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Abstract	When a diaphragm of drum-type vibrates in a fluid, it emits sound waves in all directions. In this paper, confining ourselves to a case in which the wave length is very long in comparison with the radius of the drum, the so-called virtual mass of the diaphragm has been estimated theoretically.
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surface) and $R_1/R_2=0$ (Cylindrical surface) has been treated in the above. Hence we could draw curves shown in Fig. 6. From the curves it is seen that the values k of the rate of reflection of impulsive pressure are considerably close to each other. The case of other rigid walls (for example, $R_1/R_2=2$) will all lie between the two curves shown, and we may easily imagine to what extent the rate of reflection is affected by the curvature and distance of wall from the point source of impulse.

On Vibration of Drum-Type Diaphragm in Water

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Fumiki Kitō: On Vibration of Drum-type Diaphragm in Water When a Diaphragm of drum-type vibrates in a fluid, it emits sound waves in all directions. In this paper, confining ourselves to a case in which the wave length is very long in comparison with the radius of the drum, the so-called virtual mass of the diaphragm has been estimated theoretically.

Section 1. Introduction The Author has shown in a recent paper (Several Examples of generation and prevention of vibration due to vortex-streets in Naval Engineering, Misc. Notes of Institution of Naval Engineers, Japan, No. 277, 1949) that it may be of some use to take up the problem of vibration of drum-type diaphragm as shown in Fig. 1, which vibrates in the water. In this report, some results of calculation made by the Author on this respect are given. Thus, the Author has shown a rough estimate on the effect of phase difference of vibration of two sides of drum upon wave propagation and virtual mass of surrounding water. In our case, the wave length of pressure wave being very large in comparison with the diameter of the drum, the equation of pressure p may be taken approximately to be $\Delta^2 p=0$, that is, the Laplace's Equation. ϕ being the amplitude of the pressure p, we have $p=\phi\cos\omega t$, and ϕ must also be a solution of Laplace's equation, viz., $\Delta^2\phi=0$. The instantaneous value of the kinettic

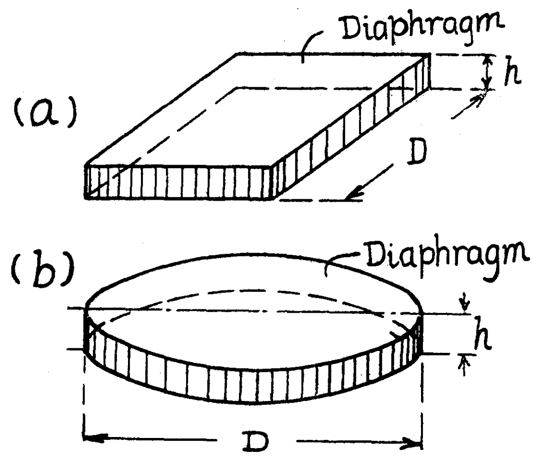

Fig. 1. Drum-type Vibrator

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energy of entire water surrounding the body such as the above mentioned drum is given by;

$$T=\frac{1}{2\rho\omega^2}\iint\phi\frac{\partial\phi}{\partial n}ds\cdot\sin^2\omega t$$

where ρ is the density of the water. $\partial\phi/\partial n$ means the derivative of the function ϕ along the direction normal to the surface of the body in question, and ds the surface element of the body. The double integral is to extend to the whole surface of the body. For a rough estimate we replace the body by an oblate spheroid as shown in Fig. 2. Using a system of spheroidal coordinates (μ, ζ, θ) we have $x=k\mu\zeta$, $y=r\cos\theta$, $z=r\sin\theta$, where k denotes the radius of focal circle. Further we have

$$r=k(1-\mu^2)^{1/2}(\zeta^2+1)^{1/2}$$

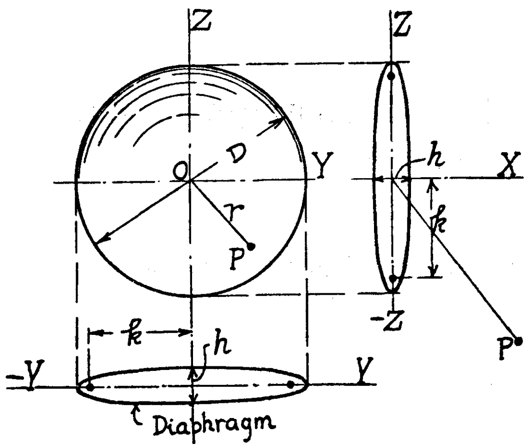

Fig. 2. Vibrating Diaphragm in form of a Oblate Spheroid

Considering only the vibration symmetrical about the axis of spheroid, the solution for ϕ can be made up of terms of the form

