{16}$$

When computing, ratios a_0, a_2 $\cdots\cdots$ of α_0, α_2 $\cdots\cdots$ and $1/\delta_0$ are made $\alpha_0=a_0/\delta_0$, $\alpha_2=a_2/\delta_0$ $\cdots\cdots$; so,

$$\alpha(x) = \frac{1}{\delta_0}\sum_{n=0}^{\infty} a_{2n}\cos\frac{2n\pi}{\tau}x \tag{16$'$}$$

This is more convenient for use. Then, Eq. (10), is transformed into

$$B(x) = \frac{\sum_k \beta_k(t)\cdot\sin k\left(\frac{\pi}{\tau}x+\gamma_k\right)}{1-\sum_m\sum_n \frac{\varepsilon_m}{\delta_0}\, a_{2n}\cos\frac{2n\pi}{\tau}x\cdot\sin m\left(\frac{\pi}{\tau}x+\frac{v\pi}{\tau}t+\kappa_m\right)} \tag{17}$$

V. Correction Formula for Induced Voltage

Eq. (17) is composed of four kinds of travelling flux $m\pm k\pm 2n$ th degree. By use of this equation instead of Eq. (5) in the preceeding report, the corrected equation for induced voltage is obtained. If k is odd number, $k+2n$ is odd number too. Then, in this case it is same as uncorrected one, that m must be even for producing ripple voltage.

Let $\varepsilon_m/\delta_0 \equiv \xi_m$, $v\pi/\tau \equiv \omega$, $\pi x_0/\tau \equiv \phi_0$, and the conditions of the *normal flux distribution* are considered; then by omitting the terms of the above second degree of

ξ_{2m}, the induced voltage will be expessed as Eq. (18).

$$
\begin{aligned}
e_g = -2L\frac{cN}{10^8}\frac{dx_0}{dt}\sum_k \beta_k(t) \Big\{ & C_{dk}\,C_{pk}\,C_{sk}\cos\phi_0 + \sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\xi_{2m}a_{2n}}{4} \\
\times\Big[& \frac{k+2n}{2m-k-2n}C_{d\overline{2m-k-2n}}C_{p\overline{2m-k-2r}}C_{s\overline{2m-k-2n}}\sin 2m\left(\omega t+\kappa_{2m}+\frac{2m-k-2n}{2m}\phi_0\right) \\
& +\frac{k+2n}{2m+k+2n}C_{d\overline{2m+k+2n}}C_{p\overline{2m+k+2n}}C_{s\overline{2m+k+2n}}\sin 2m\left(\omega t+\kappa_{2m}+\frac{2m+k+2n}{2m}\phi_0\right) \\
& +\frac{k-2n}{2m-k+2n}C_{d\overline{2m-k+2n}}C_{p\overline{2m-k+2n}}C_{s\overline{2m-k+2n}}\sin 2m\left(\omega t+\kappa_{2m}+\frac{2m-k+2n}{2m}\phi_0\right) \\
& +\frac{k-2n}{2m+k-2n}C_{d\overline{2m+k-2n}}C_{p\overline{2m+k-2n}}C_{s\overline{2m+k-2n}}\sin 2m\left(\omega t+\kappa_{2m}+\frac{2m+k-2n}{2m}\phi_0\right)\Big]\Big\}
\end{aligned}
\qquad \cdots\cdots\cdots\cdots(18)
$$

$\beta_k(t)$ is expressed by Eq. (19) when we omit the terms which satisfy the condition $m\pm k\pm n=0$, because these terms have no influence on ripple voltage,

$$
\beta_k(t) = \frac{f(t)\cdot\beta_{k0}}{1-\frac{k}{2}\sum\limits_m\sum\limits_n\left(\frac{k+2n}{2m^2-k+2n^2}+\frac{k-2n}{2m^2-k-2n^2}\right)\xi_{2m}a_{2n}\sin 2m(\omega t+\kappa_{2m})} \qquad (19)
$$

In Eqs. (18) and (19), if we put $n=0$, then we get the equations of the preceeding report only multiplied by correction coefficient of fundamental term a_0. The values of a_0, $a_2\cdots\cdots$ are less than unity, and the sign of a_0 is opposite to that of a_2 and a_4; so, the computed value becomes very small compared with uncorrected value, and close to the experimental value.[3] Below a_6 values are small, but they must be considered to the differnce of a_0 and a_2 or a_4. Therefore they must not be neglected.

VI. Conclusions

The process of corrections for non-uniform air-gap and saturation of teeth, and the corrected formula of the theoretical formula reported in the preceeding report have been discussed. Treatments on air-gaps of salient-pole machines may not have, heretofore, been proposed. Most recently, the auther read in Bewley's book [4] a very short description on the treatment of air-gap permeance of synchronous machine. The method presented here will be useful in considering the problems of D. C. machines also.

3) T. HORII, Autumn Meeting of I. E. E. of Japan, 1952, No. 4−15

4) L. V. Bewley, "Alternating Current Machinery", P. 306(1949)