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Bridge Type Oscillator Analysis

(Received Feb. 9, 1952)

Toshio TSUNOGAE*

Abstract

Theory of a bridge type oscillator and its application to a wide band oscillator are described in this paper. These contents have an intimate relation with RC oscillator, about which some treaties have been reported.[1) 2)]

I. General Treatment (a)

The schematic diagram of the bridge type oscillator under consideration is shown by Fig. 1.

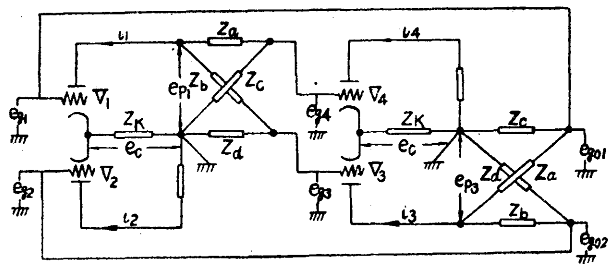

Fig. 1

In which V_1 V_2 V_3 and V_4 are vacuum tubes of same charactor, V_1 V_3 have bridge type coupling impedances Z_p consisting of Z_a Z_b Z_c and Z_d, V_2 V_4 have ordinary impedances, and Z_k represents the common cathode impedance of V_1 V_2 and V_3 V_4. Notations i, e_p and e_g respectively means alternating component of anode current, anode voltage and grid voltage, and they are individualized by numbering with the peculiar numbers of vacuum tubes to which they are belonging.

The following analytical treatment is done under the assumptions that the internal resistance R_i of every vacuum tube is so much greater than its load impedance, and that the elements have linear properties. Further the grid current of the tube is omitted. These assumptions and this omission are permissible for the purpose of this paper. The influence of smaller internal resistances will be considered afterwards.

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1) F. B. Anderson , Seven-League Oscillator. Proc. I. R. E., 39, Aug. 881 (1951)

2) P. G. Sultzer , Wide Range R-C Oscillator. Electronics. 23, Sept. 88 (1950)

Now regarding the vacuum tube as a constant current generater, let it be presumed as follows.

$$i_1 = g_m (e_{g1} - e_c) \tag{1}$$

$$i_2 = g_m (e_{g2} - e_c) \tag{2}$$

where e_c is the produced voltage between cathode and ground return by the product of $(i_1 + i_2)$ and Z_k. Then it is defined by

$$e_c = \frac{g_m Z_k}{1+2g_m Z_k} \cdot (e_{g1} + e_{g2}) \tag{3}$$

Through substitution of Eq. (3) into Eq. (1), it becomes

$$i_1 = g_m (\alpha e_{g1} - \beta e_{g2}) \tag{4}$$

where

$$\alpha = \frac{1 + g_m Z_k}{1 + 2g_m Z_k} \quad , \qquad \beta = 1 - \alpha$$

The anode voltage is given by the product of i_p and Z_p, and has an opposite sign with e_g. Then from Eq. (4) it becomes

$$e_{p1} = - g_m Z_p (\alpha e_{g1} - \beta e_{g2}) \tag{5}$$

There are the following relations between the input voltages of V_3 V_4 and the anode voltage of V_1

$$\begin{aligned} &\text{input of } V_3 = \delta.\ \text{output of } V_1 \\ &\text{input of } V_4 = \sigma.\ \text{output of } V_1 \end{aligned} \tag{6}$$

where

$$\delta = \frac{Z_d}{Z_b + Z_d} \quad , \qquad \sigma = \frac{Z_c}{Z_a + Z_c}$$

That is, from Eq. (5), Eq. (6)

$$\begin{aligned} e_{g3} &= -g_m Z_p (\alpha e_{g1} - \beta e_{g2}) \cdot \delta \\ e_{g4} &= -g_m Z_p (\alpha e_{g1} - \beta e_{g2}) \cdot \sigma \end{aligned} \tag{7}$$

