Title	Vertical pressure of sand column acting on the bottom in a vertical glass pipe
Sub Title	
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Publisher	慶應義塾大学藤原記念工学部
Publication year	1951
Jtitle	Proceedings of the Fujihara Memorial Faculty of Engineering Keio University Vol.4, No.15 (1951.) ,p.87(1)- 94(8)
JaLC DOI	
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Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00040015-0001

Vertical Pressure of Sand Column Acting on the Bottom in a Vertical Glass Pipe*

(Received Oct. 28, 1952)

Makoto SAKATA**

Abstract

Vertical pressure, σ, of a column of dry sand in an open cylindrical glass pipe, acting on the bottm is measured. It increases almost linearly with height. But increasing the height of the column in the same pipe, vertical pressure upon the bottom increases asymptotically and attains a certain final value, σ_f, which cahnges with a cross sectional arer, A, of the pipe. The relation of them can be formulated experimentally as follows, $\sigma_f = 1.15A^{0.52}$.

Considering frictional force of sand acting on the sides of the glass pipe, the theoretically calculated formula is $\sigma_f = 1.22A^{0.50}$

When the dried sand is soaked with water, the relation of them becomes more complicated. The relation of σ_f and percentage of moisture content is investigated. In the case of a pipe with larger diameter (above 5.5-6 cm), σ_f takes a minimum value between 8 and 9% of moisture content. In a smaller pipe (less than 5.5-6cm) σ_f becomes zero at a certain percentage below 10. These reasons are also discussed.

I. Introduction

Taking in account of the frictional force of sand grains acting on the sides of the glass pipe, the relation between the sectional areas of the pipe and final vertical pressure of dry sand is calculated. We obtained good agreement between the calculated and the experimental values. When sand which is dried up in an oven is soaked with water, it has mechanical and physical property corresponding to its moisture content. The relation between final vertical pressure of wet sand and percentage of moisture content is explained expereimentally.

II. Experimental Apparatus

Vertical pressure of sand is measured by the balancing method. In Fig. 1, A is the barance to measure vertical pressure, B is a cylindrical glass pipe which contains sand, C is a guiding pipe to pour sand into B, and D is a supporter of B and C. At first, heavy weights which can sufficiently prevail over the vertical pressure of the sand to be measured are placed on the left arm of A. An empty cylindrical glass pipe B is placed on the right arm of A and supported by D as the needle of the balance indicates zero. Afterwards sand is poured into B from a certain height (for dry sand, above 2m ; for wet sand, about 40cm) through the guiding pipe C. The pouring is stopped at a certain proper height of the sand

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* A part of this report was puclished in Journal of Applied Physics, Japan **21**, 124,(1952)

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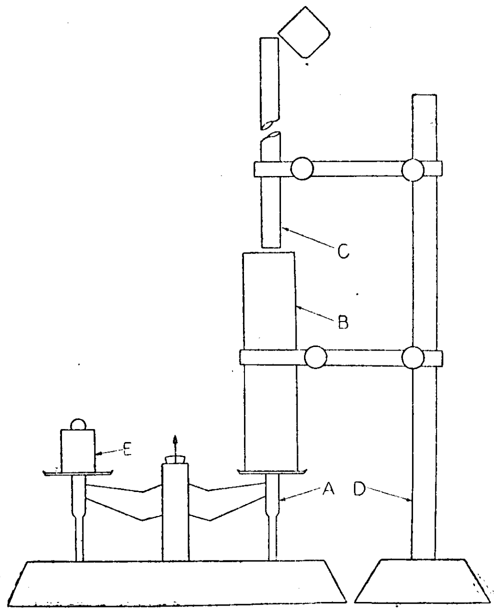

Fig. 1 Experimental apparatus. A is a balance. B is an open glass pipe which contains sand grains.
C is a guiding glass pipe pour sand into B. D is a supporter of B and C. E is the weights.

column. No artificial vibration is given the sand in the pipe. When a little vibration is given, the condition of the experiment is entirely changed and the object of this experiment is lost.

The whights on the left arm are then gradually decreased. At the instant the pressure of sand prevails a little over the gradually decreased recedual weights, the balance of them are broken, and sand pours out from the bottom of *B*. This recedual weights are really the true total vertical pressure of the sand.

