SUMMARY

of

Nonlinearity, Bounded Rationality, and Heterogeneity:

Some Aspects of Market Economies as Complex Systems

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Summary

Economics has a long history of various efforts to understand why and how economic activities fluctuate, and whether the main driving forces of economic fluctuations come from inside or outside an economy has been and continues to be one of the most important and most controversial problems. When formal model analyses of the business cycle were initiated in the 1930s, a clearly defined dichotomy between *exogenous* and *endogenous* business cycle theories emerged in the literature. According to the former view, fluctuations are caused by random exogenous shocks to an otherwise inherently stable economy. An economy is recognized to be stable in the sense that it intrinsically converges to a stationary equilibrium (path) through cyclical movements, and therefore this view is called equilibrium business cycle theory. Such a view is the source of recent neoclassical economics such as the rational expectation hypothesis, real business cycle theory, and dynamical stochastic general equilibrium models. By contrast, according to the latter view, fluctuations are generated by the endogenous interactions of economic factors. Economic fluctuations are recognized as essentially inherent or endogenous phenomena of capitalistic or market economies. This approach, which relies upon nonlinear dynamical models, has become the source of chaotic economic dynamics since the 1980s. Indeed, more recently, it has created an upsurge of interest in complex system approaches. Research on economic complexity has been carried out into two fundamental directions. One is the introduction of stochastic or statistical mechanical frameworks in terms of economic variables, while the other is the development of agent-based models that use computer simulations. The former approach has enticed many statistical

physicists to join, forming new streams of research named *econophysics* and *sociophysics*. Both approaches are, however, compatible and have been striving to move beyond the neoclassical paradigm.

This book pursues a nonlinear approach in considering both chaotic dynamical models and agent-based simulation models of economics, as well as their dynamical behaviors. Three key concepts arising in this context are "nonlinearity," "bounded rationality" and "heterogeneity," which also make up the title of the book. Nonlinearity is the warp that runs throughout all models because systems that exhibit chaotic or other complex behavior in the absence of any exogenous disturbances are absolutely nonlinear. Bounded rationality constitutes the woof, because economic systems do not exhibit complex behavior if all agents are perfectly rational, as is usually assumed in neoclassical economics. Agents who are boundedly rational have to struggle to do their best with limited information and tend to adapt to their economic environment without knowing what is the best. Furthermore, the heterogeneity of firms or consumers dyes the fabric of complex dynamics woven from the warp and woof.

Chap. 1 deals with the following basic questions from the historical and methodological perspectives: what is nonlinear economic dynamics and how has it been developed? After introducing some of the basic concepts of nonlinear economic dynamics in Sect. 1.1, we discuss how economic dynamics was formed in the 1930s and the extent to which it is closely related to the equilibrium paradigm, which is synonymous with the neoclassical paradigm, in Sect. 1.2. In Sect. 1.3, by comparing two approaches in economic dynamics, namely exogenous and endogenous business cycle theories, we state that the exogenous business cycle theory relied on linear dynamical models and was suitable for the equilibrium paradigm, while the endogenous business cycle theory relied on nonlinear dynamical models and was the starting point of nonlinear economic dynamics. We also discuss the significance of nonlinear economic dynamics, whose original standpoint was to make a clean break from the equilibrium paradigm. In Sect. 1.4, we discuss the three reasons why economic dynamics adhered to linear models from its earliest stage. Sect. 1.5 critically discusses the neoclassical doctrine that all models should be derived from economic optimization principles. In Sect. 1.6, after briefly reviewing the formation of chaos theory, we discuss how chaotic economic dynamics developed under the influence of chaos theory within the tradition of endogenous business cycle theory. We also state that chaotic economic dynamics has led to recent research on various branches of nonlinear economic dynamics including economics of complexity. In Sect. 1.7, we consider the significance of chaotic economic dynamics.

Chap. 2 provides an introductory exposition of nonlinear discrete dynamics, which takes readers into the vast ocean outside the equilibrium paradigm, by presenting a one-dimensional nonlinear cobweb model. The model is a simple extension of the standard cobweb model. The new ingredients are an iso-elastic inverse demand function (i.e., an inverse demand function with constant price elasticity) and boundedly rational producers who gradually adjust their production toward the target levels based upon naive price expectations. Thus, the key parameters of the model are the price elasticity of demand and the production adjustment speed of producers. With the aid of mathematical analysis and numerical simulations, it is shown that for a large set of parameter values, the cobweb market exhibits observable chaos (a strange attractor) as well as topological chaos (a horseshoe) associated with homoclinic points. We detect chaotic behavior in a theoretically large and empirically relevant region of the price elasticities of demand and adjustment speeds. The faster producers adjust their production, and the more inelastic demand is, the more likely the market behaves chaotically.

