

Title	On vibration of drum-type diaphragm in water
Sub Title	
Author	鬼頭, 史城(Kito, Fumiki)
Publisher	慶應義塾大学藤原記念工学部
Publication year	1949
Jtitle	慶應義塾大学藤原記念工学部研究報告 (Proceedings of Faculty of Engineering, Keiogijuku University). Vol.2, No.6 (1949. 9) ,p.97(13)- 102(18)
JaLC DOI	
Abstract	When a diaphragm of drum-type vibrates in a fluid, it emits sound waves in all directions. In this paper, confining ourselves to a case in which the wave length is very long in comparison with the radius of the drum, the so-called virtual mass of the diaphragm has been estimated theoretically.
Notes	
Genre	Departmental Bulletin Paper
URL	<a href="https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00020006-0013">https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00020006-0013</a>

慶應義塾大学学術情報リポジトリ(KOARA)に掲載されているコンテンツの著作権は、それぞれの著作者、学会または出版社/発行者に帰属し、その権利は著作権法によって保護されています。引用にあたっては、著作権法を遵守してご利用ください。

The copyrights of content available on the Keio Associated Repository of Academic resources (KOARA) belong to the respective authors, academic societies, or publishers/issuers, and these rights are protected by the Japanese Copyright Act. When quoting the content, please follow the Japanese copyright act.

surface) and  $R_1/R_2=0$  (Cylindrical surface) has been treated in the above. Hence we could draw curves shown in Fig. 6. From the curves it is seen that the values  $k$  of the rate of reflection of impulsive pressure are considerably close to each other. The case of other rigid walls (for example,  $R_1/R_2=2$ ) will all lie between the two curves shown, and we may easily imagine to what extent the rate of reflection is affected by the curvature and distance of wall from the point source of impulse.

## On Vibration of Drum-Type Diaphragm in Water

Received Jan. 20, 1949

Fumiki Kitō\*

**Fumiki Kitō: On Vibration of Drum-type Diaphragm in Water** When a Diaphragm of drum-type vibrates in a fluid, it emits sound waves in all directions. In this paper, confining ourselves to a case in which the wave length is very long in comparison with the radius of the drum, the so-called virtual mass of the diaphragm has been estimated theoretically.

**Section 1. Introduction** The Author has shown in a recent paper (Several Examples of generation and prevention of vibration due to vortex-streets in Naval Engineering, Misc. Notes of Institution of Naval Engineers, Japan, No. 277, 1949) that it may be of some use to take up the problem of vibration of drum-type diaphragm as shown in Fig. 1, which vibrates in the water. In this report, some results of calculation made by the Author on this respect are given. Thus, the Author has shown a rough estimate on the effect of phase difference of vibration of two sides of drum upon wave propagation and virtual

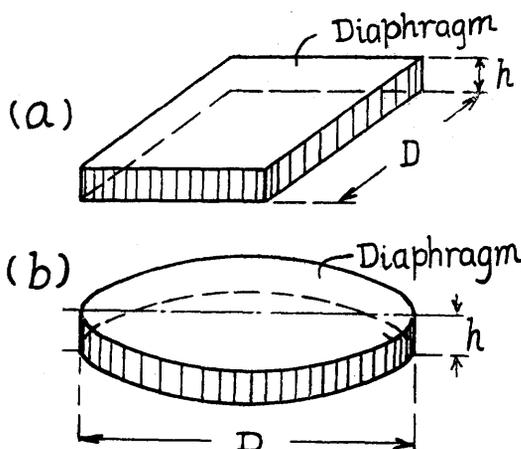


Fig. 1. Drum-type Vibrator

mass of surrounding water. In our case, the wave length of pressure wave being very large in comparison with the diameter of the drum, the equation of pressure  $p$  may be taken approximately to be  $\nabla^2 p=0$ , that is, the Laplace's Equation.  $\phi$  being the amplitude of the pressure  $p$ , we have  $p=\phi \cos \omega t$ , and  $\phi$  must also be a solution of Laplace's equation, viz.,  $\nabla^2 \phi=0$ . The instantaneous value of the kinetic

\* Dr. Eng., Prof. of Keio University.

