Title	Orbit assignment for nuclear shell model (identical nucleon numbers 1-50)
Sub Title	
Author	Ota, Toyomi
Publisher	慶應義塾大学藤原記念工学部
Publication year	1950
Jtitle	Proceedings of the Fujihara Memorial Faculty of Engineering Keio University Vol.3, No.11 (1950.) ,p.91(1)- 94(4)
JaLC DOI	
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Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00030011-0001

Orbit Assignment for Nuclear Shell Model (Identical Nucleon Numbers 1 ～ 50)

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(Received May 20, 1952)

Abstract

M. Mayer's recent theory concerning the nuclear spin and magnetic moment based mainly on one particle model with strong spin orbit coupling has been recognized as a satisfactory one. The author, in order to accomplish her table and to check her theory, has assigned the orbits for nuclei containing 1 ～ 50 identical nucleons and has found that one particle model is not suitable in several cases. In these cases many particle model seems to be more preferable, though no decisive results have been gained as yet.

I. Introduction

The remarkable abundance of atomic nuclei in nature containing certain numbers, such as 2, 8, 20, 28, 50, ········ of protons or neutrons has long been considered as the proof of their stable structure. Many authors have tried to account for the stability in an analogical way such as that adopted in quantum theory of atomic structure. But explanations along this line failed to arrange the sum of degrees of degeneracy of each successive energy levels to agree with the proton or neutron numbers of stable nuclei containing more than 20 protons or neutrons. Accordingly, the modification of usual energy levels obtained as the eigen-value solutions of non-relativistic Schrödinger wave equation ought to have preceded the appearance of a satisfactory theory.

Recently M. Mayer noticed this point. She divided one energy level belonging to the azimuthal quantum number l into two levels, namely $l+1/2$ and $l-1/2$. This means, in atomic theory, taking the so-called spin orbit coupling energy into consideration. This coupling term comes out as the small perturbation of Dirac's relativistic wave equation. If one assumes this coupling energy strong enough, he can assign two separated shells for two levels with the same l and different j.

M. Mayer, in her paper[1], made four assumptions based on one particle model with strong spin orbit coupling in order to explain nuclear configuration. They are :

(1) that strong spin orbit coupling caused by the second perturbation of tensor interaction leads to inverted doublet,

(2) that an even number of identicl nucleons couple to zero angular momentum,

(3) an odd number coupled to the angular momentum which the single odd particle has in a square well pontential of infinite depth,

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1) M. Mayer, Phys, Rev. 78, 16 (1950)

(4) that a negative pairing energy, increasing with the j value of the orbit exists.

Using newly measured values of spin and magnetic moment, we assigned the energy levels for odd-even nuclei in order to accomplish her table and to check her theory. The results are shown in table 1.

Table 1

No. of odd Nucleons	Element (Neutrons)	Spin	Element (Protons)	Spin	Orbit Assigned
1	n	1/2	H^1	1/2	$s_{1/2}$
3	He^5	$p_{1/2}$(scat.exp.)	Li^7	3/2	$p_{3/2}$
5	Be^9	3/2	B^{11}	3/2	
7	C^{13}	1/2	N^{15}	1/2	$p_{1/2}$
9	O^{17}	5/2	F^{17}	5/2	$d_{5/2}$
			F^{19}	$1/2(S_{1/2})$	
11	Ne^{21}	$3/2(P_{3/2})$	Na^{23}	$3/2(P_{3/2})$	
13	Mg^{25}	5/2	$Al^{25,27}$	5/2	
15	Si^{29}	1/2	P^{31}	1/2	$s_{1/2}$
17	S^{33}	3/2	Cl^{35}	3/2	$d_{3/2}$
19	S^{35}	3/2	$K^{39,41}$	3/2	
21			Sc^{45}	7/2	$f_{7/2}$
23	Ca^{43}	$(f_{7/2})$	V^{51}	7/2	
25	Ti^{47}	$(f_{7/2})$	Mn^{55}	$5/2(D_{5/2})$	
27	Ti^{49}	$(f_{7/2})$	Co^{59}	7/2	
29	Cr^{53}	$(p_{3/2})$	Cu^{63}	3/2	$p_{3/2}$
31	Fe^{57}	$(p_{3/2})$	$Ga^{69,71}$	3/2	
33	As^{75}	$3/2(p_{3/2})$	Ni^{61}	$(p_{3/2})$	$f_{5/2}$
35			$Br^{79,81}$	$3/2(p_{3/2})$	
37	Zn^{67}	5/2	Rb^{85}	5/2	
			Rb^{87}	$3/2(p_{3/2})$	
39			Y^{89}	1/2	$p_{1/2}$
41	Ge^{73}	9/2	Nb^{93}	9/2	$g_{9/2}$
43	Se^{77}	$7/2\pm1$	Tc^{99}	(9/2 or 7/2)	
		$(1/2)(p_{1/2})$			
45	Se^{79}	$7/2(F_{7/2})$	Rh^{103}	$(1/2)(p_{1/2})$	
47	Kr^{83}	9/2	$Ag^{107,109}$	$1/2(p_{1/2})$	
49	Sr^{87}	9/2	$In^{113,115}$	9/2	

II. Detailed Discussion

The succession of energy levels calculated from the experimental data by us agrees very well with that of Feenberg's square well type potential levels[2] except the two cross-overs between $2s_{1/2}$ and $1d_{3/2}$; $2p_{3/2}$ and $1f_{5/2}$.

