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# Swinging of Wires of Overhead Transmission Line due to Short Circuit Current, ( II ) Transient State

( Received June 10, 1949 )

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## Adstract

Swinging motion of wires of overhead transmission line, when a heavy short circuit current is flowing along them has been studied theoretically. ( I ) Non-linear oscillations of wires has been solved in a form of successive harmonic oscillations. ( II ) Also in form of a power series of time  $t$ , which is suited to the case of monotonic motion of wires.

## I. Introduction

In a long distance overhead transmission line, when a heavy short-circuit current-flow occur along its wires, there acts large attractive or repulsive electromagnetic force between the wires, and the swinging motion of wires is set up. Due to this swinging, there is a danger of the second short circuit or flashover between the wires to occur. About this problem, the Author has made some theoretical

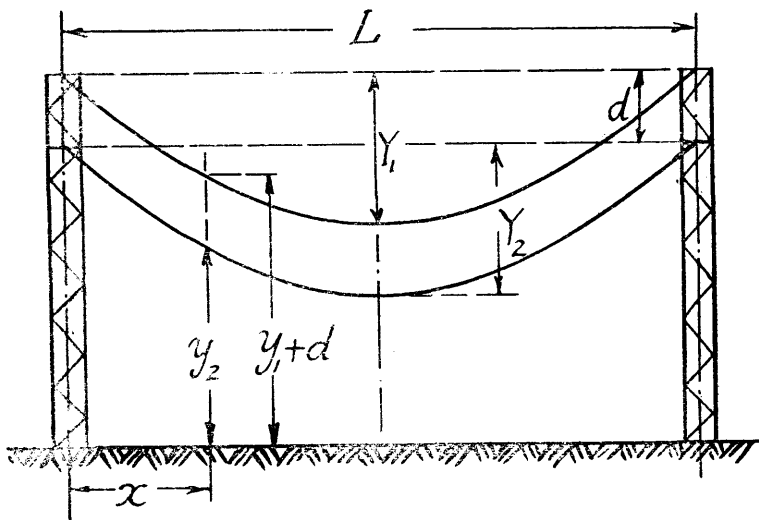


Fig. 1.

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study and reported in this Journal.<sup>1)</sup> In the present report, the result of further study on this problem is given. The main result obtained is as follows:

a) Static Problem. In the previous paper, we first obtained tensions  $T_1$ ,  $T_2$  of wires and deduced from them the values of sags  $Y_1$ ,  $Y_2$ . But here we made formula giving directly the values of  $Y_1$ ,  $Y_2$ , thus simplifying the calculation.

b) Dynamical Problem. We made approximate solution for the dynamical equation of swinging motion of the wires, which is a system of non-linear differential equations. The result is represented as a sum of series of simple harmonic motions. We have also obtained the solution in a form of power series of time  $t$ , which is suited in treating the case of monotonous (non-oscillatory) motion of the wires.

## II. Fundamental Equations

Here we use the same notation as in the previous paper, viz.;

$T_1$ ,  $T_2$  = Horizontal tension of wires (kg).  $F_1$ ,  $F_2$  = attractive (repulsive) electromagnetic force due to short circuit current. (kg/m)  $L$  = Span (m)  $d$  = vertical spacing of wires (m).  $w$  = weight of wire itself (kg/m)  $A_0$  = cross-sectional area of the wire (m<sup>2</sup>)  $E_0$  = Modulus of elasticity of the wire. (kg/m<sup>2</sup>)  $\eta$  = Spacing factor =  $(Y_1 - Y_2)/d$ ,  $Y_1$ ,  $Y_2$  = Sag of wires. The suffix '1' attached to any letter shows that it represents the value for upper conductor, whereas the suffix '2' indicates that it refers to the lower conductor. The suffix '0' shows the value when short circuit current is zero, and is the same for upper and lower wires. The equation of motion may be written in the form:—

$$\left. \begin{aligned} -\frac{w}{g} \frac{\partial^2 y_1}{\partial t^2} + T_1 \frac{\partial^2 y_1}{\partial x^2} + F_1 - w &= 0 \\ -\frac{w}{g} \frac{\partial^2 y_2}{\partial t^2} + T_2 \frac{\partial^2 y_2}{\partial x^2} + F_2 - w &= 0 \\ -F_1 = F_2 &= \frac{a}{d + y_1 - y_2} \\ a &= 0.10 \left( \frac{I_0}{1000} \right)^2 \end{aligned} \right\} \quad (1)$$