energy of entire water surrounding the body such as the above mentioned drum. is given by;

$$T = \frac{1}{2\rho\omega^2} \iint \phi \frac{\partial\phi}{\partial n} ds \cdot \sin^2\omega t$$

where  $\rho$  is the density of the water.  $\partial\phi/\partial n$  means the derivative of the function  $\phi$  along the direction normal to the surface of the body in question, and  $ds$  the

surface element of the body. The double integral is to extend to the whole surface of the body. For a rough estimate we replace the body by an oblate spheroid as shown in Fig. 2. Using a system of spheroidal coordinates  $(\mu, \zeta, \theta)$  we have  $x = k\mu\zeta$ ,  $y = r \cos\theta$ ,  $z = r \sin\theta$ , where  $k$  denotes the radius of focal circle. Further we have

$$r = k(1 - \mu^2)^{1/2}(\zeta^2 + 1)^{1/2}$$

Considering only the vibration symmetrical about the axis of

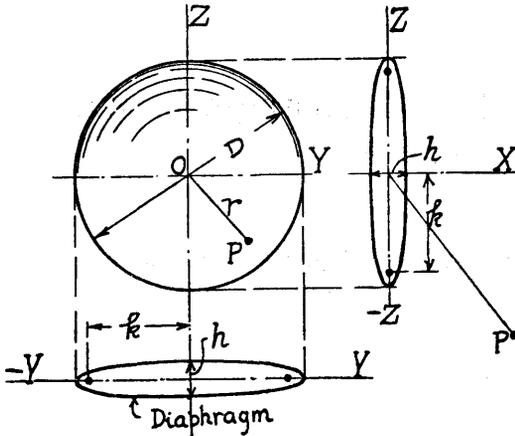


Fig. 2. Vibrating Diaphragm in form of a Oblate Spheroid

spheroid, the solution for  $\phi$  can be made up of terms of the form

$$\phi_n P_n(\mu) q_n(\zeta) \quad n = (0, 1, 2, \dots)$$

where  $P_n(\mu)$  is the Legendre's polynomial of degree  $n$ , and  $q_n(\zeta)$  the function defined by:—

$$q_0(\zeta) = \cot^{-1}\zeta$$

$$q_1(\zeta) = 1 - \zeta \cot^{-1}\zeta$$

$$q_2(\zeta) = \frac{1}{2}(3\zeta^2 + 1) \cot^{-1}\zeta - \frac{3}{2}\zeta$$

$$q_3(\zeta) = \frac{5}{6}(3\zeta^2 + 1) - \frac{1}{6} - \frac{1}{2}(5\zeta^3 + 3\zeta) \cot^{-1}\zeta$$

.....

$$q_n(\zeta) = (-1)^n \left[ p_n(\zeta) \cot^{-1}\zeta - \frac{2n-1}{1 \cdot n} p_{n-1}(\zeta) + \frac{2n-5}{3 \cdot (n-1)} p_{n-3}(\zeta) - \frac{2n-9}{5 \cdot (n-2)} p_{n-5}(\zeta) + \dots \right]$$

This type of solution  $\phi_n$  is fitted to solve problems outside of the spheroid. Now, taking the surface of given body (diaphragms) to be given by  $\zeta = \zeta_0$ , the thickness  $h$  and the diameter  $D$  of the drum will be given by

$$h = 2k\zeta_0, \quad D = 2k(1 + \zeta_0^2)^{1/2}$$

When the ratio  $h/D$  is small, we may put, approximately;  $\zeta_0 = h/D$ ,  $D = 2k$ . For a rough estimate, however, we may regard the surface to be  $\zeta_0 = 0$ . The upper

surface corresponds to  $0 < \mu < 1$ , whereas on the lower surface we have  $-1 < \mu < 0$ . The line element  $dn$  drawn normal to the surface being given by  $dn = k|\mu|d\zeta$

$$\frac{\partial \phi_n}{\partial n} = \frac{P_n(\mu)}{k|\mu|} q'(0) = N_n$$

$$N = \delta \left[ 1 - \left( \frac{r}{k} \right)^2 \right] = \delta \mu^2$$

**Section 2. Solution for Case A**

Of course, the upper and lower surface vibrates with the same frequency  $f$ , but their phases may not coincide with each other. As the Case A, we take the case in which the upper and lower surfaces vibrate in phase with each other. In this case we must have, at the surface

