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34 A Report on Euler Diagrammatic Reasoning System: Theory and Experiment Design

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Abstract

In this paper we are concerned with logical and cognitive aspects of reasoning with Euler circles. We give a proof-theoretical analysis of diagrammatic reasoning with Euler circles involving unification and deletion rules. Diagrammatic *syllogistic* reasoning is characterized as a particular class of the general diagrammatic proofs. Given this proof-theoretical analysis, we propose an experiment for a cognitive psychological study.

I. Introduction

The purpose of this paper is to give a summary of our theory presented in [7] and give an additional material for the further experimental studies based on our theory, which updates and revises the section 4 of [7].[1] The result of the experimental study will be announced elsewhere in a forthcoming paper [10].

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[1] This report is prepared for the annual report of Center for Advanced Research on Logic and Sensibility, Keio University.

We study diagrammatic reasoning with Euler circles composed of unification and deletion inferences. A primitive unification inference step is specified as a unification of two Euler diagrams.[2] We define the notion of diagrammatic *proof* (*d-proof*, in short), which is considered as a (possibly long) chain of unification and deletion steps. A major difficulty in the Euler-style diagrammatic proofs consists in the fact that the complexity of the diagrams increases during the processes of diagrammatic proof constructions; especially, unification often multiplies *disjunctive ambiguity* (i.e., the ambiguity described by duplicating a point, say x, at many different positions in a unified diagram with linking them, or, instead, by multiplying diagram-pages; cf. Peirce [8]). By contrast, when the Euler-style diagrammatic reasoning is restricted to the *syllogistic* inferences, essentially no disjunctive ambiguity appears during any diagrammatic proof construction process. We shall present a diagrammatic proof system which has essentially no disjunctive ambiguity, and which includes the syllogistic proofs as special cases.

In Section II, we consider a diagrammatic representation system for Euler circles in which disjunctive ambiguity is not allowed. (One may call disjunction-free diagrams "one-page diagrams", and disjunctive diagrams "multiple-page diagrams.") We give a definition of an Euler diagrammatic syntax and a set-theoretical semantics for it.

In Section III, we provide a diagrammatic inference system consisting of unification and deletion rules, where any conclusion of a (possibly long) d-proof is always representable on a one-page diagram. The *syllogistic* d-proof system is characterized as a specific subsystem of our d-proof system (where unification and deletion appear alternately without repeating in a proof). Compared with linguistic syllogistic reasoning, the diagrammatic reasoning with Euler diagrams in our system has some distinctive features: linguistic syllogistic reasoning involves explicit operations with logical negation and with the "subject-predicate" distinction, whereas the reasoning with Euler diagrams in our system does not.

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2 Some Euler-style diagrammatic reasoning systems (e.g. Hammer [4]) do not have unification rule, hence cannot deal with syllogistic reasoning. But we do not consider such a simple case in this paper.

In Section IV, we introduce the instruction for the experiments we are planning ([10]).

II. Diagrammatic representation system EUL for Euler circles and its (set-theoretical) semantics

In this section, we introduce a graphical representation system EUL for Euler diagrams in which disjunctive ambiguity is not allowed. We define a syntax of EUL (Section II.1) and its formal semantics (Section II.2).

1. Diagrammatic syntax of EUL

Let us start by defining the diagrams of EUL.

Definition 1 (EUL-diagrams)
An EUL-*diagram* is a 2-dimensional ($\mathbb{R}^2$) plane with a finite number (at least two) of *named simple closed curves*[3] (denoted by A, B, C, ...) and *named points* (denoted by x, y, z, ...), where

- each named simple closed curve or named point has exactly one name;
- any two distinct named simple closed curves have different names;
- any two distinct named points have different names.

EUL diagrams are denoted by $\mathcal{D}$, $\mathcal{E}$, $\mathcal{D}_1$, $\mathcal{D}_2$, ...