In Sect. 2.1, a preliminary nonlinear cobweb model is studied. Although the model is so simple that its qualitative behavior coincides with that of a linear cobweb model, this provides an introduction to the stability analysis of one-dimensional discrete dynamical systems. In Sect. 2.2, the preliminary model is extended to incorporate adaptive adjustment by boundedly rational agents and the stability analysis of the model is presented. The model presented in Sect. 2.2 is still simple, but its behavior becomes complex including chaotic motions when the fixed point loses its stability. Sect. 2.3 studies the complex behavior of the model. After outlining the complex behavior of the model with an unstable fixed point, we study the occurrence of bifurcations and the chaotic property of the model by introducing some important definitions of chaos and relevant notions, and finally present the numerical simulation results with the two key parameters varied. In Sect. 2.4, we present a more rigorous mathematical analysis of the model by introducing symbolic dynamics and the notion of homoclinic bifurcation. We also present two propositions that state that our model exhibits not only topological chaos but also observable chaos. Sect. 2.5 concludes and the appendix includes the mathematical proofs of the three propositions presented in the main text.

Chap. 3 extends the one-dimensional model studied in Chap. 2 to include two different types of producers in order to investigate whether slight behavioral heterogeneity could be a factor that drastically changes the dynamical properties of a market. The two types of producers are naive optimizers and cautious adapters. A naive optimizer produces the profit-maximizing quantity instantaneously, while a cautious adapter adjusts his/her output toward the profit-maximizing quantity as a target. We obtain a two-dimensional model, which is more difficult to analyze because mathematical theories of higher-dimensional nonlinear dynamical systems are underdeveloped compared with those of one-dimensional systems. With the aid of mathematical analysis as well as numerical calculations, we show that a single agent may change the complexity of market behavior. In a market of naive optimizers, a single cautious adapter stabilizes the otherwise exploding market. In a market of cautious adapters, a single naive optimizer may destabilize the market. Without him/her, there exists at most one periodic attractor in the market. However, with him/her, many (and even infinitely many) coexisting periodic attractors may appear.

In Sect. 3.1, we first discuss the importance of heterogeneity in the study of economic dynamics. In Sect. 3.2, we derive a two-dimensional nonlinear cobweb model including these two different types of producers from a general N-dimensional model including N types of producers. It is shown that the one-dimensional models considered in Chap. 2 can also be derived from the N-dimensional model as a special case. Sect. 3.3 studies the complex behavior of the model: after seeing that a prerequisite for a two- or higher-dimensional model to exhibit complex behavior is having a saddle-type fixed point, we study the occurrence of bifurcations and the chaotic property of the model by using bifurcation diagrams. In particular, light is shed on phenomena intrinsic to a two-dimensional model such as the occurrence of fishhooks (a complex type of local bifurcation structure) and emergence of strange attractors and their coexistence. In Sect. 3.4, we present a more rigorous mathematical analysis of the model by introducing the notion of homoclinic bifurcation in a general form, show the rich variety of the complex dynamical behavior of a two-dimensional model, and finally offer two propositions that state that a single heterogeneous agent may drastically change the qualitative dynamical feature of an otherwise homogeneous market, implying that heterogeneity matters. Sect. 3.5 concludes and the appendix includes the mathematical proofs of one of the lemmas and two of the propositions presented in the main text.

In Part I, we discussed the significance of nonlinear economic dynamics and investigated two simple models that exhibit chaotic behavior. In Chap. 4, we take a moment to pause and reflect on the status quo and the future of nonlinear economic dynamics. Nonlinear economic dynamics originated in the 1930s, led into chaotic economic dynamics at the end of the 1970s, and continues today. However, research on nonlinear economic dynamics has thus far suffered from the serious restriction on mathematical analytics. We discuss this restriction in Sect. 4.1. In Sect. 4.2, we consider where nonlinear economic dynamics should be headed and state that it should aim to use computationally oriented research methods against the background of complex system theory. This statement underpins the studies presented in Chaps. 5, 6, and 7. Furthermore, Sect. 4.3 points out that two fundamental directions exist in which research on economic complexity has been carried out: the econophysics approach and agent-based model approach. We concentrate on the latter approach in Chaps. 6 and 7. This short chapter also serves as a manifestation of our methodological standpoint in Part II.

One of the important conclusions of Chap. 3 is that heterogeneity matters decisively in the complex behavior of a nonlinear economy when at least two different types of agents exist. The question then arises: what happens if there are many different agents in a nonlinear economy? Chap. 5 investigates such a problem by concentrating on synchronization among producers' chaotic behavior. For the sake of simplicity, behavioral heterogeneity is ignored and producers are considered to be identical in the model; however, they can be deemed to be heterogeneous in the sense that the initial conditions of producers are randomly selected. Recall that owing to sensitive dependence on initial conditions, a chaotic map can generate completely different orbits for different initial conditions.

