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Title	Ripple voltage of d. c. generator, (I) theoretical formura for low-frequency ripple voltage of two-pole machine
Sub Title	
Author	堀井, 武夫(Horii, Takeo)
Publisher	慶應義塾大学藤原記念工学部
Publication year	1950
Jtitle	Proceedings of the Fujihara Memorial Faculty of Engineering Keio University Vol.3, No.11 (1950.) ,p.103(13)- 120(30)
JaLC DOI	
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Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00030011-0013

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Ripple Voltage of D. C. Generator, (I)
Theoretical Formura for Low-Frequency Ripple Voltage of Two-Pole Machine

(Received April 20, 1952)

Takeo HORII *

Abstract

Ripple Voltage of d. c. generator can be classified according to the origin of generation:

(i) Low-frequency ripple (ii) Slot ripple (iii) High-frequency ripple. As the first step of studying ripple voltage, we have perfomed theoretical and exprimental research of low-frequency voltage, because, due to its complexity, no quantitative and theoretical researches have been made yet. In this paper derived theoretical equation for two-pole machine, with which we can calculate ripple voltage, is discussed on the individual origin of ripple.

Comparison of the calculated and measured ripple voltage will be explainen in detail in succeeding reports.

The propriety of the assumptions which we used to derive the equation will also be studied in them.

I. Introduction

Ripple voltage of d. c. generator, which appears superposed upon d. c. output voltage, con be classified according to the origin of generation :

(i) Low-frequency ripple (ii) Slot ripple

(iii) High-frequency ripple

Hitherto, it has bee a well-known fact that the low-frequency ripple, lower-frequency than slot ripple, has different magnitude and wave-form for individual machine of same design. But, due to its complexity, no quantitative and theoretical researches have been made on it yet.

The purpose of this paper is to derive theoretical equation for low-frequency ripple, with which we can calculate ripple voltage, and to discuss on the influence of individual origin of the ripple. Further, it is indicated that we can discriminate between the good and the bad manufacture of some generators by means of the low-frequency ripple generated by them.

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II. Origins of Low-frequendy Ripple

a) Variation of flux.···· If magnetic reluctance is varied in accordance with the rotation of armature, the flux will vary in its distribution and magnitude, and cosequently, induce ripple voltage. In case we consider the reluctance of the air gap and core independently, then the air gap variation due to rotation and the orientation of silicon steel will be the origin of reluctance variation.

b) Periodical variation of the relative velocity between flux and coil.···· For example, when the peripheral velocity of rotor is not constant, but pulsates periodically. Remarkable increase of ripple voltage in case of wrong coupling is mainly due to this origim.

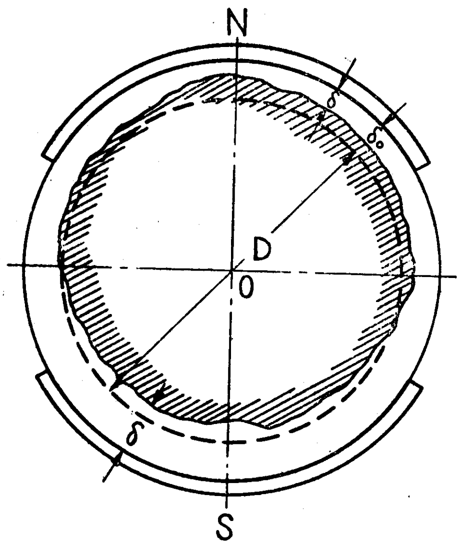

Fig. 1. Air gap variation

c) Periodical variation of the relative position between flux and coil. ···· For example, in case of axial vibration of rotor. However in such a case, the periodical variation of retative velocity will occur simultaneously.

Among these three groups of the origins, b) and c) are not originated by the gnerator, but are due to the driving machines and coupling conditions mainly, whereas, a) is the origin of of ripple produced out of the generator itself and is therefore the principal object of this paper.

Now, according to Fig. 1, we represent variable air gap due to rotation by Eq.(1),

$$\delta = \delta_0 - \sum_{m=1}^{\infty} \varepsilon_m \sin m\left(\frac{\pi}{\tau}x + \frac{v\pi}{\tau}t + \kappa_m\right)$$
$$= \delta_0\left\{1 - \sum_{m=1}^{\infty} \frac{\varepsilon_m}{\delta_0}\sin m\left(\frac{\pi}{\tau}x + \frac{v\pi}{\tau}t + \kappa_m\right)\right\} \qquad (1)$$

where τ = wave length of the fundamental air gap variation = pole pitch.
m = order of the space harmonics.
x = coordinate of of any point measured from brush.
v = peripheral velocity of rotor.
κ_m = phase angle of the mth harmonics.
ε_m = mean amplitude of the mth harmonics in axial direction.
δ_0 = mean air gap leogth under pole arc.

Eq. (1), includes every cause of air gap variation. For example, the effect of bending of the axis is shown in Fig. 2, where centre and radius of the pole arc and that of rotor are O, O′ and R, r respectively, and OO′ ≡ ξ, then

$$\rho = \xi\cos(\theta-\alpha) \pm \sqrt{r^2 - \xi^2\sin^2(\theta-\alpha)} \cong \xi\cos(\theta-\alpha) \pm r$$

$$\text{or} \quad \delta = R - \rho = \delta_0 - \xi\cos(\theta-\alpha) \qquad (2)$$

where θ = position angle $= \frac{\pi}{\tau}x$

α = time angle $= \frac{v\pi}{\tau}t$

then Eq. (2), is resolved into Eq. (1), when m = 1.

The effect of the orientation of silicon steel may be assumed as follows; as the permeabillty of silicon steel is greater in the direction of roll than in the orthogonal direction, the silicon steel might be longer in the direction of roll; accordingly air gap is less than orthogonal direction. Then we can substitute this effect by equivalent air gap variation. As Fig. 3. (a). shows that the direction of roll is orientated in one direction, the order of the fundamental of the eqivalent air gap variation is m = 2, whilst if it is divided into two directions the order is m = 4. (see Fig. 3. (b))