III. Dry Sand

Sand taken from a certain river side is cleaned and sifted. The average diameter of sand grains is 0.072mm and the true density is 2.46. Microscopically viewed, the shape of sand grains are comparatively long and rugged. Sand is dried up in an oven at about 200°C. for 4-5 hours and sealed in a desiccator, and this is dry sand. The apparent density of sand, frictional force of sand grains acting on the sides of the glass pipe and the vertical pressure are changed by the temperature the grains are dried and the height we pour the sand mass. In this report, however, these relations are not explained.

A. Frictional Coefficient between Dry Sand and Glass Plate

Pressure of sand grains is affected by statical frictional coefficient between grains and sides of a cylindrical glass pipe. Frictional coefficient is measured by sand grains and a glass plate.

The dry sand grains are piled up on a glass plate and are flattened at the top of the sand pile. Next the plate is gradully inclined. Then the upper sand layer slips off leaving the under layer unchanged at first, and when we incline the plate more and more, the residual under layer slips off from the glass plate. The tangent of the former angle of inclination is frictional coefficient between the sand grains, that of the latter angle is one between sand and glass. These frictional coefficients are changed according to the average diameter of the sand grains as shown in Fig. 2. The upper curve is frictional coefficient between the sand grains and the glass plate, the under curve is that between the grains.

Statical frictional coefficienent is independent of the contact area and the thickness of sand layer which is piled up on a glass plate.

B. Vertical Pressure of Dry Sand

The more the diameter of glass pipe increases, the more the vertical pressure increases. Moreover, the higher the sand column is, the more vertical pressure increases as shown in Fig. 3 in which there are many curves, because each of them corresponds to each diameter of the glass pipe respectively. Each vertical pressure gradually increases with the height at first, but from a ceatain height (about 20-30cm) does not increase and attains a certain final value. All these final values are named " Final Vertical Pressure ", σ_f.

These relations are caliculable. Vertical pressure acting on a certain cross sectional disk of sand in a glass pipe is equal to the residue which is obtained by subtracting frictional force acting on the periphery of the sand column from the weight of the sand disk. Let us use the following notations :

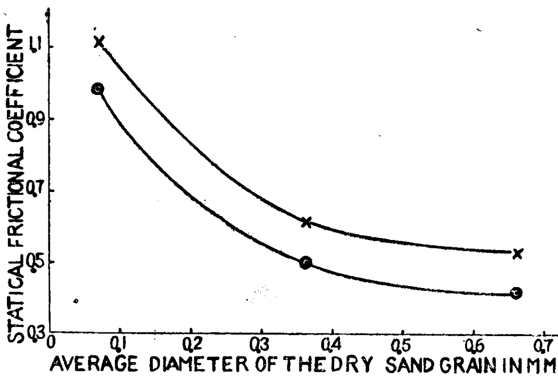

Fig. 2. Statical frictional coefficient for the average diameter of the dry sand grains. The upper curve is the one between grains and a glass plate, the under is the one between the sand grains.

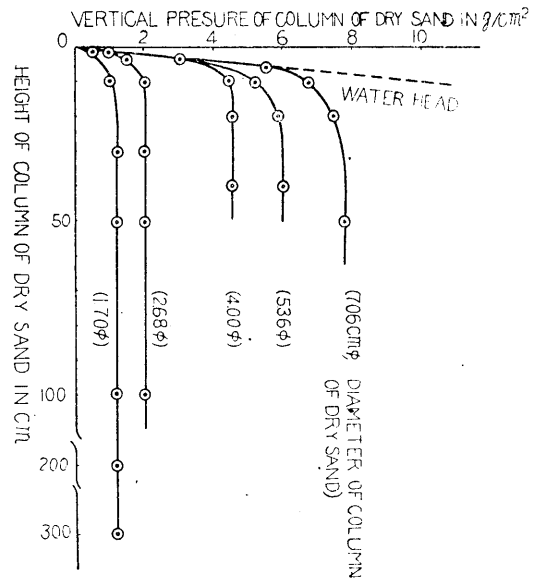

Fig. 3. Vertical pressures of columns of dry sand which change with their heights. There are many curves for many different diameters of the columns. A dotted line presents water head.

