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On Virtual Mass of a Grating of Flat Plates Vibrating in Water*

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Abstract

In connection with the torsional vibration of screw propellers etc., there arose the problem of finding amount of virtual mass of a grating of flat-plates which is vibrating in a water region. In this report, this problem is solved by using the two-dimensional potential function, and the result of calculation is shown in a chart.

I. Fundamental Expressions

It is known, in theory of two-dimensional potential flow in Hydrodynamics, that a grating of flat plates as shown in ζ-plane of Fig. 1 can be related conformally to a circle of radius c in the z-plane, by means of a function of complex variable z;

$$\zeta = \frac{t}{2\pi}\left[e^{i\beta} \log \frac{z-a}{z+a} + e^{-i\beta} \frac{z-c^2/a}{z+c^2/a} \right] \tag{1}$$

where $z = x + iy$, $\zeta = \xi + i\eta$, and t is the pitch, β the stagger angle, of grating and l is the breath of each flat plate. a is a positive real number greater than c.

When there exist a flow of water with angle of incidence λ to this grating, the corresponding complex velocity potential w_a is given by

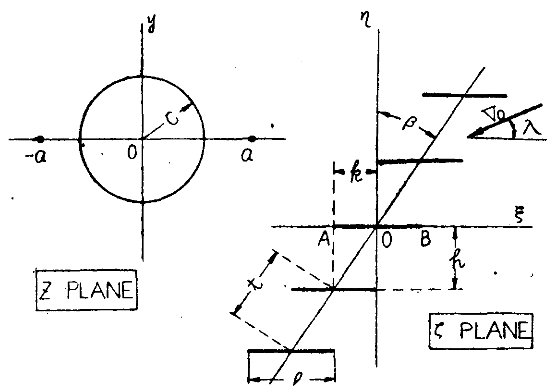

Fig. 1.

$$w_a = \frac{tV}{2\pi}\left[e^{i(\beta-\lambda)} \log \frac{z-a}{z+a} + e^{-i(\beta-\lambda)} \log \frac{z-c^2/a}{z+c^2/a} \right] \tag{2}$$

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The amount of circulation around each flat plate is here taken to be zero. If there exist no grating, the uniform flow with velocity V and direction λ, is given by

$$w_b = Ve^{-i\lambda}\zeta .$$

Therefore, the complex velocity potential $w=w_a-w_b$ represents the flow caused by the grating of flat plates when it advances with velocity V to direction of angle λ.

Thus we have

$$w = w_a - w_b = \frac{tV}{2\pi} e^{-i\beta} \cdot 2i \sin\lambda \log \frac{z-c^2/a}{z+c^2/a}$$

The complex velocity potential, when the grating advances in the direction normal to the face of each flat plate is obtained by putting $\lambda = 90°$ into the above expression, and we have

$$w = \frac{tV}{\pi} \cdot ie^{-i\beta} \log \frac{z-c^2/a}{z+c^2/a} \qquad (3)$$

The velocity potential ϕ corresponding to this state of flow can be obtained by taking real part of the above expression (3).

II. Amount of virtual mass for grating of flat plates when there exist no flow throgh the grating.

The motion of water, when the above-mentioned grating makes a vibratory motion, is represented by a potential $p = \phi \cos \omega t$. When angular frequency ω of vibration is not too high, and the breadth l is small in comparison with the wave length of pressure-wave in water, the wave equation

$$\nabla^2\phi = \frac{1}{c^2} \frac{\delta^2\phi}{\delta t^2}$$

may approximately be replaced by a potential equation $\nabla^2\phi = 0$. And the expression for ϕ can be obtained from (3). In order to obtain the amount of virtual mass of water, we should find the value of kinetic energy of water contained in a region CDEF as shown in Fig. 2, surrounding one member AB of the grating.

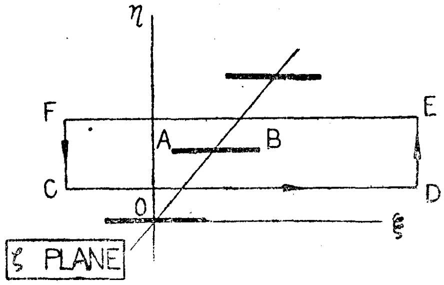

Fig. 2.

According to a known theorem in Hydrodynamics, it can be expressed as

$$T_1 = \frac{1}{2} \rho \int \phi \frac{\delta\phi}{\delta n} ds \qquad (4)$$

where ρ is the density of water, $\delta\phi/\delta n$ the normal derivative of ϕ, and the integral is to extend to entire boundary of the region. But, since the value of integral around the outer-boundary CDEF cancells out, we have only to integrate the expression (4) over the both faces of flat plate AB.