$I_0$  being the short circuit current. The equation (1) is a system of nonlinear partial differential equations. In the previous report, we have seen that when  $y_1$ ,  $y_2$  are independent of time  $t$  (namely the static case), the form of wires may approximately be represented by parabolas:—

$$y_i = Y_i \frac{4x}{L^2} (x - L) + C \quad (i = 1, 2) \quad (2)$$

When wires are swinging, it may also be permissible to assume that the wires form at every instant nearly parabolic curves. So we assume tentatively the rela-

1) This Journal Vol. 1, No. 2, 1948.

tion ( 2 ) to hold at any instant  $t$ ,  $Y_i$  being now functions of  $t$  which are to be determined hereafter. Now integrate both sides of the equation ( 1 ) from  $x = 0$  to  $x = L$ . Then

$$-\frac{w}{g} \frac{d^2}{dt^2} \left[ \int_0^L y_1 dx \right] + 2T_1 \left[ \frac{dy_1}{dx} \right]_0^L + \int_0^L [F - w] dx = 0 \quad (3)$$

Putting expressions ( 2 ) into ( 3 ) we have

$$\left( \frac{2wL}{3g} \right) \frac{d^2 Y_i}{dt^2} + \frac{8T_i}{L} Y_i - wL \mp \frac{La}{d} \varphi(\eta_0) = 0 \quad (i = 1, 2) \quad (4)$$

where we have put as an approximate estimate ( see Rep. I ) :—

$$\int_0^L F_1 dx = -\frac{La}{d} \varphi(\eta_0)$$

### III Approximate Values for Static Case

When the form of wire is given by eq. ( 2 ), the length  $S$  of the wire span will be given approximately by

$$S = L + \frac{8}{3} \left( \frac{Y_i}{L} \right)^2$$

whence we have

$$T_i = T_0 + \frac{8}{3} A_0 E_0 \left[ \left( \frac{Y_i}{L} \right)^2 - \left( \frac{Y_0}{L} \right)^2 \right] \quad (i = 1, 2) \quad (5)$$

For a static case, the terms  $d^2 Y_i / dt^2$  in the eq. ( 4 ) disappear, and putting expressions ( 5 ) into them we have

$$\frac{8}{L} \left[ T_0 + \frac{8}{3} A_0 E_0 \left\{ \left( \frac{Y_i}{L} \right)^2 - \left( \frac{Y_0}{L} \right)^2 \right\} \right] Y_i - wL \mp \left( \frac{La}{d} \right) \varphi(\eta_0) = 0$$

whence, by taking differences and rearranging;

$$\eta_0 = \frac{(La/d)(L/d)\varphi(\eta_0)}{4 \left[ \frac{8}{3} (A_0 E_0 / L^2) (Y_1^2 + Y_1 Y_2 + Y_2^2 - Y_0^2) - T_0 \right]} \quad (6)$$

Putting ( as in Rep. I ),  $T_1/T_0 = t_1$ ,  $T_2/T_0 = t_2$ , we have approximately

$$t_1 = \frac{1 + \gamma \varphi(\eta)}{\sqrt{1 + \delta(t_1 - 1)}} = \frac{1 + \gamma \varphi(\eta_0)}{\sqrt{1 + \delta \gamma \varphi(\eta_0)}}$$

$$t_2 = \frac{1 - \gamma \varphi(\eta_0)}{\sqrt{1 - \delta(t_2 - 1)}} = \frac{1 - \gamma \varphi(\eta_0)}{\sqrt{1 - \delta \gamma \varphi(\eta_0)}}$$

where we have put

$$\gamma = \frac{a}{dw} \quad \delta = 24 \left( \frac{T_0}{A_0 E_0} \right) \left( \frac{T_0}{Lw} \right)^2$$

Values of sags  $Y_1$ ,  $Y_2$  will then be given by

$$Y_i = \left[ wL \pm \left( \frac{La}{d} \right) \varphi(\eta_0) \right] \frac{L}{8T_i} = Y_0 \left[ 1 \pm \frac{1}{2} \delta \gamma \varphi(\eta_0) \right] \quad (7)$$

Whence we have

$$(9)$$

$$\eta_0 = \frac{Y_0}{d} \delta \gamma \varphi(\eta_0) = 3 \left( \frac{a}{dw} \right) \left( \frac{T_0}{A_0 E_0} \right) \left( \frac{T_0}{dw} \right) \varphi(\eta_0) \quad (8)$$