In what follows, we sometimes call a named simple closed curve a *named circle*. Moreover, named circles and named points are collectively called *objects*. We use a rectangle to represent a plane for an EUL diagram.

The binary relations $A \sqsubset B$, $A \vdash\!\dashv B$, $A \bowtie B$, $x \sqsubset A$, $x \vdash\!\dashv A$, and $x \vdash\!\dashv y$ mean, respectively, "the interior[4] of A is inside of the interior of B," "the interior

3 See [2, 13] for a formal definition of simple closed curves on $\mathbb{R}^2$.
4 Here, the interior of a named circle A means the region strictly inside of A. Cf. [2].

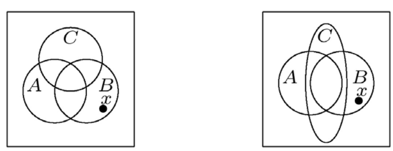
of A is outside of the interior of B," "there is at least one crossing point between A and B," "x is inside of the interior of A," "x is outside of the interior of A," and "x is outside of y (i.e. x is not located at the point of y)".

Proposition 1

Given an EUL*-diagram* $\mathcal{D}$,

1. *for any distinct named simple closed curves A and B , exactly one of* $A \sqsubset B$, $B \sqsubset A$, $A \mathrel{\vdash\!\dashv} B$, *and* $A \bowtie B$ *holds;*
2. *for any named point x and any named simple closed curve A, exactly one of* $x \sqsubset A$ *and* $x \mathrel{\vdash\!\dashv} A$ *holds;*
3. *for any distinct named points x and y,* $x \mathrel{\vdash\!\dashv} y$ *holds.*

For example, consider the EUL diagram $\mathcal{D}_1$ below, composed of A, B, C, and x.



$\mathcal{D}_1$ $\mathcal{D}_2$

Figure 1

The relations $A \bowtie B$, $A \bowtie C$, $B \bowtie C$, $x \mathrel{\vdash\!\dashv} A$, $x \sqsubset B$, and $x \mathrel{\vdash\!\dashv} C$ hold on $\mathcal{D}_1$. The same relations also hold for $\mathcal{D}_2$.

An EUL diagram which has only two objects is called a *minimal diagram* (or an *atomic diagram*).

Given an EUL diagram $\mathcal{D}$ and two objects, say s and t, on $\mathcal{D}$, a diagram obtained from $\mathcal{D}$ by deleting all objects other than s and t is called a *component minimal diagram* of $\mathcal{D}$. Given $\mathcal{D}$ and objects s and t, the component minimal diagram thus obtained is determined uniquely up to isomorphism. Given an EUL diagram $\mathcal{D}$, then, the set of component minimal diagrams is determined (up to isomorphism). The set of component minimal (atomic) diagrams of $\mathcal{D}$ is called the *decomposition set* of $\mathcal{D}$. As a direct corollary of Proposition 1, for any minimal (atomic) diagram, say $\mathcal{D}$, composed of two objects s and t, exactly one of $s \sqsubset t$, $t \sqsubset s$, $s \mathrel{\vdash\!\dashv} t$, $s \bowtie t$ holds. For example, the set $\{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6\}$ of Fig. 2 is, up to isomorphisms, the decomposition set of $\mathcal{D}_1$ in Fig.1 above.

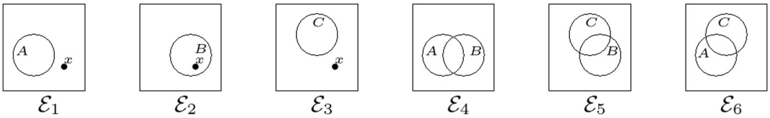


Figure 2: The decomposition set of D_1

2. Set-theoretical semantics of EUL

Diagrams of EUL can be used as an auxiliary device to supplement linguistic representations such as formulas of predicate logic. A named circle plays the role of a predicate, and a named point plays the role of a constant symbol. In order to capture such roles of diagrams in a precise way, we give a formal semantics for EUL. Here, we adopt the standard set-theoretical semantics.[5]

Definition 2 (Model)

A model $\mathcal{M}$ of EUL is a pair (U, I), where U is a non-empty set (the domain of $\mathcal{M}$), and I is an interpretation function such that

- $I(x) \in U$ for any named point x;
- $I(A) \subseteq U$ and $I(A) \neq \emptyset$ for any named simple closed curve A.