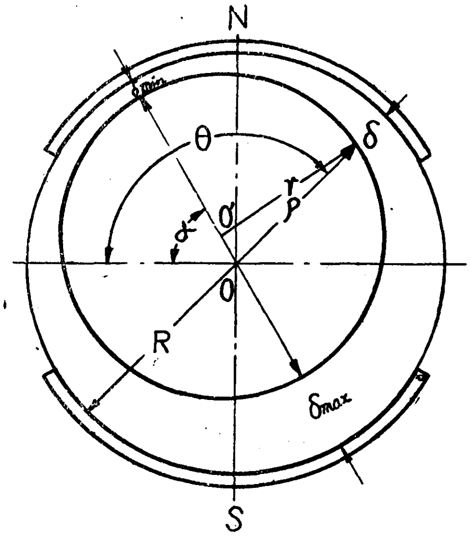

Fig. 2. Effect of the bending of shaft

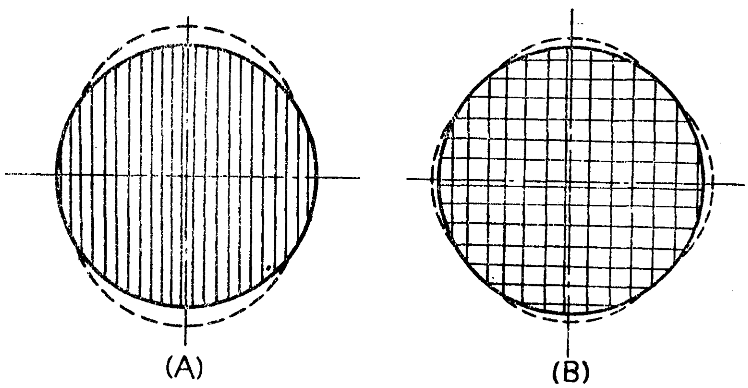

Fig. 3. Equivalent air gap variation

As stated above, using equivalent air gap variation, we can consider that all causes of flux variation are involved in Eq. (1). Regarding the most powerful cause producing ripple, it depends on the kinds of the generators,[1] In **Fig. 4**, A-curve represents the fundamental low-frequency ripple voltage of a small

1) T. HORII, I. E. E. of Japan, May 1951, No. 4 – 7.

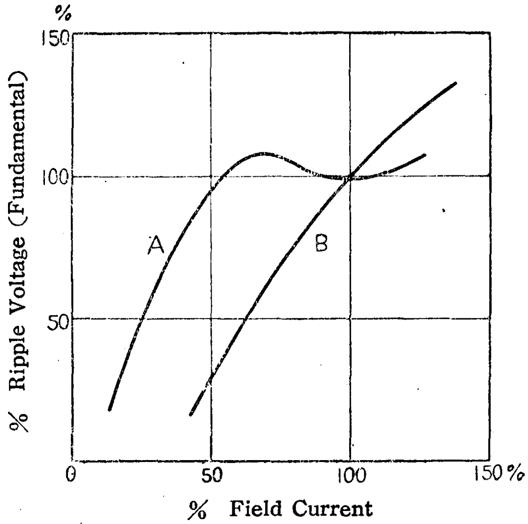

Fig. 4. Low-frequency ripple voltage

dynamomotor, 0.3 mm. air gap, 7700 r. p. m., 1000 V, and B-curve is of 2.5 mm. air gap, 1500 r. p. m., 35 V, 3 KW generator. The former, clearly, shows the influence of air gap variation, while, the latter is mainly influenced by the orientation and undue coupling. In succeeding reports, these experimental data will be explained in detail.

III. Variation of Flux

Considering "*smooth rotor*", namely: in Eq. (1), every $\varepsilon m = 0$, the distribution of flux may be expressed by the Fourier series as follows;

$$B_0 = \sum_{k=1}^{\infty} \beta k_0 \sin k(\frac{\pi}{\tau}x + \gamma_k) \qquad (3)$$

When pole arc and rotor are concentric, and brushes are situated on neutral point, k must be odd numbers and $\gamma_k = 0$. As every d. c. generator assumes these conditions for no-load, hereafter we will call this distribution "*normal flux distribution*".

In general, because of the assymmetry of the stator side, the air gap under poles do not remain uniform, even with the "*smooth rotor*". However, if we take in consideration this effect of the assymmetry as component of Eq. (3), we can assume the air gap to be uniform. From Eq. (3),

$$\delta_0 B_0 = \sum_k \delta_0 \beta k_0 \sin k(\frac{\pi}{\tau}x + \gamma_k) \qquad (4)$$

then δ_0 is the air gap magnetic reluctance per unit area, and $\beta k_0 \sin k(\frac{\pi}{\tau}x + \gamma_k)$ represents flux density at any point, so, we can assume that the superposed harmonic m. m. f., expressed on the right-hand side of Eq. (4), exists on all round the rotor surface.

With "*smooth rotor*", the air gap does not vary by rotation, and these m. m. f. yield flux distribution expressed in Eq. (3). But, if the air gap varies according to Eq. (1), the flux distribution must be

$$B = \frac{\sum_k \delta_0 \beta_k(t) \cdot \sin k\left(\frac{\pi}{\tau}x + \gamma_k\right)}{\delta_0\left\{1 - \sum_m \frac{\varepsilon_m}{\delta_0} \sin m\left(\frac{\pi}{\tau}x + \frac{v\pi}{\tau}t + \kappa_m\right)\right\}} \tag{5}$$

From the condition $\varepsilon_m/\delta_0 \ll 1$, Eq. (5), can be expressed

$$B \cong \sum_k \beta_k(t) \cdot \sin k(\frac{\pi}{\tau}x + \gamma_k)\left\{1 + \sum_m \frac{\varepsilon_m}{\delta_0} \sin m(\frac{\pi}{\tau}x + \frac{v\pi}{\tau}t + \kappa_m)\right\}$$

$$= \sum_k \beta_k(t) \cdot \sin k(\frac{\pi}{\tau}x + \gamma_k)$$
$$+ \sum_k \sum_m \frac{\varepsilon_m \beta_k(t)}{2\delta_0} \left\{ \cos\left(\frac{m-k}{\tau}\pi x + \frac{mv\pi}{\tau}t + m\kappa_m - k\gamma_k\right) \right.$$
$$\left. - \cos\left(\frac{m+k}{\tau}\pi x + \frac{mv\pi}{\tau}t + m\kappa_m + k\gamma_k\right) \right\} \tag{6}$$

According to Eq. (6), the flux distribution is composed of (i) standing harmonic fluxes with variable amplitude $\beta_k(t)s$, and (ii) infinite numbers or revolving fluxes with amplitude $\varepsilon_m\beta_k(t)/2\delta_0$, where $m \pm k$ are odd numbers.

As stated before, we took ε_m as a mean value towards axial direction, so, the relations between total fluxes can be compared with fluxes per unit axial length. Integrating Eq. (6), from 0 to τ, and τ to 2τ, represented by Φ_n, and Φ_s respectively, then

$$\Phi_n = -\Phi_s = f(t) \cdot \Phi_0 \tag{7}$$

$$\Phi_0 = \int_0^\tau B_0\, dx = \sum_k \frac{2\tau}{k\pi}\beta k_0 \equiv \sum_k \Phi k_0 \tag{8}$$

where $f(t)$ represents time variation of Φ_0 due to rotation of the rotor shown in Fig. 1.