#### IV Approximate Equation of Dynamic Case, (Swinging Motions)

When the wires are making swinging motions, we may assume that the relations (5) to hold approximately at every instant. Putting (5) into the eq. of motion (4) we have

$$\left( \frac{2wL}{3g} \right) \frac{d^2 Y_i}{dt^2} + \frac{8}{L} Y_i \left[ T_0 + \frac{8}{3} \frac{A_0 E_0}{L^2} (Y_i^2 - Y_0^2) \right] - wL \mp \left( \frac{La}{d} \right) \varphi(\eta_0) = 0 \quad (9)$$

Now, before any short circuit current flows, sags were  $Y_0$ . Under short circuit current  $I$ , the static values of sags are to be found by the above mentioned method. We denote them by  $Y_{1s}$ ,  $Y_{2s}$ , and put

$$Y_i = Y_{is} + u_i \quad (i = 1, 2) \quad (10)$$

where  $u_1, u_2$  may be understood to represent so called "transient" terms. We have also

$$\eta_0 = (Y_1 - Y_2)/d = (Y_{1s} - Y_{2s})/d + (u_1 - u_2)/d = \eta_{00} + \zeta \quad (11)$$

where  $\zeta$  represent transient term of  $\eta_0$ . On the other hand, the function  $\varphi(\eta_0)$ , being nearly represented by a parabola (see Fig. 4 of Report I), we may put

$\varphi(\eta_0) = 1 + k\eta_0^2$  at least for a limited range of values of  $\eta_0$ . Putting these values into the equation of motion (9) and rearranging, we have

$$\begin{aligned} & \left( \frac{wL}{g} \right) \frac{d^2 u_i}{dt^2} + \frac{8}{L} \left[ T_0 + \frac{8}{3} \frac{A_0 E_0}{L^2} (3Y_{is}^2 - Y_0^2) \right] u_i \\ & \mp \left( \frac{La}{d} \right) 2k\eta_{00}\zeta - \left( \frac{64}{3L} \right) \left( \frac{A_0 E_0}{L^2} \right) [3Y_{is}u_i^2 + u_i^3] \\ & \pm \left( \frac{La}{d} \right) k\zeta^2 = 0 \end{aligned} \quad (12)$$

where  $i = 1, 2$ . This is a system of nonlinear differential equation with regard to unknown quantities  $u_1, u_2$ , the time  $t$  being the independent variable.

When the short circuit current is not very near to critical value  $I_0$ , nonlinear terms in the equation (12) have smaller values than linear terms. In order to see this fact, we estimate values of the ratios

$$A_1 = \left[ \frac{64}{3L} \frac{A_0 E_0}{L^2} \cdot 3Y_{is} u_i^2 \right] \div \left[ \frac{8}{L} T_0 u_i \right]$$

$$A_2 = \left[ \frac{La}{d} k\zeta^2 \right] \div \left[ \frac{8}{L} T_0 u_i \right]$$

These ratios can also be written in the form

$$A_1 = 8 \left( \frac{A_0 E_0}{T_0} \right) \left( \frac{Y_{is}}{L} \right) \left( \frac{u_i}{L} \right)$$

(10)

$$A_2 = \frac{L^2 a}{8T_0} k \left( \frac{\zeta}{d} \right) \left( \frac{\zeta}{u_t} \right)$$

This  $A_1$  has value of order of the difference of tensions when sag has changed from  $Y_{is}$  to  $Y_{is} + u_i$  to initial tension  $T_0$ . Taking  $L = 300$  m,  $T_0 = 200$  kg,  $a = 0.1$  ( these values being the same as numerical example given in Rep. I ), we have  $(L^2 a) / (8T_0) = 5.625$ . Further,  $\gamma = a / (dw) = 0.10 / (2.5 \times 0.43) = 0.093$  and  $T_1/T_0 = t_1 = 1 + \gamma \varphi(\gamma_0) = 1.093$ . Now, if we wish to include values of the ratio  $A_1$  up to 0.20, we may cover the value of  $a$  up to 0.22 into our range of treatment. This means that the value of short circuit current under 1400 amps comes into this range, while the critical short circuit current has value of 3160 amps.

