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Sticks That Want Measuring in an Integral Fashion

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I. Introduction

In this brief report, I propose that the proper semantic analysis of certain existential statements in natural language requires the mathematical notion of integration, and in particular, that of Lebesgue integration. This discovery leads to a new semantic theory where existential statements are generally analyzed by virtue of Lebesgue integration.

II. Temporal Interpretation of Measure Phrases

1. Musan's Generalization

Since Enç (1981), it has been known that the evaluation time of a noun phrase in a sentence may be different from the evaluation time of the sentence's main predicate. For instance, if the noun *fugitive* and the main predicate *be in jail* in (1) were evaluated with respect to the same time, the sentence would be claiming that some individuals are simultaneously fugitives and in jail, a contradiction.

(1) Every fugitive is now in jail.

Instead, (1) should be understood as 'every individual who was a fugitive is now in jail.' *Fugitive* can thus be evaluated with respect to some past time, even though the main predicate is evaluated with respect to the present time.

Following Enc's work, Musan (1995) has made the generalization that such temporally independent readings are possible only with presuppositional noun phrases and that the temporal interpretation of a non-presuppositional noun phrase is always dependent on that of the main predicate. The relevant concept of (non-)presuppositionality is due to Dieasing (1992). Noun phrases with a "strong" determiner (e.g. *every rabbit*, *most rabbits*) and partitive noun phrases (e.g. *some of the rabbits*) are presuppositional. Noun phrases with a "weak" determiner (e.g. *some rabbits*) are ambiguous as to their presuppositionality, but their syntactic position or stress pattern may help disambiguate it in some cases. For instance, in Musan's German data (2), the subject noun phrase in (2a) is presuppositional because the determiner, rather than the head noun, is stressed and it in effect means 'some of the professors', while the subject noun phrase in (2b) is non-presuppositional because the head noun is stressed.

- (2) a. EINIGE Professoren waren in den sechziger Jahren glücklich.
 some professors were in the sixties happy
 'Sóme professors were happy in the sixties.'
- b. Einige PROFESSOREN waren in den sechziger Jahren glücklich.
 some professors were in the sixties happy
 'Sm professors were happy in the sixties.'

What Musan has observed is that while (2a) may be talking about current professors who were not necessarily professors in the sixties, (2b) only asserts the existence of people who were simultaneously professors and happy in the sixties. Hence her generalization.

2. Extension of Musan's Generalization to Measure Phrases

Let us now turn to the temporal interpretation of measure phrases such as *three liters*, *fifty kilograms*, etc. Consider (3):

- (3) Exactly sixty-six tons of foreigners were transported through the tunnel in the first week of May 2009.

Here, *exactly sixty-six tons of foreigners* is in principle ambiguous as to its presuppositionality, but when (3) is uttered out of the blue, i.e., in a discourse where never has the existence of any students been asserted or alluded to, one can safely assume that this noun phrase is non-presuppositional. Now, imagine that there is a railroad in some town in Japan with only two stops, which are connected via a tunnel and thus trains on the railroad run through it. (3) can then be a statement reporting the amount of certain kind of passengers in the first week of May 2009. Since (3) may easily be uttered in a discourse that does not presuppose the existence of foreigners

in the town, the subject noun phrase can be non-presuppositional. In that case, as Musan's generalization predicts, the temporal interpretation of *foreigners* indeed depends on that of the main predicate. That is, those people whom (3) reports to have been transported were foreigners at the very time they were transported on the railroad. Therefore, if some passengers in the first week of May 2009 held Japanese citizenship at that time but only either lost it afterwards or had not yet obtained it before that time, they do not count as foreigners in this utterance.

Let us now turn our attention to the temporal interpretation of the measure phrase *exactly sixty-six tons*. Here, we should be aware that People's weights generally keep changing. Even in a single week, one's weight might fluctuate significantly. To make the story simple, let us assume that no foreigner used the railroad more than once in the first week of May 2009. Suppose that a foreigner named John weighed 70 kgs on Monday, 72 kgs on Wednesday, and 71 kgs on Friday in this week, and that he used the railroad on Wednesday. When (3) is uttered without presupposing the existence of foreigners, John's weight is included in the 66 tons, and what is more, what is counted is obviously John's weight at the time of his train ride, viz. 72 kgs, and not his weight at any other time. Thus, the figure *exactly sixty-six tons* must have been calculated by looking at each foreign passenger's weight at the time of his/her train ride and then summing up all those figures. One can then say that the time with respect to which the measure phrase of a non-presuppositional noun phrase is evaluated always coincides with the evaluation time of the main predicate.

By contrast, the evaluation time of the measure phrase of a presuppositional noun phrase may be distinct from that of the main predicate. Consider (4), where the subject noun phrase is designated as presuppositional by the strong determiner *the*:

- (4) The exactly sixty-six tons of foreigners were transported through the tunnel in the first week of May 2009.

As Musan's generalization predicts, unlike the previous case, (4) may be talking about people who held Japanese citizenship in the first week of May 2009 but who, say, are now foreigners. Moreover, the figure *exactly sixty-six tons* should then be the result of summing up those people's current weights, rather than their weights at the time of their train rides. One can then see that Musan's generalization extends parallelly to the temporal interpretation of measure phrases.

