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Measurement of The Distribution of Reservation Wage Using Household Data: Price of Labor From Preference Maps for Income and Leisure*

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and
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1. Introduction

The labor supply probability of a person or a group of persons (labor supplier or suppliers) is given by the definite integral of the (density) distribution function of reservation wage, (a kind of latent variable), which can not be directly observed. Reservation wage can be mathematically described using the parameters (denoted by Γ's) of labor suppliers' preference maps (income-leisure indifference function), because reservation wage is, by definition, the value of the marginal rate of substitution at hours worked equal to zero.

(1)This definition of reservation wage was employed by Heckman(1974). However, we will use a modified definition of reservation wage.

Contrary to self-employed workers, employees' hours of work (h) tend to be assigned ($h = \bar{h}$; where $\bar{h}$ stands for assigned hours) by employers. In this case, reservation wage is defined as the average rate of substitution where the working hours are assigned as $h(\neq 0)$. We shall call this type of reservation wage the "minimum supply price of labor (MSPL)"(Obi[1969][1980]). We will clarify the factors and mechanism , which determine the size distoribution function of MSPL ; and we will estimate the values and size distribution (among the suppliers) of the income-leisure prefernce function. These parameters are the most basic determinant of labor supply probability function.

(2) Usually regression method (probit or logit etc., a kind of regression) is employed to estimate parameters of the supply function. These estimated parameters of regression equation can be interpreted as estimates of the reduced from equation, since the independent variables included in the supply function are the exogenuous variables in the context of labor supply behavior. It is well known that we will not only need to estimate reduced form parameters,but we also need to obtain structural paramters (in this case the suppliers' preference parameters among income and leisure).

As was shown by the rigorons analysis done by the pioneers of positive economic analysis [R. Frisch(1948), and J. Marshak(1950)], we have to estimate structural parameters because it is indispensable to know the values of structural parameters in order to analysis the effects of expected structural change on the values of reduced form parameters.

In this contex, we will estimate the structural parameters, which are the parameters of income-leisure preference function, using Japanese household data.

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(3) The third point of this paper is to clarify the interaction of labor supply behavior of both self-employed workers and employees, throngh the construction of a comprehensive labor supply model. That is, by applying conditional utility maximization, we derive the probability function of labor supply for employees (denoted by μ^e), self-employed (μ^d), and for both (μ^{ed}).

(4) In the following, firstly, the parameters of utility function are estimated. Secondly, the labor supply probability functions which describe μ^e, μ^d and μ^{ed} are derived. Lastly, some simulations are carried out using these results (see section 4).

(5) By the above analysis, we show that [1] the redefined reservation wage, that is, minimum supply price of labor (MSPL) is useful and [2] it lead to a positive model providing a comprehensive description of self-employed workers and employees.

(6) This comprehensive labor supply model is both applicable and indispensable in analysing economic development. This is because the development process can be seen, from the labor suppliers' point of view, as the evolution of a modern employee labor market from one of self-employed workers working in the indigenous industries.

(7) In order to set out a comprehensive model of household labor supply, it is appropriate to begin with a simple case. Hence, we would like to consider the type of household whose members consist of a husband (principal earner) who is an employee, a wife (non- principal potential earner) and an unspecified number of children under fifteen years of age. We shall call this type of household "type A" hereafter.

(8) Remarks on RW, MSPL and an alternative notion, $H(d)$.

As we here discussed above , MSPL is usefull for analysing the supply behavior when the supplier faces working opportunity of employee work where the working hours are assigned by employer. RW is convenient for analysing supply probability when the hours of work is completely adjustable to optimum hours of work. However , when we symultaneously analyse the supply probability for these two kinds of employments, that is, employee and self-employed, it will be shown in the following that an alternative notion which we tentatively call the distribution of $H(d)$, optimal hours of work for self-employed opportunity, is more convinient and suitable . The value of MSPL can be analytically transformed to $H(d)$ and vice versa. This is shown in the appendix.

2. Labor supply model of type A households

In type A household (husband being employee), the wife's labor supply behavior makes the labor supply probability of the household change. Hence, the synthetic model of labor supply for type A households should clarify the conditions by which the wife (non principal potential earner) in a given household chooses to belong to either of the following four patterns:

(1) She (or non principal potential earner) is neither an employee nor self-employed.

(2) She is not an employee but self-employed.

(3) She is an employee but is not self-employed.

(4) She is both an employee and self-employed.

We shall construct a model describing the mechanism in which above participation patterns occur. In the following for the simplicity of analysis, let (1) the income leisure preference function be quadratic [1] and (2) wives' income generating function (production function) for self-employed work be linear, i.e., the marginal earning rate (marginal value productivity) with respect to hours of labor is a constant. Proposition (2) is introduced for the sake of simplicity and does not impair characteristics of the model. In the following, we shall refer to the income leisure preference contours of type A household. However, as was discussed above, it would be appropriate to assume that these preference curves stand for the preference curves of wives in type A household.

2.1. The determinants of wife's pattern of labor supply

Let us consider a group of type A households with a common level of principal earner's income, I (Fig.1). Let the marginal earning rate (marginal value productivity of wife's self-employed work) be v which is assumed to be common to all the households considered. The wage rate offered by firms to the wives of the households and the assigned hours of work are denoted by w and $\bar{h}$ respectively which are also assumed to be common to all the households considered.

In Fig.1 $tan\theta_w$ and $tan\theta_v$ stand for w and v respectively. When the wife accepts an employee opportunity, her income leisure position is given by point k. CD is the line passing through point k and parallel to aB, aB being a line of self-employed income. If the wife accepts the employee opportunity and further works as self-employed, the household income will be augmented and lie along the line kD.

Now consider a contour passing through point a. The gradient of the contour at point a, $|dx/d\Lambda|_a$, will vary among the households considered due to the difference in income-leisure preference among them. Let us call the sub group of households i with

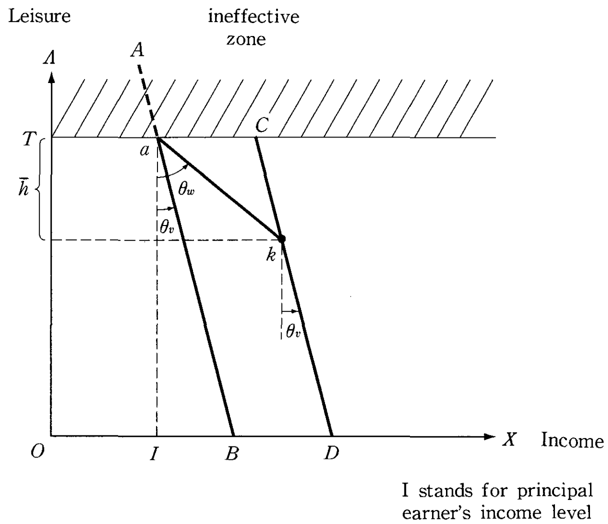

Figure 1

[1] Allen-Bowley type utility function

$$\left|\frac{dx}{d\Lambda}\right|_a^i > v \qquad (2-1)$$

group I, and the sub group of households j with

$$\left|\frac{dx}{d\Lambda}\right|_a^j < v \qquad (2-2)$$

group II.

It is clearly seen that for any household i in group I there is no tangency point on the line aB, while there is a tangency point on the line aB for a household included in group II. Needless to say, for a household with $|dx/d\Lambda|_a = v$, the tangency point lies just on point a.

As to the households in group II, tangency points lie below point a on the line aB. On the other hand, for the households of group I, the tangency point will be situated at some point on the dotted line Aa which is in an ineffective zone of the indifference map.

2.1.1. Wives' participation behavior in households in group I.

In Fig.2, a contour ω_a (a contour passing through point a)of a household in group I is depicted. The tangency point of aB and the contour is shown by point d in the ineffective zone of the indifference map. Let the intersection point of ω_a and ak be m. In Fig.2 point m is situated above point k on the line ak. First, we shall examine the behavior of a wife of a household with such a contour ω_a as is shown in Fig. 2. When the wife accepts an employee opportunity, her income-leisure situation is given by point k. Her situation is shown by point a if she neither accepts the opportuniy nor works to earn her self-employed income. When the wife earns both a wage and self-employed income, her situation is shown by some point between k and D on the line kD (By the definition of group I, a household wife does not choose self employment and is therefore not situated between a and B). Among those three situations, point a is clearly the optimum because point a lines on the contour with the highest utility indicator compared to point k and any points between k and D.

In Fig.3 the indifference curve of a household in which the intersection point, m, of contour passing through point a, ω_a, and the extention of line ak, aE, lies below point k is depicted.

By the examination mentioned elsewhere, when ω_a is quadratic any points between k and J lie on the indifference curves with inferior values of the utility indicator in comparison with the indifference curve passing through point k, and it is clearly seen that point k is preferable to point a. [2]

Hence, the wife of a household with such an indifference map as is shown in Fig.3 accepts the employee opportunity and does not earn an additional self-employed income.

