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On Pressure-Rise in a Penstock of Hydro-Electric Power Station Equipped with a Surge Tank

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Abstract

Taking up case of hydro-electric power station equipped with a surge tank of special arrangement (Fig. 1), the Author has made a theoretical calculation of pressure rise in the penstock, when the guide-vanes of water-turbine are closed. The result shows us that, when length of penstock is comparable with that of surge tank, the effect of existence of surge tank is to make the amount of pressure rise in penstock somewhat larger than the amount of pressure rise when the effect of surge tank is not taken into account. A comparison between calculated and observed values, about the Hiraoka Power Station is also given.

I. Introductiou

We consider a Hydro-electric Plant as sketched in Fig. 1. It consists of pres-

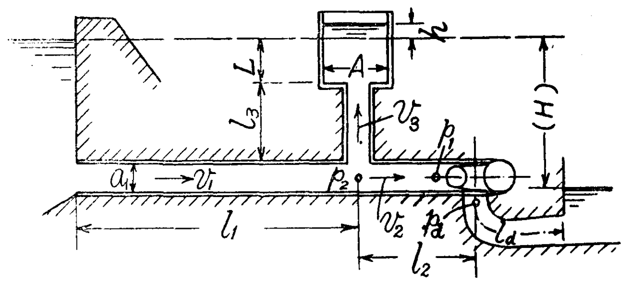

Fig. 1. An arrangement of hydro-electric power station equipped with a special type of surge tank

sure tunnel, penstock, surge tank with a riser stand pipe, water-turbine and a draft-tube. An example of this type of power station is the Hiraoka Power Station of Tenryu River (3x 26,000 kw). But there are several power plants of this type now in existence in Japan.

In order to culculate the amount of pressure rise when the guide-vanes are closed with a closing time T, let us use the following notations:

l_1 = length of pipe line from intake to branch point p_2, m.

l_2 = ditto, from branch point p_2 to guide vane of water turbine, m.

l_3 = length of riser of surge tank, the equivalent length of surge tank body being taken into account, m.

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p_1 = water pressure at a point directly in front of guide vane, (take varying part only)(m in water column.)

p_2 = pressure at the branch point, ditto, m.

A_0 = area of surge tank body, m^2

L = height of ditto, m

h = water level at suge tank, m. H = effective head, m,

a_4 = equivalent value of passage area of guide-vanes, m^2

$\psi(t)$ = guide vane closure curve, if the closure is made by straight line law,

$\psi(t) = \left(1 - \frac{t}{T}\right)$ T = closing time, sec.

We use notations v_1, v_2, v_3 and a_1, a_2, a_3 to denote mean water velocities and areas of three pipe lines, as shown in the Figure 1. If, as usually occur at actual power plants, the cross section of pipe lines are not uniform but varies gradually along their lengths, we take equivalent values of areas a_e by the formula

$$\frac{l_e}{a_e} = \sum \frac{l_i}{a_i}$$

and regard them to have uniform areas a_e.

II. Equation of Motion

Neglecting loss heads. the equation of motion for pipe lines as shown in Fig. 1 may be written :

$$\left.\begin{aligned} p_2 &= -\frac{l_1}{g}\frac{dv_1}{dt} = h + \frac{l_3}{g}\frac{dv_3}{dt} \\ p_1 - p_2 &= \frac{l_2}{g}\frac{dv_2}{dt} \\ v_1 a_1 &= v_2 a_2 + v_3 a_3 \end{aligned}\right\} \qquad (1)$$

The relation between the head and the discharge through the guide-vane passsage may be written,

$$\sqrt{2g(H + p_1)}\, a_4 \psi(t) = v_2 a_2$$

But, since value of (p_1/H)^2 is usually small in comparison wlth 1, we use, as an approximate relation,

$$\sqrt{2gH}\left[1 + \frac{1}{2}\frac{p_1}{H}\right] a_4 \psi(t) = v_2 a_2 \qquad (2)$$