Definition 3 (Truth-conditions)

(I) For any minimal (atomic) diagram $\mathcal{D}$ and for any model $\mathcal{M}$,

1. when $x \vdash\!\dashv y$ holds on $\mathcal{D}$, $\mathcal{M} \models \mathcal{D}$ if and only if $I(x) \neq I(y)$;
2. when $x \sqsubset A$ holds on $\mathcal{D}$, $\mathcal{M} \models \mathcal{D}$ if and only if $I(x) \in I(A)$;
3. when $x \vdash\!\dashv A$ holds on $\mathcal{D}$, $\mathcal{M} \models \mathcal{D}$ if and only if $I(x) \notin I(A)$;
4. when $A \sqsubset B$ holds on $\mathcal{D}$, $\mathcal{M} \models \mathcal{D}$ if and only if $I(A) \subseteq I(B)$;
5. when $A \vdash\!\dashv B$ holds on $\mathcal{D}$, $\mathcal{M} \models \mathcal{D}$ if and only if $I(A) \cap I(B) = \emptyset$;
6. when $A \bowtie B$ holds on $\mathcal{D}$, $\mathcal{M} \models \mathcal{D}$.[6]

[5] For similar set-theoretical approaches to the semantics of Euler diagrams, see Hammer [4], Hammer and Shin [5], and Swoboda and Allwein [15]. Compared to them, our semantics is distinctive in that diagrams are interpreted as a set of binary relations, and thus that not every region in a diagram has a meaning.

[6] Informally speaking, $A \bowtie B$ may be understood as $I(A) \cap I(B) = \emptyset \vee I(A) \cap I(B) \neq \emptyset$, which is true as stated in 6 here.

(II) For any EUL diagram $\mathcal{D}$, where $\{\mathcal{D}_1, \mathcal{D}_2, ..., \mathcal{D}_n\}$ is the decomposition set of $\mathcal{D}$, $\mathcal{M} \models \mathcal{D}$ if and only if $\mathcal{M} \models \mathcal{D}_1$ and $\mathcal{M} \models \mathcal{D}_2$ ... $\mathcal{M} \models \mathcal{D}_n$.

Note that well-definedness of Definition 3 (II) of the truth-conditions follows from Proposition 1, which assures that the decomposition set is uniquely determined for a given diagram $\mathcal{D}$.

Definition 4 (Validity)
Euler diagram $\mathcal{E}$ is a *semantically valid consequence* of $\mathcal{D}_1, ..., \mathcal{D}_n$ (written as $\mathcal{D}_1, ..., \mathcal{D}_n \models \mathcal{E}$) when for any model $\mathcal{M}$ such that $\mathcal{M} \models \mathcal{D}_1, ..., \mathcal{M} \models \mathcal{D}_n$ hold, $\mathcal{M} \models \mathcal{E}$ holds.

III. A diagrammatic inference system for generalized syllogistic reasoning

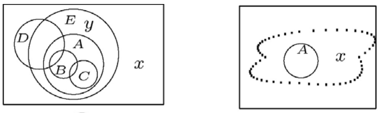
In this section, based on the graphical representation system EUL, we introduce a diagrammatic inference system for EUL, called Generalized Diagrammatic Syllogistic inference system GDS. In Section III.1, we give the definition of GDS which is sound with respect to the formal semantics of EUL given in Section II.2. In Section III.2, we show that the diagrammatic inferences for Aristotelian categorical syllogisms are characterized as a specific subclass of the diagrammatic proofs of GDS.