$$\sum N_n = N = \delta \mu^2$$

We find as the solution satisfying these conditions;

$$\phi_a = -\frac{1}{n} \delta k \rho \omega^2$$

$$\left[ \frac{6}{5} P_1(\mu) q_1(\zeta) + \frac{8}{15} P_3(\mu) q_3(\zeta) \right]$$

$$\left[ \frac{\partial \phi_a}{\partial n} \right]_0 = \frac{\delta \rho \omega^2}{|\mu|}$$

$$\left[ \frac{3}{5} P_1(\mu) + \frac{2}{5} P_3(\mu) \right]$$

To draw some conclusion from this result, we note first that for a point  $R$  very far away from the origin we have approximately  $OR = k\zeta$ . Also we have approximately, for very large value of  $\zeta$ ;

$$q_n(\zeta) = \frac{n!}{1 \cdot 3 \cdot 5 \dots (2n+1)} \frac{1}{\zeta^{n+1}} \quad (n=0, 1, 2, \dots)$$

Thus we infer that for a point very distant from the origin, we have approximately

$$\phi_a = -\frac{1}{20\pi} \delta D \rho \omega^2 \left( \frac{D}{R} \right)^2 \mu$$

where  $R$  is the distance  $OR$ . Therefore the pressure wave propagates with amplitude inversely proportional to the square of the distance  $R$ . Also it has directional property which is represented by the factor  $\mu = \cos \theta$ . Lastly, calculating total kinetic energy  $T_a$  of the surrounding water, we have,

$$2T_a = 0.520 \delta^2 \rho \omega^2 \left( \frac{D}{2} \right)^3 \sin^2 \omega t$$

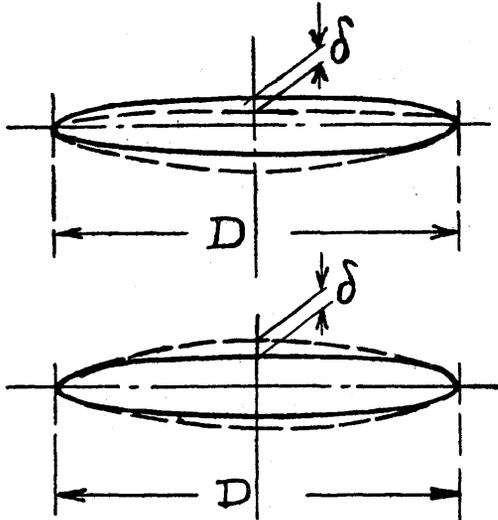


Fig. 3. Vibration Modes of Diaphragms.

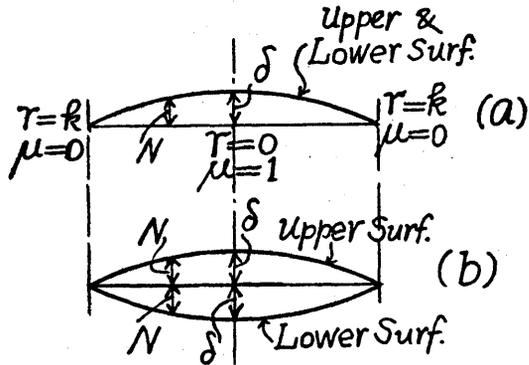


Fig. 4. Distribution of Vibration Amplitudes

**Section 3. Solution for Case B** As an alternative to the above, we take the case in which the phase of vibration of two faces are opposite to each other. For this case, taking as the solution  $\phi_b = \sum C_n q_n(\zeta) P_n(\mu)$ , we must have at the boundary surface;

$$\delta k \rho \omega^2 \mu^2 |_{\mu=1} = \sum C_n q_n'(0) P_n(\mu)$$

This means that the unknown constants  $C_n$  must be so chosen as to satisfy the above equation, which is done by the usual method of Fourier-series expansion. Thus we have  $C_1 = C_3 = C_5 = \dots = 0$ . Also, for even values of  $n$ ,

$$C_n q_n'(0) = \delta k \rho \omega^2 (2n+1) \int_0^1 \mu^3 P_n(\mu) d\mu$$

and the value of this last definite integral is given by

$$a_n = \int_0^1 \mu^3 P_n(\mu) d\mu = \frac{3 \cdot 1(-1)(-3) \dots (5-n)}{4 \cdot 6 \cdot 8 \cdot 10 \dots (n+2)(n+4)}$$