When a flat plate is vibrating with an amplitude A normal to its own face, the normal displacement η at any instant is given by $\eta = A \sin \omega t$ Hence, its normal velocity v_η is

$$v_\eta = \omega A \cos \omega t .$$

According to previous section, the motion of surrounding water can be expressed approximately by

$$\phi = (Real\ pt.)\ \frac{t}{\pi} A\, \omega\, i e^{-i\beta} \left[\log \frac{z-c^2/a}{z+c^2/a} \right] \cos \omega t \tag{5}$$

Now, on a point of flat plate we have $ds = d\xi,\ d\zeta = d\xi + i d\eta = d\xi$. Also we have at there $\delta\phi/\delta n = \omega A \cos \omega t$. Putting these values into (4) we have

$$T_1 = \frac{1}{2} \rho\, \omega\, A \int \phi\, d\xi \cos \omega t = \frac{1}{2} \rho\, \omega\, A \cos \omega t \int \phi\, (d\xi + i d\eta)$$

where the integral is to extend to both faces of the plate.

And this expression is equivalent to the real part of a contour integral

$$U = \oint (\phi + i\psi)\, d\xi = \oint (\phi + i\psi)(d\xi + i d\eta)$$

taken over both faces of the plate. By (1) and (5), it can also be written: —

$$U = \frac{t}{\pi} A\, \omega\, i \cos \omega t \oint \left[\log \frac{z-c^2/a}{z+c^2/a} \right] e^{-i\beta}$$
$$\times \frac{t}{2\pi} \left[e^{i\beta} \left(\frac{1}{z-a} - \frac{1}{z+a} \right) + e^{-i\beta} \left(\frac{1}{z-c^2/a} + \frac{1}{z+c^2/a} \right) \right] dz$$

and the integration may be carried out once around the circumference of the circle c in z-plane. The integrand in U has, over a region outside of the circle c in z-plane, two singular points at $z = +a$ and $z = -a$. Therefore we have, by a theorem on residue: —

$$U = \frac{t}{\pi} A \omega i \times \frac{t}{2\pi} \times 2\pi i e^{-i\beta} \cos \omega t$$
$$\times \left[e^{i\beta} \log \frac{a^2-c^2}{a^2+c^2} - e^{i\beta} \log \frac{a^2+c^2}{a^2-c^2} \right]$$
$$= \left(\frac{t}{\pi} \right)^2 2\pi A \omega \log \frac{a^2+c^2}{a^2-c^2} \cos \omega t$$

Hence the value of T_1 is found to be

$$T_1 = \frac{1}{2} \rho \left[\omega A \cos \omega t \right]^2 \left[\frac{t^2}{\pi} 2 \log \frac{a^2+c^2}{a^2-c^2} \right]$$

For a member of flat-plate gratings, the virtual mass M is found to be

$$M = \rho l^2 \frac{\pi}{4} K \tag{6}$$

per unit length, where we have put

$$K = \frac{8}{\pi^2} \left(\frac{t}{l} \right)^2 \log \frac{a^2+c^2}{a^2-c^2} \tag{7}$$

Also we have

$$\frac{l}{t} = \frac{2}{\pi}\left[\cos\beta \log \frac{\sqrt{a^4+c^4+2a^2c^2\cos 2\beta} + 2ac\cos\beta}{a^2-c^2} + \sin\beta \tan^{-1}\frac{2ac\sin\beta}{\sqrt{a^4+c^4+2a^2c^2\cos 2\beta}}\right] \tag{8}$$

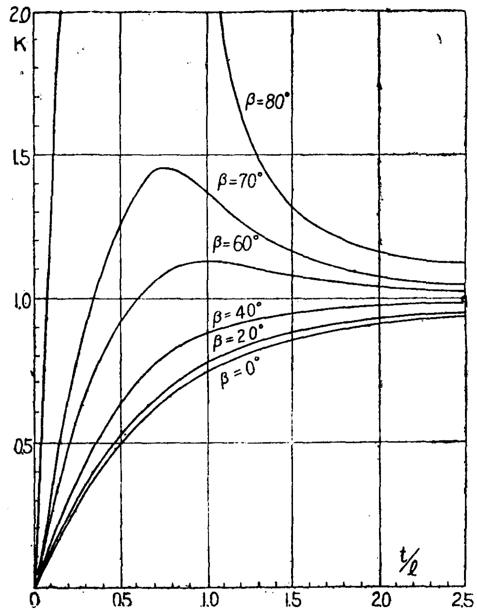

Fig. 3. Value of the factor K

For $a/c \to \infty$, we have

$$\frac{t}{l} = \frac{\pi}{4}\frac{a}{c},$$

$$\log\frac{a^2+c^2}{a^2-c^2} = 2\left(\frac{c}{a}\right)^2$$

so that we have in this case $K = 1$. Therefore for a simple isolated plate vibrating in water we have

$$M_0 = \rho l^2 \frac{\pi}{4} \tag{9}$$

When it is a member of grating, the value M_0 is multiplied by the factor K. In Fig. 3, the values of K for various values of t/l and β are shown in a chart, thus making easy the actual estimation..