1. Generalized diagrammatic syllogistic inference system GDS

In this subsection, we introduce *Unification* and *Deletion* of GDS. In the following definition, in order to indicate occurrence of some objects in a context on a diagram, we write the indicated objects explicitly and the context by "dots " as in $\mathcal{D}_1$ below.[7] For example, when we need to indicate only A and x on $\mathcal{D}_1$ of the left-hand side, we could write $\mathcal{D}_1$ in the manner

[7] Note that the dots notation is used only for abbreviation of a given diagram. For a formal treatment of such "backgrounds" in a diagram, see Meyer [6].

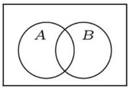
shown in the right-handside.



$\mathcal{D}_1$ $\mathcal{D}_1$

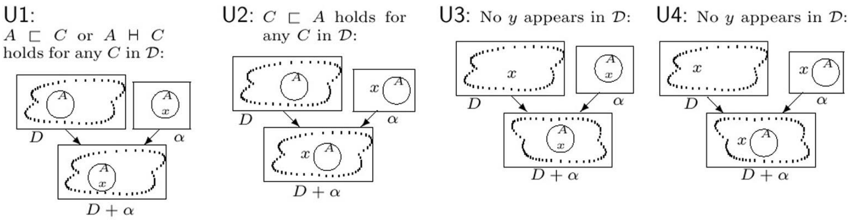
Using this notation, we describe general patterns of each rule and present some examples of its application.

Definition 5 (Inference rules of GDS)
Axiom, Unication and *Deletion* of GDS are defined as follows:
Axiom: For any circle *A* and *B*, the following form of atomic diagram is an axiom:



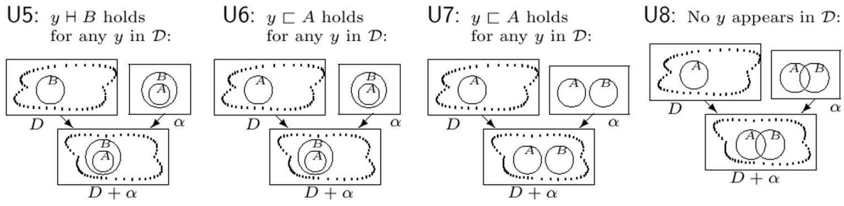
Unification: Atomic diagrams are denoted by α, β,... for readability. The unified diagram of $\mathcal{D}$ with α is denoted as $\mathcal{D} + \alpha$.
When the relation which holds on α also holds on $\mathcal{D}$, $\mathcal{D} + \alpha$ is $\mathcal{D}$ itself.
The other unification rules with premises $\mathcal{D}$ and α to obtain $\mathcal{D} + \alpha$ are listed as U1-U10 below. In each rule, the first sentence attached to the two premise diagrams expresses the constraint which should be satisfied before performing the unification, and the sentence attached to the conclusion diagram $\mathcal{D} + \alpha$ expresses the actual operation to be performed at the unification step. We distinguish the following two cases: (I) $\mathcal{D}$ and α share one object; (II) $\mathcal{D}$ and α share two circles.
Moreover, there is another unification rule called point insertion rule (III).

(I) $\mathcal{D}$ and α share one object:



In U1 and U2, fix $x \dashv\vdash y$ for any y in $\mathcal{D}$.

In U3 and U4, fix $C \bowtie A$ on $\mathcal{D} + \alpha$ for any circle C in $\mathcal{D}$.

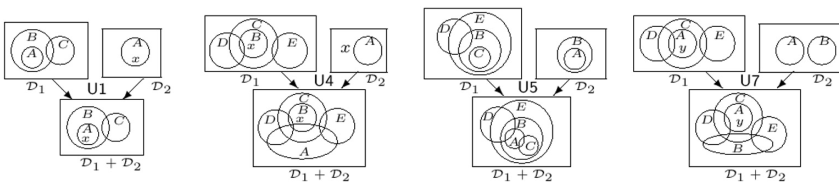


In U5, fix $C \bowtie A$ in $\mathcal{D}+\alpha$ for any C on $\mathcal{D}$ such that $C \sqsubset B$ or $C \bowtie B$ holds.