Through similar deduction and substitution the output voltages of the whole system are expressed by Eq. (8) such as following

$$\begin{aligned} e_{g01} &= (g_m Z_p)^2 (\alpha e_{g1} - \beta e_{g2})(\alpha\sigma - \beta\delta) \cdot \sigma \\ e_{g02} &= (g_m Z_p)^2 (\alpha e_{g1} - \beta e_{g2})(\alpha\sigma - \beta\delta) \cdot \delta \end{aligned} \tag{8}$$

In order to produce the self-excited oscillation the grid voltage fed back from the anode voltage regulated by Eq. (8) must be able at least to keep constant anode voltage through controlling anode current. Then let the following relations be introduced as the codition required for the existence of persistent oscillation just above mentioned.

$$e_{g01} \geqq e_{g1} \qquad\qquad e_{g02} \geqq e_{g2} \tag{9}$$

After simple calculation with Eq. (8), Eq. (9) the following relation is obtained

$$(g_m Z_p)^2 (\alpha\delta - \beta\sigma)(\alpha\sigma - \beta\delta) \geqq 1 \tag{10}$$

Furthermore through substituting Z_p defined such as follows

$$Z_p = \frac{(Z_a+Z_c)(Z_b+Z_d)}{(Z_a+Z_c)+(Z_b+Z_d)}$$

into Eq. (10), it becomes

$$C(Z_a+Z_c)(Z_b+Z_d)-A(Z_a+Z_c)^2-B(Z_b+Z_d)^2 \geqq 0 \qquad (11)$$

where

$$\begin{aligned} C &= \{g_m^2(\alpha^2+\beta^2)Z_cZ_d-2\} \\ A &= \{g_m^2\alpha\cdot\beta\cdot Z_d^2+1\} \\ B &= \{g_m^2\alpha\cdot\beta\cdot Z_c^2+1\} \end{aligned} \qquad (11)'$$

It is plain enough that all of these coefficients A, B and C are nondimentional functions, but others in Eq. (11) are complex quantities. Substituting newly introduced notations such as are expressed by relation (11)″ for Eq. (11), it can be parted into two relations so-called (a) the condition of amplitude and (b) the condition of frequency of oscillation.

$$\text{(a)} \qquad C\dot{R}_1 - A\dot{R}_2 - B\dot{R}_3 \geqq 0 \qquad (12\text{ a})$$

$$\text{(b)} \qquad C\dot{X}_1 - A\dot{X}_2 - B\dot{X}_3 = 0 \qquad (12\text{ b})$$

$$\begin{aligned} (Z_a+Z_c)(Z_b+Z_d) &= \dot{R}_1 + j\dot{X}_1 \\ (Z_a+Z_c)^2 &= \dot{R}_2 + j\dot{X}_2 \\ (Z_b+Z_d)^2 &= \dot{R}_3 + j\dot{X}_3 \end{aligned} \qquad (11)''$$

Thus characteristics of the oscillation occurring in the bridge type oscillator are regulated by Eq. (12 a) and Eq. (12 b). Now the effect of the internal resistances of the vacuum tubes will be considered additionally in a few words. If R_i should be taken into account, α and β in Eq. (4) become complex numbers. Accordingly the coefficients A, B and C may also have complex quantities, and the frequency of oscillation and the allowable region of existence of harmonic oscillation must be shifted to some extent.

II. General Treatment (b)

Many useful and interesting systems are found through simplifying the oscillator circuit shown by Fig. 1. One of the most worthy and simplified cases is stated in the sequel. In the previous general treatment the plate impedances of V_1 and V_3 are composed of four impedance elements. From now, let them be transfomed such that the three elements of them excepting impedance Z_a are all pure resistances, and the ratio of Z_d and Z_b+Z_d are mechanically adjustable elements. Substituting r_1, r_2, r_3 and $R+jX$ for Z_b, Z_d, Z_c, and Z_a respectively, Eq. (12) is deduced to the following two relations.