[2] If the wife in a household of this kind of household accepted both an employee opportunity and self employed work, her income-leisure situation would be given by the tangency point of contour and line kD somewhere between k and J as the hours of work for earning self-employed income can be adjusted as the supplier (wife) desires. It should be noted, however, that there does not exist any tangency point on the indifference curve and line between the points k and J on the line cD. If there were a tangency point, g, which is not shown in Fig.3, it would be said that, when the principal earners income is Tf (in

the case where a is selected

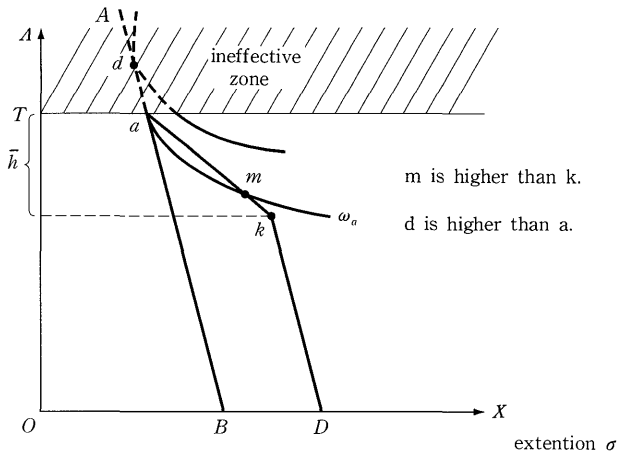

Figure 2

2.1.2. Wives' supply behavior in group *II* household

Household in which the tangency point, d, lies between points a and P. For this type of household, let the crossing point of ω_a and ak be denoted by m'.

Firstly, consider a household in which point m' lies above point k as is shown in Fig.4. The wife (non-principal potential earner) in this kind of household prefers point d, because d is situated on the indifference curve with the highest indicator among the points k, a, and all the points between k and D. Hence, she works as self-employed only and does not accept employee opportunities.

Let the extention of line ak be kF (dotted line) in Fig.5. The intersection point of kF and contour ω_d is denoted by m'. Consider a household in which point m' lies below point k as shown in Fig.5. The wife (non-principal potential earner) in this type of household will never choose any points between k and J. If she chooses those points it would mean that she is both an employee and self-employed. However, such a point would have to be a tangency point. There could not be any tangency points between k and J because of the requirement mentioned in footnote 2. Hence, d is preferred to a, and any points between k and J are preferred to d. Therefore k is preferred to the points between k and J. That is, the wife will be an employee and will not work as self-employed.

Fig.3), the non principal earner's (wife's) optimal hours of work for the earning rate $v(\tan\theta_v)$ is given by the ordinate difference of point f and g. If such a case occurred, it would be clear, by comparing points d and g, that the larger is the principal earner's income, the longer is the nonprincipal earner's (wife's) optimal hours of work, the nonprincipal earning rate v being given. This means, under the assumption of a quadratic preference function, that the locus of point d (Fig.3 or Fig.4) on the $X \sim \Lambda$ plane is downward sloping. However, the downward sloping locus is evidently inconsistent with the observed facts(Obi 1987 vol.1 pp.36–41.) Hence, it is proved that, under the assumption of a quadratic preference function, there should be no tangency point between points k and J.

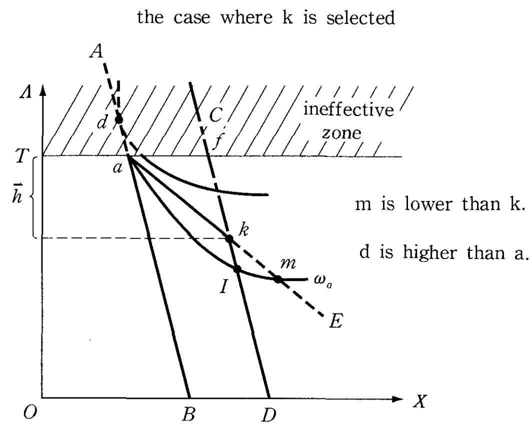

Figure 3

Household in which point d lies between points p and B. For this kind of household two types of households are further differentiated from each other.

In Fig.6 point e is a tangency point of the indifference curve and the line fk which is the extention of line kD. Now, consider a household indifference map which has such characteristics that there exists a tangency point between the indifference curve and the line fk. For this kind of household all the points between k and D on the line kD are situated on indifference curves with smaller indicators compared to the indifference curve passing through point k , because the gradient of contour at point k to the vertical axis ,$|dx/d\Lambda|_k$, is larger than that of line kD to the vertical axis.

Hence, among points a, d, k and all the points between k and D, d is preferred to a, all the points between k and D are preferred to d, and k is preferred to all the points between k and D; thus k is preferred. This means that the wife (non principal potential earner) of this household accepts the employee opportunity only and has no self-employed income.

Consider a household in which point e lies below point k. The indifference map of this kind of household is depicted in Fig.7.

It is clearly seen that e is preferred to a, d and k. Hence, the wife (non principal potential earner) will accept the employee opportunity and at the same time she will work as self-employed.

2.2. Labor Supply Model of type A Households

2.2.1. Summary on the patterns of wives' labor supply

Let us denote the coordinates of point d, m, m' and e in Fig.2 through 7 with regard to hours of work by $H(d), H(m), H(m')$, and $H(e)$ respectively. The coordinates of both points k and p with respect to hours of work are $\bar{h}$ (hours of work assigned by firms). The coordinates of both points B and D with respect to hours of work are T which stands

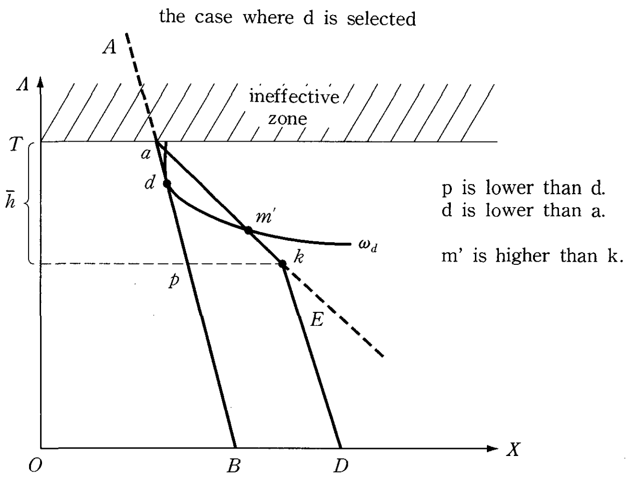

Figure 4

for the wife's (non principal potential earner's) total disposable time (composed of leisure and hours of work if any). Hours of work for earning self-employed income and that for income from the employee opportunity are denoted by H_{self} and H_{emp} respectively. The coordinates of point a with respect to hours of work is zero.

Making use of these notations, the conditions mentiond in 2.1 are rewritten as shown in Table.1.

2.2.2. The Relation between $H(m)$ and $H(d)$ for households with $H(d) < 0$.

In order to construct a synthetic model for type A households, we shall first consider a group of households with $H(d) < 0$. With regard to the determinants of participation behavior of this kind of household, the position of point m in relation to the position of point d in Fig.2 is fundamentally important. Let the relation of $H(m)$ to $H(d)$ be

$$H(m) = \phi[H(d)] \qquad ; \qquad H(d) < 0 \qquad \cdots \phi function$$

A concrete analytical form of ϕ is given in the subsequent section.

2.2.3. The Relation between $H(m')$ and $H(d)$ for the households with $\bar{h} > H(d) > 0$.

For the households where $H(d) > 0$ holds, the position of point m' in Fig.4 and 5 is important. Let the relation between $H(m')$ and $H(d)$ be denoted by

$$H(m') = f[H(d)] \qquad ; \qquad \bar{h} > H(d) > 0 \qquad \cdots f function$$

An analytical form of f is given in the subsequent section.

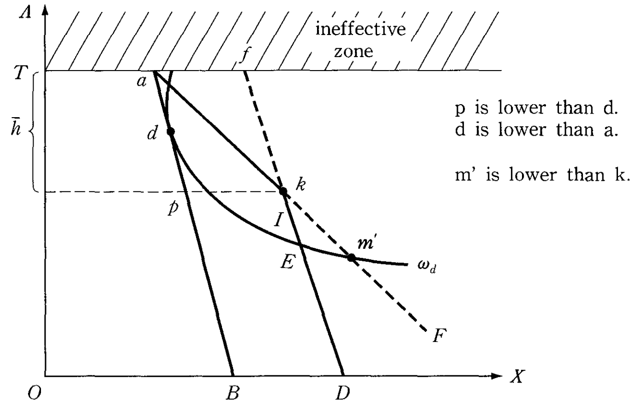

Figure 5

2.2.4. The Relation between $H(d)$ and $H(e)$ for the Households with $H(d) > \bar{h}$.

For this kind of household the position of e is also important. Let the relation between $H(e)$ and $H(d)$ be

$$H(e) = \psi[H(d)] \qquad ; \qquad H(d) > \bar{h} \qquad \cdots \psi function$$

The analytical form of ψ is given in the subsequent section.