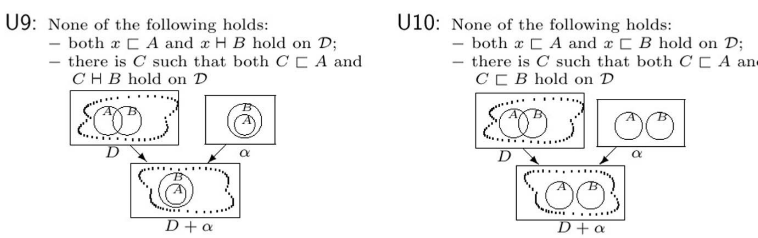
In U6 and U7, fix $C \bowtie B$ on $\mathcal{D}+\alpha$ for any C in $\mathcal{D}$ such that $A \sqsubset C$ or $A \vdash\!\dashv C$ or $A \bowtie C$ holds.

In U8, fix $C \bowtie B$ on $\mathcal{D}+\alpha$ for any C in $\mathcal{D}$.

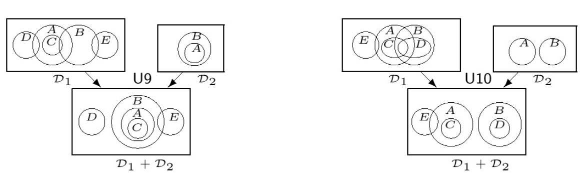
For example, U1, U4, U5, and U7 rules are applied as follows:



(II) $\mathcal{D}$ and α share two circles:

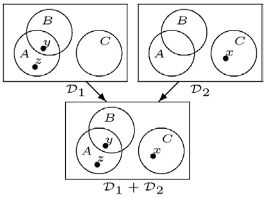


For example, U9 and U10 rules are applied as follows:



(III) Point insertion: When $\mathcal{D}_1$ and $\mathcal{D}_2$ have the same information about circles and in addition $\mathcal{D}_2$ contains one point x, we may add x to $\mathcal{D}_1$ in such a way that the relationship between x and the circles in $\mathcal{D}_2$ is preserved in $\mathcal{D}_1 + \mathcal{D}_2$.

For example, the point insertion rule is applied as follows:



Deletion: There are two deletion rules: the circle-deletion rule and the point-deletion rule. Let t be an object, i.e., a named circle or a named point. For any EUL diagram $\mathcal{D}$ and for any object t in $\mathcal{D}$, applying deletion rule results in $\mathcal{D} - t$ under the constraint that $\mathcal{D} - t$ has at least two objects.

We give an inductive definition of diagrammatic proofs (d-proofs) of GDS.

Definition 6 (Diagrammatic proofs of GDS)
A diagrammatic proof (d-proof, for short) π of GDS is defined inductively as follows:

1. A diagram $\mathcal{D}$ is a d-proof from the premise $\mathcal{D}$ to the conclusion $\mathcal{D}$.
2. Let π_1 be a d-proof from $\mathcal{D}_1, \dots, \mathcal{D}_n$ to $\mathcal{F}$ and π_2 be a d-proof from $\mathcal{E}_1, \dots, \mathcal{E}_m$ to $\mathcal{E}$, respectively. If $\mathcal{D}$ is obtained by an application of Unification of $\mathcal{F}$ and $\mathcal{E}$, then the following (i) is a d-proof π from $\mathcal{D}_1, \dots, \mathcal{D}_n, \mathcal{E}_1, \dots, \mathcal{E}_m$ to $\mathcal{D}$ in GDS.
3. Let π_1 be a d-proof from $\mathcal{D}_1, \dots, \mathcal{D}_n$ to $\mathcal{E}$. If $\mathcal{D}$ is obtained by an application of Deletion to $\mathcal{E}$, then the following (ii) is a d-proof π from $\mathcal{D}_1, \dots, \mathcal{D}_n$ to $\mathcal{D}$ in GDS.

