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Author	鬼頭, 史城(Kito, Fumiki)
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On Swinging of a Wire of Transmission Line with Suspension Insulators

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Fumiki KITŌ*

Abstract

The Author has made a theoretical study on the swinging motion in the vertical plane of the wire of over-head electric power transmission line. We take the case of several consecutive spans each suspended with suspension insulators, the two extreme ends being dead-ended. In this Report, the fundamental equations and characteristic equation of free oscillation together with a numerical example is given.

I. Introduction

There are many cases under which the wire of an over-head electric power transmission line makes oscillatory or swinging motions. Sudden flow of a heavy short-circuit current through the wire, or sudden drop of coating of sleet from the wire causes the wire to swing in nearly the vertical plane. When there are consecutive several spans each suspended by suspension insulators, the mathematical treatment of the swinging motion becomes considerably complicated. In the present paper, the Author has made some theoretical study on such a swinging motion of wire in vertical plane, for the case in which there are n consecutive spans with suspension insulators, the two ends of the whole system being dead-ended. In order to make the analysis suitable to practical applications, the Author has made a simple assumption as to the relation between tension and deflection of the wire.

II. Fundamental Equations

Referring to Fig. 1, let us use the following notations;

L = span m, l = length of a string of suspension insulators m, α = angle of deflection of a string of suspension insulators rad., T = horizontal tension of a wire kg, x = horizontol distance (abscissa) m, E = Young's modulus of the wire kg/cm^2, w = weight/m of the wire kg/m, W = weight of a span of wire, kg. S = whole length of a span of wire m, D = sag of the wire m, Q = vertical load which hangs on the wire clamp kg, Y = ordinate (measured down-wards) of the wire at the position x m, y = value of the varied part of Y m,

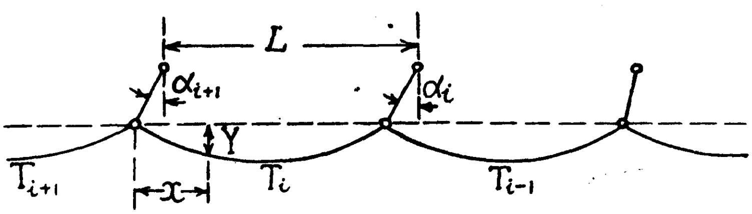

Fig. 1.

* Dr. Eng., Professor of Keio Univ.

In order to specify to which span the quantities α, T, D, etc. beloug, we use suffixes and denote α_i, T_i, etc. (see Fig. 1). For simplicity we assume in what follows that the successive spans have the same span length L, but similar treatment can be made for the case in which span lengths L are not the same to each other. The suffix 0 is used to denote the value at final, or stable state.

Using these notations, the equation of motion of the wire is given by

$$-\frac{w}{g}\frac{\partial^2 Y_i}{\partial t^2} + T_i\frac{\partial^2 Y_i}{\partial x^2} = -w. \qquad (1)$$

The relation between tension and elongation is taken to be approximately given by

$$\frac{T-T_0}{AE} = \frac{S-S_0}{L} \qquad (2)$$

During the swinging motion, the span is increased by an amount ΔS, where

$$\Delta S = l(\sin\alpha_{i+1} - \sin\alpha_i) \qquad (3)$$

so that the whole length S of the wire will be given by

$$S = \int_0^{L+\Delta s}\sqrt{1+\left(\frac{\partial Y}{\partial x}\right)^2}dx$$

or, approximately by

$$S = L + \frac{1}{2}\int_0^L\left(\frac{\partial Y}{\partial x}\right)^2 dx + \Delta S \qquad (4)$$

Equation of force acting at a suspension clamp is (Fig. 2)

$$\frac{T_{i+1}-T_i}{1/2(W_i+W_{i+1})} = \tan\alpha_{i+1} \qquad (5)$$

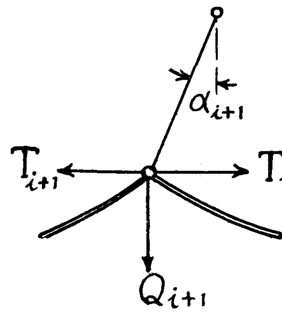

Fig. 2.

At the initial state, which corresponds to $t<o$, we suppose that $Y_i = Y_o$, $\alpha_i = o$, $T_i = T_o$, which means that the transmission line is in equilibrium under uniform tension. At $t = o$,the loading is suddenly changed, and from $t = o$ onwards, the wire begin to make swinging motion in the vertical plane. The final state of equilibrium, if the change of loading took place very gradually, will be denoted by $Y_i = Y_{io}$, $\alpha_i = \alpha_{io}$, $T_i = T_{io}$.