2.2.5. On the graphs of functions ϕ, f and ψ.

The functions ϕ, f and ψ are assumed to be monotonic and are depicted by the curves $\alpha\alpha', \alpha'\beta$ and $\gamma\gamma'$ in Fig.8. It should be noted that curve $\alpha\alpha'$ standing for ϕ and $\alpha'\beta$ standing for f have a point of conjunction, α', because when $H(d) = 0, f[H(d)] = \phi[H(d)]$ holds, as can be seen in Fig.3 and 4.

In Fig.8 the numbers attached to the curves in first and second quadrant correspond to those in the column of Table 1. It should be remarked that the participation pattern denoted by ③ does not occur when point α' lies above point $\bar{h}$ in Fig.8. While, when α' lies below $\bar{h}$, pattern ③ doeds occur as shown in Fig.8.

Pattern ③ is for self-employed wives only, not wives who are employees. However according to observed facts, a pattern such as ③ does exist. Hence, point α' should be below point $\bar{h}$ as is shown in Fig.8. Although pattern ②in Table 1, does not appear in Fig.8, patterns ④ and ⑤ are quite the same as ②. Hence all the participation patterns observed for type A household appear in Fig.8. In this sense the shapes of functions (curves) of ϕ, f and ψ in Fig.8 are consistent with observation. [3]

[3] Taking into account the results in section 2.2.5, it can be seen that the participation patterns generated from Fig.3 and 4 are exclusive of each other. This stems from that we assumed the curves $\alpha\alpha'$ and $\alpha'\beta$ are upward sloping monotonic curves. This specific characteristic of the curves results from the postulate that the preference function is approximated by a quadratic function.

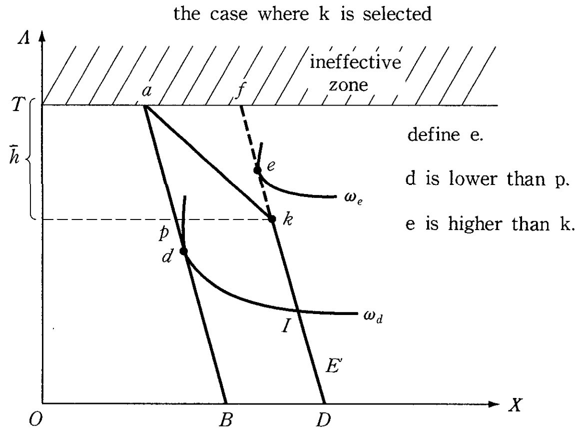

Figure 6

2.2.6. Probabilities of generating various participation patterns in type A households, —Latent variable $H(d)$ and its density distribution—.

In this section the determination of the probabilities of generating four patterns of participation in type A household will be clarified when the principal earner's (husband's) income, I, the wage rate, w, the hours of work assigned by firms, $\bar{h}$, and the earning rate of self employed work, v, for non-principal potential earner (wife) are given.

The density distribution curve of $H(d)$ is depicted in the third and the forth quadrants in Fig.8. This distribution reflects the differences in magnitudes of preference parameters among households where the common values of I, w, v and $\bar{h}$ are given respectively.

Taking into account the results summarized in Table 1, it will clearly be seen that area S_1 under the distribution curve, gives the probability that the wife (non-principal potential earner) is neither an employee nor self-employed. This is the probability that pattern ① in Table 1 occurs. Let us call area S_1 , the probability of non-participation.

Area S_2 in Fig.8 gives the probability that the participation pattern ③ in Table 1 occurs. This is the probability that the wife engages in self-employed work only without accepting an employee opportunity. Let us call this probability the probability of self-employment participation, μ_d,

where

$$\mu_d \equiv \frac{\text{number of self employed wives not accepting employee opportunity}}{\text{number of wives}}$$

Area S_3 gives the probability that either participation pattern ④ or ⑤ in Table 1 occurs. Here it should be noted ④ and ⑤ are the same pattern. Let us call this probability

Contrary to the upward sloping monotonic curves, if the shape of curve $\alpha\alpha'\beta$ is not monotonic, both cases ② and ③ in Table 1 (or the cases shown in Fig.3 and 4) could coexist. Precise discussion on this point is given in [Obi 1987, 88].

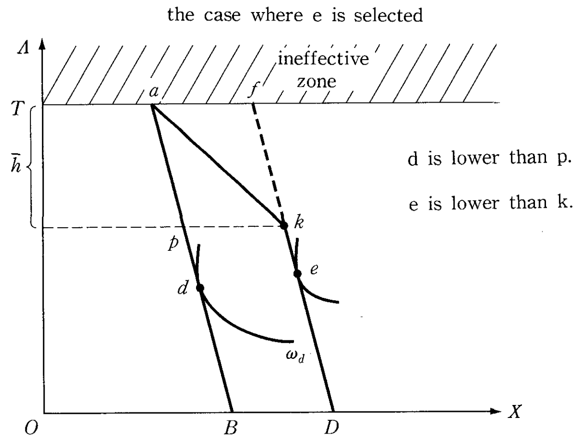

Figure 7

the probability of accepting an employee opportunity and not self-employed work, or in short, the probability of beeing an employee, μ_e (probability of employee participation), where

$$\mu_e \equiv \frac{\text{number of wives accepting employee opportunity and not self-employed work}}{\text{number of wives}}$$

Area S_4 stands for the probability that the participation pattern ⑥ in Table 1 occurs. Let us call this probability the probability of double participation, μ_{ed}, where

$$\mu_{ed} \equiv \frac{\text{number of wives participating in both self-employed work and as an employee}}{\text{number of wives}}$$

Of course,

$$\text{non-participation probability } + \mu_d + \mu_e + \mu_{ed} = 1$$

Prior to drawing the curves in Fig.8 the values of I, w, h, and v have to be given. That is, when these conditions change, the shape of all the curves change simultaneously and, in effect, the areas $S_i (i = 1, 2, 3, 4)$ or magnitude of $\mu_d;\mu_e$ and μ_{ed} change. Hence analytical forms of the function ϕ, f, ψ and the size distribution funtion of $H(d)$ have to be known in order to describe the changes in participation probabilities corresponding to the changes in I, w, and v. This will be discussed in the following section .

2.2.7. Analytical Forms of Functions ϕ, f, and ψ

Analytical Form of ϕ

Table 1

(1) Households with $H(d) < 0$	(1.1) households with $H(m) < \overline{h}$	$H_{emp} = 0, H_{self} = 0$	case ① Fig.2
	households with $H(m) > \overline{h}$	$H_{emp} = \overline{h}, H_{self} = 0$	case ② Fig.3
(2)Households with $H(d) > 0$	(2.1)$H(d) < \overline{h}$ $H(d) < \overline{h}$	(2.1.1) households with $H(m') < \overline{h}$ $H_{emp} = 0, H_{self} > 0$	case ③ Fig.4
		(2.1.2) households with $H(m') > \overline{h}$ $H_{emp} = \overline{h}, H_{self} = 0$	case ④ Fig.5
	(2.2) households with $H(d) > \overline{h}$	(2.2.1) households with $H(e) < \overline{h}$ $H_{emp} = \overline{h}, H_{self} = 0$	case ⑤ Fig.6
		(2.2.2) households with $H(e) > \overline{h}$ $H_{emp} = \overline{h}, H_{self} > 0$	case ⑥ Fig.7

In order to obtain the concrete form of ϕ it is necessary to calculate the coordinates of point d in Fig.2 or 3. The equation of line aB is given by

$$X = I + vh \tag{1}$$

where h and X stand for hours of work (for the employee opportunity and/or self-employed work) and household's income respectively. v stands for the earning rate of self-employed work.

The preference function ω is given by

$$\omega = \frac{1}{2}\gamma_1 \cdot X^2 + \gamma_2 \cdot X + \gamma_3 \cdot X \cdot \Lambda + \gamma_4 \cdot \Lambda + \frac{1}{2}\gamma_5 \cdot \Lambda^2 \tag{2}$$

where

$$\Lambda \equiv T - h.$$

We assume magnitude of the preference parameter γ_4 differs among the households considered. [4]

Under the constraint of (1), (2) is maximized with respect to h. When the value of h maximizing ω is negative, that value of h stands for $H(d)$ in the function ϕ. This stems from the fact that the indifference maps shown in Fig.2 or 3 are the maps of households with such γ_4 that places tangency point , d, on AB in the ineffective range.