III. Effect of Flow upon the Virtual Mass.

In the above calculation, we assumed that there is no flow through the grating. In an actual case of screw propeller which is working and is making vibration at the same time, there exist a flow through each blades.

Let us assume that there exist a parallel flow attacking the grating as shown in Fig. 1 with an angle of incidence λ. Also we assume the existense of circulation around each flat plate, the value of circulation being determined by usual condition of finiteness of flow-velocity at trailing edge of plate. The state of flow can be expressed by the following complex velocity potential W: —

$$W = \frac{tV_0}{2\pi}\left[e^{i(\beta-\alpha)}\log\frac{z-a}{z+a} + e^{-i(\beta-\alpha)}\log\frac{z-c^2/a}{z+c^2/a} + iH\log\frac{z+a}{z+c^2/a}\right] = \Phi + i\Psi \tag{10}$$

where we have put

$$H = \frac{2act\sin(2\beta-\alpha)}{\pi[2ac\cos\beta + \sqrt{(a^2+c^2)(a^2+c^2\cos 2\beta)}]}$$

When each plates of the grating is vibrating in this flow, the corresponding velocity potential ϕ_s is given by

$$\phi_s = \Phi + \phi \tag{11}$$

where ϕ is the value in previous section and Φ is the real part of the above expression (10). The value of pressure p of water at any instant is given by

$$p = -\rho\frac{\partial\phi}{\partial t} - \frac{\rho}{2}\left[\left(\frac{\partial\Phi}{\partial\xi}+\frac{\partial\phi}{\partial\xi}\right)^2+\left(\frac{\partial\Phi}{\partial\eta}+\frac{\partial\phi}{\partial\eta}\right)^2\right]$$

Write this expression thus: –

$$p = p_1 + p_2 + p_3$$

where

$$p_1 = -\frac{1}{2}\rho\left[\left(\frac{\partial\Phi}{\partial\xi}\right)^2+\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]$$

$$p_2 = -\rho\frac{\partial\phi}{\partial t} - \rho\left[\frac{\partial\Phi}{\partial\xi}\frac{\partial\phi}{\partial\xi}+\frac{\partial\Phi}{\partial\eta}\frac{\partial\phi}{\partial\eta}\right]$$

$$p_3 = -\frac{1}{2}\rho\left[\left(\frac{\partial\phi}{\partial\xi}\right)^2+\left(\frac{\partial\phi}{\partial\eta}\right)^2\right]$$

p_1 is the value for flow only, and has no concern with the vibration. p_3 has values of second order in ϕ, so when ϕ is a small quantity of first order, we may neglect p_3. In the expression for p_2, the term $-\rho\partial\phi/\partial t$ is the same when there was no flow, its effect being explained in the previous section. The remaining term, giving effect of flow, is

$$p_2' = -\rho\left[\frac{\partial\Phi}{\partial\xi}\frac{\partial\phi}{\partial\xi}+\frac{\partial\Phi}{\partial\eta}\frac{\partial\phi}{\partial\eta}\right]$$

On the face of flat we have, $\partial\Phi/\partial\eta = 0$, so that

$$p_2' = -\rho\frac{\partial\Phi}{\partial\xi}\frac{\partial\phi}{\partial\xi} \tag{12}$$

The resultant of this pressure p_2' acting upon the face of the flat plate has direction of η-axis, and the amount is: –

$$F' = \int -\rho\frac{\partial\Phi}{\partial\xi}\frac{\partial\phi}{\partial\xi}\,d\xi \tag{13}$$

Now on a point of plate we have

$$\frac{\partial\Phi}{\partial\xi} = V_\xi = V_\xi - iV_\eta\ , \qquad d\xi = d\zeta$$

so we have, also on the plate

$$\frac{\partial\Phi}{\partial\xi}\frac{\partial\phi}{\partial\xi}\,d\xi = [Real\ pt.][V_\xi - iV_\eta][v_\xi - iv_\eta]\,d\xi$$

$$= [Real\ pt.]\frac{dW}{d\zeta}\frac{dw}{d\zeta}\,d\zeta$$

Where W is given by (10) and w by (5).