From Eqs. (6), (7), and (8), thereafter substituted by conditions of *normal flux distribution*, then

$$f(t)\sum_k \frac{2\tau}{k\pi}\beta k_0 = \sum_k \left[\frac{2\tau}{k\pi}\beta_k(t) \pm \frac{\tau}{2\delta_0}\beta_k(t)\varepsilon_k \cos k\left(\frac{v\pi}{\tau}t + \kappa_k\right) \right.$$
$$\left. - \frac{2\tau}{\pi\delta_0}\beta_k(t)\sum_m \frac{k}{m^2 - k^2}\varepsilon_m \sin m\left(\frac{v\pi}{\tau}t + k_m\right)\right] \tag{9}$$

(where k = odd, m = even
+ sign, $0 \rightarrow \tau$, − sign, $\tau \rightarrow 2\tau$,)

If the flux distribution has odd harmonics alone, the effects of odd harmonics of air gap variation towards flux harmonics tend to cancel each other under each pole, except $m = k$ = odd. In such cases, as shown in Fig. 5, i. e. $m = k = 3$, the effects of each halfwave towards fiux wave are in the same phase, and if the directions of main flux under N & S poles are as shown by arrows, their effects are opposide under each poles. Because of this reason, Eq. (9), has ±signs.

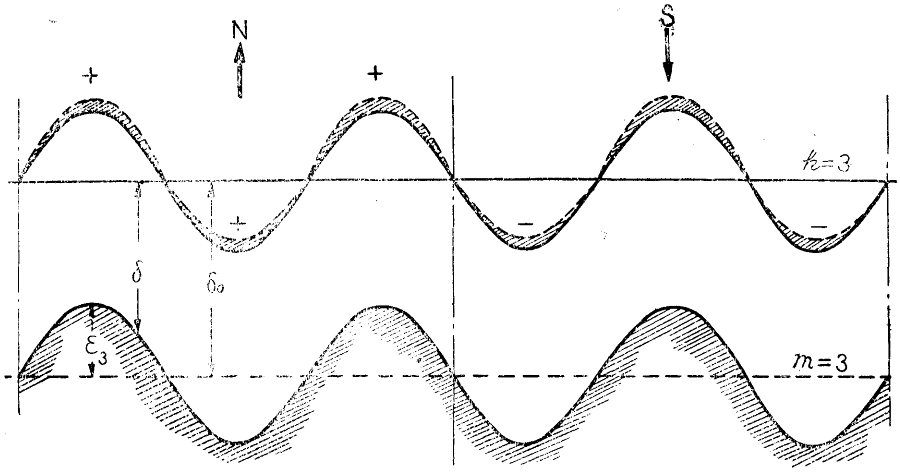

Fig. 5. Effect of odd harmonics of air gap variation ($m = k =$ odd)

Neglecting more than the 1st order terms of ε_m/δ_0 in Eq. (9), we can equate every kth harmonics on both sides of the Equation, as follows;

$$\frac{2\tau}{k\pi}f(t)\cdot\beta k_0 = \beta_k(t)\left[\frac{2\tau}{k\pi} \pm \frac{\tau}{2\delta_0}\varepsilon_k \cos k\left(\frac{vt}{\tau}t + \kappa_k\right)\right.$$
$$\left. - \frac{2\tau}{\pi\delta_0}\sum_m \frac{k}{m^2-k^2}\varepsilon_m \sin m\left(\frac{v\pi}{\tau}t + \kappa_m\right)\right]$$

or

$$\beta_k(t) = \frac{f(t)\cdot\beta k_0}{1 - \frac{k}{\delta_0}\sum_m \frac{k}{m^2-k^2}\varepsilon_m \sin m\left(\frac{v\pi}{\tau}t+\kappa_m\right) \pm \frac{k\pi}{4\delta_0}\varepsilon_k \cos k\left(\frac{v\pi}{\tau}t+\kappa_k\right)} \tag{10}$$

From Eqs. (5), and (10), the flux distribution, in case of variable air gap, can be expressed appoximately;

$$B \simeq f(t)\cdot\sum_k \beta k_0 \frac{\sin k\left(\frac{\pi}{\tau}x+\gamma_k\right)\left\{1+\sum_m \frac{\varepsilon_m}{\delta_0}\sin m\left(\frac{\pi}{\tau}x+\frac{v\pi}{\tau}t+\kappa_k\right)\right\}}{1-\frac{k}{\delta_0}\sum_m \frac{k^2}{m^2-k^2}\varepsilon_m \sin m\left(\frac{v\pi}{\tau}t+\kappa_m\right) \pm \frac{k\pi}{4\delta_0}\varepsilon_k \cos k\left(\frac{v\pi}{\tau}t+\kappa_k\right)} \tag{11}$$

If the rotor is located in the position shown in Fig. 1, the air gap reluctance under N pole is less than that of S pole, and the total flux under both poles must be equal. Consequently the m. m. f. needed for N pole is less than that of S pole. The existence of opposite sign terms in Eq. (11), under each poles, expresses this fact. As the denominater of the Equation is the integration of the numerater, it can satisfy Eq. (7), at any instant.

IV. Determination of $f(t)$

As defined in Eq. (7), $f(t)$ is the time variation of the total flux, and is not concerned with k and x.

Therefore, it can be calculated by converting the air gap reluctance variation into the flux variation.

In Fig. 6, construct equivalent rectangle, having the same area as the flux distribution curve, and determine equivalent pole arc length $(\tau - 2\alpha)$, then the air gap permeance per pole for smooth rotor is:

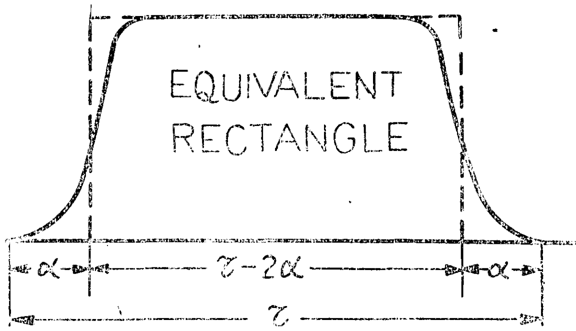

Fig. 6. Equivalent pole arc

$$P_0 = \int_{\alpha}^{\tau-\alpha} \frac{L}{\delta_0}\, dx = \frac{L(\tau - 2\alpha)}{\delta_0} \tag{12}$$

where L is the equivalent axial length of the air gap under pole given as above.

