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Swinging of Wires of Overhead Transmission Line due to Short Circuit Current, (II) Transient State

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Adstract

Swinging motion of wires of overhead transmission line, when a heavy short circuit current is flowing along them has been studied theoretically. (I) Non-linear oscillations of wires has been solved in a form of successive harmonic oscillations. (II) Also in form of a power series of time t, which is suited to the case of monotonic motion of wires.

I. Introduction

In a long distance overhead transmission line, when a heavy short-circuit current-flow occur along its wires, there acts large attractive or repulsive electromagnetic force between the wires, and the swinging motion of wires is set up. Due to this swinging, there is a danger of the second short circuit or flashover between the wires to occur. About this problem, the Author has made some theoretical

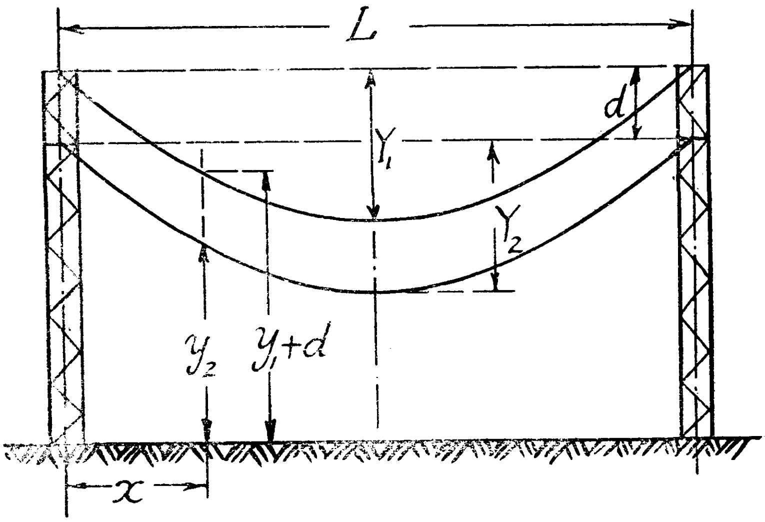

Fig. 1.

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study and reported in this Journal.[1] In the present report, the result of further study on this problem is given, The main result obtained is as follows:

a) Static Problem. In the previous paper, we first obtained tensions T_1, T_2 of wires and deduced from them the values of sags Y_1, Y_2. But here we made formula giving dirctly the values of Y_1, Y_2, thus simplifying the calculation.

b) Dynamical Problem. We made approximate solution for the dynamical equation of swinging motion of the wires, which is a system of non-linear differential equations. The result is represented as a sum of series of simple harmonic motions. We have also obtained the solution in a form of power series of time t, which is suited in treating the case of monotonous (non-oscillatory) motion of the wires.

II. Fundamental Equations

Here we use the same notation as in the previous paper, viz.;

T_1, T_2 = Horizontal tension of wires (kg). F_1, F_2 = attractive (repulsive) electromagnetic force due to short circuit current. (kg/m) L = Span (m) d = vertical spacing of wires (m). w = weight of wire itself (kg/m) A_0 = cross-sectional area of the wire (m²) E_0 = Modulus of elasticity of the wire. (kg/m²) η = Spacing factor = $(Y_1 - Y_2) / d$, Y_1, Y_2 = Sag of wires. The suffix "1" attached to any letter shows that it represents the value for upper conductor, whereas the suffix "2" indicates that it referrs to the lower conductor The suffix "0" shows the value when short circuit current is zero, and is the same for upper and lower wires. The equation of motion may be written in the form:—

$$\left.\begin{aligned} &-\frac{w}{g}\frac{\partial^2 y_1}{\partial t^2} + T_1\frac{\partial^2 y_1}{\partial x^2} + F_1 - w = 0 \\ &-\frac{w}{g}\frac{\partial^2 y_2}{\partial t^2} + T_2\frac{\partial^2 y_2}{\partial x^2} + F_2 - w = 0 \\ &-F_1 = F_2 = \frac{a}{d + y_1 - y_2} \\ &a = 0.10\left(\frac{I_0}{1000}\right)^2 \end{aligned}\right\} \qquad (1)$$