(a) Amplitude condition

$$A\left[(R+r_3)^2 + X^2\right] \geqq Br^2 \qquad (13\text{ a})$$

(b) Frequency condition

$$\left(\frac{cr}{2A} - r_3\right) = R \tag{13 b}$$

where $r = r_1 + r_2$. For brevity, notation $F(\delta)$ is used in place of the left hand side of Eq. (13. b).

Evidently, $F(\delta)$ is a function of δ, on the contrary the opposite hand side doesn't posses any relation to δ and is decided by the elements included in the impedance Z_a. Consequently through inspecting both hand sides independently,

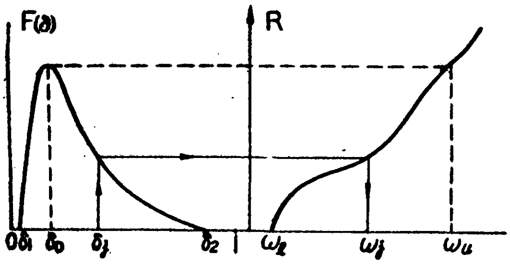

Fig. 2. Fig. 3

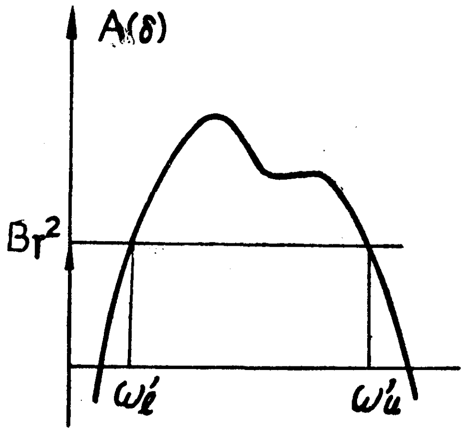

Fig. 4

the frequency charactor of the oscillator can be known. The most typical aspect of $F(\delta)$ drawn in accordance with the variation of δ from zero to unity is shown by Fig. 2. In which δ_1, δ_2 are the roots of $F(\delta) = 0$, δ_0 is given by $\frac{d}{d\delta}F(\delta) = 0$, and its value is

$$\delta_0 = \frac{2}{U} + \sqrt{\left(\frac{2}{U}\right)^2 + \frac{1}{V}}$$

where

$$U = \kappa\lambda g_m^2 \alpha\beta r^2 \ , \qquad V = g_m^2 \alpha\beta r^2$$

$$\kappa = \frac{1}{\alpha\beta} - 2$$

$$\lambda = r_3/r$$

Let Fig. 3 be representing R as an example. Arranging Fig. 2 and Fig. 3 the frequency of oscillation ω_j corresponding with δ_j is shown by the arrow, the upper and the lower limits of the oscillating frequency (ω_u, ω_l) are also decided.

Next, let us consider the range where a harmonic oscillation can be produced with Eq. (13. a). Since the left hand side, expressed by the notation $A(\delta)$ is dependent upon δ but the right hand side is not, it is possible to draw the figure of $A(\delta)$ according to the value of δ from zero to unity in Fig. 4. In which ω_u', ω_l' represents respectively the upper and the lower limit of frequency restricted by the amplitude condition.

Thus as were mentioned above, the frequency of oscillation and its allowable existence range are decided graphically with Fig. 2, Fig. 3 and Fig. 4. And this may give us great facility in anticipating the characteristics of the oscillator without any troublesome treatment. Some graphical representations are illustrated in Fig. 5.

