Title	On two kinds of fluid-flow through bent-pipes
Sub Title	
Author	鬼頭, 史城(Kito, Fumiki)
Publisher	慶應義塾大学藤原記念工学部
Publication year	1949
Jtitle	慶應義塾大学藤原記念工学部研究報告 (Proceedings of Faculty of Engineering, Keiogijuku University). Vol.2, No.5 (1949. 7) ,p.71(27)- 76(32)
JaLC DOI	
Abstract	Two kinds of Vortex motions occurring in bent-pipe has been studied theoretically. The first is the secondary vortex flow produced in a bent pipe of elliptical cross-section ; the flow being supposed to be in the state of so-called laminar flow. The second case concerns with the whirling flow through a bent-pipe of circular cross section, where there exist a vortex core along the center line of the pipe, the fluid being taken to be an ideal fluid. The amound of deviation of the center of vortex core has been estimated.
Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00020005-0027

ferent that for the backward curved blades impeller, and as is clear in fig. 29, Stodola's formula or Eck's formula shows coincidence no longer, while the new formula (19) coincides approximately with experimental results. The comparison of the results of another blower of straight radial blades, the number of which being 4, 8 and 16, with the caluclated value by (19) is shown in fig. 30, which shsws less deviations than in fig. 29. Therefore, the new formula is applicable for the straight redial blades impeller to some extent.

V. Conclusioni.

The conclusions obtained by these experiments and considerations can be abstracted as follows.

(1) The performance characteristics of a entrifugal blower show optimum at certain number of blades. When the number blades is less than this optimum value, the performances become inferior on account of partial flow within the impeller channel, wrile when the number of blades exceeds the optimum value, the performances also show lower value on account of the increase of flow resistances in impeller channel. Although the optimum number of blades of the centrifugal blower with straight radial blades is found to be $z=10\sim12$ in this case, this figure is not applicable in general, even if the shape of blades be the same. The optimum number of blades seems to be affected intensively by the diameter ratio of the impeller, and the larger the diameter ratio, i.e. the longer the blades, the less the optimum number of blades, from the experiments by the author himself and from the references already cited to. But it is unable to formulate this effect completely on account of shortness of experimental data.

(2) The ration of the ship coefficient of impeller to the number of blades for the straight radial blades impeller is quite different to the generally used impeller of backward curved blades type. The results of graphical calculation coincide with experimental results, by means of the correction of the hitherto used Eck's method by the assumptions obtained from the photographic study of the flow patterns within the impeller channel. Stodola's formula or Eck's formula is no longer applicable to the cases of high speed straight radial blades imdeller as treated in this paper, while the now formula obtained shows some coincidence with experimental results.

On Two Kinds of Fluid-Flow through Bent-Pipes.

Recieved Jan. 15, 1949.
Fumiki Kito*

Two kinds of Vortex motions occurring in bent-pipe has been studied theoretically. The first is the secondary vortex flow produced in a bent pipe of elliptical cross-section; the flow being supposed to be in the state of so-called laminar flow. The second case concerns

* Dr. of Eng. Prof. of Keiogijuku University.

with the whirling flow through a bent-pipe of circular cross section, where there exist a vortex core along the center line of the pipe, the fluid being taken to be an ideal fluid. The amound of deviation of the center of vortex core has been estimated.

Section I. Secondary Vortex produced in a Bent-pipe of elliptical cross-section.