γ = apparent density of the sand mass

σ = vertical pressure of sand

τ = lateral pressure of sand

y = variable height of the sand column

A = sectional area of the sand column

d = diameter of the sand column

l = perimeter of the sand column

$$C \equiv \tau/\sigma$$
$$f \equiv A/l$$

Using these notations, the equation is

$$\frac{d\sigma}{dy} + \frac{\mu C}{f}\sigma = \gamma \qquad (1)$$

The solution of the eq. (1) which satisfies the boundary condition at $y=0, \sigma=0$ is

$$\sigma = \frac{f\gamma}{\mu C}\left[1 - exp\left(-\frac{\mu C}{f}y\right)\right] \qquad (2)$$

This equation represents the relation between y and σ as shown in Fig. 3.

C. Final Vertical Pressure of Dry Sand

In the eq. (2), we put

$$\sigma \rightarrow \sigma_f$$

when

$$y \rightarrow \infty$$

then we have

$$\sigma_f = \frac{\gamma f}{\mu C} \qquad (3)$$

In the eq. (3), we put

$$\left.\begin{aligned} f &= \sqrt{A/4\pi} \\ &= d/4 \end{aligned}\right\}$$

then

$$\left.\begin{aligned} \sigma_f &= \frac{\gamma}{2\mu C}\sqrt{\frac{A}{\pi}} \\ &= \frac{\gamma}{2\mu C}d \end{aligned}\right\} \qquad (4)$$

For the sand grains whose average diameter is 0.072mm the experimental values are

$$\left.\begin{aligned} &\gamma = 1.20\ g/cc, \qquad \mu = 1.11 \\ &C = 0.25 \end{aligned}\right\} \qquad (5)$$

(the value of C will be explained later)

Substituting the values (5) into (4)

$$\sigma_f = 1.22A^{0.50} \qquad (6)$$

or

$$\sigma_f = d+0.1d \qquad (7)$$

The dotted lines are the calculated ones, and the real lines are the experimental ones as shown in Fig. 4 and Fig. 5.

Then the experimental formulas are

$\sigma_f = 1.15A^{0.52}$ (8)

or

$\sigma_f = d + 0.8$(between 4 and 18cm. in dia.) (9)

D. Lateral Pressure of Dry Sand

Using the experimetnal data, lateral pressure of dry sand is obtained.

Let us use the following notations:

W = total weiget of the sand column

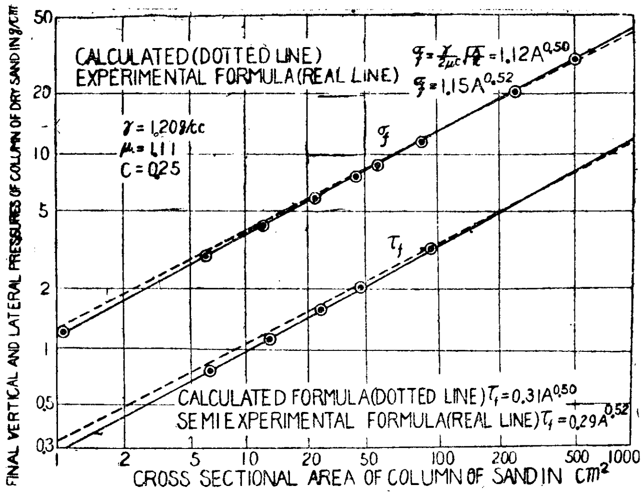

Fig. 4. Final vertical and lateral pressure of the column of dry sand changes with its diameter. The calculated formulas are presented by dotted lines.

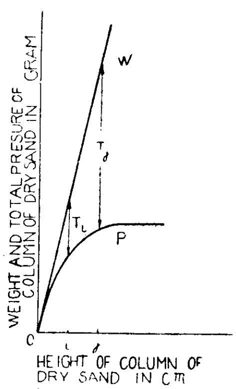

Fig. 6. The weight and the total pressure of the column of dry sand for its height.