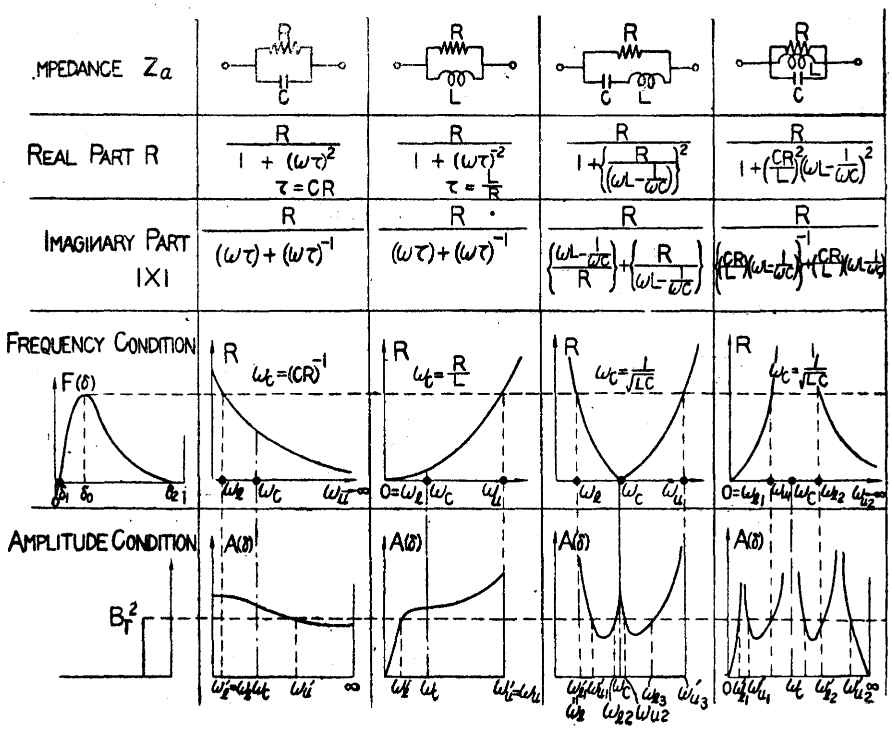

Fig. 5

III. Application of Theory : RC Oscillator

In this part, the characteristics of the RC oscillator are expressed as an instance of application of the general treatment described in the previous part. Of course, there may be many kinds of physical examples suitable to be analized through this theory, but as can be easily supposed through Fig. 5, the RC oscillator is available as a wide band oscillator. That is why this oscillator is described rather in detail in the sequel.

The principal connection diagram is same with Fig. 1, and the construction is simplified by the condition considered in part II. The load impedance of V_1 and V_3 are illustrated by Fig. 6. Then R and X of impedance Z_a are expressed by the following relations.

$$R = \frac{R_1}{1+(\omega\tau_1)^2} \quad , \qquad |X| = \frac{(\omega\tau_1)}{1+(\omega\tau_1)^2}\cdot R_1$$

where $\tau_1 = C_1R_1$, and let $\tau_1^{-1} = \omega_c$ be called in terms of the center frequency of the RC network.

Substituting these relations for Eq. (13), the two conditions become

$$\text{(a)} \qquad A\left[\left\{\frac{R_1}{1+(\omega\tau_1)^2}+\lambda r\right\}^2+\left\{\frac{(\omega\tau_1)}{1+(\omega\tau_1)^2}\cdot R_1\right\}^2\right] \geqq Br^2 \qquad (14.\ a)$$

$$\text{(b)} \qquad r\left(\frac{C}{2A}-\lambda\right)=\frac{R_1}{1+(\omega\tau_1)^2} \qquad (14.\ b)$$

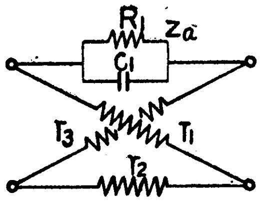

Fig. 6.

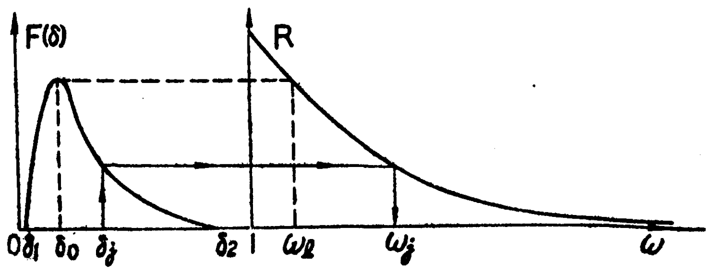

Fig. 7-1

After the same treatment described before, Fig. 7-1 is obtained from Eq. (14. b) as the demonstration of frequency condition of oscillation.