### V. The First Approximate Solution

The differential equation ( 12 ) can be written in the form

$$\begin{aligned} \frac{d^2 u_1}{dt^2} + A u_1 - B d\zeta &= -C (e u_1^2 + u_1^3) + G \zeta^2 d^2 \\ \frac{d^2 u_2}{dt^2} + A u_2 + B d\zeta &= -C (e u_2^2 + u_2^3) - G \zeta^2 d^2 \end{aligned} \quad (13)$$

where  $\zeta = (u_1 - u_2)/d$ . As a first approximation let us take only linear terms in the eq. ( 13 ). In order that it may have solution containing  $\sin (\cos) pt$  as factor, we must have

$$\begin{vmatrix} -p^2 + (A - B) & B \\ B & -p^2 + (A - B) \end{vmatrix} = 0$$

whence we have two values of  $p$ , namely,

$$p_1 = \sqrt{A}, \quad p_2 = \sqrt{A - 2B}$$

and the solution must be of the form:—

$$\begin{aligned} u_1 &= K \cos (p_1 t + \theta) + M \cos (p_2 t + \varphi) \\ u_2 &= K \cos (p_1 t + \theta) - M \cos (p_2 t + \varphi) \end{aligned}$$

The initial condition is, at  $t = 0$ ,  $u_1 = Y_0 - Y_{1s}$ ,  $u_2 = Y_0 - Y_{2s}$ ,  $du_1/dt = 0$ ,  $du_2/dt = 0$ .

Determinining the arbitrary constants  $K$ ,  $M$ ,  $\theta$  and  $\varphi$  by this condition, we have for the first approximate solution,

$$\left. \begin{aligned} u_{11} &= K \cos p_1 t + M \cos p_2 t \\ u_{21} &= K \cos p_1 t - M \cos p_2 t \end{aligned} \right\} \quad (14)$$

where we put

$$\begin{aligned} K &= \frac{1}{2} (2Y_0 - Y_{1s} - Y_{2s}) \\ M &= \frac{1}{2} (Y_{2s} - Y_{1s}) \end{aligned} \quad (15)$$

### VI. The Successive Approximate Solutions

In order to form the second approximate solution, we substitute the values (14)

into the right-hand side of eq. (13), thus

$$\begin{aligned}
 & \frac{d^2 u_{12}}{dt^2} + Au_{12} - B(u_{12} - u_{22}) \\
 &= -Ce [K \cos p_1 t + M \cos p_2 t]^2 \\
 &\quad - C [K \cos p_1 t + M \cos p_2 t]^3 + G [2M \cos p_2 t]^2 \\
 & \frac{d^2 u_{22}}{dt^2} + Au_{22} + B(u_{12} - u_{22}) \\
 &= -Ce [K \cos p_1 t - M \cos p_2 t]^2 \\
 &\quad - C [K \cos p_1 t - M \cos p_2 t]^3 - G [2M \cos p_2 t]^2 \quad (16)
 \end{aligned}$$

Now, we have for example,

$$\begin{aligned}
 & [K \cos p_1 t \pm M \cos p_2 t]^2 \\
 &= \frac{1}{2} (K^2 + M^2) + \frac{1}{2} K^2 \cos 2p_1 t + \frac{1}{2} M^2 \cos 2p_2 t \pm KM \cos (p_1 + p_2)t \\
 &\quad \pm KM \cos (p_1 - p_2)t
 \end{aligned}$$

Thus, to solve the system of differential equations (16), we only require to solve the following system of equations (17), the solution for eq. (16) being easily deduced from that of eq. (17).

$$\begin{aligned}
 & \frac{d^2 u_1}{dt^2} + Au_1 - B(u_1 - u_2) = H \cos qt \\
 & \frac{d^2 u_2}{dt^2} + Au_2 + B(u_1 - u_2) = \pm H \cos qt \quad (17)
 \end{aligned}$$

For the case of + sign in eq. (17), the solution of (17) may be written

$$\begin{aligned}
 u_1 &= \frac{H}{A - q^2} \cos qt + k \cos (p_1 t + \theta) + m \cos (p_2 t + \varphi) \\
 u_2 &= \frac{H}{A - q^2} \cos qt + k \cos (p_1 t + \theta) - m \cos (p_2 t + \varphi)
 \end{aligned}$$

while for the case of - sign in eq. (17), we have,

$$\begin{aligned}
 u_1 &= \frac{H}{A - 2B - q^2} \cos qt + k \cos (p_1 t + \theta) + m \cos (p_2 t + \varphi) \\
 u_2 &= -\frac{H}{A - 2B - q^2} \cos qt + k \cos (p_2 t + \theta) - m \cos (p_2 t + \varphi)
 \end{aligned}$$

Thus we see that the second approximate solution of eq. (13) can be expressed as a sum of terms of sines and cosines of  $p_1 t$ ,  $p_2 t$ ,  $(p_1 + p_2)t$ ,  $(p_1 - p_2)t$ , etc.