Using cylindrical coordinates r, θ, z and denotidg velocity components by W_r, W_n and W_z, the equation of motion of a viscous fluid is given by;

$$-\frac{g}{\rho}\frac{\partial p}{\partial r}+\frac{\mu g}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial rW_r}{\partial r}\right)+\frac{\partial^2 W_r}{\partial z^2}\right]$$
$$=W_r\frac{\partial W_r}{\partial r}+W_n\frac{\partial W_r}{r\partial\theta}+W_z\frac{\partial W_r}{\partial z}-\frac{W_n{}^2}{r}$$
$$-\frac{g}{\rho}\frac{\partial p}{r\partial\theta}+\frac{\mu g}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial rW_n}{\partial r}\right)+\frac{\partial^2 W_n}{\partial z^2}\right]$$
$$=W_r\frac{\partial W_n}{\partial r}+W_n\frac{\partial W_n}{r\partial\theta}+W_z\frac{\partial W_n}{\partial z}+\frac{W_nW_r}{r}$$
$$-\frac{g}{\rho}\frac{\partial p}{\partial z}+\frac{\mu g}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial W_z}{\partial r}\right)+\frac{\partial^2 W_z}{\partial z^2}\right]$$
$$=W_r\frac{\partial W_z}{\partial r}+W_n\frac{\partial W_z}{r\partial\theta}+W_z\frac{\partial W_z}{\partial z}$$
$$\frac{1}{r}\frac{\partial rW_r}{\partial r}+\frac{\partial W_n}{r\partial\theta}+\frac{\partial W_z}{\partial z}=0$$

where μ is the coefficient of viscosity, p the pressure, ρ the density of the fluid, and we consider the steady flow only.

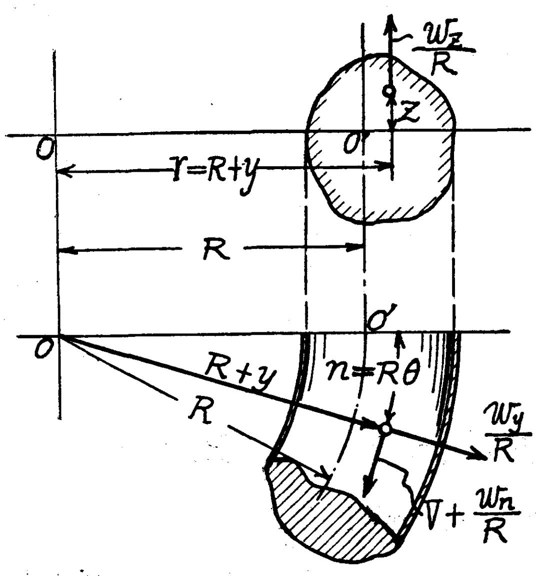

Fig. 1.
Flow of Viscous Fluid through a Curved Pipe.

Taking the case in which the radius of curvature R is considerably large in comparison with the diameter of the bent-pipe, we put $r=R+y$, $R\theta=n$, and we assume that the value of $(y/R)^2$ is so small that it can be neglected in comparison with unity.

Further we put $p=\Pi+p_1/R$ $W_r=w_y/R$, $W_z=w_z/R$,

$$W_n=V+w_n/R$$

and substitute these values into the above mentioned equation of motion. Neglecting terms of order $1/R^2$, and re-arranging, our equation reduces to the following equation

$$\frac{\partial^4 F}{\partial y^4}+2\frac{\partial^4 F}{\partial y^2\partial z^2}+\frac{\partial^4 F}{\partial z^4}=-\frac{\rho}{\mu g}\frac{\partial V^2}{\partial z} \quad \cdots(A)$$

where F is a stream function defined by $w_y=\partial F/\partial z,\ w_z=-\partial F/\partial y$.

In the case of a bent-pipe of elliptical cross-section, we may put

$$V=-\frac{1}{2}K\frac{a^2b^2}{a^2+b^2}\left(1+\frac{y^2}{a^2}-\frac{z^2}{b^2}\right)$$

$$F=\left(1-\frac{y^2}{a^2}-\frac{z^2}{b^2}\right)^2\left[A_0+A_1\frac{y^2}{a^2}+B_1\frac{z^2}{b^2}\right]z$$

where A_0, A_1 and B_1 are unknown constants. These constants must be chosen so as to satisfy the equation (A). The boundary condition for the boundary wall in form of an ellipse whose equation is

$$\frac{y^2}{a^2}+\frac{z^2}{b^2}=1,$$

is satisfied in itself by the above mentioned expression of F.

