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主　論　文　要　旨

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主 論 文 題 名：

Variational Convergence and Their Applications to Econometrics and Statistics

（内容の要旨）

In this thesis, as a reasonable choice of the topology we choose the mosco-convergence, that is the "weakest" notion of convergence for sequences of convex functional which allows to approach the limit on corresponding minimization problems. On this way, various limit problems are analyzed: some, such as a functional linear quantile regression, generalized method of moments estimate of diffusion processes, a kernel density estimate by partial differential equation method, convergence of invariant measure of computed dynamics with unbounded shocks and a relation between admissibility of statistical estimator and recurrence of Markov processes. For all these examples Mosco-convergence provides a flexible tool and a deep insight.

In chap:VariationalConvergence, we present the general set-up and main results of a fully abstract model. We describe the Mosco convergence and introduce the narrow convergence in the Mosco topology. We derive the quadratic approximation of a convex objective function in an infinite-dimensional Hilbert space. We also provide the asymptotic distribution of the optimal value.

In chap:Dirichlet-Forms, we present the general notion of Dirichlet form and their relationship with the symmetric Markov process. There is one-to-one correspondence between the Dirichlet form and semi-group of the symmetric Markov process. We apply the Mosco convergence to the perturbated Dirichlet form and describe the Mosco convergence of the Dirichlet form. We derive the Mosco convergence of the Dirichlet form implies the narrow convergence of the corresponding symmetric Markov process and vise versa.

In chap:QuantileRegression, we study an asymptotics of functional linear quantile regression in which the dependent variable is scalar while the covariate is a function. We apply a roughness regularization approach of a reproducing kernel Hilbert space framework. In the above circumstance, narrow convergence with respect to uniform convergence fails to hold, because of the strength of its topology. A new approach we propose to the lack-of-uniform-convergence is based on Mosco-convergence that is weaker topology than uniform convergence. By applying narrow convergence with respect to Mosco topology, we develop an infinite-dimensional version of the convexity argument and provide a proof of an asymptotic normality of argmin processes. Our new technique also provides the asymptotic confidence intervals and the generalized likelihood ratio hypothesis testing in fully nonparametric circumstance.

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In chap:GMM, we adopt the approach of , and provide an asymptotics of this approach. We begin by considering a Markov process specified in terms of its infinitesimal generator. Formally, this generator is defined as an operator on a function space, and, in effect, this operator stipulates the local evolution of the process. For the fully identification, one must estimate the second largest eigenvalue of the infinitesimal generator G which involves an optimization problem including differential operator. That is beyond an usual asymptotics of empirical process theory. We deal with this problem by the introduced Mosco topology.
In chap:PDE, We extend 's adaptive kernel density estimation method based on the smoothing properties of linear diffusion processes in two ways. First, we extend their proposed diffusion kernel method to kernel density estimators based on Lévy processes, which have the diffusion estimator as a special case. The kernels constructed via a Lévy process could be tailored for data for which smoothing with the diffusion estimator is not optimal. Second, we consider an asymptotics of the estimated diffusion differential operator that has a random fluctuate due to the estimated pilot density. This problem induces a variational problem, and in fact can be addressed by a straightforward application of Mosco convergence of Dirichlet form.
In chap:Convergence-of-Computed, we provide the conditions for the convergence of invariant measure obtained from numerical simulations to the exact invariant measure. Most dynamic economic models do not have a closed-form solution. Model's policy functions are approximated by numerical methods. Therefore, the researcher can only evaluate an approximated invariant measure associated with the approximated transition function rather than the exact invariant measure implied by the exact transition function. However, previous study assumed that the state space is compact and therefore, the support of the shock of dynamical system is assumed to be bounded. We relax the compactness assumption for the convergence of the approximated invariant measure.
In chap:Admissibility, we generalize and reformulate idea of the admissibility question for more general distributions, for more general Bayesian decisions and for more general variational form, i.e., Dirichlet form. This connection goes far beyond the diffusion processes case that consider. The relation between admissibility of a general Bayesian decision which is based on general distributions and recurrence of the other symmetric Markov processes is established. Since general distributions include Levy type(infinitely divisible) distributions as a special case of a much more general phenomenon, we give a striking result on a maximum likelihood estimate(MLE) of Cauchy distribution that MLE of Cauchy distribution with dimension d=1 is admissible but is inadmissible with $d \geq 2$. This phenomenon is compatible with the transiency of Cauchy processes with the division between dimensions m=1 and m=2