I_0 being the short circuit current. The equation (1) is a system of nonlinear partial differential equations. In the previous report, we have seen that when y_1, y_2 are independent of time t (namely the static case), the form of wires may approximately be represented by parabolas :—

$$y_i = Y_i\frac{4x}{L^2}(x - L) + C \qquad (i = 1, 2) \qquad (2)$$

When wires are swinging, it may also be permissible to assume that the wires form at every instant nearly parabolic curves. So we assume tentatively the rela-

1) This Journal Vol. 1, No. 2, 1948.

tion (2) to hold at any instant t, Y_i being now functions of t which are to be determined hereafter. Now integrate both sides of the equation (1) from $x = 0$ to $x = L$. Then

$$-\frac{w}{g}\frac{d^2}{dt^2}\left[\int_0^L y_1 dx\right] + 2T_1\left[\frac{dy_1}{dx}\right]_0^L + \int_0^L\left[F - w\right]dx = 0 \qquad (3)$$

Putting expressions (2) into (3) we have

$$\left(\frac{2wL}{3g}\right)\frac{d^2Y_i}{dt^2} + \frac{8T_i}{L}Y_i - wL \mp \frac{La}{d}\varphi(\eta_0) = 0 \qquad (i = 1, 2) \qquad (4)$$

where we have put as an approximate estimate (see Rep. I) :—

$$\int_0^L F_1\, dx = -\frac{La}{d}\varphi(\eta_0)$$

III Approximate Values for Static Case

When the form of wire is given by eq. (2), the length S of the wire span will be given approximately by

$$S = L + \frac{8}{3}\left(\frac{Y_i}{L}\right)^2$$

whence we have

$$T_i = T_0 + \frac{8}{3}A_0E_0\left[\left(\frac{Y_i}{L}\right)^2 - \left(\frac{Y_0}{L}\right)^2\right] \qquad (i = 1, 2) \qquad (5)$$

For a static case, the terms d^2Y_i/dt^2 in the eq. (4) disappear, and putting expressions (5) into them we have

$$\frac{8}{L}\left[T_0 + \frac{8}{3}A_0E_0\left\{\left(\frac{Y_i}{L}\right)^2 - \left(\frac{Y_0}{L}\right)^2\right\}\right]Y_i - wL \mp \left(\frac{La}{d}\right)\varphi(\eta_0) = 0$$

whence, by taking differences and rearranging;

$$\eta_0 = \frac{(La/d)(L/d)\varphi(\eta_0)}{4[8/3(A_0E_0/L^2)(Y_1^2 + Y_1Y_2 + Y_2^2 - Y_0^2) - T_0]} \qquad (6)$$

Putting (as in Rep. I), $T_1/T_0 = t_1$, $T_2/T_0 = t_2$, we have approximately

$$t_1 = \frac{1 + \gamma\gamma(\eta)}{\sqrt{1 + \delta(t_1 - 1)}} = \frac{1 + \gamma\varphi(\eta_0)}{\sqrt{1 + \delta\gamma\varphi(\eta_0)}}$$

$$t_2 = \frac{1 - \gamma\varphi(\eta_0)}{\sqrt{1 - \delta(t_2 - 1)}} = \frac{1 - \gamma\varphi(\eta_0)}{\sqrt{1 - \delta\gamma\varphi(\eta_0)}}$$

where we have put

$$\gamma = \frac{a}{dw} \qquad \delta = 24\left(\frac{T_0}{A_0E_0}\right)\left(\frac{T_0}{Lw}\right)^2$$

Values of sags Y_1, Y_2 will then be given by

$$Y_i = \left[wL \pm \left(\frac{La}{d}\right)\varphi(\eta_0)\right]\frac{L}{8T_i} = Y_0\left[1 \pm \frac{1}{2}\delta\gamma\varphi(\eta_0)\right] \qquad (7)$$

Whence we have

$$\eta_0 = \frac{Y_0}{d}\delta\gamma\varphi(\eta_0) = 3\left(\frac{a}{dw}\right)\left(\frac{T_0}{A_0E_0}\right)\left(\frac{T_0}{dw}\right)\varphi(\eta_0) \tag{8}$$