The Author has made the calculation for unknown constants A_0, A_1 and B_1, but the detail of it is omitted here. From the result of calculation it is seen that a state of secondary vortex flow as sketched in Fig. 2 occurs about the cross section of elliptical bent-pipe. One way of comparing the strength of this vortex flow is to estimate the value of secondary-flow velocity w_y at the center of cross-section viz., at $y=0, z=0$. Denoting this velocity by w_{00}, we have on ground of above mentioned calculation:—

$$w_{00}=\frac{2\pi\rho ab}{3\mu gR}(V_m)^2\frac{\alpha}{\pi}f(\alpha)$$

where $\alpha=b/a$, and

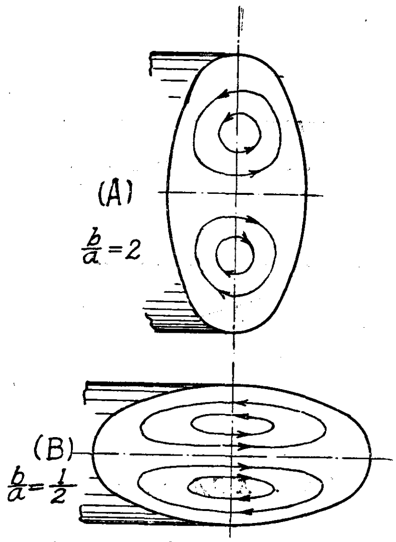

Fig. 2.
Secondary Vortex Flow in Bent-Pipe of Elliptical Cross-section.

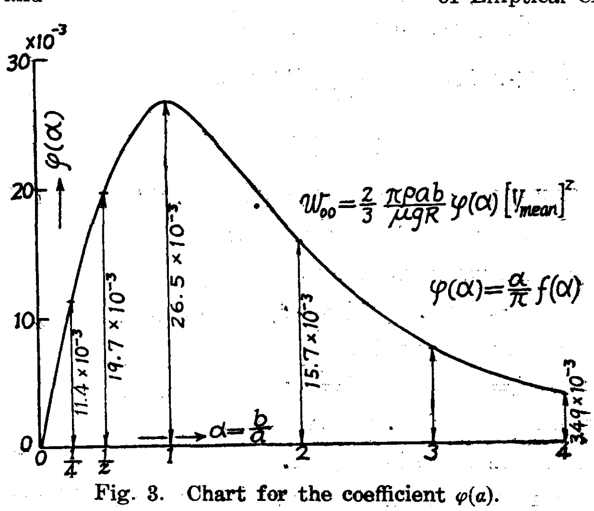

Fig. 3. Chart for the coefficient $\varphi(\alpha)$.

$$f(\alpha)=\begin{vmatrix} 13\alpha^4+10\alpha^2+5 & 4\alpha^2 \\ 4\alpha^2 & 3\alpha^4+14\alpha^2+75 \end{vmatrix} \div \Delta$$

$$\Delta=(\alpha^4+2\alpha^2+5)\begin{vmatrix} 15\alpha^4+12\alpha^2+5 & 6\alpha^2+10 \\ 6\alpha^4+10\alpha^2 & 3\alpha^4+20\alpha^2+105 \end{vmatrix}$$

and V_m is the mean value of V. A chart for w_{00} is shown in Fig. 3, where we see for example that for bent-pipe of the same elliptical cross section, the value of secondary-flow velocity w_{00} is somewhat larger when it is oblong (A of Fig. 2) than when it is oblate (B of Fig. 2).

Section II. Whirling Flow through a Bent-pipe of circular cross-section.