We also have

$$A_0 \frac{dh}{dt} = v_3 a_3 \qquad (3)$$

These six equations (1), (2) and (3) determine six quantites v_1, v_2, v_3, p_1, p_2 and h as functions of the time t. Further, if we put

$$\left[1 + \frac{p_1}{2H} \right] \psi(t) = \xi \tag{4}$$

then, we have

$$v_1 = \frac{A_0}{a_1}\frac{dh}{dt} + \frac{a_4}{a_1}\sqrt{2gH}\ \xi$$

$$v_2 = \frac{v_4}{v_2}\sqrt{2gH}\ \xi \qquad\qquad v_3 = \frac{A_0}{v_3}\frac{dh}{dt}$$

Puttiug these values into (1), and rearranging, we have,

$$\left.\begin{aligned} &\alpha \frac{d^2h}{dt^2} + h + \beta\frac{d\xi}{dt} = 0 \\ &\lambda \frac{d^2h}{dt^2} + \mu \frac{d\xi}{dt} + \frac{\xi}{\psi(t)} - 1 = 0 \end{aligned}\right\} \tag{5}$$

where

$$\left.\begin{aligned} &\alpha = \frac{l_1}{g}\frac{A_0}{a_1} + \frac{l_3}{g}\frac{A_0}{a_3} \\ &\beta = \frac{l_1}{g}\frac{a_4}{a_1}\sqrt{2gH} = \frac{l_1}{ga_1}Q_0 \end{aligned}\right\} \tag{6}$$

$$\lambda = \frac{l_1}{2gH}\frac{A_0}{a_1}$$

$$\mu = \left(\frac{l_1}{a_1} + \frac{l_2}{a_2} \right) \frac{a_4}{\sqrt{2gH}}$$

Moreover, eliminating h from equatious (5), we have

$$\frac{1}{\lambda}\left[\alpha \frac{d^2}{dt^2} + 1 \right]\left[1 - \frac{\xi}{\psi(t)} \right]\left(\beta - \frac{\alpha\mu}{\lambda} \right)\frac{d^3\xi}{dt^3} - \frac{\mu}{\lambda}\frac{d\xi}{dt} = 0 \tag{7}$$

Or, returning, by (4), to the original variable p_1, we have

$$-\frac{1}{2H\lambda}\left[\alpha\frac{d^2p_1}{dt^2} + p_1 \right] + \left(\beta - \frac{\alpha\mu}{\lambda} \right)\left[\frac{\psi(t)}{2H}\frac{d^3p_1}{dt^3} + \frac{3\psi'(t)}{2H}\frac{d^2p_1}{dt^2} + \frac{3\psi''(t)}{2H}\frac{dp_1}{dt} \right.$$

$$\left. + \left(1 + \frac{p_1}{2H} \right)\psi'''(t) \right] - \frac{\mu}{\lambda}\left[\frac{\psi(t)}{2H}\frac{dp_1}{dt} + \left(1 + \frac{p_1}{2H} \right)\psi'(t) \right] = 0 \tag{8}$$

This is the linear ordinary differential equation about the unknown quantity p_1. When the guide-vane closure is made according to a straight line law with respect to the time t, we have $\psi(t) = 1 - (t/T)$, in such a case, the above equation (8) reduces to :—

$$x\frac{d^3p_1}{dx^3} + A\frac{d^2p_1}{dx^2} + Bx\frac{dp_1}{dx} + Cp_1 = D \tag{9}$$

where we have put,

$$x = 1 - (t/T), \qquad A = 3 + \frac{\alpha T}{(\beta\lambda - \alpha\mu)},$$

$$B = -\frac{\mu T^2}{(\beta\lambda - \alpha\mu)}, \qquad C = \frac{T^3}{(\beta\lambda - \alpha\mu)}, \qquad D = \frac{\mu T^2}{(\beta\lambda - \alpha\mu)}\cdot 2H$$

III. The Solution

Our problem was thus reduced to the solution of ordinary differential equation (9). This differential equation is linear, and coefficients are linear functions of the independent variable x.

