Title	The fundamental motivation for mathematics lies in "beauty" : what is the attraction of the pursuit of pure mathematics?
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Associate Professor Takeshi Katsura is Featured in this issue,
whose field of research focuses on the theory of C*-algebras, a field of pure mathematics.

The fundamental motivation for mathematics lies in "Beauty"

What is the attraction of the pursuit of pure mathematics?

Of the many academic disciplines, mathematics is one of the most indispensable as the foundation of our civilized society but is perhaps the least understood by people due to its breadth and depth. Particularly, the field known as pure mathematics – which is in the opposite end of practical sciences – is a world extremely hard to see and a world that only a handful of specialists can deal with. For this issue, we asked Dr. Takeshi Katsura about the attractiveness of and his approach to pure mathematics, and some of the theory of C*-algebras which is his specialty.

Solving problems is not all about pure mathematics

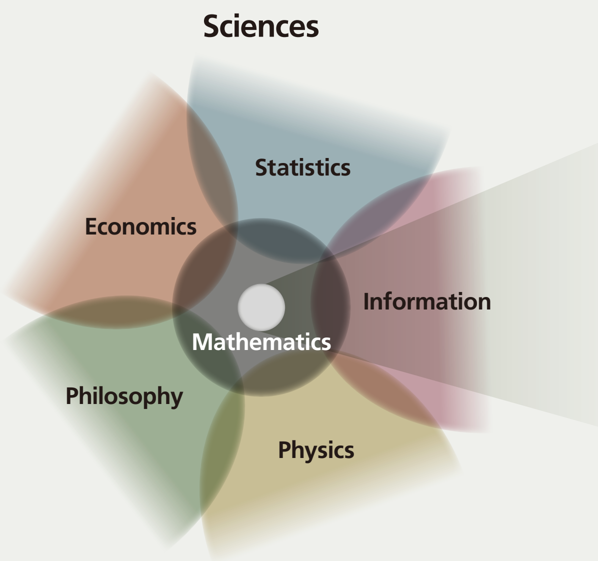
We sometimes hear of news bandy around like this, "A mathematical problem, which has been unsolved for centuries, has been solved at last!" But it is often the case that understanding how such a difficult problem was solved is extremely difficult even for specialists and can take years to examine whether the proof is correct. Such is a hard-to-understand aspect of the world of pure mathematics. Then, how are mathematicians solving problems?

"Of course I admit that solving problems is the mainstream of endeavors for studying pure mathematics. On the other hand, in pursuing studies we mathematicians rarely experience solving a problem truly worth solving. This is because many of the problems that remain unsolved by humans are either totally worthless or ones too difficult to solve. It is also true that too difficult a problem to solve is not always well worth solving. Many scientists then scrutinize the value of a solved problem and its results over a period of many years. It can also happen that results of abstract pure mathematics studies are unexpectedly applied to practical uses after a century or two. That's why I think we, researchers of pure mathematics, must establish and maintain values and an aesthetic sense of our own that are not affected by others and yet sympathized by them," emphasizes Dr. Katsura.

If solving problems is very difficult as he says, then what are mathematicians advancing their own research for, and how?

"In the study of pure mathematics, I think much of our efforts are directed toward finding curious phenomena and understanding them, rather than simply solving problems and finding answers. To do so, we must thoroughly investigate the object, build a hypothesis and verify it through experiments. This approach seems similar to approaches taken in the engineering field. What characterizes pure mathematics is that our targets are abstract mathematical objects. Another characteristic is that we basically take an approach that we term 'thought experiments' although experiments often depend on computers."

He continues, "No matter how acceptable an answer appears, it cannot be considered to be the 'result' unless it is proven properly. This is still another characteristic of mathematics. Suppose you have found a phenomenon to be proven but are unable to prove it. In that case you may publish it in the form of a conjecture, but even so it is regarded as an important research achievement in mathematics. Some day in the future, I'd like to publish a conjecture that would lead mathematical studies of the world. Of course, it would be best if I could prove it."