The total air gap reluctance is:

$$R_0 = \frac{2}{P_0} = \frac{2\delta_0}{L(\tau - 2\alpha)} \tag{13}$$

But, for uneven rotor, because the permeance will be varied according to rotation, they must be expressed as follows;

$$\left.\begin{aligned} P_N &= \int_{\alpha}^{\tau-\alpha} \frac{L\, dx}{\delta_0 - \sum_m \varepsilon_m \sin m\left(\frac{\pi}{\tau}x + \frac{v\pi}{\tau}t + \kappa_m\right)} \\ P_S &= \int_{\tau+\alpha}^{2\tau-\alpha} \frac{L\, dx}{\delta_0 - \sum_m \varepsilon_m \sin m\left(\frac{\pi}{\tau}x + \frac{v\pi}{\tau}t + \kappa_m\right)} \end{aligned}\right\} \tag{14}$$

The total air gap reludtance may be calculated by

$$R_a = \frac{1}{P_N} + \frac{1}{P_S}$$

As shown in Fig. 7, even harmonics of the air gap variation effect inphase under each poles, and odd harmonics have phase difference of π, so, the effects of odd harmonics tend to cancel each other under both poles and the 1st order terms of ε_m / δ_0 (when m = odd) vanish in Ra. Taken into consideration the fact that ε_1, is greater than others, we took the 2nd order terms of ε_m / δ_0 for calculation of P_N, and P_S; then R_a is expressed as follows;

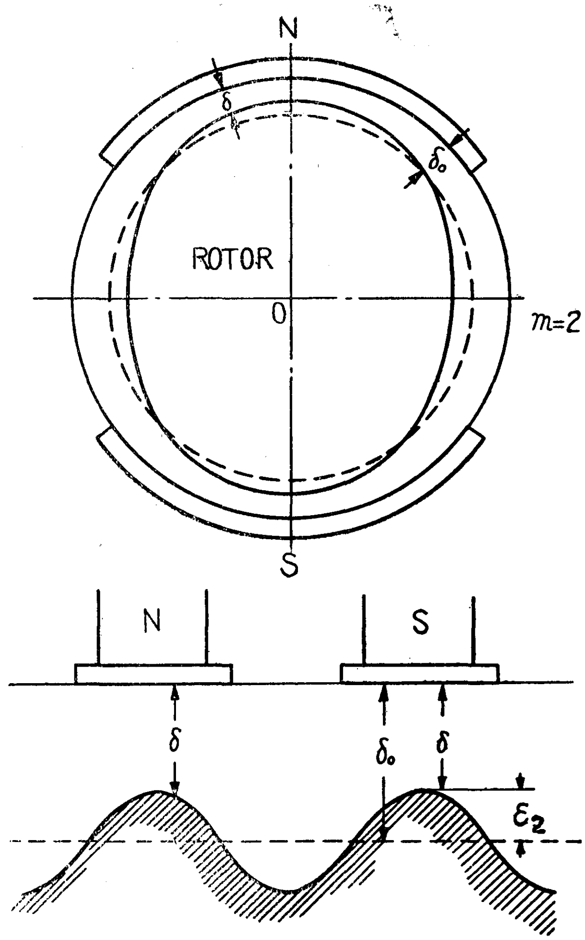

Fig. 7. Effect of the even harmonics of air gap variation

$$\frac{R_a}{R_0} = 1 + \frac{\tau}{\pi(\tau-2\alpha)} \sum_{m=1}^{\infty} \frac{1}{m} \frac{\varepsilon_{2m}}{\delta_0} \sin 2m\theta \cdot \sin 2m\left(\frac{v\pi}{\tau}t + \kappa_{2m}\right)$$

$$+ \left\{\frac{\tau}{\pi(\tau-2\alpha)}\right\}^2 \frac{1}{2} \left[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4}{\overline{2n-1}\cdot\overline{2m-1}} \cdot \frac{\varepsilon_{2n-1}}{\delta_0} \cdot \frac{\varepsilon_{2m-1}}{\delta_0} \cos\overline{2n-1}\theta \cdot \cos\overline{2m-1}\theta \right.$$

$$\times \left\{ \cos\left(2\cdot\overline{m+n-1}\,\frac{v\pi}{\tau}t + \overline{2n-1}\,\kappa_{2n-1} + \overline{2m-1}\,\kappa_{2m-1}\right) \right.$$

$$\left. + \cos\left(2\cdot\overline{m-n}\,\frac{v\pi}{\tau}t + \overline{2m-1}\,\kappa_{2m-1} - \overline{2n-1}\,\kappa_{2n-1}\right) \right\}$$

$$+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{nm} \cdot \frac{\varepsilon_{2m}}{\delta_0} \cdot \frac{\varepsilon_{2n}}{\delta_0} \sin 2m\theta \cdot \sin 2n\theta$$

$$\times\left\{\cos\left(2\cdot\overline{m-n}\frac{v\pi}{\tau}t+2m\kappa_{2m}-2n\kappa_{2n}\right)\right.$$

$$\left.-\cos\left(2\cdot\overline{m+n}\frac{v\pi}{\tau}t+2m\kappa_{2m}+2n\kappa_{2m}\right)\right\}\Bigg]$$

$$+\left\{\frac{\tau}{\pi(\tau-2\alpha)}\cdot\frac{1}{2}\right\}\left[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\frac{1}{m-n}\cdot\frac{\varepsilon_{2m-1}}{\delta_0}\cdot\frac{\varepsilon_{2n-1}}{\delta_0}\sin 2\,\overline{m-n}\,\theta\right.\right.$$

$$\cdot\cos\left(2\cdot\overline{m-n}\frac{vt}{\tau}t+\overline{2m-1}\,\kappa_{2m-1}-\overline{2n-1}\,\kappa_{2n-1}\right)$$

$$-\frac{1}{m+n-1}\cdot\frac{\varepsilon_{2m-1}}{\delta_0}\cdot\frac{\varepsilon_{2n-1}}{\delta_0}\sin 2\cdot\overline{m+n-1}\,\theta$$

$$\cdot\cos\left(2\cdot\overline{m+n-1}\frac{v\pi}{\tau}t+\overline{2m-1}\,\kappa_{2m-1}+\overline{2n-1}\,\kappa_{2n-1}\right)$$

$$+\frac{1}{m-n}\cdot\frac{\varepsilon_{2m}}{\delta_0}\cdot\frac{\varepsilon_{2n}}{\delta_0}\sin 2\cdot\overline{m-n}\,\theta\cdot\cos\left(2\,\overline{m-n}\frac{v\pi}{\tau}t+2m\kappa_{2m}-2n\kappa_{2n}\right)$$

$$\left.\left.-\frac{1}{m+n}\cdot\frac{\varepsilon_{2m}}{\delta_0}\cdot\frac{\varepsilon_{2n}}{\delta_0}\sin 2\cdot\overline{m+n}\,\theta\cdot\cos\left(2\,\overline{m+n}\frac{v\pi}{\tau}t+2m\kappa_{2m}+2n\kappa_{2n}\right)\right\}\right] \quad (15)$$

(where $\theta=\alpha\pi/\tau$)

In Eq. (15), pick up necessary terms of the same order, and add the terms one after another taking into account their phase angles, we can calculate the amplitude of any air gap reluctance variation with this order of frequency, in the ratio to R_0. Convert the amplitude into variation of Ampere Turns, then the amplitude of the flux variation may be calculated as follows (see Fig. 8)