$$\text{(i)}\quad \begin{tikzcd}[row sep=small] \pi_1 & & \pi_2 \\ \mathcal{F} \arrow[dr] & & \mathcal{E} \arrow[dl] \\ & \mathcal{D} & \end{tikzcd} \qquad\qquad \text{(ii)}\quad \begin{tikzcd}[row sep=small] \pi_1 \\ \mathcal{E} \arrow[d] \\ \mathcal{D} \end{tikzcd}$$

Here $\overset{\pi}{\mathcal{D}}$ means a d-proof π with $\mathcal{D}$ as the conclusion. The *length* of a d-proof is defined as the number of the applications of inference rules.

The following Fig. 3 is an example of d-proof from $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{D}_4$, $\mathcal{D}_6$, $\mathcal{D}_7$ to

$\mathcal{D}_{11}$.

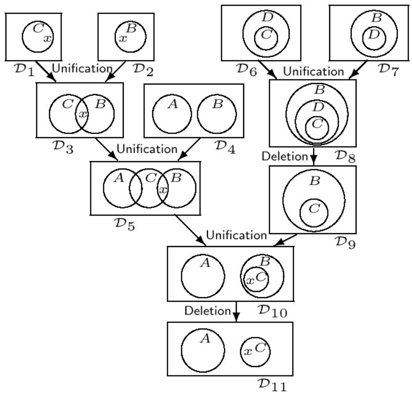


Figure 3: A diagrammatic proof (d-proof) in GDS.

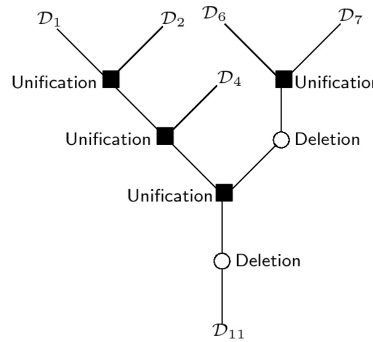


Figure 4: An underlying tree structure of a d-proof in GDS.

The inference rules of GDS are well-defined.

Proposition 2 (Well-definedness of inference rules)
For any EUL*-diagrams $\mathcal{D}_1, ..., \mathcal{D}_n$, if there is a d-proof from $\mathcal{D}_1, ..., \mathcal{D}_n$ to $\mathcal{E}$, then $\mathcal{E}$ is an* EUL*-diagram.*

The soundness theorem of GDS holds with respect to the formal semantics given in Section II.2. It is shown by induction on the length of a given d-proof.

Theorem 1 (Soundness of GDS)
For any EUL*-diagrams $\mathcal{D}_1, .., \mathcal{D}_n$, $\mathcal{E}$, if there is a d-proof from $\mathcal{D}_1, ..., D_n$ to $\mathcal{E}$ in* GDS*, then $\mathcal{E}$ is a semantically valid consequence of $\mathcal{D}_1, ..., \mathcal{D}_n$.*

Our completeness theorem of GDS will be appeared in a forthcoming paper.

2. Aristotelian categorical syllogisms

In this subsection, we show that the diagrammatic inferences for Aristotelian categorical syllogisms are characterized as specific diagrammatic proofs of GDS.

We first introduce, in Fig. 5, a correspondence between the statements of Aristotelian categorical syllogisms and a class of EUL diagrams.

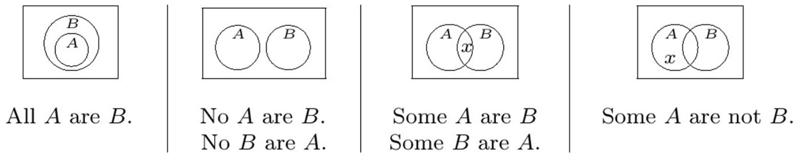


Figure 5: Syllogistic diagrams.