Sh that we could apply the method of Laplace Integral for solution of it. But it was found that power-series solution by the method of Frobenius is more adapted to practical solution. Thus we have, for a system of fundamental solutions of the homogeneous differential equation with regard to the equ. (9):

$$p_1 = A(x) = x^{2-A}\Big[1 - \frac{C+(2-A)B}{2(4-A)(3-A)}x^2 + \frac{[C+(2-A)B][C+(4-A)B]}{8(6-A)(5-A)(4-A)(3-A)}x^4 - \cdots\Big]$$

$$p_1 = B(x) = x - \frac{B+C}{6(A+1)}x^3 + \frac{(B+C)(3B+C)}{120(3+A)(1+A)}x^5 + \cdots$$

$$p_1 = C(x) = 1 - \frac{C}{2A}x^2 + \frac{C(2B+C)}{24A(A+2)}x^4 + \cdots$$

As to the initial condition, we must have for $t = 0$.

$$p_1 = 0,\quad p_2 = 0,\quad v_3 = 0,\quad v_2 = v_{20},\quad v_1 = v_{10},\quad h = 0,$$

Consequently we have, by equations (1) etc.,

at $t = 0$.

$$\frac{dv_1}{dt} = 0,\quad \frac{dv_3}{dt} = 0,\quad \frac{d^2h}{dt^2} = \frac{dv_2}{dt} = 0,$$

$$\frac{dh}{dt} = \frac{v_3 a_3}{A_0} = 0,$$

Differentiating equ. (2) and utilizing the relation (5), we have at $t = 0$ (or' at $x = 1$)

$$\frac{dp_1}{dx} = -2H,\qquad \frac{d^2p_1}{dx^2} = 2HK$$

where

$$K = 2 + \frac{\alpha T}{(\lambda\beta - \alpha\mu)}$$

Now, the general solution of diff. equ. (9) can be written

$$p_1 = \frac{D}{C} + C_1A(x) + C_2B(x) + C_3C(x)$$

wher $A(x)$, $B(x)$ and $C(x)$ are functions defined by the above mentioned power series and C_1, C_2, C_3 are arbitrary constants. According to the initial condition these arbitrary constants must be determined by the following system of equations

$$\left.\begin{aligned} C_1A(1) + C_2B(1) + C_3C(1) &= -\frac{D}{C} \\ C_1A'(1) + C_2B'(1) + C_3C'(1) &= -2H \\ C_1A''(1) + C_2B''(1) + C_3C''(1) &= 2KH \end{aligned}\right\} \qquad (10)$$

thus our problem has, at least formally, been solved.

IV. Approximate Value of Pressure-rise and Comparison with Observd Values

As an example, let us take up the case of Hiraoka Power Station

In this case we have $l_1 = 500\text{m}$, $l_2 = 40\text{m}$, $l_3 = 80\text{m}$, $a_1 = 34.2\text{m}^2$, $a_2 = 15.9\text{m}^2$, $a_3 = 19.6\text{m}^2$, $a_4 = 2.02\text{m}^2$ (assumed) $H = 45\text{m}$, $A_0 = 200\text{m}^2$.

Consequently, $\alpha = 322$, $\beta = 89.5$, $\mu = 1.16$, $A = -9.2$, $B = 0.103$, $C = -0.296$ $D = -16.7$, $K = -9.40$.

For such a case, we see by actual calculation that we may take approximately at least for practical purpose:

$$A(x) = x^{2-A}, \qquad B(x) = x, \qquad C(x) = 1$$

and corresponding approximate values of constants C_1, C_2, C_3 are found to be

$$C_1 = \frac{2KH}{(2-A)(1-A)} \qquad C_2 = -2H\frac{(K+1-A)}{(1-A)}$$

$$C_3 = -\frac{D}{C} + 2H\left[1 + \frac{K}{2-A}\right]$$

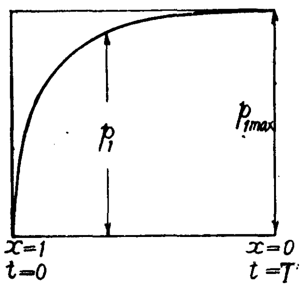

Fig. 2. Curve of pressure rise

We have also $K = A - 1$. Putting these values into the general solution for p_1, we see that the approximate solution is given by

$$p_1 = \frac{2H}{2-A}\left[1 - x^{2-A}\right]$$

The mode of variation of this value of pressure p_1 is sketched in Fig. 2.