Hence, we obtain

$$H(d) = \frac{-(\gamma_1 \cdot v - \gamma_3)I - v(\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} \quad ; \quad H(d) < 0 \tag{3}$$

The value of $H(d)$ varies among households with given I, w, h and v owing to the difference in γ_4 of each household. Hence the size distribution of γ_4 can be easily transformed to that of $H(d)$ by using equation (3).

[4]We adopt this assumption by taking into account some consistency between observational facts and the model.Precise discusstion is given in [Obi,1987].

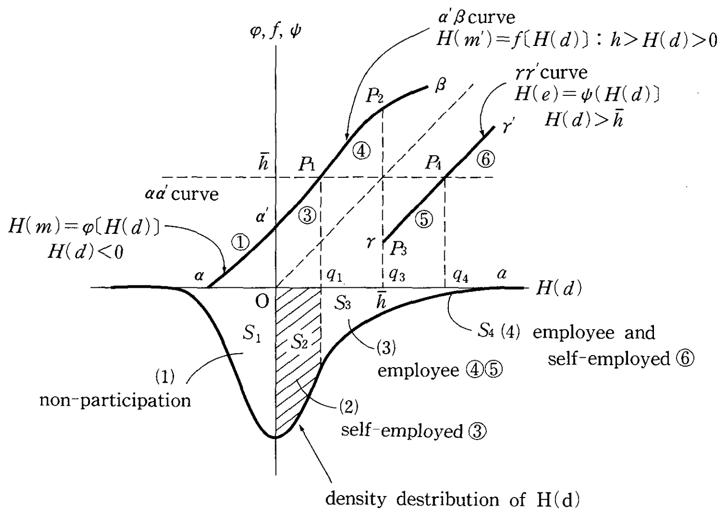

Figure 8

The equation of the indifference curve ω_a in Fig.2 and 3 can be obtained as follows. By inserting the values of the ordinates of point a in Fig.2 and 3,

$$X = I \qquad (4) \quad ; \qquad \Lambda = T \qquad (5)$$

into the left hand side of prefernce function (2), we obtain the value of indicator ω_a at point a,

$$\omega_a = \frac{1}{2}\gamma_1 \cdot I^2 + \gamma_2 \cdot I + \gamma_3 \cdot I \cdot T + \gamma_4 \cdot T + \frac{1}{2}\gamma_5 \cdot T^2 \tag{6}$$

I and T being given. Hence, the equation of the indifference curve ω_a can be written as

$$\omega_a = \frac{1}{2}\gamma_1 \cdot X^2 + \gamma_2 \cdot X + \gamma_3 \cdot X \cdot \Lambda + \gamma_4 \cdot \Lambda + \frac{1}{2}\gamma_5 \cdot \Lambda^2 \tag{7}$$

where ω_a is given by (6).

Finally let us obtain the ordinate of point m in Fig.2 and 3.
The equation of line ak is given by

$$X = I + wh. \tag{8}$$

We can solve (8) together with (7) for h. The solution is the coordinate of point m with respect to hours of work, $H(m)$, that is,

$$H(m) = \frac{(-\gamma_1 \cdot w + \gamma_3)I - w(\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\frac{1}{2}(\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5)} \tag{9}$$

It can be seen that the magnitude of $H(m)$ varies among households considered owing to differences in γ_4 of each household.

Now, we are ready to obtain a concrete form of the function ϕ . The parameter γ_4, the magnitude of which is supposed to vary among households, is included both in equations (9) and (3). Hence, by eliminating common parameter γ_4 both in (9) and (3) we obtain a relation between $H(m)$ and $H(d)$,

$$H(m) = \frac{2(\gamma_1 \cdot v - \gamma_3 v + \gamma_5)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5} H(d) + \frac{2(v-w)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5}$$

$$\cdots \text{function} \quad \phi \qquad (10)$$

where $H(d) < 0$.

Analytical Form of funcution f. Function f stands for a relation between point m' and d in Fig.4 and 5. The coordinate of point d, $H(d)$, is previously given by (3),

$$H(d) = \frac{-(\gamma_1 \cdot v - \gamma_3)I - v(\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} \quad ; \quad H(d) > 0 \qquad (3)$$

We shall obtain the equation of ω_a in Fig.4 and 5.
The coordinates of point d are given by

$$X = I + v \cdot H(d) \qquad (11) \quad ; \qquad \Lambda = T - H(d) \qquad (12)$$

where $H(d)$ is given by (3). Inserting (11) and (12) into (2) we have

$$\omega_d = \frac{1}{2}\gamma_1[I + vH(d)]^2 + \gamma_2[I + v \cdot H(d)] + \gamma_3[I + v \cdot H(d)][T - H(d)]$$

$$+ \gamma_4[T - H(d)] + \frac{1}{2}\gamma_5[T - H(d)]^2 \qquad (13)$$

Given I and v, the value of ω_d in (13) is specific to each household with specific value of γ_4.

The equation of contour ω_d in Fig.4 and 5 is given by

$$\omega_d = \frac{1}{2}\gamma_1 \cdot X^2 + \gamma_2 \cdot X + \gamma_3 \cdot X \cdot \Lambda + \frac{1}{2}\gamma_5 \cdot \Lambda^2 \qquad (14)$$

where ω_d is given by (13).

The equation of the segment ak or that of the extention of the segment is given by

$$X = I + w \cdot h \quad ; \qquad T - \Lambda = h \qquad (15)$$

where h stands for hours of work for the employee opportunity and/or self-employed work. Hence, we can obtain the ordinate of point m' by solving (14) and (15) simultaneously with respect to h. By denoting this solution $H(m')$ we have

$$H(m') = \frac{-1}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5}[I(\gamma_1 w - \gamma_3) + (\gamma_2 + \gamma_3 T)w - \gamma_4 - \gamma_5 T]$$

$$\pm \Big[[(\gamma_1 w - \gamma_3) + (\gamma_2 + \gamma_3 T)w - \gamma_4 - \gamma_5 T]^2 - 2(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)$$

$$\times\left\{\frac{1}{2}\gamma_1 I^2+(\gamma_2+\gamma_3 T)I+\gamma_4 T+\frac{1}{2}\gamma_5 T^2-\left[\frac{1}{2}\gamma_1(I+vH(d))^2\right.\right.$$

$$+\gamma_2(I+vH(d))+\gamma_3(I+vH(d))(T-H(d))+\gamma_4(T-H(d))$$

$$\left.\left.\left.+\frac{1}{2}\gamma_5(T-H(d))^2\right]\right\}\right]^{\frac{1}{2}}\cdot\frac{1}{\gamma_1 w^2-2\gamma_3 w+\gamma_5} \tag{16}$$

where $H(d)$ is given by (3). By examining Fig.4 and 5, the algebraically larger root among the two given by (16) is adopted as the value of $H(m')$.

Finally we shall obtain the function f. By eliminating the common parameter γ_4 included in both (16) and (3), we have

$$H(m')=\frac{-K-\sqrt{D}}{\gamma_1\cdot w^2-2\gamma_3\cdot w+\gamma_5}\quad\cdots\quad\text{function}\quad f \tag{17}$$

and,

$$K\equiv(w-v)(\gamma_1\cdot I+\gamma_2+\gamma_3\cdot T)-(\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5)h_d^*$$

$$D\equiv(w-v)\{(w-v)(\gamma_1\cdot I+\gamma_2+\gamma_3\cdot T)^2-2(\gamma_1\cdot I+\gamma_2+\gamma_3\cdot T)$$

$$\times(\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5)h_d^*$$

$$+(\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5)\left[2\gamma_3-\gamma_1(w+v)\right](h_d^*)^2\}$$

where h_d^* is the abbreviation of $H(d)$ given by (3). Equation (17) is the function f when the preference function ω is quadratic.

Analytical Form of function ψ. Function ψ stands for the relation between point d and e in Fig.6 and 7.

Firstly the coordinate of $H(d)$ is given by

$$H(d)=\frac{(\gamma_1\cdot v-\gamma_3)I-v(\gamma_2+\gamma_3\cdot T)+\gamma_4+\gamma_5\cdot T}{\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5} \tag{3}$$

as previously shown in 2.1.2.7.1.1. However,

$$H(d)>\overline{h} \tag{3'}$$

must hold here in order that point d lies below p in Fig.6 and 7.

In the second place we shall obtain the coordinate of point e. Taking into account that the coordinates of point k is given by

$$X = I + w \cdot \overline{h} \quad (18) \quad ; \quad \Lambda = T - \overline{h} \quad (19)$$

the equation of line fD passing through point k is written as

$$X = I + (w - v)\overline{h} + v \cdot h_{fD} \quad (20)$$

where h_{fD} stands or the coordinate of hours of work on the line fD.