IV Approximate Equation of Dynamic Case, (Swinging Motions)

When the wires are making swinging motions, we may assume that the relations (5) to hold approximately at every instant. Putting (5) into the eq. of motion (4) we have

$$\left(\frac{2wL}{3g}\right)\frac{d^2Y_i}{dt^2} + \frac{8}{L}Y_i\left[T_0 + \frac{8}{3}\frac{A_0E_0}{L^2}(Y_i^2 - Y_0^2)\right] - wL \mp \left(\frac{La}{d}\right)\varphi(\eta_0) = 0 \tag{9}$$

Now, before any short circuit current flows, sags were Y_0. Under short circuit current I, the static values of sags are to be found by the above mentioned method. We denote them by Y_{1s}, Y_{2s}, and put

$$Y_i = Y_{is} + u_i \qquad (i = 1, 2) \tag{10}$$

where u_1, u_2 may be understood to represent so called "transient" terms. We have also

$$\eta_0 = (Y_1 - Y_2)/d = (Y_{1s} - Y_{2s})/d + (u_1 - u_2)/d = \eta_{00} + \zeta \tag{11}$$

where ζ represent transient term of η_0. On the other hand, the function $\varphi(\eta_0)$, being nearly represented by a parabola (see Fig. 4 of Report I), we may put $\varphi(\eta_0) = 1 + k\eta_0^2$ at least for a limited range of values of η_0. Putting these values into the equation of motion (9) and rearranging, we have

$$\left(\frac{wL}{g}\right)\frac{d^2u_i}{dt^2} + \frac{8}{L}\left[T_0 + \frac{8}{3}\frac{A_0E_0}{L^2}\left(3Y_{is}^2 - Y_0^2\right)\right]u_i \mp \left(\frac{La}{d}\right)2k\eta_{00}\zeta - \left(\frac{64}{3L}\right)\left(\frac{A_0E_0}{L^2}\right)\left[3Y_{is}u_i^2 + u_i^3\right] \pm \left(\frac{La}{d}\right)k\zeta^2 = 0 \tag{12}$$

where $i = 1, 2$. This is a system of nonlinear differential eqaution with regard to unknown quantities u_1, u_2, the time t being the independent variable.

When the short circuit current is not very near to critical value I_0, nonlinear terms in the equation (12) have smaller values than linear terms. In order to see this fact, we estimate values of the ratios

$$A_1 = \left[\frac{64}{3L}\frac{A_0E_0}{L^2}\cdot 3Y_{is}u_i^2\right] \div \left[\frac{8}{L}T_0u_i\right]$$

$$A_2 = \left[\frac{La}{d}k\zeta^2\right] \div \left[\frac{8}{L}T_0u_i\right]$$

These ratios can also be written in the form

$$A_1 = 8\left(\frac{A_0E_0}{T_0}\right)\left(\frac{Y_{is}}{L}\right)\left(\frac{u_i}{L}\right)$$

$$A_2 = \frac{L^2a}{8T_0} k \left(\frac{\zeta}{d} \right) \left(\frac{\zeta}{u_i} \right)$$

This A_1 has value of order of the difference of tensions when sag has changed from Y_{is} to $Y_{is} + u_i$ to initial tension T_0. Taking $L = 300$ m, $T_0 = 200$ kg, $a = 0.1$ (these values being the same as numerical example given in Rep. I), we have $(L^2a) / (8T_0) = 5.625$. Further, $\gamma = a / (dw) = 0.10 / (2.5 \times 0.43) = 0.093$ and $T_1/T_0 = t_1 = 1 + \gamma\varphi(\eta_0) = 1.093$. Now, if we wish to include values of the ratio A_1 up to 0.20, we may cover the value of a up to 0.22 into our range of treatment. This means that the value of short cicuit current under 1400 amps comes into this range, while the critical short circuit current has value of 3160 amps.