And the maximum value is found to be

$$p_{1max} = \frac{2H}{2-A} = \frac{2H}{-1 + \frac{2gHT}{Q}\left(\frac{F_3}{F_3F_2 - F_1^2}\right)} \qquad (11)$$

where Q = discharge m^3/sec. T = closing time, F_1, F_2 and F_3 are so ealled pipe line constannts defined respectively by

$$F_1 = \frac{l_1}{a_1}, \qquad F_2 = \frac{l_1}{a_1} + \frac{l_2}{a_2}, \qquad F_3 = \frac{l_1}{a_1} + \frac{l_3}{a_3}$$

(For an ordinary simple surge tank without riser stand pipe, we have to put $l_3 = 0$) A comparison between the calculated and observed values of the maximum amount of pressure rise p_{1max} is shown in Table 1. In this table last two colmns marked by * is the case of two units cut off simultaneously.

The values of column (a) were obtained by calculation of so called Allievi's approximate formula for pipe line l_2. Those for column (b) are values obtained by calculation of the Author's formula (11). The observed values were obtained from trial running data, by taking the amount $a - c$ in pressure variation curve as

Table 1. Comparison beween the observed and calculated values of pressure rise in Hiraoka Power Station

Turbine No.	Load cut off Kw	Discharge m^3/sec	Closing time sec.	Pressure Rise by Calculation (a) m	Pressure Rise by Calculation (b) m	Pressure Rise by Observation m
1	24,800	59.4	3.9	4.29	6.72	7.50
1	5,200	17.4	1.5	1.52	2.30	2.50
1	10,300	27.9	1.3	2.35	3.70	5.50
1	15,200	38.2	1.35	3.21	4.94	6.00
1	20,400	49.2	1.25	4.14	6.52	6.50
* 1	20,600	49.5	3.25	4.28	8.73	8.50
* 2	20,200	48.7	3.4	4.05	8.17	10.00
* 1	22,800	54.5	3.15	4.90	10.1	8.50
* 2	22,600	54.0	3.6	4.24	8.57	9.50

shown in Fig. 3.

The above table gives only a part of the data obtained by trial running of the Kiraoka Power Plant. From Table 1, we see that the formula (11) coincides roughly with the observed values, while the result of usual formula gives only about half the amount of observed pressure sise. So that the effect of riser of surge tank upon the pressure rise is quite notable.

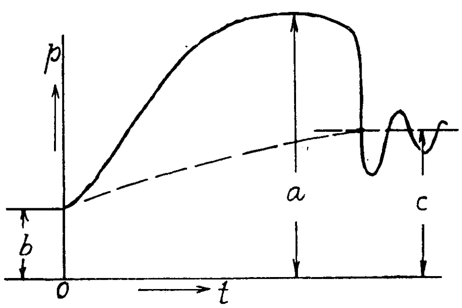

Fig. 3. Pressure rise of a penstock

V. Effect of Draft Tube

When we take the effect of draft tube into account, we have, iu addition to equation (1'),

$$p_d = \frac{l_d}{g}\frac{dv_2}{dt} = -\frac{l_d}{l_2}(p_1 - p_2)$$

where p_d is the pressure (in head, m) at top of draft tube, and l_d is equivalent length of draft tube as referred to cross-sectional area a_2. And the equation (2) is replaced by the following equ.

$$\sqrt{2gH}\left[1 + \frac{1}{2H}(p_1 - p_d)\right]a_4\psi(t) = v_2 a_2$$

Treating these equations quite similarly as above, we obtain for the maximum pressure rise :

$$p_{1max} = \frac{2H}{-1 + \frac{2gHT}{Q}\left\{\frac{F_1 + F_3}{(F_1 + F_3)(F_1 + F_2 + F_d) - (F_1)^2}\right\}} \times \frac{(F_1 + F_2)(F_1 + F_3) - (F_1)^2}{(F_1 + F_2 + F_d)(F_1 + F_3) - (F_1)^2}$$

where $F_d = l_d / a_2$. In the case of Hiraoka Power Station, this effect of draft tube was seen to be considerably small.