Now let us consider a contour integral

$$Q = \oint\frac{dW}{d\zeta}\frac{dw}{d\zeta}\,d\zeta \tag{14}$$

taken around a circuit enclosing one of flat plate. This can be regarded as a contour integral

$$Q = \oint \frac{dW}{dz} \frac{dw}{dz} \left(\frac{dz}{d\zeta} \right)^2 d\zeta = \oint \frac{dW}{dz} \frac{dw}{dz} \left(\frac{dz}{d\zeta} \right) dz$$

taken round a circuit enclosing the circle c in the z-plane. But we have

$$\begin{aligned} \frac{dW}{dz} \frac{dw}{dz} \left(\frac{dz}{d\zeta} \right) dz &= Z\, dz \\ &= \Big[\frac{tV_0}{2\pi} e^{i(\beta-\alpha)} \left(\frac{1}{z-a} + \frac{1}{z+a} \right) \\ &\qquad + \frac{tV_0}{2\pi} e^{-i(\beta-\alpha)} \left(\frac{1}{z-c^2/a} + \frac{1}{z+c^2/a} \right) \\ &\quad + iH \left(\frac{1}{z+a} - \frac{1}{z+c^2/a} \right) \Big] \\ &\quad \times \left[\frac{t}{\pi} A\omega i e^{-i\beta} \cos \omega t \left(\frac{1}{z-c^2/a} + \frac{1}{z+c^2/a} \right) \right] \\ \times \frac{2\pi}{t} \Big[e^{i\beta} \left(\frac{1}{z-a} + \frac{1}{z+a} \right) &+ e^{i\beta} \left(\frac{1}{z-c^2/a} - \frac{1}{z+c^2/a} \right) \Big]^{-1} \cdot dz . \end{aligned} \tag{15}$$

Singular points of this expression are

(C) $z_c = \dfrac{c^2}{a}$, (B) $z_b = -\dfrac{c^2}{a}$

(A) leading edge $z_A = Ce^{i\theta}$, trailing edge $z_D = -Ce^{i\theta}$

where

$$\theta = \tan^{-1} \left[\frac{a^2-c^2}{a^2+c^2} \tan \beta \right]$$

The last one, that is the trailing edge, is not actually the singular point, since the circulation was so chosen so as to assure the finite value of flow-velocity at there.

Let us draw a small circle of radius δ with center at A, as shown in Fig. 4. We understand, as the contour for (14), the contour line made up of the circumference of circle C exepting the arc ab, and inside half of the circle δ. (make afterwards $\delta \to 0$)

The value of contour integral taken over a circle of radius $C+\varepsilon$ (ε being a small positive number) is zero, because there exist no singularity outside of this circle.

If we donote by R the residue of the function Z in (15) for point A, we have for the value of (14):

$$Q_0 = \frac{1}{2} \cdot 2\pi i R$$

On the other hand, Z being merely an algebraic fractional expression having singular points at A, B and C, the sum of residues at B and C is equal to $-R$. Therefore we have, by calculation of residues at points $z_C = c^2/a$ and $z_B = -c^2/a$,

$$\begin{aligned} -R &= \frac{tV_0}{2\pi} e^{-i(\beta-\alpha)} \left\{ \frac{t}{\pi} A\omega i e^{-i\beta} \cos \omega t \right\} \frac{2\pi}{t} e^{i\beta} \\ &\quad + \left[-\frac{tV_0}{2\pi} e^{-i(\beta-\alpha)} - iH \right] \left\{ \frac{t}{\pi} A\omega i e^{-i\beta} \cos \omega t \right\} \frac{2\pi}{t} e^{i\beta} \end{aligned}$$

$$= -iH2A\omega i \cos\omega t = 2HA\omega \cos\omega t$$

Thus we have

$$Q = -2\pi i HA\omega \cos\omega t$$

and this is pure imaginary. So that its real part is zero, and consequently we have in (13), $F' = 0$. This shows us that, at least when we confine to ideal fluid, the effect of flow has nothing upon the value of virtual mass.

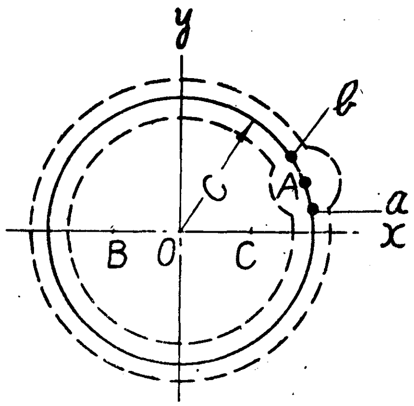

Fig. 4

Note: The Author has shown afterwards, that in general the value of virtual mass is not affected by the presence of a flow around a vibrating body, so long as the flow is a potential flow.