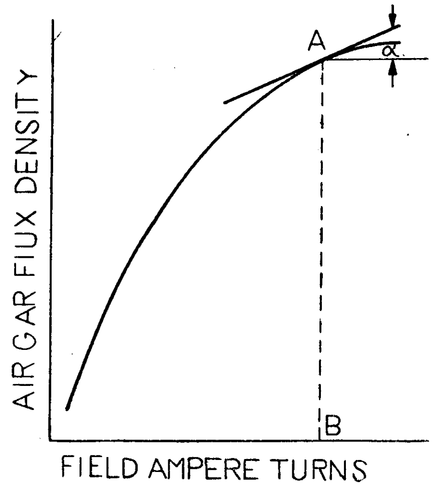

Fig. 8.

$$\Delta B=\frac{dB}{dA.T.}\times\Delta A.T.=\tan\alpha\times\Delta A.T. \quad (16)$$

Therefore, $f(t)$ is

$$f(t)=1+\zeta_2\sin 2\left(\frac{v\pi}{\tau}t+\lambda_2\right)+\zeta_4\sin 4\left(\frac{v\pi}{\tau}t+\lambda_4\right)+\cdots\cdots \quad (17)$$

where ζ_2, ζ_4 ········ means amplitudes of the total flux variation of the 2nd and the 4th ········ time harmonics, expressed in the ratio to Φ_0.

V. Induced Voltage

In Eq. (10), put

$$\left.\begin{aligned} F_k(t) &\equiv \frac{k}{\delta_0}\sum_m{}' \frac{k}{m^2-k^2}\varepsilon_m \sin m\left(\frac{v\pi}{\tau}t+\kappa_m\right) \\ S_k(t) &\equiv \frac{k\pi}{4\delta_0}\varepsilon_k \cos k\left(\frac{v\pi}{\tau}t+\kappa_k\right) \end{aligned}\right\} \tag{18}$$

then

$$\beta_k(t) = \frac{f(t)\,.\,\beta k_0}{\{1-F_k(t)\}\pm S_k(t)}$$
$$\equiv f(t)\beta k_0[\,\boldsymbol{H}_k(t)\mp \boldsymbol{D}_k(t)\,] \tag{19}$$

where, $\boldsymbol{H}_k(t)$ represents all terms having same sign under N, S poles, and $\boldsymbol{D}_k(t)$ represents terms which have opposite signs. Then,

$$\boldsymbol{H}_k(t) = 1 + F_k(t) + F_k^2(t) + S_k^2(t) + \cdots\cdots\cdots \qquad \cong 1 + F_k(t) \tag{20}$$
$$\boldsymbol{D}_k(t) = S_k(t) + 2F_k(t)\cdot S_k(t) + \cdots\cdots\cdots \qquad \cong S_k(t)$$

From Eqs (20), and (5),

$$B = f(t)\sum_k \beta k_0[\,\boldsymbol{H}_k(t)\mp \boldsymbol{D}_k(t)\,]\sin k\left(\frac{\pi}{\tau}x+\gamma_k\right)$$
$$\times\left\{1+\sum_m{}' \frac{\varepsilon_m}{\delta_0}\sin m\left(\frac{\pi}{\tau}x+\frac{v\pi}{\tau}t+\kappa_m\right)\right\} \tag{21}$$

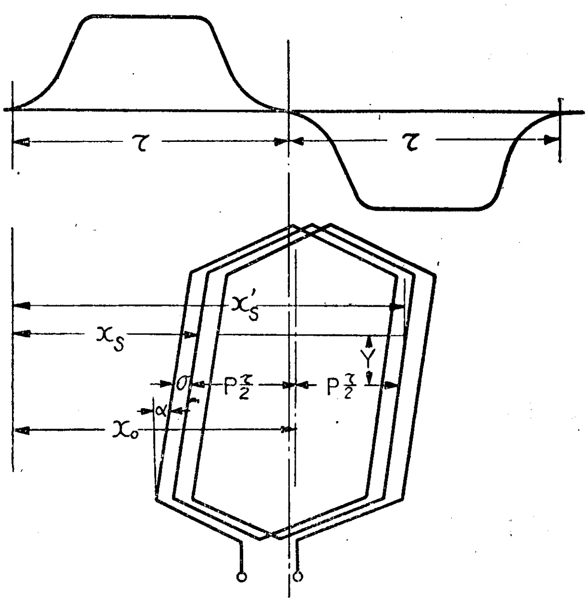

Fig. 9. Arrangement of flux and coil

The induced voltage originated according to the terms multiplied by $\boldsymbol{D}_k(t)$, tend to cancel each other under N, S poles as to be stated later, and do not appear out of brushes. Therefore, concerning induced voltage we can neglect $\boldsymbol{D}_k(t)$, and may proceed calculation to to take $\boldsymbol{H}_k(t)$ only.

Now, in Fig. 9,

σ = slot pitch, x_0 = coordinate of the midpoint of the coil group,
α = angle of skew,
y = coordinate at right angles to the direction of motion, reckoned from midpoint of the coil side,
x_s, x'_s = coordinate of the trailing and leading coil sides of the sth coil, counting from the trailing end of the group of coils respectively.

The total flux included in the sth coil is, by Eqs. (18), (20), and (21),

$$\phi_s = \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{x_s}^{x'_s} B\,dx\,dy$$

$$= \iint_{Sth\ coil} f(t) \sum_k \beta k_0 \sin k\left(\frac{\pi}{\tau}x + \gamma_k\right) dx\,dy \qquad \cdots\cdots (a)$$

$$+ \iint_{Sth\ coil} f(t) \sum_k \beta k_0 F_k(t) \sin k\left(\frac{\pi}{\tau}x + \gamma_k\right) dx\,dy \qquad \cdots\cdots (b)$$

$$+ \iint_{Sth\ coil} \Sigma \left\{ f(t)\, \beta k_0 H_k(t) \sin k\left(\frac{\pi}{\tau}x + \gamma_k\right) \right.$$

$$\left. \times \sum_m \frac{\varepsilon_m}{\delta_0} \sin m\left(\frac{\pi}{\tau}x + \frac{v\pi}{\tau}t + \kappa_m\right) \right\} dx\,dy \qquad \cdots\cdots (c)$$

(22)

Calculate the induced voltage of the sth coil of N turns, and sum up for the group of c such coils, then the induced voltage between brushes is obtained.[2)]