In which, δ_0 is derived from the relation $\dfrac{dF(\delta)}{d\delta}=0$, and means the transmission ratio from V_1 to V_3 and from V_3 to V_2 at the lowest frequency of oscillation, δ_1, δ_2 are the roots of $F(\delta)=0$. Since the right hand side of Eq. (14. b) unquestionably has some positive real quantities the left hand side, that is $F(\delta)$, must have positive value. Through substituting the relation (11)′ for A and C of $F(\delta)$, the condition just above mentioned can be described such as follows.

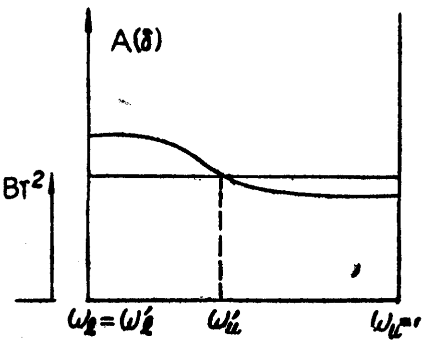

Fig. 7-2.

$$r\left[\frac{\{g_m^2(\alpha^2+\beta^2)\lambda r^2\delta-2\}}{2\{g_m^2\alpha\beta r^2\delta^2+1\}}-\lambda\right] \geqq 0 \qquad (14.\ b)'$$

Since the denominator of the first term of this inequality has always positive value, multiplication of it to both hand sides does not change the sign of inequality. Then adjusting this inequality with respect to δ, it becomes the following form.

$$a\delta^2+b\delta+c \leqq 0 \qquad (15)$$

where
$$a = 2\lambda V$$
$$b = -\kappa\lambda V$$
$$c = 2(\lambda + 1)$$

Thus the roots δ_1 δ_2 are obtained with this relation, and it becomes

$$(\delta - \delta_1)(\delta - \delta_2) \leqq 0 \tag{16}$$

If $\delta_2 > \delta_1$, the relation (14. b)′ is satisfied in the region of $\delta_1 \leqq \delta \leqq \delta_2$. The state of the right hand side of Eq. (14. b) can be easily known with the calculated table of $\frac{1}{1+n^2}$ illustrated by Fig. 8.

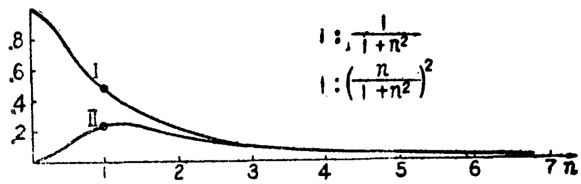

Fig. 8.

Then the correspondence of the transmission ratio δ, and the frequency of of harmonic oscillation ω can be shown such as in the following.

Transmission ratio	Frequency of oscillation
$0 \leqq \delta \leqq \delta_1$	harmonic oscillation doesn't occur
$\delta = \delta_1$	$\omega = \infty$
$\delta_1 < \delta < \delta_0$	from $\omega = \infty$ to $\omega = \omega_0$
$\delta_0 < \delta < \delta_2$	from $\omega = \omega_0$ to $\omega = \infty$
$\delta_2 < \delta < 1$	harmonic oscillation doesn't occur

When we get $\delta_2 > 1$ from Eq. (16), the upper limit of the oscillation frequency is restricted by R which corresponds to $F(\delta=1)$.

In Eq. (14. a), the square value of imaginary part of Z_a can be drawn with the calculated table of $\left(\frac{n}{1+n^2}\right)^2$ illustrated by Fig. 8. Thus the left hand side of the amplitude condition, that is the figure of $A(\delta)$, can be mapped out through simple calculation with Eq. (14. a). Of course, it must be done in the range of $\delta_1 \leqq \delta \leqq \delta_2$ defined just above. Fig. 7—2 shows the state of the amplitude condition.