$$\phi_n P_n(\mu) q_n(\zeta) \quad n=(0, 1, 2, \cdots)$$

where $P_n(\mu)$ is the Legendre's polynomial of degree n, and $q_n(\zeta)$ the function defind by:—

$$q_0(\zeta)=\cot^{-1}\zeta$$

$$q_1(\zeta)=1-\zeta\cot^{-1}\zeta$$

$$q_2(\zeta)=\frac{1}{2}(3\zeta^2+1)\cot^{-1}\zeta-\frac{3}{2}\zeta$$

$$q_3(\zeta)=\frac{5}{6}(3\zeta^2+1)-\frac{1}{6}-\frac{1}{2}(5\zeta^3+3\zeta)\cot^{-1}\zeta$$

$$\cdots\cdots\cdots \quad \cdots\cdots\cdots \quad \cdots\cdots\cdots$$

$$q_n(\zeta)=(-1)^n\Big[p_n(\zeta)\cot^{-1}\zeta-\frac{2n-1}{1\cdot n}p_{n-1}(\zeta)+\frac{2n-5}{3\cdot(n-1)}p_{n-3}(\zeta)-\frac{2n-9}{5\cdot(n-2)}p_{n-5}(\zeta)+\cdots\Big]$$

This type of solution ϕ_n is fitted to solve problems outside of the spheroid. Now, taking the surface of given body (diappragms) to be given by $\zeta=\zeta_0$, the thickness h and the diameter D of the drum will be given by

$$h=2k\zeta_0, \quad D=2k(1+\zeta_0^2)^{1/2}$$

When the ratio h/D is small, we may put, approximately; $\zeta_0=h/D$, $D=2k$. For a rough estimate, however, we may regard the surface to be $\zeta_0=0$. The upper surface corresponds to $0<\mu<1$, whereas on the lower surface we have $-1<\mu<0$. The line element dn drawn normal to the surface being given by $dn=k|\mu|d\zeta$

$$\frac{\partial\phi_n}{\partial n}=\frac{P_n(\mu)}{k|\mu|}q'(0)=N_n$$

$$N=\delta\left[1-\left(\frac{r}{k}\right)^2\right]=\delta\mu^2$$

Section 2. Solution for Case A

Of course, the upper and lower surface vibrates with the same frequency f, but their phases may not coincide with each other. As the Case A, we take the case in which the upper and lower surfaces vibrate in phase with each other. In this case we must have, at the surface

$$\Sigma N_n=N=\delta\mu^2$$

We find as the solution satisfying these conditions;

$$\phi_a=-\frac{1}{n}\delta k\rho\omega^2$$
$$\left[\frac{6}{5}P_1(\mu)q_1(\zeta)+\frac{8}{15}P_3(\mu)q_3(\zeta)\right]$$
$$\left[\frac{\partial\phi_a}{\partial n}\right]_0=\frac{\delta\rho\omega^2}{|\mu|}$$
$$\left[\frac{3}{5}P_1(\mu)+\frac{2}{5}P_3(\mu)\right]$$

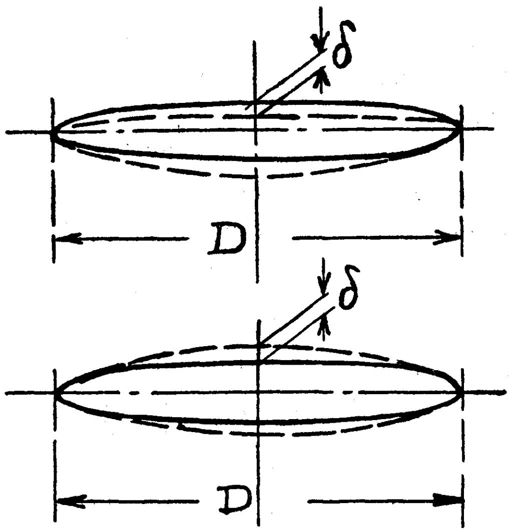

Fig. 3. Vibration Modes of Diaphragms.

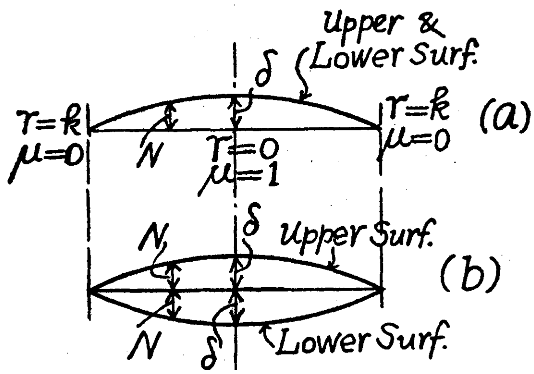

Fig. 4. Distribution of Vibration Amplitudes

To draw some conclution from this result, we note first that for a point R very far away from the origin we have approximately $OR=k\zeta$. Also we have approximately, for very large value of ζ;—

$$q_n(\zeta)=\frac{n!}{1\cdot3\cdot5\cdots(2n+1)}\frac{1}{\zeta^{n+1}}\quad(n=0,1,2,\cdots)$$