V. The First Approximate Solution

The differential equation (12) can be written in the form

$$\begin{aligned} \frac{d^2u_1}{dt^2} + A\,u_2 - Bd\zeta &= - C\,(eu_1^2 + u_1^3) + G\zeta^2d^2 \\ \frac{d^2u_2}{dt^2} + Au_2 + Bd\zeta &= - C\,(eu_2^2 + u_2^3) - G\zeta^2d^2 \end{aligned} \qquad (13)$$

where $\zeta = (u_1 - u_2)/d$. As a first approximation let us take only linear terms in the eq. (13). In order that it may have solution containing sin (cos) pt as factor, we must have

$$\begin{vmatrix} -p^2 + (A - B) & B \\ B & - p^2 + (A - B) \end{vmatrix} = 0$$

whence we have two values of p, namely,

$$p_1 = \sqrt{A}, \quad p_2 = \sqrt{A - 2B}$$

and the solution must be of the form :-

$$\begin{aligned} u_1 &= K\cos (p_1t + \theta) + M\cos (p_2t + \varphi) \\ u_2 &= K\cos (p_1t + \theta) - M\cos (p_2t + \varphi) \end{aligned}$$

The initial condition is, at $t = 0$, $u_1 = Y_0 - Y_{1s}$, $u_2 = Y_0 - Y_{2s}$, $du_1/dt = 0$, $du_2/dt = 0$.

Determinining the arbitrary constants K, M, θ and φ by this condition, we have for the first approximate solution,

$$\left.\begin{aligned} u_{11} &= K\cos p_1t + M\cos p_2t \\ u_{21} &= K\cos p_1t - M\cos p_2t \end{aligned}\right\} \qquad (14)$$

where we put

$$\begin{aligned} K &= \frac{1}{2} (2Y_0 - Y_{1s} - Y_{2s}) \\ M &= \frac{1}{2} (Y_{2s} - Y_{1s}) \end{aligned} \qquad (15)$$

VI. The Successive Approximate Solutions

In order to form the second approximate solution, we substitute the values (14) into the right-hand side of eq. (13), thus

$$\frac{d^2u_{12}}{dt^2} + Au_{12} - B(u_{12} - u_{22})$$

$$= -Ce[K\cos p_1t + M\cos p_2t]^2$$

$$-C[K\cos p_1t + M\cos p_2t]^3 + G[2M\cos p_2t]^2$$

$$\frac{d^2u_{22}}{dt^2} + Au_{22} + B(u_{12} - u_{22})$$

$$= -Ce[K\cos p_1t - M\cos p_2t]^2$$

$$-C[K\cos p_1t - M\cos p_2t]^3 - G[2M\cos p_2t]^2 \qquad (16)$$

Now, we have for example,

$$[K\cos p_1t \pm M\cos p_2t]^2$$

$$= \frac{1}{2}(K^2 + M^2) + \frac{1}{2}K^2\cos 2p_1t + \frac{1}{2}M^2\cos 2p_2t \pm KM\cos(p_1 + p_2)t$$

$$\pm KM\cos(p_1 - p_2)t$$

Thus, to solve the system of differential equations (16), we only require to solve the following system of equations (17), the solution for eq. (16) being easily deduced from that of eq. (17).

$$\left.\begin{aligned} &\frac{d^2u_1}{dt} + Au_1 - B(u_1 - u_2) = H\cos qt \\ &\frac{d^2u_2}{dt^2} + Au_2 + B(u_1 - u_2) = \pm H\cos qt \end{aligned}\right\} \qquad (17)$$

For the case of + sign in eq. (17), the solution of (17) may be written

$$u_1 = \frac{H}{A - q^2}\cos qt + k\cos(p_1t + \theta) + m\cos(p_2t + \varphi)$$

$$u_2 = \frac{H}{A - q^2}\cos qt + k\cos(p_1t + \theta) - m\cos(p_2t + \varphi)$$

while for the case of − sign in eq. (17), we have,

$$u_1 = \frac{H}{A - 2B - q^2}\cos qt + k\cos(p_1t + \theta) + m\cos(p_2t + \varphi)$$

$$u_2 = -\frac{H}{A - 2B - q^2}\cos qt + k\cos(p_2t + \theta) - m\cos(p_2t + \varphi)$$

Thus we see that the second approximate solution of eq. (13) can be expressed as a sum of terms of sines and cosines of p_1t, p_2t, $(p_1 + p_2)t$, $(p_1 - p_2)t$, etc.