III. Treatment of Linear Equation for y_i.

During the swinging motion, we put

$$Y_i = Y_{io} + y_i. \qquad (6)$$

suppose that the angle α_i of inclination of insulator string is sufficiently small so that we can take α_i instead of tan α_i.

Substituting the value (6) and combining the equations (1) to (5) we have, after neglecting y_i^2 in comparison with Y_i^2;

$$-\frac{1}{a_i^2}\frac{\partial^2 y_i}{\partial t^2} + \frac{\partial^2 y_i}{\partial^2 x^2} - \frac{w_i AE}{T_{io}^2}\left[\frac{w_i}{T_{io}L}\int_0^L y_i dx + \frac{l}{L}(\alpha_{i+1} - \alpha_i)\right] = o, \qquad (7)$$

where

$$v_i = \sqrt{\frac{gT_{io}}{w_i}}. \qquad (8)$$

In order to solve these system of linear differential equations, we put

$$y_i = \sum \eta_{in} \cos\omega_n t,$$
$$\alpha_i = \sum a_{in} \cos\omega_n t. \qquad (9)$$

where η_{in} are functions of x. Thus the solution for y_i consist of aggregate of harmonic oscillations of angular frequencies ω_n. In what follows we shall find each separate component oscillations. We may omit the suffix n without causing any confusion. Then we have

$$\frac{\omega_n^2}{v_i^2}\eta_{in} + \frac{d^2\eta_{in}}{dx^2} - \frac{w_i AE}{T_{i0}^2}\left[\frac{w_i}{T_{i0}L}\int_0^L \eta_{in}dx + \frac{l}{L}(a_{i+1,n} - a_{i,n})\right] = 0. \qquad (10)$$

The initial condition is satisfied by putting

$$\sum_n \eta_{in} = y_{in} = Y_i - Y_{i0},$$
$$\sum_n a_{in} = -\alpha_{i0}.$$

The solution of the eq. (10) which satisfy the boundary condition that at $x = 0$ or L, $\eta_{io} = 0$ is given by;

$$\eta_i = -\frac{B_i}{\Delta_i}\left[\sin\frac{\omega}{v_i}(x + \varphi_i) - \sin\frac{\omega}{v_i}\varphi_i\right] \qquad (11)$$

where

$$B_i = \frac{w_i AE}{T_{io}^2}\frac{l}{L}(a_{i+1} - a_i),$$
$$C_i = \left(\frac{w_i}{T_{io}}\right)^2 \frac{AE}{LT_{io}}$$
$$\Delta_i = \left[\left(\frac{\omega}{v_i}\right)^2 - C_i L\right]\cos\left(\frac{\omega}{v_i}\frac{L}{2}\right) + 2\left(\frac{v_i}{\omega}C_i\right)\sin\left(\frac{\omega}{v_i}\frac{L}{2}\right),$$

For this solution, we have

$$\int_0^L \eta_i dx = -\frac{B_i K_i}{\Delta_i} = -E_i(a_{i+1} - a_i),$$

where

$$E_i = \frac{B_i}{\Delta_i}\left[2\frac{v_i}{\omega}\sin\left(\frac{\omega}{v_i}\frac{L}{2}\right) - L\cos\left(\frac{\omega}{v_i}\frac{L}{2}\right)\right].$$

IV. Characteristic Equation for the Swinging Motion in the Vertical Plane

Putting the above solution (9) (11) into the equation (7) we have

$$a_{i+1} = u_i\left[-F_i\{E_{i+1}(a_{i+2} - a_{i+1}) - E_i(a_{i+1} - a_i)\} + \frac{l}{L}\{(a_{i+2} - a_{i+1}) - (a_{i+1} a_i)\}\right], \qquad (12)$$

where

$$F_i = \frac{w_i}{T_{io}L}, \qquad u_i = \frac{AE}{1/2(W_{i+1} + W_i)}$$

In this equation we have to take $i = 1, 2, 3, \cdots$ with $a_i = 0$ for $i \gtreqqless 6$ or for $i \leqq 1$.

The system of equations (12) for amplitudes of oscillation a_i, are in form of a system of difference equations. Eliminating a_i's from these equations there results a determinant equation which is the characteristic equation for a component oscillation. There are an infinite number of solutions of this characteristic equation, which gives values of angular frequencies ω_1, ω_2, $\cdots\cdots$ For practical purpose, only some lowest of them will be required.

As an application of this theory, let us take the case in which there are consecutive five spans, and let us assume that the state of equilibrium and of swinging motion all occur symmetrically about the central point of the middle span.