Thus we infer that for a point very distant from the origin, we have approximately

$$\phi_a=-\frac{1}{20\pi}\delta D\rho\omega^2\left(\frac{D}{R}\right)^2\mu$$

where R is the distance OR. Therefore the pressure wave propagates with amplitude inversely proportional to the square of the distance R. Also it has directional property which is represented by the factor $\mu=\cos\theta$. Lastly, calculating total kinetic energy T_a of the surrounding water, we have,

$$2T_a=0.520\,\delta^2\rho\omega^2\left(\frac{D}{2}\right)^3\sin^2\omega t$$

Section 3. Solution for Case B As an alternative to the above, we take the case in which the phase of vibration of two faces are opposite to each other. For this case, taking as the solution $\phi_b=\Sigma C_n q_n(\zeta)P_n(\mu)$, we must have at the boundary surface;

$$\delta k\rho\omega^2\mu^2\,|\,\mu\,|=\Sigma C_n q_n'(0)P_n(\mu)$$

This means that the unknown constants C_n must be so chosen as to satisfy the above equation, which is done by the usual method of Fourier-series expansion. Thus we have $C_1=C_3=C_5=\cdots=0$. Also, for even values of n,

$$C_n q_n'(0)=\delta k\rho\omega^2(2n+1)\int_0^1\mu^3 P_n(\mu)d\mu$$

and the value of this last definite integral is given by

$$a_n=\int_0^1\mu^3 P_n(\mu)d\mu=\frac{3.1(-1)(-3)\cdots(5-n)}{4,6,8,10\cdots(n+2)(n+4)}$$

Moreover we have $q_n'(0)=-1/P_n(0)$ by virtue of the formula;

$$P_n(\zeta)q_n'(\zeta)-P_n'(\zeta)q_n(\zeta)=-1/(1+\zeta^2)$$

And we have, for an even number n,

$$C_n=\delta k\rho\omega^2(2n+1)a_n/q_n'(0)$$

Some values of the coefficients a_n etc. are shown in Table I.

Table I

n	a_n	$q_n'(0)$	$q_n(0)$
0	1/4	-1	$\pi/2$
2	1/8	-2	$\pi/4$
4	1/64	$-8/3$	$3\pi/16$
6	$-1/640$	$-16/5$	$5\pi/32$
8	1/2560	$-128/35$	$35\pi/256$

The total kinetic energy T_b of the whole surrounding water, being given by

$$2T_b=\frac{2\pi k}{\rho\omega^2}\Sigma C_n^2\left(\frac{-2}{2n+1}\right)q_n(0)q_n'(0)\sin^2\omega t$$

is found to be;

$$T_b=0.810\,\delta^2\rho\omega^2\left(\frac{D}{2}\right)^3\sin^2\omega t$$

For a point far away from the body we have approximately

$$\phi_b=\frac{1}{16}(\delta D\rho\omega^2)\left(\frac{D}{R}\right)$$

Section IV. Concluding Remarks From the above results of calculation, it is seen that at a point very far away from the diaphragm, the amplitude of the propagated pressure wave varies inversely as the square R^2 of the distance R for Case A, whereas for Case B, it varies as the inverse $1/R$ of the distance R. The total kinetic energy of the surrounding water for Case B is, however, only 1.6 times that for Case A.

Denoting by w the amplitude of vibration at the center of the diaphragm, we have

$$w=\delta\cos\omega t,\quad dw/dt=-\omega\delta\sin\omega t$$

Therefore the kinetic energies of surrounding water may be expressed in the following form;

$$T_a=\frac{1}{2}M_a\left(\frac{dw}{dt}\right)^2,\quad M_a=1.040\,\rho\left(\frac{D}{2}\right)^3$$

$$T_b=\frac{1}{2}M_b\left(\frac{dw}{dt}\right)^2, \quad M_b=1.62\,\rho\left(\frac{D}{2}\right)^3$$

M_a and M_b may be termed "virtual mass" of the surrounding water. The value M_b coincides with that obtained by MacLachlan (The Accession to Inertia of Flexible Discs vibrating in a Fluid, Proc. Phys. Soc., 1932).