Under the constraint of (20), we shall obtain the value of h_{fD} maximizing ω in (2). This value of h_{fD} is $H(e)$. Hence we have

$$H(e) = \frac{-(\gamma_1 \cdot v - \gamma_3)[I + (w - v)\overline{h}] - v(\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} \quad (21)$$

We are now ready to obtain the analytical form of ψ:

That is, by eliminating γ_4 in both (3) and (21), the relation between $H(d)$ and $H(e)$ is derived.

$$H(e) = H(d) - \frac{(\gamma_1 \cdot v - \gamma_3)(w - v)\overline{h}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} \quad \cdots \mathit{function} \quad \psi \quad (22)$$

where

$$H(d) > \overline{h}, \quad and \quad w > v,$$

is obtained. This is the function ψ when the preference function ω is quadratic.

2.3. Calculation of supply probability

2.3.1. The coordinates of points q_1 and q_4

It can be seen that function f contains preference parameters, γ_1, γ_2, γ_3 and γ_5, and exogenous variables, $v, w, \overline{h}$ and I, respectively; that is, f is rewritten as

$$H(m') = f\left[H(d), \gamma_1, \gamma_2, \gamma_3, \gamma_5 |\ v, w, \overline{h}, I\right] \quad (23)$$

where $H(d) > o$.

In the same fashion function ψ can be rewritten as

$$H(e) = \psi\left[H(d), \gamma_1, \gamma_3, \gamma_5 | v, w, \overline{h}\right] \quad (24)$$

where $H(d) > \overline{h}$.

Applying $H(m') = \overline{h}$ to the left hand side of equation (23), we have

$$\overline{h} = f\left[H(d), \gamma_1, \gamma_2, \gamma_3, \gamma_5 | v, w, \overline{h}, I\right] \quad (25)$$

This equation can be solved for $H(d)$. Let us denote the solution for $H(d)$ by $H(d)q_1$. Hence

$$H(d)q_1 = f^{-1}\left[\gamma_1, \gamma_2, \gamma_3, \gamma_5 | v, w, \overline{h}, I\right] \quad (26)$$

where f^{-1} stands for the inverse function of f. $H(d)q_1$ given by (26) is the coordinate of point q_1 on the $H(d)$ axis in Fig.8. Applying quadratic utility function (2), (26) can be written as [5]

$$H(d)q_1 = \overline{h} - \sqrt{(\overline{h}^2 - \frac{\overline{h}[r_1 w^2 - 2r_3 w + r_5)\overline{h} + 2(w-v)(r_1 I + r_2 + r_3 T)]}{r_1 v^2 - 2r_3 v + r_5}} \tag{26$'$}$$

Now we shall obtain the coordinate of point q_4 in Fig.8. Replacing $H(e)$ on the left hand of equation (24) by $\overline{h}$ we have

$$\overline{h} = \psi\left[H(d), \gamma_1, \gamma_3, \gamma_5 | v, w, \overline{h}, I\right] \tag{27}$$

We can solve (27) with respect to $H(d)$ and let us denote the solution by $H(d)q_4$. Hence we have

$$H(d)q_4 = \psi^{-1}\left[\gamma_1, \gamma_3, \gamma_5 | v, w, \overline{h}\right] \tag{28}$$

where ψ^{-1} is the inverse function of ψ. Equation (28) gives the coordinate of point q_4 in Fig.8. Applying quadratic utility function (2), (28) can be written as [6]

$$H(d)q_4 = \overline{h} + \frac{(r_1 v - r_3)(w-v)\overline{h}}{r_1 v^2 - 2r_3 v + r_5} \tag{28$'$}$$

It can be seen that $H(d)q_4$ is invariant with the principal earner's income level, I, because (28$'$) does not contain I as an argument. This stems from the characteristics of quadratic utility function (2). [7]

2.3.2. Density distribution function of $H(d)$

Finally we shall discuss the density distribution function of $H(d)$. $H(d)$ has been given by (see 2.1.2.7.1.1)

$$H(d) = \frac{-(\gamma_1 \cdot v - \gamma_3)I - v(\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} \tag{3}$$

Where the magnitude of γ_4 varies among households considered. With respect to a household i, the value of γ_4^i is given by

$$\gamma_4^i = \overline{\gamma}_4 \cdot u_i + \gamma_4^0 \tag{29}$$

where $\overline{\gamma}_4$ and γ_4^0 are constants which are common to all the households considered and u_i is a random variable, the distribution of which is log-normal [8] with mean $E(u_i)$, and variance σ_u^2, where

$$E(u_i) = 1,$$

[5] Procedure of calculation is given in [Obi, 1987, 88].

[6] Procedure of calculation is given in [Obi, 1987, 88].

[7] Several additional constraints can be obtaines with respect to (26$'$) and (28$'$). These contraints are useful for the estimation of preference paramters. Precise description on these points are given in [Obi, 1987, 88].

[8] Precise discussion is given in [Obi, 1987, 88].

σ_u^2 being a constant. Let the density distribution of u_i be

$$\ell(u|\sigma_u^2) \tag{30}$$

where the suffix i is deleted. By considering (29), (3) can be reduced to

$$H(d) = \frac{-(\gamma_1 \cdot v - \gamma_3)I - v(\gamma_2 + \gamma_3 \cdot T) + \overline{\gamma}_4 \cdot u + \gamma_4^0 + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} \tag{3}$$

Solving this equation with respect to u, we have

$$u = \frac{1}{\overline{\gamma}_4}\{(\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5)H(d)$$

$$+ (\gamma_1 \cdot v - \gamma_3)I + v(\gamma_2 + \gamma_3 T) - \gamma_4^0 - \gamma_5 T\Big\} \tag{31}$$

or in short,

$$u = u(H(d), \gamma_1, \gamma_2, \gamma_3, \overline{\gamma}_4, \gamma_4^0, \gamma_5 | v, I) \tag{32}$$

From (31) we have

$$d_u = \frac{1}{\overline{\gamma}_4}(\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5) \cdot dH(d) \tag{33}$$

From (32), (30) and (33), we have

$$\ell(u)d_u = \ell[u(H(d), \gamma_1, \gamma_2, \gamma_3, \overline{\gamma}_4, \gamma_4^0, \gamma_5 | v, I)\sigma_u] \left|\frac{d_u}{dH(d)}\right| dH(d)$$

$$= \ell_H(d)|\gamma_1, \gamma_2, \gamma_3, \overline{\gamma}_4, \gamma_4^0, \gamma_5, v, I\sigma_u) \left|\frac{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}{\overline{\gamma}_4}\right| dH(d) \tag{34}$$

This is the function which transforms the distribution function of $u, \ell(u)$, to that of $H(d), \ell[H(d)]$. The right hand side of equation (34) (except for $dH(d)$) is the density distribution function of $H(d)$ depicted in Fig.8. For the sake of brevity, let us denote the distribution function, the right hand side of (34), (except for $dH(d)$) by

$$\ell^*(H(d)|\gamma_1, \gamma_2, \gamma_3, \overline{\gamma}_4, \gamma_4^0, \gamma_5 | v, I), \sigma_u) \tag{35}$$

It can be seen from (34) that the distribution of $H(d)$ is invariant with respect to changes in w.

2.3.3. Supply Probabilities

By using (35), μ_d shown by area S_2 in Fig.8 is given by the definite integration of $\ell^*, i, e.$

$$\mu_d = \int_0^{H(d)q_1} \ell^*(H(d)|\gamma_1, \gamma_2, \gamma_3, \overline{\gamma}_4, \gamma_4^0, \gamma_5 | v, I, \sigma_u) dH(d) \tag{36}$$

where $H(d)q_1$ is given by (26).

In the same manner, u_e shown by area S_3 in Fig.8 is given by

$$\mu_e = \int_{H(d)q_1}^{H(d)q_4} \ell^*(H(d)|\gamma_1,\gamma_2,\gamma_3,\overline{\gamma}_4,\gamma_4^0,\gamma_5|v,I,\sigma_u)dH(d) \tag{37}$$

where $H(d)q_4$ is given by (28). The value of u_{ed} shown by area S_4 in Fig.8 is given by

$$\mu_{ed} = \int_{H(d)q_4}^{\infty} \ell^*(H(d)|\gamma_1,\gamma_2,\gamma_3,\overline{\gamma}_4,\gamma_4^0,\gamma_5|v,I,\sigma_u)dH(d) \tag{38}$$

2.3.4.

It can be seen from (36), (37) and (38) that the values of three kinds of supply probabilities μ_e, μ_d and μ_{ed}, are respectively determined by the values of $\gamma_1,\gamma_2,\gamma_3,\gamma_4^0,\overline{\gamma}_4,\gamma_5,\sigma_u,v,\ w$ and I. It should be noted that the magnitude of w affects the probabilities via limits of integration, $H(d)q_1$ and $H(d)q_4$, as well, because these are functions of w respectively.