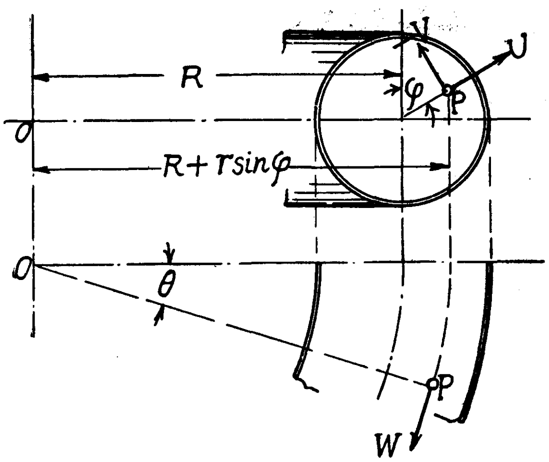

Fig. 4. A System of Toroidal Coordinates.

Using a system of Toroidal-coordinates as shown in Fig. 4, and denoting by U, V and W the component velocity of flow of an ideal fluid in r-, φ- und θ directions respectively, the equation of motion may be written in the form;

$$U\frac{\partial U}{\partial r}+\frac{V}{r}\frac{\partial U}{\partial \varphi}+\frac{W}{R+r\sin\varphi}\frac{\partial U}{\partial \theta}-\frac{V}{r}\frac{W^2\sin\varphi}{R+r\sin\varphi}=-\frac{\partial P}{\partial r}$$

$$U\frac{\partial V}{\partial r}+\frac{V}{r}\frac{\partial V}{\partial \varphi}+\frac{W}{R+r\sin\varphi}\frac{\partial V}{\partial \theta}+\frac{VU}{r}-\frac{W^2\cos\varphi}{R+r\sin\varphi}=-\frac{\partial P}{r\partial \varphi}$$

$$U\frac{\partial W}{\partial r}+\frac{V}{r}\frac{\partial W}{\partial \varphi}+\frac{W}{R+r\sin\varphi}\frac{\partial W}{\partial \theta}+\frac{UW\sin\varphi}{R+r\sin\varphi}+\frac{VW\cos\varphi}{R+r\sin\varphi}=-\frac{1}{R+r\sin\varphi}\frac{\partial P}{\partial \theta}$$

$$\frac{\partial}{\partial r}[r(R+r\sin\varphi)U]+\frac{\partial}{\partial \varphi}[(R+r\sin\varphi)V]+\frac{\partial}{\partial \theta}(rW)=0$$

(A) A State of Whirling Flow through a Straight Pipe of Circular Section, is considered to consist of the following two parts;

Outside Whirling Zone, where we have

$$W_0=\text{Const.} \qquad V_0=\omega a^2/r \qquad (a<r<b)$$

Inside Vortex Core, in which we have

$$W_0=0, \qquad V_0=\omega r \qquad (0<r<a)$$

and this corresponds to the case in which $R=\infty$.

(B) Whirling Flow through Bent-pipe.

In order to obtain an approximate estimate of the state of flow when the whirling flow as described above (A) is sent into a bent-pipe of radius of curvature R, we put

$$U=u, \quad V=V_0+v, \quad W=W_0+w, \quad P=P_0+p,$$

into the equation of motion, and we neglect squares and products of u, v, w. This means that we take the case in which the ratio R/a is large. In this way, we obtain equations for additional velocities and pressures u, v, w, p, with respect to inside and outside zones. The boundary conditions to be satisfied are taken approximately to be;

at $r=a$; u's and p's coincide,

at $r=b$; $u=0$

A system of solutions satisfying all of these requirements is seen to be given by;