Chap. 5 extends the one-dimensional nonlinear model studied in Chap. 2 to a high-dimensional nonlinear model to study multi-regional business cycle synchronization. Since little is known about the mathematical theory of high-dimensional nonlinear dynamical systems, we rely on numerical simulations to inspect the behavior of a high-dimensional model. Empirical studies often conclude that multi-regional business cycles exhibit intermittent transition between the synchronization and desynchronization of each regional fluctuation. In this chapter, we robustly observe this behavior (called chaotic itinerancy) in a model of multi-regional business cycle synchronization, in which all the regions of a national economy are homogeneous and connected through producers' behavior based on the average level announced by the government. One of the main findings is that although a producer slowly adjusts his/her output toward the average level, regional business cycles begin to synchronize because of the entrainment effect. Moreover, when a producer emphasizes profit maximization more and places more weight on the average level in his/her decision-making, the economy is more likely to exhibit such intermittent transition. Further, it is clarified that behind intermittent transition exist cycles among periodic orbits with a different number of unstable directions.

In Sect. 5.1, the motivation of the chapter and outline of our model are presented, and Sect. 5.2 describes a regional business cycle model. In Sect. 5.3, we inspect the long transient behavior of the model, which contributes toward understanding the important phenomena in high-dimensional nonlinear dynamical systems called chaotic itinerancy. Sect. 5.4 discusses chaotic itinerancy occurring in the model and Sect. 5.5 characterizes chaotic itinerancy from a mathematical point of view. Sect. 5.6 concludes.

In Chaps. 6 and 7, we investigate the time evolution of the market structure in terms of market share dynamics, by employing agent-based models with boundedly rational firms and consumers interacting with each other. The common basic feature of boundedly rational firms is that since they do not know the shape of their demand functions they face, they adaptively revise production levels and prices so as to raise their profits based on the reaction by consumers. The main difference lies in consumers' behavior.

Chap. 6 investigates a monopolistically competitive market with boundedly rational consumers: the model presented in this chapter considers product differentiation and boundedly rational consumers in the sense that each of them has a preference for the product of a particular firm and exhibits habitual purchasing behavior unless the relative price differences exceed a certain critical level. Each firm revises its production decisions and prices to raise its profit based on the reaction by consumers. We do not take into consideration mergers and acquisitions or the game-theoretic settings among firms. Hence, this kind of consumer brand loyalty is the only source of the emergence of monopoly and oligopoly. The simulation results show that as the control parameter of a consumer's brand loyalty increases, monopoly and oligopoly emerge led by the invisible hand. In addition, oligopoly rather than monopoly is the final state of a market economy.

Sect. 6.1 introduces Chaps 6 and 7 simultaneously, indicating one possible direction for new microeconomics theory beyond the neoclassical paradigm, based on the criticism of the methodology of neoclassical economics. The model is presented in Sect. 6.2, the simulation results are discussed in Sect. 6.3, and Sect. 6.4 concludes.

In Chap. 7, we continue to investigate a pseudo-perfectly-competitive market as a complex adaptive system consisting of mutually interacting, boundedly rational firms and consumers. In the model presented in Chap. 6, product differentiation was considered, showing that a consumer's brand loyalty plays an important role in the emergence of oligopoly. By contrast, in this chapter, we consider a market of homogeneous goods. In this market, boundedly rational consumers decide from which firm to purchase goods to increase their utility, and we employ, as a first step, a statistical description to represent firms' distribution of consumer share because the number of consumers is very large. Firms' distribution is characterized by a single parameter representing how "rationally" the mass of consumers pursues higher utility. Aggregate consumer behavior is described by the Boltzmann distribution, which indicates how rationally the consumer seeks to increase his/her utility. Since the boundedly rational firms do not know the shape of the demand function they face, they adaptively revise their production levels and prices to raise their profits with the aid of a simple reinforcement learning algorithm (i.e., by learning through experience).

We do not take into consideration mergers and acquisitions or the game-theoretic settings among firms as in Chap. 6. Instead, we focus on the dynamical phases that emerge as the rationality of consumer changes and characterize their statistical properties such as the probability distribution of firms' size and growth rates. The simulation results show that the three market structure phases, namely the uniform share phase, oligopolistic phase, and monopolistic phase, arise depending upon how rational consumers. In the oligopolistic phase, the market share distribution of firms follows Zipf's law and the growth rate distribution of firms follows Gibrat's law. This phase is the best state in the sense that it maximizes consumers' utility. However, it minimizes firms' profits because the moderate rationality of consumers generate severe competition.

In Sect. 7.1, we present our model and demonstrate that it includes neoclassical competitive equilibria (the Cournot equilibrium and perfectly competitive equilibrium) as special stationary states. In Sect. 7.2, we discuss the simulation results. First, an artificial monopoly case is examined in order to verify that the learning process is workable in the model. Second, the time evolution of the competitive model is examined and it is demonstrated that all firms come to face approximately the same demand curves through a learning process. Third, it is demonstrated that our model exhibits three phases appears as the degree of consumer's rationality varies. Finally, the market structure dynamics are characterized from various aspects such as the Herfindahl index, variances, probability distributions, and averaged utility and profit. Sect. 7.3 concludes.