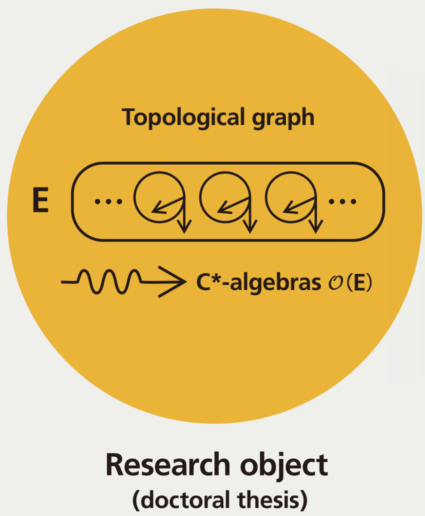


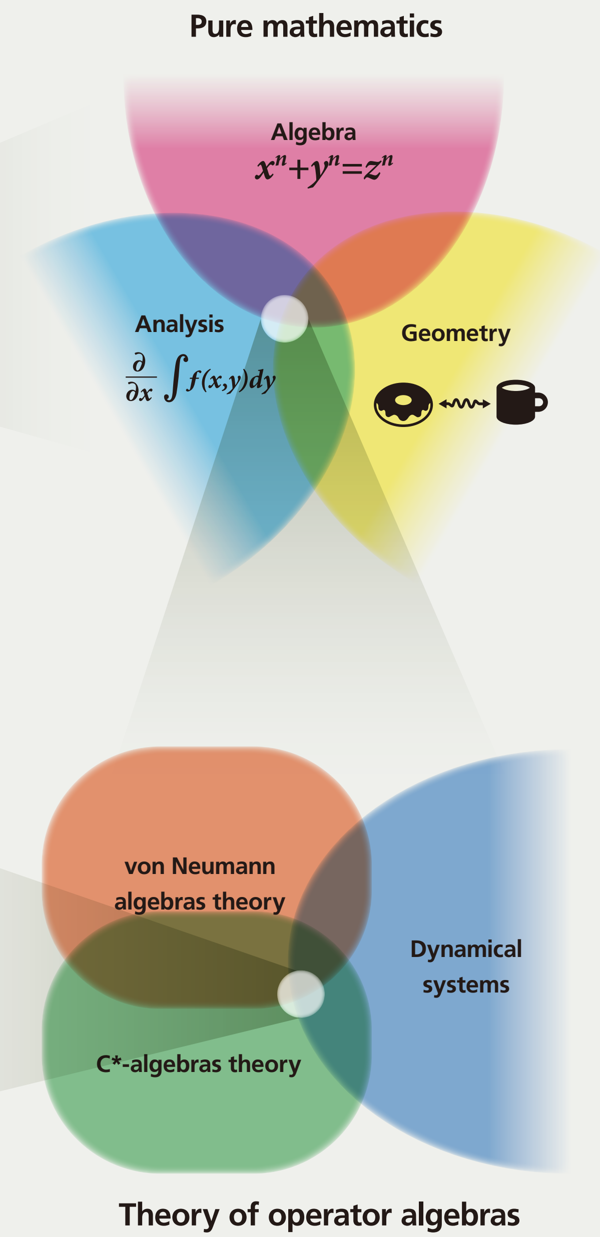
Fig. The positioning of Dr. Katsura's doctoral thesis and related fields

Generally speaking, mathematics is related to diverse fields, but Dr. Katsura's specialty is "pure mathematics" seemingly having little to do with the other fields. Yet, he maintains an interest in aspects of pure mathematics that are concerned with other fields. In his doctoral thesis, he introduced a new perspective into an area where "C*-algebras" in the theory of operator algebras meet dynamical systems related to physics. He is now becoming more interested in set-theory-oriented mathematics, a field said to be close to philosophy.

Noticing structures of sets and examining their relationships

How is it possible to investigate into mathematical objects that are both invisible and intangible? The keywords are sets and structures, according to Dr. Katsura.