Outside Zone

$$U=u=-\frac{W_0^2}{2\omega R}\left(1-\frac{b^2}{r^2}\right)\cos\varphi$$

$$V=V_0+v=\frac{\omega a^2}{r}+\frac{W_0^2}{2\omega R}\left(1+\frac{b^2}{r^2}\right)\sin\varphi$$

$$W=W_0+w=W_0-\frac{W_0}{R}r\sin\varphi$$

Inside Vortex Core

$$U=u=\frac{W_0^2}{2\omega R}\left(\frac{b^2}{a^2}-1\right)\cos\varphi$$

$$V=V_0+v=\omega r-\frac{W_0^2}{2\omega R}\left(\frac{b^2}{a^2}-1\right)\sin\varphi$$

$$W=w=0$$

According to this state of flow, the boundary surface between inside and outside zones becomes

$$r=a+\frac{W_0}{2\omega^2 R}\left(\frac{b^2}{a^2}-1\right)\sin\varphi$$

so that the boundary surface is shifted outwards by the amount;

$$\eta=\frac{W_0}{2\omega^2 R}\left(\frac{b^2}{a^2}-1\right)\sin\varphi$$

Taking, as an example, $R=3$ m, $b=1$ m, $a=0.44$ m and $\omega a/W_0=\tan 54°=1.37$, we obtain $\eta=0.075$ m. Of course, the value $R/b=3$ not being sufficiently large, this estimate is a very rough one.

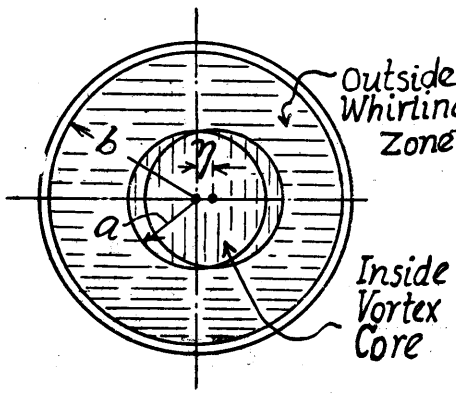

Fig. 5. Cross Section of a Bent Pipe.

In the above treatment, we assumed the flow to be independent of the value of the variable θ. For example, when an elbow is connected to a straight pipe, and whirling water is sent into the other end of straight pipe, the flow through the elbow will be something like that sketched in Fig. 6.

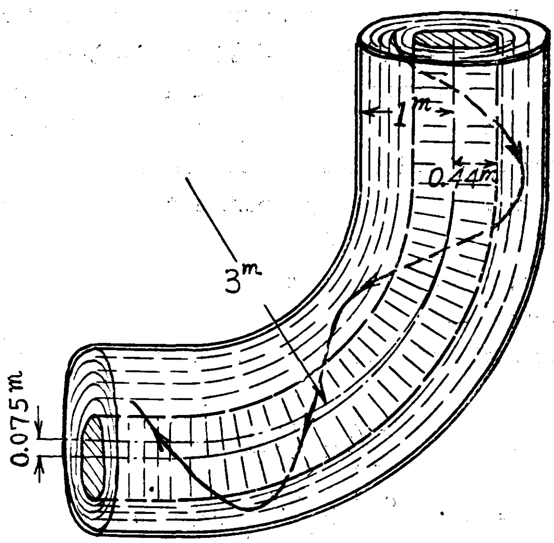

Fig. 6.
Sketch of flow of Whirling Water
through an Elbow.

Note:—Some copies giving the main feature of this paper has already been distributed by the Author in Oct. 1941.

Bending of a Circular Plate of Non-Uniform Thickness.

Recieved Jan. 15, 1949.
Fumiki Kito*

A circular plate whose thickness varies linearly with the radius r is subjected to non-uniform load which is a specified function of r. This problem of bending of plate has been studied theoretically, and the result obtained is shown as charts, so that the designer could make estimates of stresses etc., in such a plate with easiness.

Section I. Preliminary Remarks.

Let us consider a circular plate of radius R, as shown in Fig. 1. Its thickness h is not uniform, but it is a function of radial distance r. On this plate lateral load of strength p/unit area is applied. This distributed load p is also a function of radial distance r. Thus, all things being symmetrical about the center O of the plate, the stresses occurring in the plate must also be functions of radius r only. The Author has made some calculation on such a state of stress, and it is reported here. A case of stress in blades of a marine propeller (especially wide-bladed one) under the action

* Dr. of Eng. Prof. of Keiogijuku University.