Now it is possible to say as the conclusion given from the study of both conditions of oscillation that the oscillator has physical reality in the frequency range where these two conditions are satisfied simultaneously. In other words, in according with the variation of transmission ratio δ the frequency of oscillation varies from ω_l to ∞, but Eq. (14. a) restricts this range and consequently it becomes from ω_l to ω_u.

It is desirable to give a constant value to impedance Z_a over broad frequency range, but this is not easy from the technical point of view. It is improved, however, through connecting several RC networks in series to extend the frequency range as shown in Fig. 8. Of course these networks must be suitably designed.

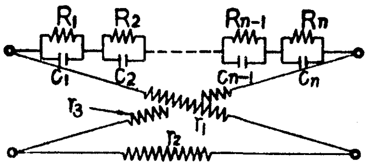

Fig. 9

Even this case analytical treatment is quite similar to the case when Z_a consists of one RC network.

With above stated theory it is possible to guess that the RC oscillator is serviceable to be used as a wide band oscillator. Then let us additionally describe one or two remarkable properties of the RC oscillator.

The load impedance Z_p of V_1 or V_3 is expressed by

$$Z_p = \frac{1}{\frac{1}{r} + \frac{1}{Z_a + r_3}} \tag{17}$$

Impedance Z_a of RC combination in this relation is defined by

$$Z_a = \sum_{j=1}^{n} \frac{R_j}{1+(\omega\tau_j)^2} + \sum_{j=1}^{n} \frac{(\omega\tau_j)}{1+(\omega\tau_j)^2} \cdot R \tag{18}$$

The real part and the imaginary part of Z_a can be figured with Fig. 8.

Then $Z_a = R+jX$ is obtained by drawing R and X separately with uniform scale of ω on the real axis and imaginary axis in Fig. 10. In which, when

$$\omega = \omega_1, \quad R = R_1 \quad X = X_1 \quad Z_a = Z_{a1}$$
$$\omega = \omega_2, \quad R = R_2 \quad X = X_2 \quad Z_a = Z_{a2}$$

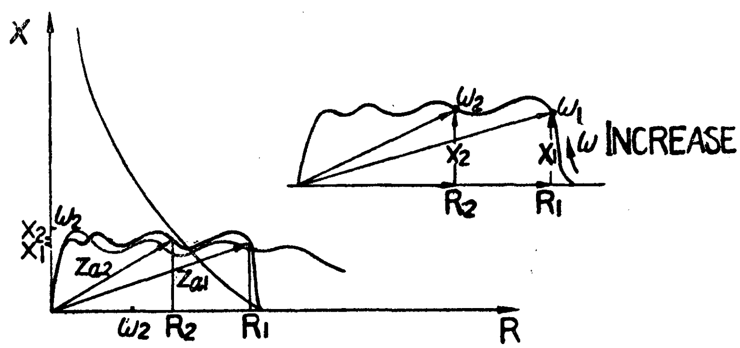

Fig. 10

Accordingly adequate construction of RC networks varies little over a pretty wide frequency range. Furthermore it is plain that the first term of the denominator in Eq. (17) is constant. Then the variation of Z_p can be made small in comparison with the change of frequency, consequently the amplitude charactor of the RC oscillator is improved.

In this theory, the frequency of oscillation ω_j at the definite transmission ratio δ_j is decided by the corresponding frequency of $F(\delta_j)$ in Fig. 7—1. But $F(\delta)$ contains the transconductance g_m of vacuum tube. Then if $F(\delta)$ fluctuates its value from $F(\delta_1)$ to $F(\delta_2)$ in response to alteration of g_m, the frequency is shifted to the corresponding frequency of $F(\delta_2)$. Accordingly the frequency range where the harmonic oscillation occurs may be somewhat changed.

There remain many important problems, for instance the stabilizing methode of oscillating frequency, design of RC networks smoothing of the transmission loop, to be discussed from the practical point of view. These problems, however, may have a chance to be reported extensively someday.

Acknowledgment

The writer whishes to acknowledge the attentive leading given by Prof. T. Suezaki in connection with the completion of this theory.