"When asked about the most familiar mathematical objects, everyone will answer they are numbers such as natural numbers, integers and real numbers. In investigating into numbers, there is an approach: to collect and examine a set of all numbers that satisfy certain properties, instead of dealing with each individual number. In mathematics, we call a collection of such mathematical objects a 'set.' For example, a set of all natural numbers is represented by N, which is an infinite set. On the other hand, a set of all real numbers, R, is often expressed as a number line."



"These sets of numbers naturally have their own 'structures.' For example, a number line as an expression of a set of all real numbers, R, is considered to express structures such as a 'relation' for two real numbers – which is large and which is small – and a 'metric' between the two numbers. Besides these structures, sets of numbers have a structure known as 'operations' that involves addition, multiplication and so on," Dr. Katsura mentions.

He says it is possible to shed light on the very structures by describing these various types of structures in set-theory languages like mapping (*1).

"Each time we notice a certain structure, the set having that particular structure becomes a research object in mathematics. For example, the structure 'metric' leads us to concepts such as metric spaces and topological spaces (*2), which are the objects of geometry. Likewise, the structure 'operations' involving addition and multiplication leads us to concepts such as "groups," "rings" and "fields" (*3), which are the objects of algebra. In today's pure mathematics study, it is common to look for instances of such mathematical objects and investigate into relationships between properties shared by such objects and those between different objects," Dr. Katsura continues.

In particular, Dr. Katsura shows a special interest in and focuses on relationships between objects of different structures (ring, metric space, etc.) and objects having multiple structures (group, topological space, etc.) at the same time.

"Up to now I have focused on the object called 'C*-algebras' (*4). In addition to structures such as metric spaces and rings mentioned earlier, C*-algebras also have structures called linear spaces and '*' (star) and satisfy various conditions (*5) among these structures. While intriguingly interrelating with other objects such as dynamical systems (*6), topological spaces and fields, C*-algebras are being studied even today. I personally regard C*-algebras as very charming, but I cannot explain it in compact, easy-to-understand expressions. Because easy explanations often come with incorrectness and falsehoods, you know," says Dr. Katsura smilingly.

I'd like to open up a new horizon of mathematics by challenging boundary areas

What meaning does Dr. Katsura find in studying the highly abstract world of mathematics? "For me, the foremost motivation is that I feel 'beauty' in there. In fact, while I'm wrestling with a question, there happens to be a moment when I find a truth that I feel truly exquisite. In the 19th century, mathematicians discovered an extremely beautiful area of mathematics called complex analysis (*7). In the 20th century, this complex analysis played vital roles in mathematical description of quantum mechanics by von Neumann and others, the results of which have supported our modern society through semiconductors. In this manner, it is often the case that truly beautiful mathematics can find unexpected applications after many years. I'd like to leave such a significant achievement to posterity myself," talks Dr. Katsura eagerly. He also expressed prospects that he would like to pioneer yet unexplored fields of mathematics by expanding his research objects beyond C*-algebras to cover boundary areas of dynamical systems, number theory and set theory studies.

(Reporter & text writer : Madoka Tainaka)

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*1: In mathematics, mapping refers to a rule to assign to each element in one set a particular element in the same set or in another. For example, addition can be described as mapping from the set of all pairs of numbers assigned the sum of each pair.

*2: The term "topology" in mathematics totally differs in concept from "phase" that concerns with waves and other phenomena.

*3: Roughly speaking, the "group" refers to objects with a structure of addition, the "ring" refers to objects with structures of addition and multiplication, and the "field" refers to objects with structures of the four arithmetic operations including division.

4: C-algebras are one of what are called operator algebras. The other type of operator algebras is von Neumann algebras named in honor of John von Neumann (1903~1957), the father of the operator algebras theory. The theory of operator algebras was created in the 20th century for the purpose of formulating quantum mechanics mathematically. Operators are something like an infinite matrix and operator algebras are rings consisting of operators.

5: Of the various conditions, the condition $\|T^*T\|=\|T\|^2$ is called the C-condition. It is a magical and important condition enabling C*-algebras to work as operator algebras.

*6: Dynamical systems are used to describe time evolution of states, which are a field of mathematics born of physics just like operator algebras.

*7: Complex analysis is something like replacing real numbers with complex numbers that we learned in high school calculus.