Moreover we have  $q_n'(0) = -1/P_n(0)$  by virtue of the formula;

$$P_n(\zeta) q_n'(\zeta) - P_n'(\zeta) q_n(\zeta) = -1/(1+\zeta^2)$$

And we have, for an even number  $n$ ,

$$C_n = \delta k \rho \omega^2 (2n+1) a_n / q_n'(0)$$

Some values of the coefficients  $a_n$  etc. are shown in Table I.

Table I

$n$	$a_n$	$q_n'(0)$	$q_n(0)$
0	1/4	-1	$\pi/2$
2	1/8	-2	$\pi/4$
4	1/64	-8/3	$3\pi/16$
6	-1/640	-16/5	$5\pi/32$
8	1/2560	-128/35	$35\pi/256$

The total kinetic energy  $T_b$  of the whole surrounding water, being given by

$$2T_b = \frac{2\pi k}{\rho \omega^2} \sum C_n^2 \left( \frac{-2}{2n+1} \right) q_n(0) q_n'(0) \sin^2 \omega t$$

is found to be;

$$T_b = 0.810 \delta^2 \rho \omega^2 \left( \frac{D}{2} \right)^3 \sin^2 \omega t$$

For a point far away from the body we have approximately

$$\phi_b = \frac{1}{16} (\delta D \rho \omega^2) \left( \frac{D}{R} \right)$$

**Section IV. Concluding Remarks** From the above results of calculation, it is seen that at a point very far away from the diaphragm, the amplitude of the propagated pressure wave varies inversely as the square  $R^2$  of the distance  $R$  for Case A, whereas for Case B, it varies as the inverse  $1/R$  of the distance  $R$ . The total kinetic energy of the surrounding water for Case B is, however, only 1.6 times that for Case A.

Denoting by  $w$  the amplitude of vibration at the center of the diaphragm, we have

$$w = \delta \cos \omega t, \quad dw/dt = -\omega \delta \sin \omega t$$

Therefore the kinetic energies of surrounding water may be expressed in the following form;

$$T_a = \frac{1}{2} M_a \left( \frac{dw}{dt} \right)^2, \quad M_a = 1.040 \rho \left( \frac{D}{2} \right)^3$$

$$T_b = \frac{1}{2} M_b \left( \frac{dw}{dt} \right)^2, \quad M_b = 1.62 \rho \left( \frac{D}{2} \right)^3$$

$M_a$  and  $M_b$  may be termed "virtual mass" of the surrounding water. The value  $M_b$  coincides with that obtained by MacLachlan (The Accession to Inertia of Flexible Discs vibrating in a Fluid, Proc. Phys. Soc., 1932).

Assuming that the diaphragm itself is made up of a material whose density is  $\rho_m$  and its thickness  $h$ , the kinetic energy of the diaphragm itself will be given by

$$T_q = \frac{1}{2} M_q \left( \frac{dw}{dt} \right)^2, \quad M_q = \frac{2\pi}{3} \rho_m \left( \frac{D}{2} \right)^2 h$$

so that the total kinetic energy will be given by.  $T_q + T_a$ , or  $T_q + T_b$ .

The factors  $\epsilon_a, \epsilon_b$  giving the rate of increase of apparent mass due to the existence of surrounding water will become:—

$$\epsilon_a = \frac{M_a}{M_q} = 0.498 \left( \frac{\rho}{\rho_m} \right) \left( \frac{D}{2h} \right)$$

$$\epsilon_b = \frac{M_b}{M_q} = 0.775 \left( \frac{\rho}{\rho_m} \right) \left( \frac{D}{2h} \right)$$

These results were obtained for two special modes  $A$  and  $B$  of the vibration of the vibration of diaphragm. The other mode can be derived by them by means of the principle of superposition. For example, if one side is vibrating with the mode as shown in Fig. 4 (a), while the other side stands still, this state can be represented by  $\phi_0 = \frac{1}{2}(\phi_a + \phi_b)$ . The function  $\phi_a$  contains  $P_n(\mu)$  of odd order only, while the function  $\phi_b$  is made up of terms of even orders. Utilizing the orthogonal property of  $P_n(\mu)$ , it is easy to show that the corresponding kinetic energy  $T_0$  will be given by

$$T_0 = \frac{1}{4} [T_a + T_b]$$

Thus the virtual mass  $M_0$  will become  $M_0 = 0.67 \rho (D/2)^3$ .