Employing an abridged formulation, (36), (37) and (38) can be rewritten as

$$\mu_d = \mu_d(\Gamma,\sigma_u,v,w,I), \tag{39}$$

$$\mu_e = \mu_e(\Gamma,\sigma_u,v,w,I), \tag{40}$$

and

$$\mu_{ed} = \mu_{ed}(\Gamma,\sigma_u,v,w,I), \tag{41}$$

where Γ stands for a set of parameters $\{\gamma_1,\gamma_2,\gamma_3,\gamma_4^0,\overline{\gamma}_4,\gamma_5\}$.

Making use of these relation we can proceed to obtain the estimates of the preference parameters, Γ, and σ_u. This procedure is shown in the following section.

3. Estimates of preference parameters

Estimates of the preference parameters, $\gamma_1(\equiv -1),\gamma_2,\gamma_3,\overline{\gamma}_4,\gamma_4^0,\gamma_5$ and σ_u, are obtained by the maximum likelihood method. Firstly let the number of non-principal potential earners(wives) in their principal earners income group be N_i. Among N_i non-principal potential earners (abridged as NPPE), let the number of employee-workers, self-employed workers, and persons engaging in both employee and self-employed work be n_i^d, n_i^e, and n_i^{ed} respectively. The probability , P_i, of occurance of n_i^d, n_i^e, and n_i^{ed} occuring at the same time in the ith income group $(i = 1,2,\cdots,m)$ can be obtained by,

$$P_i = \frac{N_i!}{n_i^d!\,n_i^e!\,n_i^{ed}!\,n_i^\lambda!}(\mu_i^d)^{n_i^d}(\mu_i^e)^{n_i^e}(\mu_i^{ed})^{n_i^{ed}}(\lambda_i)^{n_i^\lambda}, \tag{42}$$

where n_i^λ stands for the number of NPPE who do not work, and $\lambda_i \equiv \frac{n_i^\lambda}{N_i}$.

By definition,

$$\mu_i^d + \mu_i^e + \mu_i^{ed} + \lambda_i \equiv 1. \tag{43}$$

The probability of occurance of $n_i^d, n_i^e, n_i^{ed} n_i^\lambda (i = 1,2,\cdots,m)$ for all the income group considered at the same time is given by $\prod_{i=1}^m P_i$.

Hence, taking into account (42), we have the likelihood function, L,

$$L \equiv \log \prod_{i=1}^{m} P_i = \sum_{i=1}^{m} [\log N_i! - (\log\ n_i^d! + \log\ n_i^e! + \log\ n_i^{ed}! + \log\ n_i^{\lambda}!)$$

$$+ n_i^d \log\ \mu_i^d + n_i^e \log\ \mu_i^e + n_i^{ed} \log\ \mu_i^{ed} + n_i^{\lambda} \log\ \lambda_i\ ] \tag{44}$$

Replacing μ^d, μ^e and μ^{ed} in (44) by (36),(37) and (38), we have

$$L = \sum_{i=1}^{m} [\log N_i! - (\log\ \ n_i^d! + \log\ \ n_i^e! + \log\ \ n_i^{ed}! + \log\ \ n_i^{\lambda}!)$$

$$+ n_i^d\ \log\{\mu_i^d(I_i, w, \overline{h}, v|\Gamma, \sigma_u)\} + n_i^e\ \log\{\mu_i^e(I_i, w, \overline{h}, v|\Gamma, \sigma_u)\}$$

$$+ n_i^{ed}\ \log\{\mu_i^{ed}(I_i, w, \overline{h}, v, |\Gamma, \sigma_u)\ \ \}] \tag{44'}$$

We used shugyo kozo Kihonchosa (Employment status survey ; type A households were selected) for the estimation of preference parameters. Sample sizes are shown in Tab.3.

In order to maximize the likelihood function L with respect to the preference parameters, we need the initial values of the preference parameters for the computation.

We tentatively employ the values of parameters obtained in the previous analyses (we shall call these old estimates). These were obtained using FIES (Family income and expenditure survey data) in 1961 through 1964. [9] However, the sample size in the above was far less than those used in this analysis. [10]

The old estimates of the preference parameters did not satisfy the following restlictions when they were applied to the new data;i, e.

$$(1)\quad \frac{\partial w}{\partial \Lambda} > 0, \qquad (2)\quad \mu^d > 0, \qquad (3)\quad \mu^{ed} > 0$$

Taking these facts into account, we sought the areas of parameters satisfying the theoretical constraints. [11] The set of parameters thus obtained is given in the first column of Table 2.

Starting from these initial values, we obtained the estimates (maximum likelihood) given in the second column of the Table 2.

The values of the preference parameters are common for all the years, 1971 through 1977. However, it was found that, using those estimates in the second column in Tab.2 as the initial values, we could remarkably revise the values of the likelihood function by allowing the parameters $\overline{\gamma}_4$ and γ_5 to change year by year. The values of the preference parameters, other than $\overline{\gamma}_4$ and γ_5, are common for the observed years 1971 through 1977. The results of maximum likelihood estimation are shown in the third column of Tab.2 [12]

[9] These old estimates of the preference paramters are given in [Obi, 1987, 88].

[10] On account of the smallness of sample size, the estimates of observational errors for μ^e, μ^d and μ^{ed} in the old data were fairly large. These are shown in [Obi, 1987, 88].

[11] More precise description is given in [Obi, 1993] forthcoming.

[12] Discussions on the identification of preference parameters are given in [Obi, 1987, 1988] and [1992].

Table 2 : Estimates and Standard Errors

	1971	1974	1977
γ_2	3947.889	3947.889	3947.889
standard error	3.0630	6.4263	4.2759
γ_3	1947.171	1947.171	1947.171
standard error	3.2822	8.5028	3.2124
$\overline{\gamma_4}$	1320608.	2432788.	2712545.
standard error	532.1113	1331.5705	982.3230
γ_5	−770900.7	−1666412.	−1979044.
standard error	460.8712	1503.9327	952.8991
γ_4^0	64669.40	64669.40	64669.40
standard error	388.9118	1217.0307	795.2554
σ_u	0.1261800	0.1261800	0.1261800
standard error	0.0001576	0.0003914	0.0001768

4. Results of simulation: the effects of principal earners' income I, wage rates w, assigned hours of work $\overline{h}$ and earning rate of self-employed work v on three kinds of supply probabilies, μ^d, μ^e and μ^{ed}.

By using the estimates of the preference parameters in Table 2, we can examine the effects of principal earners' income, I, NPPE'S wage rate w, the essigned hours of work $\overline{h}$, and the earning rate of self-employed work, v, or the supply probabilities, μ^d, μ^e and μ^{ed}.

The effects are shown by the elasticities in Table 3. The results can be summarized as follows;

4.1.

The elasticities of the exogenous variables, $w, \overline{h}$ and v respectively, on the supply probabilities, μ^d, μ^e and μ^{ed} are different from each other. However, the order of their magnitudes are quite stable during the observed years, 1971 through 1977.

4.2.

(a) With respect to the employee probability μ^e, the absolute values of the elasticity of w, the wage rate, is the largest, following $\overline{h}$ and v in order.

(b) The augmentation of w increases μ^e.

(c) Reducing(increasing) the assigned hours of work, $\overline{h}$, increases (decreases) μ^e. This is the economic foundation of the so called "part time workers", or "short time

Table 3

elastisity of μ^e, μ^d, μ^{ed} to $w, \bar{h}, v$ and I

unit of income: 1,000 yen per month (1961 constant price)

			1971	1974	1977
household heads' income			47.7	49.3	49.6
sample size			6881	8136	7589
μ^e	·wage rate of work	(w)	7.20	4.76	4.03
	·assigned hour of work	$(\bar{h})$	−1.71	−1.47	−1.36
	·earning rate of self-employed work	(v)	−0.69	−0.38	−0.36
	·principal earners income	(I)	−1.12	−0.58	−0.48
μ^d	·wage rate of work	(w)	−23.90	−22.17	−18.37
	·assigned hour of work	$(\bar{h})$	5.83	7.17	6.60
	·earning rate of self-employed work	(v)	23.98	19.60	15.49
	·principal earners income	(I)	−0.44	−0.16	−0.09
μ^{ed}	·wage rate of work	(w)	−2.09	−1.40	−1.21
	·assigned hour of work	$(\bar{h})$	−9.83	−8.82	−8.51
	·earning rate of seslf-employed work	(v)	16.90	10.53	8.83
	·principal earners income	(I)	−3.58	−1.87	−1.60

workers" in Japan; employers can recruit more workers by reducing the assigned working hours without increasing the wage rate.

(d) Increase (decrease)in v reduces (increase) μ^e . However, the elasticity of v is far less than unity.

4.3.