And we obtain, by actual calculation;-

$$\frac{1}{2}(u_1 + u_2) = -\frac{1}{2}\frac{Ce}{A}(K^2 + M^2)$$

$$-\frac{1}{2}Ce\,K^2\frac{1}{A - 4p_1^2}\cos 2p_1t$$

$$-\frac{1}{2}Ce\,M^2\frac{1}{A - 4p_2^2}\cos 2p_2t$$

$$- C\left[\frac{3}{4}K^3 + \frac{3}{2}K^2M\right]\frac{1}{A - p_1^2}\cos p_1 t$$

$$- C\left[\frac{3}{4}K^2 M\right]\frac{1}{A - (2p_2 - p_1)^2}\cos(2p_2 - p_1)t$$

$$- C\left[\frac{3}{4}K^2M\right]\frac{1}{A - (2p_2 + p_1)^2}\cos(2p_2 + p_1)t$$

$$- C\left[\frac{1}{4}K^3\right]\frac{1}{A - 9p_1^2}\cos 3p_1 t$$

$$+ k\cos(p_1 t + \theta)$$

$$\frac{1}{2}(u_1 - u_2) = - CeKM\frac{1}{A - 2B - (p_1 + p_2)^2}\cos(p_1 + p_2)t$$

$$- CeKM\frac{1}{A - 2B - (p_1 - p_2)^2}\cos(p_1 - p_2)t$$

$$- C\left[\frac{3}{2}K^2M + \frac{3}{4}M^3\right]\frac{1}{A - 2B - p_2^2}\cos p_2 t$$

$$- \frac{3}{4}CK^2M\frac{1}{A - 2B - (2p_1 - p_2)^2}\cos(2p_1 - p_2)t$$

$$- \frac{3}{4}CK^2M\frac{1}{A - 2B - (2p_1 + p_2)^2}\cos(2p_1 + p_2)t$$

$$- \frac{1}{4}CM^3\frac{1}{A - 2B - 9p_2^2}\cos 3p_2 t$$

$$+ 2GM^2\frac{1}{A - 2B}$$

$$+ 2GM^2\frac{1}{A - 2B - 4p_2^2}\cos 2p_2 t$$

$$+ m\cos(p_2 t + \varphi)$$

where k, θ, φ are arbitrary constants as before.

VII. The Solution as the power series of time t

When the wire makes a monotonous motion, instead of swinging motion, or when we wish to know values of Y_1, Y_2 for very short period of the swinging, it is preferrable to obtain solutons for Y_1 and Y_2 in forms of power series of time t. Thus assuming the power series

$$Y_1 = Y_0 + mt^2 + nt^4 + st^6 + \cdots$$

$$Y_2 = Y_0 + m't^2 + n't^4 + s't^6 + \cdots$$

and putting these expressions into (19) we obtain

$$m = \frac{3ga}{4wd}$$

$$n = -\frac{T_0 g}{wL}\left[1 + \frac{16}{3}\left(\frac{A_0E_0}{T_0}\right)\left(\frac{Y_0}{L}\right)^2\right]m$$

$$s = -\frac{2g}{5wL^2}\left[\left(\frac{8A_oE_o}{3L^2} + 2Y_o\right)Y_om^2\right.$$

$$\left.+\left(T_o + \frac{16A_oE_o}{3L}Y_o^2\right)n\right] + \frac{akg}{wd^3}\times\frac{1}{5}m^2$$

$$m' = -m, \qquad n' = -n,$$

$$s' = -\frac{2g}{5wL^2}\left[\left(\frac{8A_oE_o}{3L^2}2Y_o\right)Y_om^2\right.$$

$$-\left[\left(T_0 + \frac{16A_0E_0}{3L^2}Y_0^2\right., n\right] - \frac{akg}{wd^3}\times\frac{1}{5}m^2$$

For practical purpose, what is required is a convenient chart or diagram to facilitate quick estimate. The Author wishes to be given opportunity of publishing them in near future.