All the above treatment are based on the assumption that the surrounding water extends to infinity in every directions. When there exist bottom surface or free surface as the boundary surface of water region, the law of propagation of the pressure wave may be different from the above mentioned results. Take for example the water region to consist of the region as shown in Fig. 5, where it extends to infinity in two directions, but it has

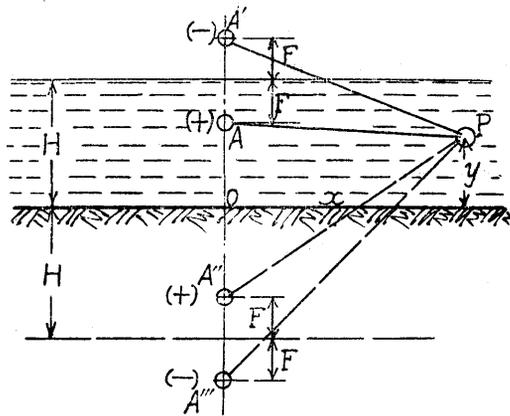


Fig. 5. Vibration Source and its Images

a finite depth  $H$ . Let a diaphragm  $A$  which is vibrating in the manner of Case  $B$  be placed at some depth  $F$  below the free surface, as shown in the same figure. Assuming that the diameter  $D$  of the diaphragm is small in comparison with  $H$  and  $F$ , the amplitude of vibration due to that at  $A$  in the figure will approximately be given by

$$\phi_0 = K/R, \quad K = \delta D^2 \rho \omega^2 / 16$$

In order to take into account the effect of bottom wall and free surface we imagine (+) image  $A''$  and (-) images  $A''$ ,  $A'''$  to be placed as shown in Fig. 5. In this way, we conclude that the propagation of pressure wave in the case of Fig. 5 may approximately be given by the formula

$$\phi_0 = K \left[ \left( \frac{1}{AP} - \frac{1}{A''P} \right) + \left( \frac{1}{A''P} - \frac{1}{A'''P} \right) \right]$$

From this expression it can be seen that in the Case of Fig. 5, it is far more difficult to transmit low frequency pressure wave in horizontal direction, than in the Case of water extending to infinity in all directions.

## 磁鐵極による鹽素酸鹽の電解製造に関する 基礎的研究 (第1報) 過鹽素酸鹽副生現象に就て

昭和24年(1949)8月10日受理

故 藤岡忠仁\*, 永井 隆\*\*

**Chūji Fujioka\* and Takashi Nagai\*\*:** Studies on the Electrolytic Manufacturing of Chlorate with Magnetite Anode (I) About Perchlorate By-Product This investigation was performed to study the effect of perchlorate by-product in electrolytic manufacturing of chlorate with Japanese magnetite anode. The perchlorate was produced even at low current densities, proportional to chlorate concentration, independent to hydrogen ion and temperature. The production of perchlorate was prevented remarkably by chlor ion. Same electrolysis of chlorate using graphite anode was studied for comparison. So the perchlorate by-product should be remarkable near the end of the electrolytic manufacturing of chlorate, especially at high current densities. From above experiments we concluded that the production of perchlorate has no direct effect to lowering of the chlorate current efficiency.

### 1 緒 言

現在本邦産磁性酸化鐵を用いて食鹽より鹽素酸曹達を電解製造する際其の成績は餘り良好ではない。此の原因として2つの要素が考えられる。其の第1は本邦産原鑛石から製造

\* 工博, 慶應義塾大學教授, Dr. Eng., Prof. of Keio University.

\*\* 慶應義塾大學工學部講師, Lecturer of Faculty of Eng., Keio University.