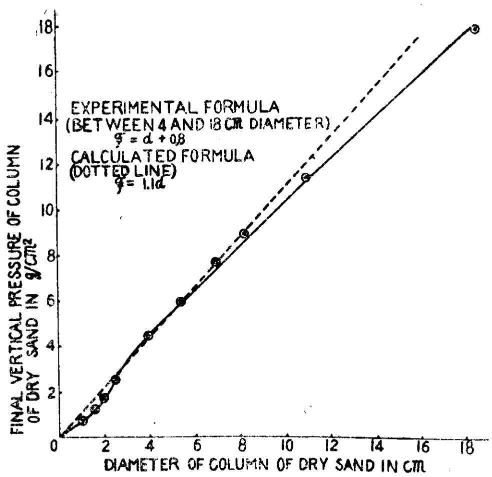

Fig. 5. Final vertical pressure of the column of dry sand for its diameter. The calculated formula is presented by the dotted line, and the experimental formula is presented by the real line.

P = total vertical pressure of sand

L = total height of the sand column

$T_L \equiv W - P$

Using these notations, T_L is given by

$$T_L = \int_0^L \tau \mu l dy \tag{10}$$

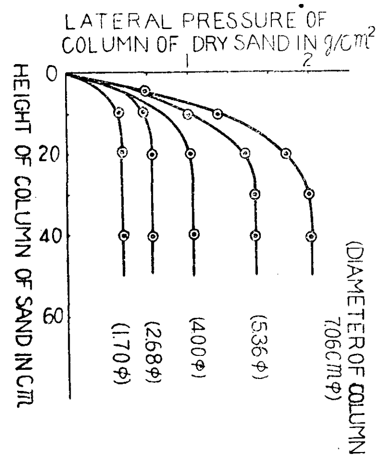

Fig. 7. Lateral pressures of columns of dry sand which change with their heights. Many curves corespond to their different diameters respectively.

Lateral prssure τ is the function of the variable height y, but its functional formula is not explicit, so we can not integrate the eq. (10). Then we rewrite (10) in the form of the difference eq.,

$$T_L = \mu l \sum_i \tau_i \Delta y = k \sum_i \tau_i \Delta y \tag{11}$$

where

$$k = \mu l = 2\mu \sqrt{\pi A}$$

For this solution, we have

$$\left.\begin{aligned} \tau_1 &= \frac{T_2}{k\Delta y} \\ \tau_i &= \frac{T_i - T_{i-1}}{k\Delta k}, \quad i=2,3,4,\cdots \end{aligned}\right\} \tag{12}$$

The value of T_i is easily given by W and P, so we obtain τ_i at any height. The relation between τ and y is presented in Fig. 7 as σ and y.

E. Final Lateral Pressure and Decision of $C\ (\equiv \tau/\sigma)$

Increasing the height of sand in the same pipe, lateral pressure acting on the sides of the glass pipe increases and aspmptotically attains a final value. This value is " Final Lateral Pressure ", τ_f.

By the experimental values σ_f and τ_f, C is calculated for many different pipes. The ratio of τ_f and σ_f or C is decided by their averages, and is given by

$$\langle C \rangle = \langle \tau_f/\sigma_f \rangle = 0.25$$

The relation between τ_f and A is presented in Fig. 4 and the semi experimental formula is

$$\tau_f = 0.29A^{0.52} \tag{13}$$

and the caluculated formula is

$$\tau_f = 0.31A^{0.50} \tag{14}$$

IV. Wet Sand

When the dry sand which is dried up in an oven is soaked with water, it has mechanical and physical property corresponding to its moisture content.

A. σ_f of Wet Sand and Percentage of Moisture Content

The weight percent of water is added to dry sand by a sprayer and they are uniformly mixed up. In the case of wet sand, the important relation is not between σ_f and A, but between σ_f and percentage of moisture content, η. Their relations are presented graphically in Fig. 8 wherein many curves correspond with different sizes of glass pipe respectively. The circles and triangles represent the cases of sand grains of 0.072 mm, and of 0.66mm in diameter (true specific whight of the latter is 2.60). In the case of a larger pipe (above 5.5-6cm), when moisture content of sand increases σ_f decreases and attains a certain minimum value at a certain η between 8 and 10%. Afterwards, σ_f increases gradually from the minimum value with increasing of η. In a samller pipe (less than 5.5-6cm) σ_f also decreases with η but becomes zero at a certain η below 10%.

The relaion between the apparent density and η is presented by a real line; moreover, the repose angle is presented by a dotted line on the upper stage of Fig. 8. (As the measurement of statical frictional coefficient of wet sand is difficult, so the repose angle is measured in place of that). The former resembles the σ_f curve. The latter exceeds over 90 degrees for 5-6 % of η. It will be supposed, however, that the repose angle will fall below 90 degrees from a certain η, perhaps 10%.

