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An Equilibrium Model of Continually Heterogeneous Labor Market*

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1. Introduction

In this paper a model of the labor market, where wage differentials among the firms of various scales exist, is presented. The term"firm of various scale" is used to indicate that the heights of marginal productivity curves for labor are different for different firms.

If the labor market is competitive, a unique wage rate prevails so long as the labor force is homogeneous from the firm's point of view. If the labor force is heterogeneous, but can be split into three groups A, B and C, where firm "a" exclusively recruits workers from group A and the members of group A exclusively apply to a, and so on, we have three independent labor markets and the notion of non-competing groups can be applied to determine wages within each market. However, if the firms a,b and c respectively recruit among all the members of the groups A,B and C simultaneously, then the notion of non-competing groups is not applicable to the labor market. Since the actual labor market we observe has such a nature, we need to construct a model which can describe the performance of a competitive and heterogeneous labor market.

By heterogenity, we mean the existence of various grades (or labor ques) among applying workers from the firm'point of view. The grades or ordering of applicants might be directly or indirectly correlated with their work experience,educational background, age, and/or sex. However, even if those characteristics or qualities are controlled, there may yet exist some ordering or differences in grades of applying workers. In fact, statistical data shows that there are wage differences among workers of firms of different sizes when controling for these characteristics of the workers.

This observed fact suggests that firms recongnize different grades among workers of the same age, sex, work experience, and/or educational backgrounds. Any reason for the paying of higher wages by large firms, whose labor productivities are higher than smaller ones, cannot be found as long as the grades of workers are the same across firms. In fact, large scale firms with higher productivities offer comparatively favorable work conditions (higher wages and shorter hours of work) and as a result attract many applicants of various grades.The firms recruit what they perceive as the most favorable ones among those who applied. Smaller firms with lower productivity can offer only less favorable terms and recruit among the residual applicants who fail to be employed by the large scale firms. This is the common experience of high school and college graduates in Japan.

In the following section we present a model of the labor market making use of the notion of grades[1] of labor in order to realistically approximate the labor market in Japan.

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[1] The notion of this kind, that is, labor que, is used in L.C.Thurow [10]

The model is suitably simplified. Although the labor supply actually consists of members of self - employed households (e.g. farmers' households) and employee households whose principal earners are employees, only the latter type of household is taken into account. As well, the investment behavior of firms is not explicitly treated. These simplifications will not impair the basic characteristics of the model which remain sufficiently autonamous. The performance of the model is tested by numerical examples and by the application to Japanese data.

Models of wage and employment determination with respect to a firm (or a group of firms) have been developed elsewhere.[2] In this kind of model, individual labor supply and labor demand functions for a firm are assumed; that is, the notion of a kind of local labor market is introduced in the models. However, the relation between the individual labor supply function for the specific firm considered and the supply function of the market as a whole is not explicitly discussed. Such an individual supply function is, to some extent, an ad hoc relation just as is the individual demand function for a firm′ s product in an oligopolistic market. The wage level of the firm considered and the average wage level of other firms appear as explanatory variables of the individual labor supply function(The ratio of the both variables is adopted insome cases).

Elasticities of labor supply with respect to those variables or coefficients of those variables for each firm change, reflecting change in the conditions of the labor market as a whole including changes in the degree of of competition, the labor suppliers' conjecture with respect to the recruitment policy of firms other than the firms to which the suppliers are applying. However, the mechanism of such interdependent changes of elasticities or coefficients of individual supply function has not been clarified. In this sense,models using individual supply functions lack autonomy.

Individual labor supply functions for each firm are not used in the model presented below. Instead, two basic relations are introduced. Instead of an individual supply function for a firm, which describes the relation between the number of applicants for the specific firm and the wage rate the firm offers, we use the labor supply function for the whole market describing the quantity of labor supplied, the wage rate being given. That is, the supply function used in the following model does not specify the distribution of the quantity of labor supplied among firms. The distribution itself is determined by the model including firm demand functions for labor.

2. Basic Equations of the Model

2.1. Distribution Function of Grades of Labor

Let the indicator of the grade of the worker be G_i, where

$i = 1, 2, ..., m$,

and m is the total number of people of working age. The range of G_i is supposed to be

$$\varepsilon \leq G_i \leq 1$$

where ε is some positive small number.

The cumulative distribution (cumulative from the top of G, where $G = 1$) function of G is designated by $\nu(G)$ and the density distribution by $\nu'(G)$.

[2] C.A.Pissarides [9]

2.2. Labor Supply Probability Function

Suppose among n persons, n' persons accept the employment opportunity at $w$age rate w, and assigned hoursof work h, offered by firms. The ratio n'/n is the supply ratio with respect to the employment opportunity.

$$\operatorname*{Plim}_{n\to\infty} n'/n \equiv \mu$$

is defined as the supply probability, which is a function of w and h.