We call the diagrams of the forms given in Fig. 5 *syllogistic diagrams*. For any statement S of Aristotelian categorical syllogism, we denote as $S^{\mathcal{D}}$ the corresponding syllogistic diagram given in Fig. 5.

Next, we define a particular class of d-proofs of GDS called *syllogistic normal d-proofs* as follows:

Definition 7 (Syllogistic normal d-proofs)
For any syllogistic diagrams $\mathcal{D}_1, ..., \mathcal{D}_n$, $\mathcal{D}$, a d-proof π from $\mathcal{D}_1, ..., \mathcal{D}_n$ to $\mathcal{D}$ of GDS is in *syllogistic normal form* if a unification rule and a deletion rule appear alternately in π.

Thus each syllogistic normal d-proof is of the form shown in Fig. 6:

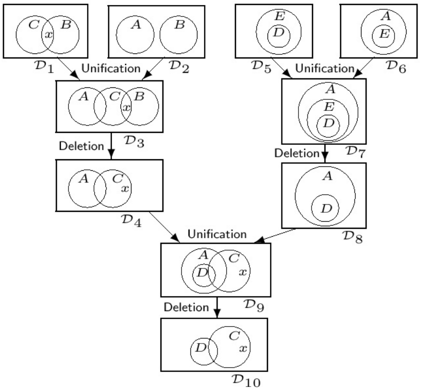


Figure 6: A syllogistic normal d-proof.

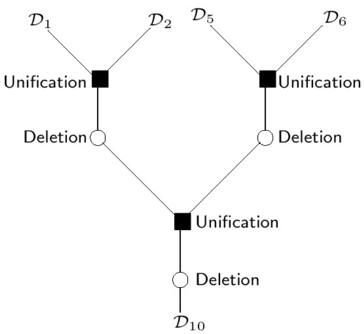


Figure 7: An underlying tree structure of a syllogistic normal d-proof.

Fig. 6 illustrates a syllogistic normal d-proof, where each pair of a unification rule and a deletion rule application corresponds to a valid pattern of syllogisms. For example, the sub-proof from $\mathcal{D}_1$ and $\mathcal{D}_2$ to $\mathcal{D}_4$ is a diagrammatic representation of a syllogism of the form: *Some C are B*. *No A are B*. Therefore *Some C are not A*. (This valid pattern is sometimes symbolized as EI2O. See [1,9] for the notation.) Indeed, each syllogistic normal d-proof can be considered as a chain of valid patterns of Aristotelian categorical syllogisms. Note that, compared with the underlying proof tree of GDS in Fig. 4 in Section III.1, the tree of the syllogistic normal d-proof has a canonical form, where a unification node, denoted by ■, and a deletion node, denoted by ○, appear one after the other.

In order to characterize the syllogistic normal d-proofs, we introduce a sub-system DS of GDS.

Definition 8 (DS)

A diagrammatic syllogistic inference system DS is a sub-system of GDS where:

1. Unification is restricted to U1-U7, and their premises are restricted to syllogistic diagrams sharing no common named point;
2. Deletion is applicable only when its conclusion is a syllogistic diagram.

Essentially, DS is a subsystem of GDS where only syllogistic diagrams are considered. It is shown that DS corresponds to the Aristotelian categorical syllogisms. Let S be a statement of syllogisms and S^D be the corresponding syllogistic diagram in Fig. 5. We have the following correspondence:

Proposition 3 (Syllogisms and DS)

Let S_1, ..., S_n, S be statements of Aristotelian categorical syllogisms. Then S is a valid conclusion of Aristotelian categorical syllogisms from the premises S_1, ...,S_n if and only if there is a d-proof of S^D in DS from the premises S_1^D, ..., S_n^D.