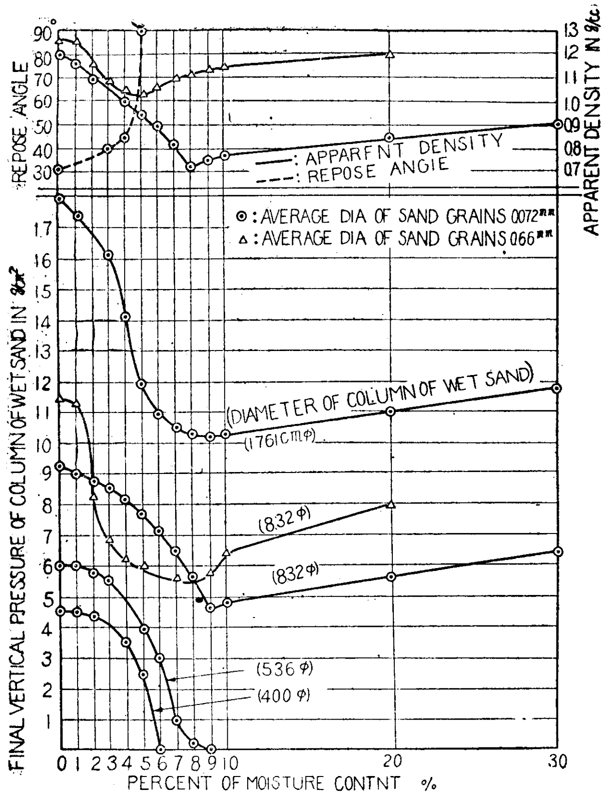

Fig. 8. Final vertical pressure of columns, the apparent density and the repose angle of the wet sand which change with percentage of the moisture content. The circles and triangles represent the cases of the sand grains of 0.072mm, and of 0.66 mm in diameter. The apparent density and the repose angle are measured by the glass pipe of 8.32cm in diameter.

B. Explanation of the Relation between σ_f of Wet Sand and η

The mechanical and physical property of sand at lower moistnre content is similar to dry sand and has only a small cohesive force acting between mutual sand grains and also between sand grains and the sides of glass pipe, but sand acquires a large cohesive force when η is large. When η is so large that the saturation ratio exceeds unity, wet sand behaves like liquid. Now we shall call the zone of 0-9% of misture content as A, of 9-30% as B and of above 30% as C for convenience.

In the zone A, it is not practically easy to mix water and sand grains uniformly.

Therefore it results that the non-uniformities thus formed in sand may become the cause of dry sand like property. When, however, water is added to sand more and more, as the choesive force becomes larger and the repose angle increases, so σ_f of wet sand decreases. Moreover, as grains coagulate with each other by moisture and behave like larger particles, apparent density decreases, and σ_f decreases. Below 5.5cm of glass pipe diameter, the cohesive force between grains and sides of pipe prevails over σ_f and it becomes more stable when compacted at the optimum moisture content, so σ_f becomes zero at a certain η. Above 5.5cm in diameter σ_f of wet sand takes a minimum value at 8 or 9%.

In the zone B, it becomes easier to mix sand grains with water and the mechanical and physical properties are different from the zone A. The moisture content is so large that water behaives like lubricant between grains and the sides of glass pipe; then the cohesive force decreases and σ_f increases gradually. Moreover, the coagulated grains are resolved again, and sand becomes muddy, so apparent density increases gradually and σ_f increases.

In the zone C, as sand behaves like muddy liquid, the value of σ is unmeasureable by our apparatus.

V. Conclusion

We obtain good agreement between the calculated and the experimental values for final vertical pressure of a column of dry sand. Lateral pressure of dry sand is obtained numerically. The relations between final vertical pressures of wet sand and percentage of moisture contents are obtained experimentally, and physical meaning of them are mentioned.

Acknowledgments

The author wishes to express his deep thanks to Dr. M.Masima, Professor of the Keio University, Dr. T. Mogami, Professor of the Tokyo University and Dr. F. Kito, Professor of the Keio University for suggesting the problem and for many valuable advices.