Assuming that the diaphragm itself is made up of a material whose density is ρ_m and its thickness h, the kinetic energy of the diaphragm itself will be given by

$$T_q=\frac{1}{2}M_q\left(\frac{dw}{dt}\right)^2, \quad M_q=\frac{2\pi}{3}\rho_m\left(\frac{D}{2}\right)^2 h$$

so that the total kinetic energy will be given by. T_q+T_a, or T_q+T_b.

The factors ε_a, ε_b giving the rate of increase of apparent mass due to the existence of surrounding water will become:—

$$\varepsilon_a=\frac{M_a}{M_q}=0.498\left(\frac{\rho}{\rho_m}\right)\left(\frac{D}{2h}\right)$$

$$\varepsilon_b=\frac{M_b}{M_q}=0.775\left(\frac{\rho}{\rho_m}\right)\left(\frac{D}{2h}\right)$$

These results were obtained for two special modes A and B of the vibration of the vibration of diaphragm. The other mode can be derived by them by means of the principle of superposition. For example, if one side is vibrating with the mode as shown in Fig. 4 (a), while the other side stands still, this state can be represented by $\phi_0=\frac{1}{2}(\phi_a+\phi_b)$. The function ϕ_a contains $P_n(\mu)$ of odd order only, while the function ϕ_b is made up of terms of even orders. Utilizing the orthogonal property of $P_n(\mu)$, it is easy to show that the corresponding kinetic energy T_0 will be given by

$$T_0=\frac{1}{4}[T_a+T_b]$$

Thus the virtual mass M_0 will become $M_0=0.67\rho(D/2)^3$.

All the above treatment are based on the assumption that the surrounding water extends to infinity in every directions. When there exist bottom surface or free surface as the boundary surface of water region, the law of propagation of the pressure wave may be different from the above mentioned results. Take for example the water region to consist of the region as shown in Fig. 5, where it extends to infinity in two directions, but it has

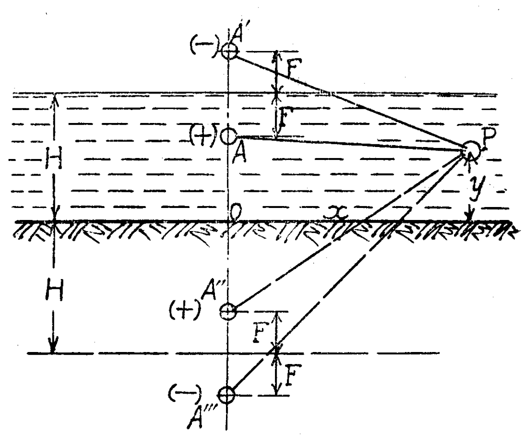

Fig. 5. Vibration Source and its Images

a finite depth H. Let a diaphragm A which is vibrating in the manner of Case B be placed at some depth F below the free surface, as shown in the same figure. Assuming that the diameter D of the diaphragm is small in comparison with H and F, the amplitude of vibration due to that at A in the figure will approximately be given by

$$\phi_b=K/R, \quad K=\delta D^2\rho\omega^2/16$$

In order to take into account the effect of bottom wall and free surface we imagine (+) image A'' and (−) images A'', A''' to be placed as shown in Fig. 5. In this way, we conclude that the propagation of pressure wave in the case of Fig. 5 may approximately be given by the formula

$$\phi_b=K\left[\left(\frac{1}{AP}-\frac{1}{A'P}\right)+\left(\frac{1}{A''P}-\frac{1}{A'''P}\right)\right]$$

From this expression it can be seen that in the Case of Fig. 5, it is far more difficult to transmit low frequency pressure wave in horizontal direction, than in the Case of water extending to infinity in all directions.

磁鐵極による鹽素酸鹽の電解製造に關する基礎的研究 (第1報) 過鹽素酸鹽副生現象に就て

昭和24年(1949)8月10日受理

故藤岡忠仁,* 永井 隆**

Chūji Fujioka* and Takashi Nagai: Studies on the Electrolytic Manufacturing of Chlorate with Magnetite Anode (I) About Perchlorate By-Product** This investigation was performed to study the effect of perchlorate by-product in electrolytic manufacturing of chlorate with Japanese magnetite anode. The perchlorate was produced even at low current densities, proportional to chlorate concentration, independent to hydrogen ion and temperature. The production of perchlorate was prevented remarkablly by chlor ion. Same electrolysis of chlorate using graphite anode was studied for comparison. So the perchlorate by-product should be remarkable near the end of the electrolytic manufacturing of chlorate, especially at high current densities. From above experiments we concluded that the production of perchlorate has no direct effect to lowering of the chlorate current efficiency.

1 緒 言

現在本邦産磁性酸化鐵を用いて食鹽より鹽素酸曹達を電解製造する際其の成績は餘り良好ではない。此の原因として2つの要素が考えられる。其の第1は本邦産原鑛石から製造

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