3. Truth Conditions

How, then, can we represent the truth conditions of the temporally dependent interpretation of (3)? For a given time interval I , $\{x \mid \text{foreigner}(I)(x) \wedge$

transported-through-the-tunnel(I)(x)} gives the set of all individuals who were foreigners in I and who were transported through the tunnel in I . Let $\xi(I)$ be the individual sum (i.e. join in a lattice of individuals) of these individuals:

$$(5) \xi(I) = \bigsqcup \{x \mid \text{foreigner}(I)(x) \wedge \text{transported-through-the-tunnel}(I)(x)\}$$

Now, let ton be a function such that for any time interval I and any individual x , $\text{ton}(I)(x)$ gives x 's weight in tons at I . Then, $\text{ton}(I)(\xi(I))$ will be the collective weight of all the individuals who were foreigners in I and who were transported through the tunnel in I . The total weight of all the foreigners who were transported through the tunnel in the first week of May 2009 is then given by taking the sum of $\text{ton}(I)(\xi(I))$ as I ranges over all time intervals in that week. Hence, the desired truth conditions of (3) are expressed as follows:

$$(6) \sum_{I \subseteq \text{first week of May 2009}} \text{ton}(I)(\xi(I)) = 66$$

III. Necessity of Mathematical Integration

1. Continuous Production/Consumption

Sentences that assert the existence of stuff continuously produced or consumed exhibit temporally dependent interpretations whose truth conditions cannot simply be analyzed as in (6) and instead require mathematical integration.

Suppose that (7) is uttered without previous mention of or allusion to any bread:

(7) John ate exactly 666 cubic centimeters of bread.

Then, just as in the case of (3) above, as predicted by the extended version of Musan's generalization, the measure phrase *666 cubic centimeters* obligatorily receives a temporally dependent interpretation.

To see this, imagine that there is a magic breadstick of the shape of a cylinder. Because of a magic spell cast on it, it continually keeps on expanding and shrinking. Its length is fixed to l cm, however, and only its radius keeps changing. Imagine further that John begins eating this breadstick at time point p_0 , starting from one end, and after a while at time point p_1 , he reaches the other end and finishes eating the whole breadstick. Letting r_0 be the initial radius at p_0 , $\pi r_0^2 l$ cm³ is the volume of the whole breadstick measured when John's meal is about to commence, but this is not what (7) is talking about. Clearly, the size of the stick never matters while it

is expanding and shrinking outside John’s mouth; it only becomes important how big it is the moment it enters the mouth. Since the eating process is continuous, we must look at every infinitesimal time span during John’s meal and measure how big the piece John eats during each infinitesimal time span is at that very moment, and collect those amounts.

Now, following (5), let $\eta(I)$ be all the bread John eats at I :

$$(8) \eta(I) = \bigsqcup \{x \mid \text{bread}(I)(x) \wedge \text{John-eats}(I)(x)\}$$

If we were to follow the analysis in Section II.3, we should take the sum of $\text{cubic-cm}(I)(\eta(I))$ as I ranges within the time interval $[p_0, p_1]$. However, as pointed out above, I must range only over infinitesimal time intervals in $[p_0, p_1]$, and for such intervals I , $\text{cubic-cm}(I)(\eta(I)) = 0$ as John cannot eat any positive amount of bread in a moment. As a result, the sum also amounts to 0, and we can never obtain a positive value like 666.

2. A Lebesgue Integral Approach

As argued above, the analysis presented in Section II.3 is insufficient for cases of continuous production/consumption. What is necessary to account for (7) instead is to calculate the rate of the volume of bread John eats per unit time and to integrate it over the interval in which John’s meal took place. As discussed in Shimada (2009), under natural assumptions, it follows from the Radon-Nikodým theorem that for almost all time point p , this rate at p is given by (9):

$$(9) \varrho(p) = \lim_{h \rightarrow 0} \frac{1}{2h} \text{cubic-cm}([p, p]) \left(\bigsqcup_{p' \in [p-h, p+h]} \eta([p', p']) \right)$$

The truth conditions of (7) can then be expressed as a condition on the Lebesgue integral of this rate over the interval $[p_0, p_1]$ as follows:

$$(10) \int_{[p_0, p_1]} \varrho d\mu = 666$$

μ is the Lebesgue measure here.

On the other hand, existential statements where the described events are not of continuous nature such as (3) can be analyzed with ordinary summation as in (6), but such truth conditions can actually be rewritten with Lebesgue integrals with respect to the counting measure. A uniform analysis can therefore be attained where all existential statements receive truth conditions that are expressed by virtue of Lebesgue integration.

IV. Summary

In this report, we first saw that Musan's generalization extends to measure phrases, that is, the temporal interpretation of the measure phrase of a non-presuppositional noun phrase obligatorily depends on that of the main predicate of the sentence. It was then argued that sentences describing continuous production or consumption where the produced/consumed stuff is denoted by a non-presuppositional noun phrase require mathematical integration in their truth conditions. In the end, a new approach to existential statements that makes use of Lebesgue integration was introduced. Due to the space limitation, I could not expose this new theory extensively here, but the interested reader is referred to my dissertation (Shimada 2009) for the details.

References

- Enç, Mürvet. (1981). Tense without scope: An analysis of nouns as indexicals. PhD Dissertation, University of Wisconsin.
- Diesing, Molly. (1992). *Indefinites*. Cambridge, MA: MIT Press.
- Musan, Renate. (1995). On the temporal interpretation of noun phrases. PhD Dissertation, MIT.
- Shimada, Junri. (2009). Measurement that transcends time: A Lebesgue integral approach to existential sentences. PhD Dissertation, MIT.