(a) With respect to the self-employed probability, μ^d , the absolute value of the elasticity of w is the largest, as in the case for μ^e (but the sign is mius). However, in contrary to the case for μ^e, the second largest is the elasticity of v, and the smallest is that of $\bar{h}$.

(b) Increase (decrease) in w decreases (increases) μ^d.

(c) Reduction (augmentation) of $\bar{h}$,the assigned hours of employee work, decreases (increases) μ^d.

(d) Increase (decrease) in v augments(reduces) μ^d. The elasticity of v is extremely large (over 23); that is, μ^d is very much affected by v as well as w.

4.4.

(a) As for μ^{ed} , the probability of being engaged in both employee and self-employed work , the absolute value of the elasticity is the largest in v , the second largest is $\bar{h}$, and the smallest is w.

(b) Increase (decrease) in w decreases (increases) μ^{ed}.

(c) Reduction (augmentation) in $\bar{h}$ increases (decreses) μ^{ed}.

(d) Increase (decrese) in v increases (decreses) μ^{ed}.

4.5.

The absolute values of the elasticities of $w, \bar{h}$ and v are larger than unity respectively, except for the effect of the change in v on μ^e.

5. Conclusions

Based on the estimation results of the preference parameters, the following can be concluded:

(1) During the observation periods, 1971, 1974, 1977, we find cross sectional and time serial variations in three kinds of supply probability μ^d, μ^e and μ^{ed}. These time serial and cross sectional variations can be completely explained by the variations in the principal earners income I, the non-principal earners' wage rate w, the assigned house of work, $\bar{h}$, and the earning rate of self-employed work v.

(2) Among the seven preference parameters $\gamma_1(\equiv -1), \gamma_2, \gamma_3, \sigma, \gamma_4^0, \overline{\gamma}_4$ and γ_5 the first four parameters are constant during the seven years, 1971 through 1977. The last two parameters, $\overline{\gamma}_4$ and γ_5 are related to the marginal utility of leisure. It was found that these two parameters shifted during the observational period. However, there seems to be some regularities with respect to the shifts in these two parameters: that is, the shifts in these parameters affect the regularity in the shifts of the center of the hyperabola, of which the locus in $X \sim \Lambda$ plane is straight line as shown in Fig.9.

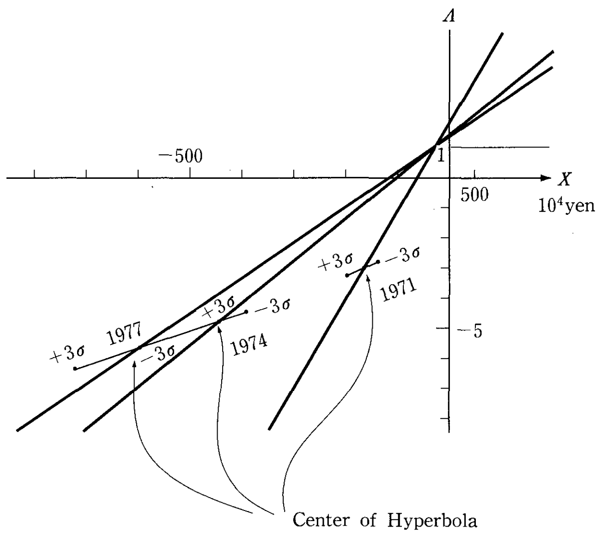

Figure 9

(3) Estimated Distribution of income-leisure preference curve is graphically exemplified in Fig.10.

(4) The redefinition of reservation wage is usefull and it helps to lead to a positive and comprehensive model of self-employed workers and employees.

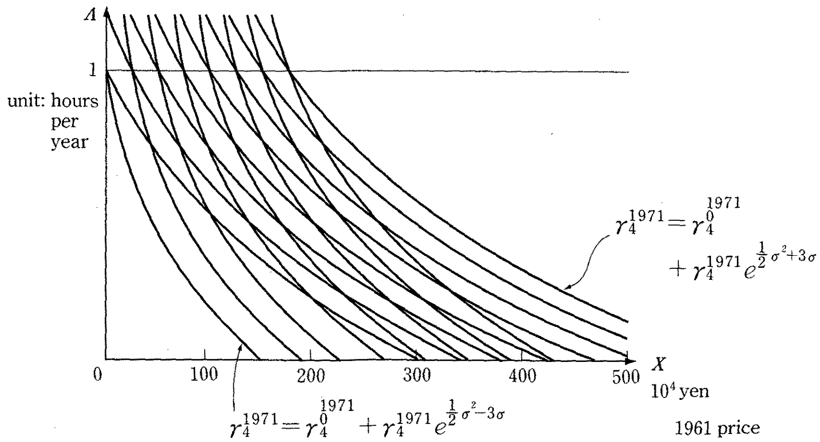

Figure 10-A

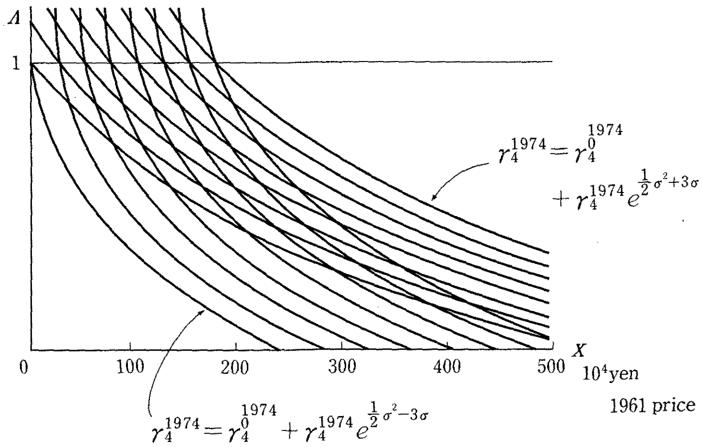

Figure 10-B

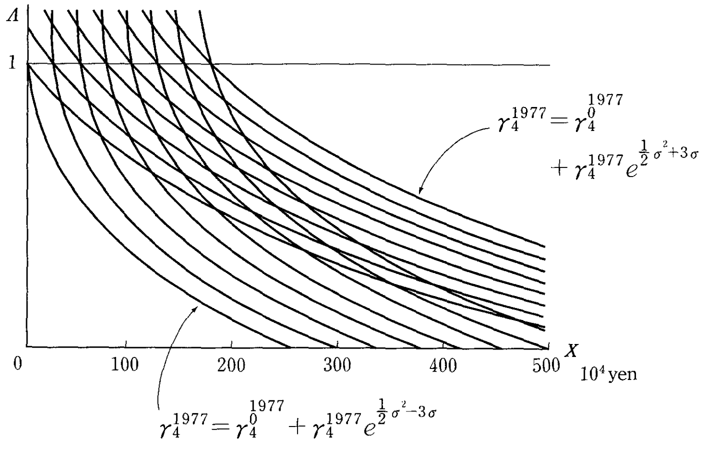

Figure 10-C

(5) We obtained the effects of principal esrners' income, I, the nonprincipal potential earners' wage rates, w, the assigned hours of work, $\bar{h}$, and the earning rate of self-employed work, v, on three kinds of supply probabilities, μ^d, μ^e and μ^{ed}. These are summarized in 4.1 $\sim$ 4.5.