The induced voltage by the flux of Eq. (22) (a), is

$$\mathrm{e}g_1 = -2L\frac{\tau}{\pi}\frac{cN}{10^8} \sum_k C_{dk} C_{sk} C_{pk} \left[\frac{1}{k} \beta k_0 \frac{df(t)}{dt} \sin k\left(\frac{\pi}{\tau}x_0 + \gamma_k\right) \right.$$

$$\left. + \frac{\pi}{\tau}\frac{dx_0}{dt} \beta k_0 f(t) \cos k\left(\frac{\pi}{\tau}x_0 + \gamma_k\right) \right] \qquad (23)$$

where

$$C_{pk} = \sin k\frac{p\pi}{2} = \text{pitch reduction factor for the } k\text{th harmonics}$$

$$C_{sk} = \frac{\sin k(\pi L \tan\alpha/2\tau)}{k(\pi L \tan\alpha/2\tau)} = \text{skew reduction factor for the } k\text{th harmonics}$$

$$C_{dk} = \frac{\sin k(c\,\sigma\pi/2\tau)}{c \sin k(\sigma\pi/2\tau)} = \text{distribution reduction facter for the } k\text{th harmonics}$$

If $f(t)$ in Eq. (23) is 1, $\mathrm{e}g_1$ gives d. c. output voltage under the condition of $\varepsilon_m = 0$ or $\delta = \delta_0$, consequently the 2nd terms of the equation represent the induced voltage of rotating conductors in case the total flux pulsate according to Eq. (17).

2) L. V. Bewley, A. I. E. E. Trans. Vol. XLIX. 1930. p. 459.

The 1st terms represent ripple voltage by transformer action of the pulsating total flux, but generally this voltage cancel each other and do not appear out of brushes.

The induced voltage by the flux of Eq. (23) (b) is

$$e_{g_2} = -2L \frac{\tau}{\pi} \frac{cN}{10^8} \sum_k C_{dk} C_{sk} C_{pk} \Big[\frac{\beta k_0}{k} \frac{df(t) \cdot F_k(t)}{dt} \sin k \left(\frac{\pi}{\tau} x_0 + \gamma_k \right)$$

$$+ \frac{\pi}{\tau} \frac{dx_0}{dt} \beta k_0 f(t) F_k(t) \cos k \left(\frac{\pi}{\tau} x_0 + \gamma_k \right) \Big] \tag{24}$$

Eq. (24) represent ripple voltage due to pulsation of each harmonic fluxes with $f(t)\ F_k(t)$.

Eq. (22) (c) composed of revolving field of different rotating speed and induce:

$$e_{g_3} = -2L \frac{\tau}{\pi} \frac{cN}{10^8} \sum_k \Big[\!\Big[\beta k_0 \frac{df(t) \cdot H_k(t)}{dt} \sum_m \Big[\frac{1}{m-k} C_{\overline{pm-k}} C_{\overline{sm-k}} C_{\overline{dm-k}}$$

$$\times \cos \left(\frac{m-k}{\tau} \pi x_0 + \frac{mv\pi}{\tau} t + m\kappa_m - k\gamma_k \right)$$

$$- \frac{1}{m+k} C_{\overline{pm+k}} C_{\overline{sm+k}} C_{\overline{dm+k}} \cdot \cos \left(\frac{m+k}{\tau} \pi x_0 + \frac{mv\pi}{\tau} t + m\kappa_m + k\gamma_k \right) \Big]$$

$$+ \beta k_0 \frac{\pi}{\tau} f(t) \cdot H_k(t) \frac{dx_0}{dt} \sum_m \frac{\varepsilon_m}{2\delta_0} \Big[\frac{k}{m-k} C_{\overline{pm-k}} C_{\overline{sm-k}} C_{\overline{dm-k}}$$

$$\times \sin \left(\frac{m-k}{\tau} \pi x_0 + \frac{mv\pi}{\tau} t + m\kappa_m - k\gamma_k \right)$$

$$+ \frac{k}{m+k} C_{\overline{pm+k}} C_{\overline{sm+k}} C_{\overline{dm+k}} \sin \left(\frac{m+k}{\tau} \pi x_0 + \frac{mv\pi}{\tau} t + m\kappa_m + k\gamma_k \right) \Big] \Big]\!\Big] \tag{25}$$

$$(m \neq k, \quad v = -\frac{dx_0}{dx})$$

where

$$C_{\overline{dm\pm k}} = \frac{\sin \overline{m\pm k}\, (c\,\sigma\pi/2\tau)}{c \sin \overline{m\pm k}\, (\sigma\pi/2\tau)} \qquad \text{etc.}$$

For 2-pole machines, when $m\pm k$ = even, $C_{\overline{dm\pm k}}$ must be zero, on account of $c\sigma = \tau$. Then as in the case of $H_k(t)$, in Eq. (25), $m\pm k$ must be odd number, or if k = odd, m must be even.

But $m = k$ (= odd) is an exceptional case, and from Eq. (21)

$$B_{m=k} = f(t) \sum_k \beta k_0 H_k(t) \frac{\varepsilon_k}{2\delta_0} \cos k \left(\frac{v\pi}{\tau} t + \kappa_k - \gamma_k \right) \tag{26}$$

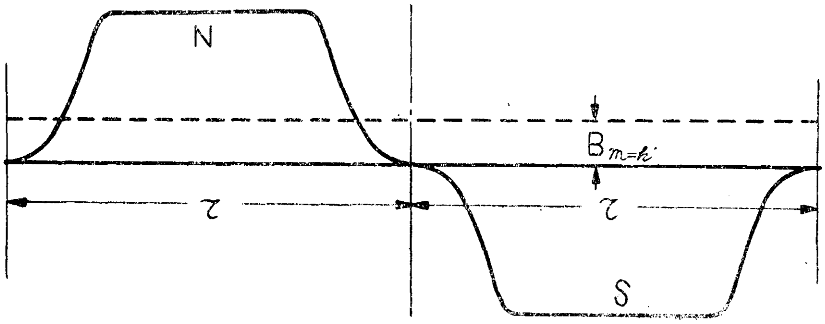

Fig. 10. Flux distribution of $B_{m=k}$

As shown in Fig. 10, this flux is composed of the kth harmonic flux irrelevant to position. Fig. 11. (A) (B), show the direction of this flux and the main flux respectively. As shown in Fig. 11. (C), the voltage induced by this flux is cancelled on both sides of any coil. But, the induced voltage by transformer action do not cancel, and the value between brushes is

$$-p\tau L \frac{Nc}{10^8} \sum_k \Big[\beta k_0 \frac{df(t) \cdot H_k(t)}{dt} \frac{\varepsilon_k}{2\delta_0} \cos k \left(\frac{v\pi}{\tau} t + \kappa_k - \gamma_k \right) - \frac{\pi k}{\tau} v\, \beta k_0 f(t)\, H_k(t) \cdot \frac{\varepsilon_k}{2\delta_0} \sin k \left(\frac{\pi v}{\tau} t + \kappa_k - \gamma_k \right) \Big] \tag{27}$$

But, because of the symmetrical arrangement of this flux, the induced voltage by transformer action has peripherally the same direction and same magnitude, and consequently, the voltage produces circulating currents and increases losses, but do not appear out of the brushes.