(A) Nuclei with less than 20 Neutrons or Protons

In this region newly obtained or revised data are concerned with O^{17} [3], Ne^{21} [4] F^{17} [5] and Al^{25} [6]. Among them, F^{17} has spin 5/2 which has proved to be in complete agreement with the the theoretically predicted spin term $d_{5/2}$, while F^{19} had

2) E. Feenberg, Phys. Rev. **75**, 1877 (1950)

3) S. Geschwind, Phys. Rev. **85**, 474 (1952)

4) J. Mack, Rev. of Mod. Phys. **22**, 64 (1950)

5) R. Laubenstein, Phys. Rev. **84**, 18 (1951)

6) L. Koester, Jr, Phys. Rev. **85**, 643 (1952)

failed to agree. The cross-over of spin terms between $2s_{1/2}$ and $1d_{3/2}$ can well be ascribed to strong spin orbit coupling. The most noteworthy is that Ne and Na, both located in the central part of $d_{5/2}$ term, have spin value 3/2 and are considered to be in $P_{3/2}$ state according to our calaculation of Landè's g factor.

(B) Odd Nuclei with N or Z between 20 ~ 50

In this region the spin and magnetic moment of Rb^{87} [8], $Se^{77,79}$ [9] and Ag^{109} [10] are added to the known data. Besides, magnetic moment of $Ti^{47,49}$ [11] and Tc^{99} [12] have been measured. We assigned the most probable orbits for these nuclei according to our calculations.

A cross-over of spin terms occurs once between $2p_{3/2}$ and $1f_{5/2}$. The spin term $f_{5/2}$ seems to be the most doubtful point in Mayer's theory, because only two out of five measured nuclei have spin 5/2 in agreement with the theoretically assigned $f_{5/2}$ term. Others belong to $p_{3/2}$ state. Mayer says the negative paring energy makes $f_{5/2}$ level descend to $p_{3/2}$ level. So to speak, Mayer has been forced to make two assumptlons to get rid of $f_{5/2}$ confusion. The pairing energy assumption is again effective in the case of Rb^{103} and $Ag^{107,109}$ which should belong to $g_{9/2}$ term. Serious deviation of spin and orbit is seen in the case of Se^{79} which must be in $F_{7/2}$ state according to Schmidt line calculation. Also $Ti^{47,49}$ and Fe^{57} are found to be reasonably assingned to $f_{7/2}$ and $p_{3/2}$ states respectively, at least as far as the measured magnetic moment are concerned. It is feared that the isomeric state has been confused with the ground state of Se^{77} which seems to have either $7/2 \pm 1$ or 1/2 spin value.

III. Conclusion

Generally speaking, Mayer's theory explains satisfactorily the nuclear structure at least in the region ranging from 1 to 50 odd identical nucleons. Such anomalies as we have seen in the spin term cross-overs and descended energy levels may well be said to have been caused by a strong spin orbit coupling and negative pairing energy increasing with j value, though the quantitative approach is yet to be made. But none of her assumptions seems to be able to account for the serious anomalies of spin occurring in $d_{5/2}$, $f_{7/2}$ and $g_{9/2}$ terms. These spin terms belong to comparatively high j values. These unclei are always located in the middle section of the nucleon numbers belonging to the same j value. For example, Se^{79} is the fifth nucleus in $g_{9/2}$ term which consists of ten nuclei.

Thus one particle model does not seem to be suitable in cases of nuclei situated in the middle section of spin terms with high j values. In these cases, the in-

7) See Rosenfeld : Nuclear Forces Appendix 2. 23
8) J. Mack, the same as 4)
9) W. Hardy, Phys. Rev. 85, 494 (1952)
10) J. Mack, the same as 4)
11) C. Jeffries, Phys. Rev. 85, 478 (1952)
12) H. Walchli, Phys. Rev. 85, 479 (1952)

teraction effect may deform the simply superposed wave functions to a degree of more than negligible order. If so, we are compeled to adopt many particle models in these special cases.

Before ending this report, it is author's greatest pleasure to express sincere thanks to Dr. M. Sasaki of Tokyo Metropolitan University for his thoughtful suggestions and discussions. The author is also deeply indebted to Dr. M. Mori of Keio University for his continued encouragement, and Mr. M. Umezawa of Tokyo University for his kind remarks.