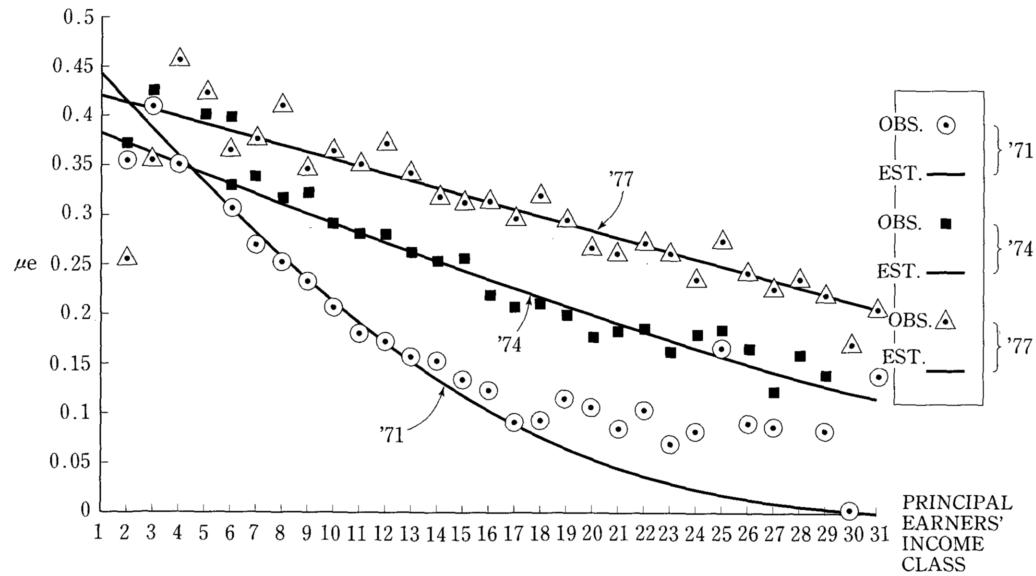

Figure 11-A

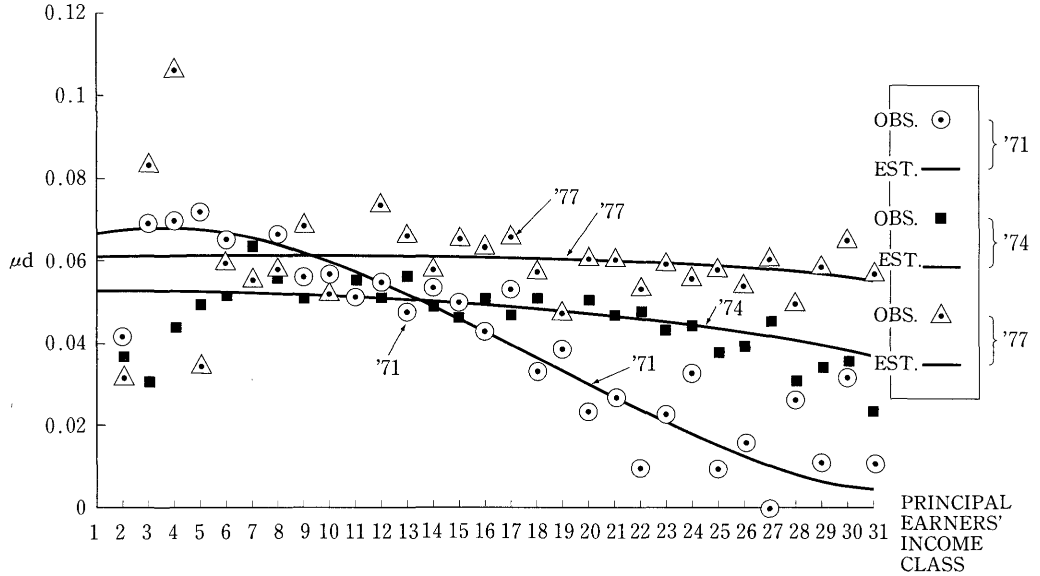

Figure 11-B

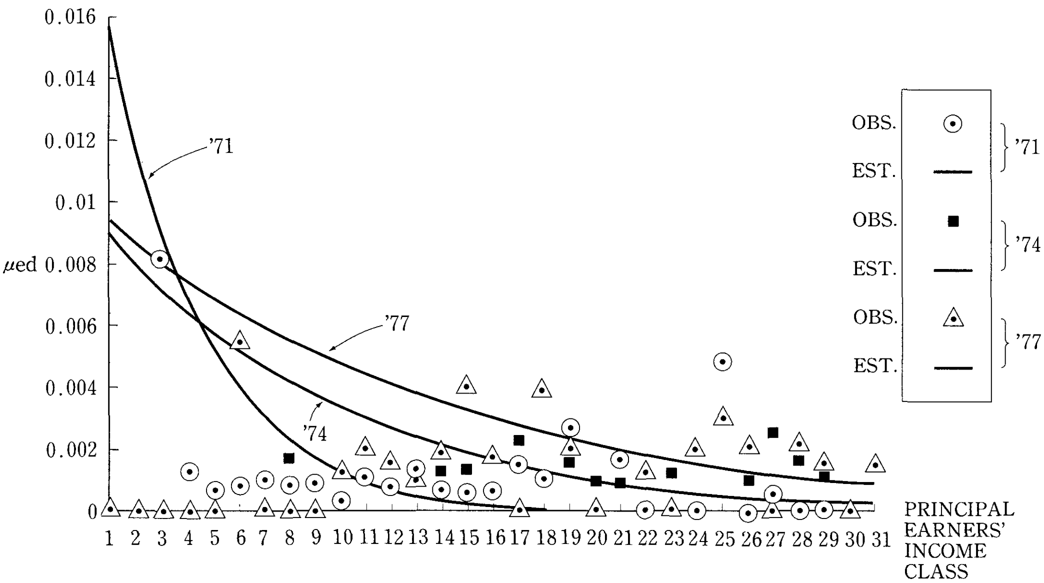

Figure 11-C

Appendix : Relation Between MSPL and $H(d)$

Let us consider a household whose indifference map is shown in Fig.A-1.
Now, imagine an experiment in which

(1) The principal earner's income is given at I_1, and, at the same time.

(2) The self-employed work with an earning rate v (shown by $\tan\theta_v$) is assigned.

As it can be seen from Fig.A-1, the optimal hours for the self-employed work, $H(d)$, of this household are shown by the ordinate of point d. $\tan\theta_k$, given by connecting point k and I_1 stands for the minimum supply price of employee work (MSPL) whose assigned working hour is given as $\overline{h}$ (as shown in Fig.A-1).

From this figure, we easily obtain the relation which transforms MSPL to $H(d)$, and vice versa; that is, the equation of the straight line I_1B is given by

$$X = I + vH \quad ,where \quad H = T - \Lambda \quad ,and \quad I = I_1 \tag{1}$$

Let the preference function of this household be

$$\omega = \omega(X, \Lambda, \Gamma), \tag{2}$$

where the set of preference parameters Γ stands for those spectfic to this this household. We can maximize ω under the constraint (1);

Define,

$$F \equiv \omega(X, T - H, \Gamma) + \lambda(X - I - vH) \tag{3}$$

where λ is the Lagrangean multiplier. By

$$\frac{\partial F}{\partial X} = \frac{\partial F}{\partial H} = 0, \tag{4}$$

we have

$$\frac{\partial F}{\partial X} = \frac{\partial \omega}{\partial X} + \lambda = 0 \quad and \quad \frac{\partial F}{\partial H} = \frac{\partial \omega}{\partial H} + \lambda(-v) = 0. \tag{5}$$

From these, we have

$$\frac{\frac{\partial \omega}{\partial H}}{v} = \frac{\partial \omega}{\partial X}. \tag{6}$$

From (1) and (6) , we can obtain the solutions for H and X,

$$H_d = H(I, v, \Gamma) \tag{7}$$

and

$$X_d = X(I, v, \Gamma). \tag{8}$$

These are the coordinates of point d in Fig.A-1.

We obtain the equation of ω_d by inserting (7) and (8) to (2).

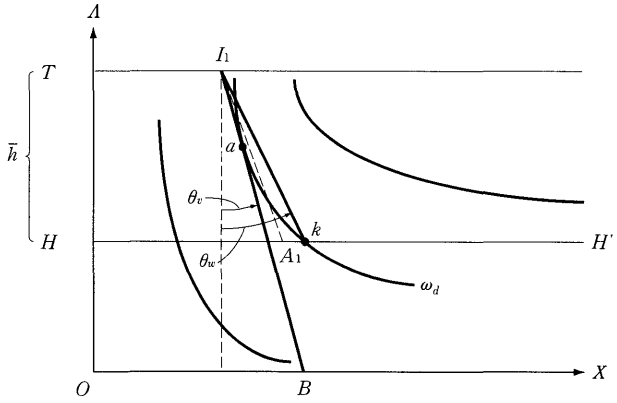

Figure A–1

$$\omega(X_d, T - H_d, \Gamma) = \omega(X, T - H, \Gamma) \tag{9}$$

This is the equation of ω_d. The intersection of ω_d and the horizonal line HH' in Fig.A-1 , can be obtained by inserting (7) into (9), and thus solving for X ; that is, firstly we have

$$\omega(X_d, T - H_d, \Gamma) = \omega(X, T - H_d, \Gamma) \tag{10}$$

and we solve (10) for X. Let us denote the solution of X by X_k. Hence, we have

$$X_k = G_k(X_d, \Gamma) \tag{11}$$

or taking into account (7) , we have

$$X_k = G_k[H(I, v, \Gamma), \Gamma] \tag{12}$$

This is the abcissa of point k in Fig.A-1. The minimum supply price for labor, MSPL $\underline{W}$ is given by

$$\underline{W} = \frac{X_k - I}{\bar{h}} \tag{13}$$

By inserting (12) to (13) , MSPL $\underline{W}$ can be obtatined ;

$$\underline{W} = \frac{G_k[H(I, v, \Gamma), \Gamma]}{\bar{h}} = \underline{W}(I, v, \bar{h}\Gamma). \tag{14}$$

This is the value of MSPL for this household when I, v, and $\bar{h}$ are given.

From (14) and (7) , we can eliminate the common variable I, and we have a transformation function $F_{\underline{w}}$ with respect to $\underline{w}$ and H_d ;

$$F_{\underline{w}}(\underline{W}, H_d | v, \bar{h}, \Gamma) = 0. \tag{15}$$

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