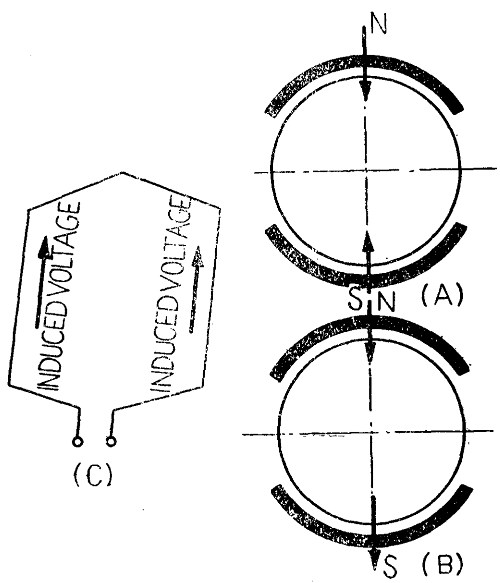

Fig. 11. Directions of fluxes and induced voltage by $B_{m=k}$

Now, the total induced voltage is

$$eg_0 = eg_1 + eg_2 + eg_3$$

Substituting the coditions of "*normal flux distribution*", and putting $f(t) \cdot H_k(t) . \beta k_0 = \beta_k(t)$, then eg is expsessed:

$$eg = -2L \frac{\tau}{\pi} \frac{cN}{10^8} \sum_k \Big[\frac{d\beta_k(t)}{dt} \Big\{ \frac{1}{k} C_{pk} C_{sk} C_{ak} \sin k \frac{\pi}{\tau} x_0$$

$$+ \sum_m \frac{\varepsilon_m}{2\delta_0} \left[\frac{1}{m-k} C_{p\overline{m-k}} C_{s\overline{m-k}} C_{d\overline{m-k}} \cos m \left(\frac{v\pi}{\tau} t + \kappa_m + \frac{m-k}{m\tau} \pi x_0 \right) \right.$$

$$\left. - \frac{1}{m+k} C_{p\overline{m+k}} C_{s\overline{m+k}} C_{d\overline{m+k}} \cos m \left(\frac{v\pi}{\tau} t + \kappa_m + \frac{m+k}{m\tau} \pi x_0 \right) \right] \Bigg\}$$

$$+ \frac{\pi}{\tau} \frac{dx_0}{dt} \beta_k(t) \Bigg\{ C_{pk} C_{sk} C_{dk} \cos k \frac{\pi}{\tau} x_0$$

$$+ \sum_m \frac{\varepsilon_m}{2\delta_0} \left[\frac{k}{m-k} C_{p\overline{m-k}} C_{s\overline{m-k}} C_{d\overline{m-k}} \sin m \left(\frac{v\pi}{\tau} t + \kappa_m + \frac{m-k}{m\tau} \pi x_0 \right) \right.$$

$$\left. + \frac{k}{m+k} C_{p\overline{m+k}} C_{s\overline{m+k}} C_{d\overline{m+k}} \sin m \left(\frac{\pi v}{\tau} t + \kappa_m \frac{m+k}{m\tau} \pi x_0 \right) \right] \Bigg\} \Bigg] \tag{28}$$

(k = odd, m = even)

Now, for the low-frequency ripple under normal flux distribution, tne midpoint of the coil group x_0 is nearly equal to τ, aud $\frac{d\beta_k(t)}{dt}$ is composed of the 1st & higher order terms of ε_m/δ_0, so, the terms multiplied by $d\beta_k(t)/dt$ are composed of the 2nd and higher order terms of ε_m/δ_0. Therefore, in calculation we can neglect these terms.

To calculate the mth frequency ripple voltage, we can pick up effective β_ks, and advance calculations comparatively easily.

V. Discussion of the Equations

(1) Frequencies of the low-frequency ripple: ...... From Eqs. (11), (15) and (28), the lowest frequency is found to be twice that of rotating frequency, and has the 2nd, the third and higher harmonics. The consequence coinsides with experimental data.

The even harmonics of air gap variation participate in producing the ripple, while, the effect of odd harmonics is only secondary.

(2) The effect of the orientation of silicon steel: ...... As stated before, the equivalent air gap variation has only even harmonics, and therefore is effective for producing ripple voltage. But, the portion in the rotor, where flux density reaches high enough to give effect of orientation, is the teeth, so, the mathematical calculation of the equivalent air gap variation will be hard, becauses of the complexity of its shape. It will be better, to estimate Its effect, by using the difference between the measured ripple voltage and the theoretical value, which is calculated by the real air gap variation.

(3) The variation of rotating speed: ...... This effect is represented by the periodical variation of $v = -\frac{dx_0}{dt}$, in Eq. (28), and d. c. output voltage pulsates in proportion to the variation.

(4) The axial vibration : ······ This effect is represented by the periodical variation of L in Eq. (28). But, unless the amplitude of the vibration is very large, the variation of the equivalent axial length will not occur. If the axial vibration produces very large ripple voltage, we must think it accompanies the variation of rotating speed.

(5) The variation of the total flux ······ As stated in IV., to calculate $f(t)$, we must convert the variation of A.T. into the variation of the flux, but ordinary d.c. generator uses fairy saturated portion of the magnetic path, so, $\tan\alpha$ in Eq.(16), is very small compared with unity. Consequently. ζ_2 and ζ_4 ······ in Fq. (17), are fairly less than ε_m/δ_0, so that the total flux actnally suffers little variation in consequence of the real air gap variation.

(6) The influence of the solt : ······ The biggest air gap variation is due to slot, but its freqvency is equal to slot-ripple, so, this effect will be stated in succeeding reports.

(7) Conclusion : — In every Equation, the air gap variatiou appears in the from of ε_m/δ_0, so that if ε_m is the same value, larger airgap length will produce less ripple voltage. To reduce the ripple voltage, we must reduce ε_ms, or adopt large air gap length, or diverge the direction of roll.

Now, as ε_ms are mainly dependent upon manufacture, it is no wonder that the low-frequency ripple is different in each generator, not withsfanding the same design. From this point of view, the measurement of the low-frequency ripple voltage provides a means for inspection of manufacture.

VI. Distorsion of the Flux Distribufion and its Effect on the Low-frequency Ripple

As stated in III, we assume " *normal flux distribution* " for the Eqs. (9), and (28), In this section, we consider many causes, which give rise to the deviation from the normality, and show how they act upon the ripple voltage.