In such a case we have

$$a_5 = u_4[-F_4\{E_5(a_6-a_5)-E_4(a_5-a_4)\}+\frac{l}{L}(a_6-2a_5+a_4)],$$

$$a_4 = u_3[-F_3\{E_4(a_5-a_4)-E_3(a_4-a_3)\}+\frac{l}{L}(a_5-2a_4+a_3)],$$

$$a_3 = u_2[-F_2\{E_3(a_4-a_3)-E_2(a_3-a_2)\}+\frac{l}{L}(a_4-2a_3+a_2)],$$

$$a_2 = u_1[-F_1\{E_2(a_3-a_2)-E_1(a_2-a_1)\}+\frac{l}{L}(a_3-2a_2+a_1)]$$

Putting the condition of symmetry viz., $E_5 = E_1$, $E_4 = E_2$, $u_4 = u_1$, $u_3 = u_2$, $-\alpha_5 = \alpha_2$, $-\alpha_3 = \alpha_4$ into these system of equations, and eliminating the unknown amplitudes $a_1, \cdot\cdot\cdot\cdot a_5$ from them, we have

$$\left[\frac{1}{u_4}+2\frac{l}{L}-F_4(E_5+E_4)\right]\cdot\left[\frac{1}{u_3}+3\frac{l}{L}-F_3(E_4+2E_3)\right]$$
$$=\left[\frac{l}{L}-F_4E_4\right]\cdot\left[\frac{l}{L}-F_3E_4\right]. \qquad (13)$$

Finding the value ω of angular freq. of this equation, the natural frequency of the swinging motion is determined.

V. Numerical Example

The Author has made numerical calculations of the above mentioned theoretical formula. Only an example will be reported here. We take the case of consecutive five spans, as shewn in Fig. 3, where initially each span is loaded with the same

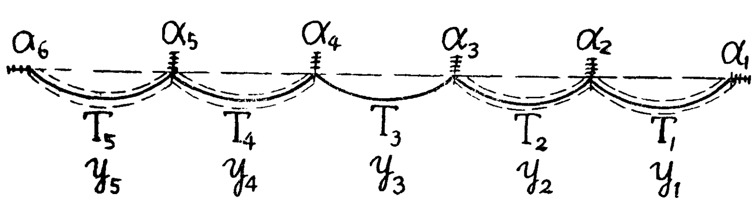

Fig. 3.

intensity $3w_0$ (kg/m) of vertical loads and is in equilibrium. At the instant $t = 0$, the load $3w_0$ of No. 3 span is supposed to be suddenly changed into w_0 (probably due to sudden fall of sleet coating) It is required the subsequent swinging motion of the wire.

Previous to consideration of the case of sudden fall, let us take the case in which the fall occurs very gradually. The angles α_i of inclination of insulator string is supposed to take the following values under which the new configuration of equilibrium is established. $\alpha_6 = 0$, $\alpha_5 = 10°$, $\alpha_4 = 20°$, $\alpha_3 = -20°$, $\alpha_2 = -10°$,

We shall take the span $L = 194$ m, weight of wire $w_0 = 1.00$ kg/m and final value of tension $T_3 = 657$ kg. According to (4), the corresponding values of T_4 and T_5 which equilibrate with the above mentioned value of T_3 is found to be $T_4 = 4.06 w_0 L$, $T_5 = 4.58 w_0 L$.

Now, turning to the case of sudden fall, we must solve the numerical equation (13), where we have

$$\mu_3 = 0.295, \quad \mu_4 = 0.740, \quad \mu_5 = 0.656,$$
$$\lambda_3 = 1520, \quad \lambda_4 = 1260, \quad \lambda_5 = 1120,$$
$$C_3 L^3 = \mu_3^2 \lambda_3 = 132, \quad C_4 L^3 = \mu_4^2 \lambda_4 = 690,$$
$$C_5 L^3 = \mu_5^2 \lambda_5 = 480.$$

If we put for shortness

$$\xi_3 = \xi = \frac{\omega}{v_3} \frac{L}{2},$$

then

$$\xi_4 = \frac{\omega}{v_4} \frac{L}{2} = 1.58\xi, \qquad \xi_5 = \frac{\omega}{v_5} \frac{L}{2} = 1.49\xi,$$

and the characteristic equation (13) becomes:

$$\Big[2 - \mu_4 \frac{\mu_4 \lambda_4 (\sin\xi_4 - \xi_4 \cos\xi_4)}{4\xi_4^3 \cos\xi_4 + C_4 L^3 (\sin\xi_4 - \xi_4 \cos\xi_4)} - \mu_4 \frac{\mu_5 \lambda_5 (\sin\xi_5 - \xi_5 \cos\xi_5)}{4\xi_5^3 \cos\xi_5 + C_5 L^3 (\sin\xi_5 - \xi_5 \cos\xi_5)} \Big]$$
$$\cdot\Big[3 - \mu_3 \frac{\mu_4 \lambda_4 (\sin\xi_4 - \xi_4 \cos\xi_4)}{4\xi_4^3 \cos\xi_4 + C_4 L^3 (\sin\xi_4 - \xi_4 \cos\xi_4)} - 2\mu_3 \frac{\mu_3 \lambda_3 (\sin\xi_3 - \xi_3 \cos\xi_3)}{4\xi_3^3 \cos\xi_3 + C_3 L^3 (\sin\xi_3 - \xi_3 \cos\xi_3)} \Big]$$
$$- \Big[1 - \mu_4 \frac{\mu_4 \lambda_4 (\sin\xi_4 - \xi_4 \cos\xi_4)}{4\xi_4^3 \cos\xi_4 + C_4 L^3 (\sin\xi_4 - \xi_4 \cos\xi_4)} \Big] \cdot\Big[1 - \mu_3 \frac{\mu_4 \lambda_4 (\sin\xi_4 - \xi_4 \cos\xi_4)}{4\xi_4^3 \cos\xi_4 + C_4 L^3 (\sin\xi_4 - \xi_4 \cos\xi_4)} \Big]$$
$$= 0$$

or,

$$\Big[2 - \frac{690 (\sin\xi_4 - \xi_4 \cos\xi_4)}{4\xi_4^3 \cos\xi_4 + 690 (\sin\xi_4 - \xi_4 \cos\xi_5)} - \frac{544 (\sin\xi_5 - \xi_5 \cos\xi_5)}{4\xi_5^3 \cos\xi_5 + 480 (\sin\xi_5 - \xi_5 \cos\xi_5)} \Big]$$
$$\cdot\Big[3 - \frac{275 (\sin\xi_4 - \xi_4 \cos\xi_4)}{4\xi_4^3 \cos\xi_4 + 690 (\sin\xi_4 - \xi_4 \cos\xi_4)} - \frac{264 (\sin\xi_3 - \xi_3 \cos\xi_3)}{4\xi_3^3 \cos\xi_3 + 132 (\sin\xi_3 - \xi_3 \cos\xi_3)} \Big]$$
$$- \Big[1 - \frac{690 (\sin\xi_4 - \xi_4 \cos\xi_4)}{4\xi_4^3 \cos\xi_4 + 690 (\sin\xi_4 - \xi_4 \cos\xi_4)} \Big] \cdot\Big[1 - \frac{275 (\sin\xi_4 - \xi_4 \cos\xi_4)}{4\xi_4^3 \cos\xi_4 + 690 (\sin\xi_4 - \xi_4 \cos\xi_4)} \Big]$$
$$= 0.$$

The root of this equation can be obtained by graphical method. In Fig, 4, the curve A shows the graph of the value of left-hand side of the numerical equation (14), the values of ξ being taken as abscissa. From the curve we infer that the lowest root of the equation (14) is given by $\xi = 143^\circ = 2.50$rad.

Corresponding to this, we find,

$$\omega = \xi v_3 \cdot \frac{2}{L} = 2\xi\sqrt{\frac{gT_3}{w_0L^2}} = 2.07,$$

so that the period of swinging motion will be $2\pi/2.07 = 3.04$ sec.

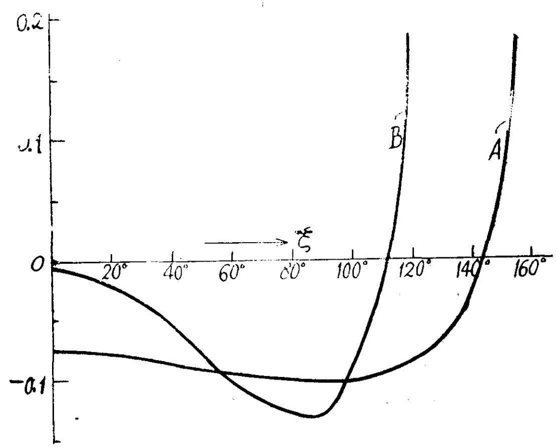

Fig. 4.

If in the above mentioned numerical example, we change the value of T_3 from 657 into $657 \times 1.5 = 986$ kg, while the other data remain quite the same, the curve A in Fig. 4 becomes changed into the curve B of the same figure. So that we have $\xi = 112^\circ = 1.95$ rad. Hence

$$\omega = 1.95 \times 2 \times \sqrt{0.171 \times 1.50} = 1.98,$$

and the period of swinging motion will be about 3.17 seconds,

Having thus determined the natural frequency of swinging motion, the amplitude can be determined by using the initial condition as mentioned in the previous article.