IV. Experiment design for syllogistic reasoning with Euler diagrams

The aim of our experimental study is to investigate the question of whether the use of Euler diagrams has any psychological advantage in syllogistic reasoning. In our experiments, we use EUL diagrams for experimental materials. We provide subjects with an instruction on the meaning of EUL diagrams, and conduct a pretest to check whether they understand the instruction correctly. The result of this experimental study will be announced in our forthcoming paper (Sato et al. [10]). Here we show the instruction used in our experiment. The sentences and instruction are given in Japanese. We used the following translation: "Subete no *A* wa *B* de aru" for *All A are B*, "Dono *A* mo *B* de nai" for *No A are B*, "Aru *A* wa *B* de aru" for *Some A are B*, "Aru *A* wa *B* de nai" for *Some A are not B*. Here we use the quantifiers "subete" and "dono" for *all* and "aru" for *some*. One remarkable difference between English and Japanese is in the translation of *No A are B*. Since in Japanese there is no negative quantifier corresponding to *no*, we use the translation "Dono *A* mo *B* de nai", which literally means *All A is not B*. Except this point, we see no essential differences between English and Japanese. So we will refer to English translation in this paper.

1. Instruction on the meaning of categorical statements

In this experiment, you are asked to solve reasoning tasks. The meaning of a sentence used in this experiment is defined as follows.

1. "All *A* are *B*" means that if there are objects which are *A*, all of them are *B*.
2. "No *A* are *B*" means that if there are objects which are *A*, none of them are *B*.
3. "Some *A* are *B*" means that there are some objects which are *A* and *B*.
4. "Some *A* are not *B*" means that there are some objects which are *A* but not *B*.

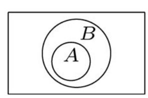
"All *A* are *B*" does not imply that there are some objects which are *A*.
Thus, "All *A* are *B*" does not imply "Some *A* are *B*".
"No *A* are *B*" does not imply that there are some objects which are *A*.
Thus, "No *A* are *B*" does not imply "Some *A* are not *B*".

2. Instruction on the meaning of EUL-diagrams

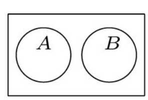
You may use diagrams in solving reasoning tasks. The meaning of a diagram used in this experiment is defined as follows.

A circle is used to denote a set of objects.

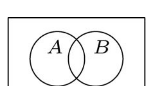
(1) The following diagram in which the region of *A* is inside of the region of *B* means that if there are objects which are *A*, all of them are *B*.



(2) The following diagram in which the region of *A* is outside of the region of *B* means that there are no objects which are *A* and *B*.



(3) The following diagram in which the regions of *A* and *B* partly overlap each other means that the relationship between the set of objects which are *A* and the set of objects which are *B* is unknown.

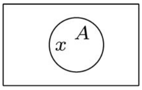


(It should be noted that this diagram says nothing about whether there are some objects which are both *A* and *B*.)

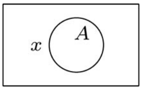
The diagrams in (1)–(3) do not mean that there are some objects which are *A* and/or *B*. These diagrams say nothing about the existence of objects.

Point *x* is used to indicate the existence of an object.

(4) The following diagram in which point x is inside of the region of A means that there is an object which is A.



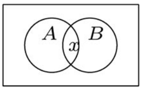
(5) The following diagram in which point x is outside of the region of A means that there is an object which is not A.



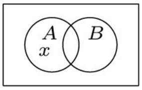
It is unknown whether there is an object in a region where point x is absent.
For example, the diagram in (4) says nothing about whether there is an object which is not A. Similarly, the diagram in (5) says nothing about whether there is an object which is A.

By the combination of the diagrams in (1)-(5), we can compose more complex diagrams. We give some examples.

The following diagram in which x is inside of the intersection of A and B means that there is an object which is A and B.



The following diagram in which x is inside of A but outside of B means that there is an object which is A but not B.



(It should be noted that this diagram says nothing about whether there is an object which is both A and B.)

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