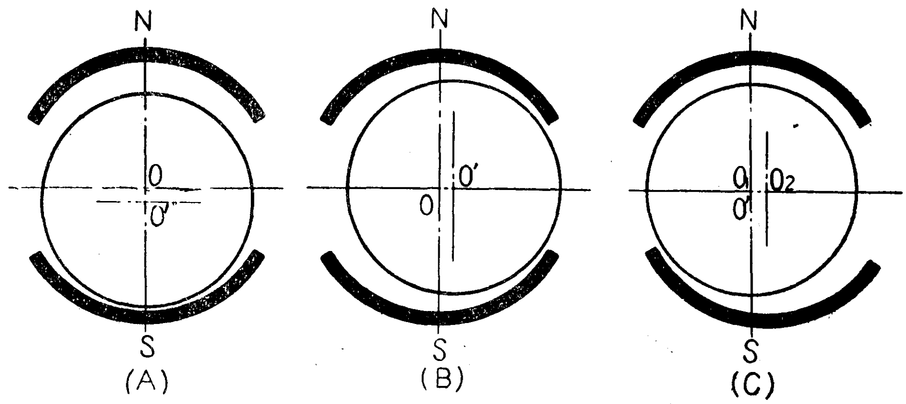

Fig. 12. Causes of the distorsion

(1) Causes of the distorsion.

i) Electrical causes.

a. Load current (Armature reaction)

b. Improper brush position

ii) Structural causes.

c. Assymmetry of the pole arc

1. Poor manufacture of the pole arc surface

2. Poor fitting of the pole. (Fig. 12.(c))

d. Deviation of both centres of the pole arc and the rotor. (Fig. 12.(a) (b))

Excessive wear of the bearing brings about this cause. When the rotor is at rest, the deviation will be toward below, but in running condition, the deviation will have angles from the perpendicular.[3)]

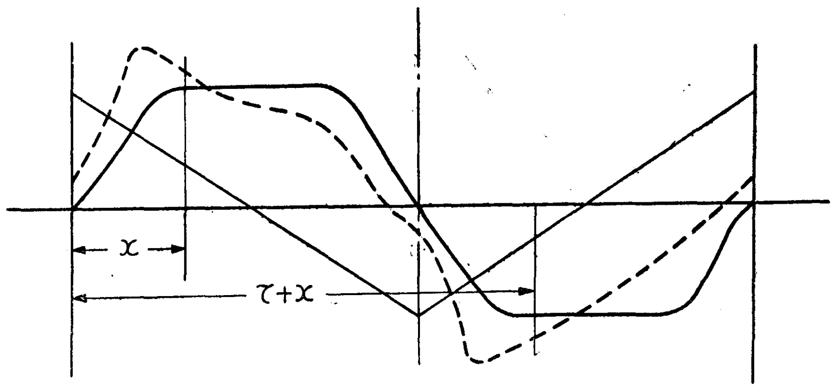

Fig. 13. Flux distorsion due to armature reaction

(2) The distorsion of the flux distribution due to each cause.

i) Electrical causes.

These causes give effect symmetrically under N, S poles, so, the flux distribution satisfies the condition (see Fig. 13)

$$f(x+\pi) = -f(x)$$

tben, in Eq. (3),

$$k = \text{odd}, \quad \gamma_k \neq 0$$

ii) The cause shown in Fig. 11. (b).

The flux distribution satisfies the condition (see Eig. 14.)

$$f(-x) = -f(x)$$

then

$$k = \text{odd and even}, \quad \gamma_k = 0$$

3) T. Sasaki, Bearing (Book) p. 17, 1948.

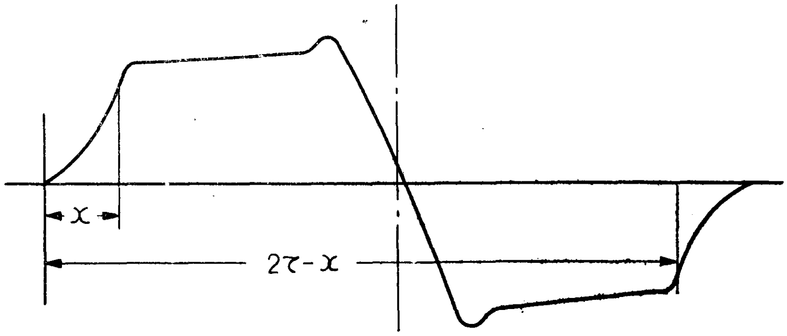

Fig. 14. Flux distorsion due to deviation of centres

iii) Causes other than mentioned above.

These causes have no regularity, and consequently,

$$k = \text{odd and even}, \quad \gamma_k \neq 0$$

(3) The effect of the electrical eauses upon the ripple.

The flux distorsion due to the armature reaction always exists when we use generators, except when they are provided with the compensating windings. But, as stated above, it only creates the phase angle of the harmonic flux, so that, we may calculate from Eqs. (23), (24), (25) and (6), taking into consideration the γ_ks.

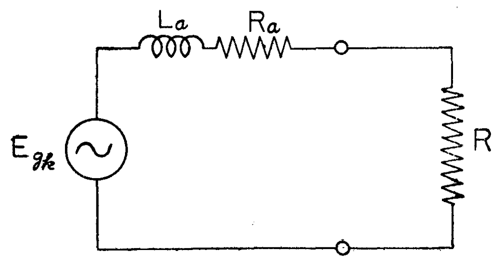

Fig. 15. Equivalent cercuit of ripple voltage

Equivalent circuit of the ripple voltage is shown in Fig. 15, where Eg_k is the kth harmonic voltage, Ra and La are resistance and inductance of the armature circuit respectively.

Then, the ripple voltage, which appears between terminals, is

$$\frac{E_{gk} \cdot R}{R + R_a + \omega_k L_a} = E_{gk} \frac{1}{1 + \dfrac{R_a + \omega_k L_a}{R}}$$

For the low-frequency ripple, ω_k is small or $R_a + \omega_k L_a \ll R$, so, it does not suffer from load resistance.

(4) The effect of the even harmonics of flux.

As stated in Eq. (25), $m \pm k =$ odd is the condition for producing the ripple. when k = odd, m must be even. But, if there are even harmonics of flux, then odd numbers of m produce the ripple too. Of cource, the even harmonics of flux will have less amplitude than the odd harmonics, but it must be noticed, that they can couple with the largest air gap variation, namely, ε_1.

Odd harmonics of tne ripple voltage can be observed in experiment, and they indicate structural assymmetry.

VII. Conclusions

As mentioned above, we derived the theoretical Equation of the low-frequency ripple, which up to the present had remained doubtfull, and discussed on the effect of every cause. Comparison of the calculated and measured ripple voltage will be explained in detail in succeeding reports. The propriety of the assumptions which we used to derive the equation will also be studied in them.

The author wishes to thank Dr. Y. Sigyo, who read the paper and gave many instructive criticisms, and Messers. A. Koizumi, K. Aoki, Y. Suzuki, and J. Hayakawa for their assistance in measurement and calculation.