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主 論 文 題 名：Essays on Asset Return Modeling with Bayesian Markov Chain Monte Carlo
(ベイズマルコフ連鎖モンテカルロ法による資産収益率のモデリング)

（内容の要旨）

Since Markowitz (1952) proposed the famous mean-variance model for optimal portfolio selection, it has remained an important foundation in research of portfolio theory as well as in practice of portfolio management. Although numerous solutions have been proposed for various problems on the mean-variance model in decades of its history, there remains some challenging tasks; (1) $p > n$ problem in case of high-dimensional data with many assets (p: the number of assets, n: the number of data points) and (2) modeling of a skewed and fat-tailed distribution with real-world data. To tackle these challenges, this doctoral dissertation consisting of three essays discusses new statistical modeling techniques with Bayesian Markov Chain Monte Carlo (MCMC) and their applications to real-world data.

The first challenge is a problem how to construct the optimal portfolio when the number of assets p exceeds the number of observations of asset returns n. In a conventional Markowitz-type portfolio, the optimal weights depend on the inverse of the covariance matrix or the precision matrix of asset returns. It is well-known that the calculation of the inverse matrix of the sample covariance matrix becomes unstable as p approaches n, and it becomes impossible when p is greater than n. Thus, to solve this $p > n$ problem, numerous dimensional compression methods have been proposed in the literature. In the field of finance, dimensional compression methods based on factor models have widely been used. In recent years, however, another type of dimension compression method that directly estimates the precision matrix of high-dimensional asset return data with a graphical model such as graphical LASSO has been developed in the literature of machine learning. This dissertation tackles the first challenge in Chapter 2 (the first essay) and Chapter 3 (the second essay) from the perspective of the second approach after explaining the purpose and the structure of this dissertation in Chapter 1.

In Chapter 2, we focus on developing the Bayesian graphical model itself that has a possibility to be used as a solution to the $p > n$ problem. In the literature, Wang (2012)

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originally proposed the block Gibbs sampler for the Bayesian graphical LASSO which has been widely applied and extended to various shrinkage priors in recent years. Our contribution is that we discover that Wang (2012)'s algorithm has a less noticeable but severe disadvantage that the positive definiteness of the precision matrix in the Gaussian graphical model is not guaranteed in each cycle of the Gibbs sampler. Specifically, if the dimension of the precision matrix exceeds the sample size, the positive definiteness of the precision matrix will be barely satisfied and the Gibbs sampler will almost surely fail. In this research, we propose modifying the original block Gibbs sampler so that the precision matrix never fails to be positive definite by sampling it exactly from the domain of the positive definiteness. To demonstrate the superiority of the proposed algorithm, we conducted the Monte Carlo experiments in which we inherited the settings from Wang (2012) and generated artificial data with $p = 30$ or $p = 100$ and six designs of the precision matrix (AR1, AR2, Block, Star, Circle, and Full) as the true structure. The results show that this modification not only stabilizes the sampling procedure but also significantly improves the performance of parameter estimation and graphical structure learning. Interestingly, the proposed algorithm also improves the performance in scenarios where Wang (2012)'s algorithm does not violate the positive definiteness. We also apply our proposed algorithm to a graphical model of the monthly return data in which the number of stocks exceeds the sample period, demonstrating its stability and scalability.

In Chapter 3, we discuss an application of graphical models to portfolio management. The previous studies already applied graphical models to portfolio optimization. Especially, Torri et al. (2019) examined performance of the global minimum variance portfolio constructed by graphical LASSO (glasso), Student's t-based graphical LASSO (tlasso), random matrix theory filtering, Ledoit-Wolf shrinkage estimation, the conventional sample covariance approach and the equal weight approach in long-term portfolio management with US stock return data. Though Torri et al. (2019) 's research was innovative, its scope of study was limited in case of $p < n$. Moreover, Torri et al. (2019) only tested non-Bayesian graphical models and Bayesian models were not included in comparison. Thus, we develop a data-driven portfolio framework based on a Bayesian graphical LASSO model proposed in Chapter 2, and try to construct the global minimum variance portfolio in case of $p > n$. In the empirical study, we constructed the global minimum variance portfolio of 100 assets for different sample lengths with the proposed Bayesian approach, variations of non-Bayesian graphical LASSO and the other approaches which have been also examined in Torri et al. (2019), and compared their out-of-sample performance in 10-year portfolio management from 2011 to 2020. We used monthly return data on 100 portfolios of US companies formed on size and book-to-market ratios provided by Kenneth French and test five scenarios: (p, n) = (100, 120), (100, 60), (100, 12), (100, 6) and (100, 3), which were corresponding to the sample period of 10 years, 5 years, 1 year, 2 quarters, and 1 quarter respectively. In this experiment, we confirmed advantages of the proposed approach over the others in terms of return-risk tradeoff and portfolio composition. Both Sharpe ratios and indices of portfolio composition were relatively stable for the proposed approach while they are either unstable for non-Bayesian graphical LASSO approaches.

The second challenge is that the normality assumption of the traditional Markowitz's approach for asset returns is not necessarily satisfied for real-world data. Actually, it is well-known that they tend to follow a fat-tailed, possibly skewed distribution. Thus, researchers have proposed numerous distributions that can express these characteristics of asset returns well. In particular, a so-called skew-t distribution is often assumed for asset return. There are many types of skew-t distribution known in the literature, but arguably the most famous one is generalized hyperbolic (GH) skew-t distribution as a special case of the GH distribution. Especially, application of the GH distribution has been recently advanced in the field of asset price volatility models. Although the GH distribution is flexible enough to model a single asset on many occasions, it has difficulty in capturing the skewness dependency among multiple assets. Fund managers would find the skewness dependency useful in particular when the financial market crashes and almost all assets suddenly go south since such sharp price co-movement may not be captured by the second moment (i.e., correlation) only.

In Chapter 4 (the third essay), we examine a skew-elliptical distribution which is another type of distribution that can express these characteristics of asset return to circumvent the aforementioned shortcoming of the GH distribution. The skew-elliptical distribution was proposed by Branco and Dey (2001) as a generalization of the multivariate skew-normal distribution by Azzalini and Valle (1996) and later improved by Sahu et al. (2003). Harvey et al. (2010) extended the Bayesian estimation method by Sahu et al. (2003) to the multivariate skew-elliptical distribution with a general skewness matrix, and applied it to Bayesian portfolio optimization with higher moments. Harvey et al. (2010)'s research is considered to be a model that is of great interest to researchers because it

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is frequently cited in papers on portfolio selection with the downward risk. In our assessment, however, their method is epochal in the sense that it can handle the skewness dependency among asset returns and incorporate higher moments into portfolio optimization, it cannot identify all elements in the skewness matrix due to label switching in the Gibbs sampler. To solve the issue, we propose a modified model in which the lower-triangular constraint is imposed upon the skewness matrix. Moreover, we devise an extended model with the horseshoe prior for both skewness matrix and precision matrix to further improve the estimation accuracy. In the simulation study, we compared the proposed models with the model of Harvey et al. (2010) in three structural designs of the skewness matrix; Diag, Sparse, and Dense. The results show that the proposed models with the identification constraint significantly improved the estimation accuracy of the skewness matrix.

In Chapter 5, as concluding remarks of this dissertation, we review key points of the thesis and comment on a future development.