And we obtain, by actual calculation,-

$$\begin{aligned}
 \frac{1}{2} (u_1 + u_2) &= -\frac{1}{2} \frac{Ce}{A} (K^2 + M^2) \\
 &\quad - \frac{1}{2} Ce K^2 \frac{1}{A - 4p_1^2} \cos 2p_1 t \\
 &\quad - \frac{1}{2} Ce M^2 \frac{1}{A - 4p_2^2} \cos 2p_2 t
 \end{aligned}$$

$$\begin{aligned}
 & - C \left[ \frac{3}{4} K^3 + \frac{3}{2} K^2 M \right] \frac{1}{A - p_1^2} \cos p_1 t \\
 & - C \left[ \frac{3}{4} K^2 M \right] \frac{1}{A - (2p_2 - p_1)^2} \cos (2p_2 - p_1) t \\
 & - C \left[ \frac{3}{4} K^2 M \right] \frac{1}{A - (2p_2 + p_1)^2} \cos (2p_2 + p_1) t \\
 & - C \left[ \frac{1}{4} K^3 \right] \frac{1}{A - 9p_1^2} \cos 3p_1 t \\
 & + k \cos (p_1 t + \theta) \\
 \frac{1}{2} (\kappa_1 - u_2) = & - CeKM \frac{1}{A - 2B - (p_1 + p_2)^2} \cos (p_1 + p_2) t \\
 & - CeKM \frac{1}{A - 2B - (p_1 - p_2)^2} \cos (p_1 - p_2) t \\
 & - C \left[ \frac{3}{2} K^2 M + \frac{3}{4} M^3 \right] \frac{1}{A - 2B - p_2^2} \cos p_2 t \\
 & - \frac{3}{4} CK^2 M \frac{1}{A - 2B - (2p_1 - p_2)^2} \cos (2p_1 - p_2) t \\
 & - \frac{3}{4} CK^2 M \frac{1}{A - 2B - (2p_1 + p_2)^2} \cos (2p_1 + p_2) t \\
 & - \frac{1}{4} CM^3 \frac{1}{A - 2B - 9p_2^2} \cos 3p_2 t \\
 & + 2GM^2 \frac{1}{A - 2B} \\
 & + 2GM^2 \frac{1}{A - 2B - 4p_2^2} \cos 2p_2 t \\
 & + m \cos (p_2 t + \varphi)
 \end{aligned}$$

where  $k, \theta, \varphi$  are arbitrary constants as before.

### VII. The Solution as the power series of time $t$

When the wire makes a monotonous motion, instead of swinging motion, or when we wish to know values of  $Y_1, Y_2$  for very short period of the swinging, it is preferable to obtain solutions for  $Y_1$  and  $Y_2$  in forms of power series of time  $t$ .

Thus assuming the power series

$$Y_1 = Y_0 + mt^2 + nt^4 + st^6 + \dots$$

$$Y_2 = Y_0 + m't^2 + n't^4 + s't^6 + \dots$$

and putting these expressions into ( 19 ) we obtain

$$m = \frac{3ga}{4wd}$$

$$n = - \frac{T_0 g}{wL} \left[ 1 + \frac{16}{3} \left( \frac{A_0 E_0}{T_0} \right) \left( \frac{Y_0}{L} \right)^2 \right] m$$



$$\begin{aligned}
s &= -\frac{2g}{5wL^2} \left[ \left( \frac{8A_oE_o}{3L^2} + 2Y_o \right) Y_o m^2 \right. \\
&\quad \left. + \left( T_o + \frac{16A_oE_o}{3L} Y_o^2 \right) n \right] + \frac{akg}{wd^3} \times \frac{1}{5} m^2 \\
m' &= -m, \quad n' = -n, \\
s' &= -\frac{2g}{5wL^2} \left[ \left( \frac{8A_oE_o}{3L^2} + 2Y_o \right) Y_o m^2 \right. \\
&\quad \left. - \left[ \left( T_o + \frac{16A_oE_o}{3L} Y_o^2 \right), n \right] - \frac{akg}{wd^3} \times \frac{1}{5} m^2 \right]
\end{aligned}$$

For practical purpose, what is required is a convenient chart or diagram to facilitate quick estimate. The Author wishes to be given opportunity of publishing them in near future.