2.3. Distribution of Minimum Supply Price of Labor

The minimum supply price of labor[3] is defined as a critical wage rate below which suppliers reject the employment opportunity, assigned hours of work h being given. The minimum supply price of labor (MSPL) is denoted by $\underline{w}$. Any supplier's level of MSPL depends on following three factors:

a) the shape of his/her income-leisure preference curve,

b) the level of his guaranteed income X_g which he/she can obtain without working (e.g. principal earner's income is a guaranteed income for non principal earners),

c) hours of work assigned by firms, h.

Hence, we have

$$\underline{w}^i = \underline{w}(x_g^i, h^i, \alpha^i) \qquad i = 1, 2, \ldots, n \tag{1.1}$$

where α^i stands for the set of preference parameters of the ith supplier. x_g^i and h^i can be regarded as exogenous variables for the ith supplier. The value of α^i is specific to ith supplier; that is, the value of α^i differs among each of the n suppliers. Hence, we have the density distribution function $\phi(\alpha)$.

Now, suppose a group of persons have the same level of guaranteed income $\overline{x}_g$; that is,

$$x_g^i = x_g^{i+1} = \overline{x}_g \tag{1.2}$$

From(1.1), (1.2) and $\phi(\alpha)$, we have

$$g_{f\phi}(\underline{w} \mid \overline{x}_g, h) \tag{1.3}$$

[3] The notion of reservation wage(RW) is given in Heckman[2]. MSPL is another definition of a kind of RW, because MSPL is defined by assuming h is a parameter which is assiged by the firm. In the modern employee labor market (in contract to self employed work), hours of work are assigned by the employer(firm). The employees have some minimal leeway as they can reduce or increase the hours worked to some extent. However, a complex array of social, psychological, and institutional factors usually produces a situation where excessive absenteeism etc. will results or dismissal. The analogus situation exists with respect to "overtime". Analysis of the MSPL of labor using an income–leisure preference fuction assuming maximization behavior is shown in Keiichiro Obi[5],[6],[7]. and T.Miyauchi[4]

which is the density distribution function of MSPL, h for brevity being assumed a common value for all persons considered. Subscript f and ϕ denote the fact that the analytical form of the function g depends on f and ϕ. Integration of g,

$$\mu = \int_{\underline{w}=0}^{w} g(\underline{w} \mid \overline{x}_g, h) d\underline{w} = \mu = \mu(w \mid \overline{x}_g, h), \tag{1.4}$$

gives the supply-probability function μ of the group of persons with $\overline{x}_g$ and h.

Multiplying by n, the number of persons in the group, we have the number of suppliers L^s , namely,

$$L^s = n\mu(w \mid \overline{x}_g, h), \tag{1.5}$$

When x_g and h are destributed as a joint density distribution

$$\psi(x_g, h), \tag{1.6}$$

we have

$$\mu(w) = \int_{\underline{w}=0}^{w} \int_{x_g=c}^{d} \int_{h=a}^{b} g(\underline{w}, x_g, h)\psi(x_g, h) \cdot dh \cdot dx_g \cdot d\underline{w} \tag{1.7}$$

where a, b, c, d and e are the values standing for regions of integration for the relevant variables, h, x_g and w.

3. The Outline of the Model

Let the production function of the ith sector (or firm) be

$$Q_i = F(L_i, \overline{G}_i, A_i) \qquad (i = 1, 2, \ldots, n) \tag{2.1}$$

where A and $\overline{G}$, respectively, stand for the set of firm parameters and the index of the grade of workers employed in the ith sector. Further, $\overline{G}_i$ can be written as

$$\overline{G}_i = \overline{G}_i(G_i^{min}, G_i^{max}) \tag{2.1$'$}$$

where G_i^{max} and G_i^{min} are indicators of the highest grade of workers (most preferable workers among applicants from the firm's point of view) and the lowest grade of workers. It is supposed that

$$\frac{\partial F}{\partial \overline{G}_i} > 0, \qquad \frac{\partial F}{\partial L_i} > 0.$$

Let the supply probability equation (1.7) be

$$\mu = \mu(w, \overline{\lambda}) \tag{2.2}$$

where $\overline{\lambda}$ is a set of parameters of individuals, and for the sake of brevity assigned hours of work, h, is excluded and the guaranteed income level x_g is supposed to be included in the set $\overline{\lambda}$.

The (cumulative) distribution function of G is denoted by

$$\nu_G = \nu(G) \tag{2.3}$$

where

$$\varepsilon \leq G \leq 1 \tag{2.4}$$

An indirect method of observation of G is discussed in section 4.1.3.

Let us suppose that the analytical form of the function ν is common to all the sectors under consideration. Hence, by letting the number of potential suppliers be N, the number of suppliers with grade G and over, N_G, is given by

$$N_G = N \cdot \nu_G = N \cdot \nu(G) \tag{2.5}$$

The number of suppliers with grade G and over going to the ith sections, L_s^i, is written as

$$L_s^i = N \cdot \nu(G) \cdot \mu(w_i, \overline{\lambda}) \tag{2.6}$$

where w_i stands for the wage rate offered by the ith sector.

3.1. Behavior of the Leader

Imagine a sector (or firm) which offers the most favorable wage in comparison to other sectors in order to attract a number of potential suppliers. This sector can recruit workers of higher grades comparing to other sectors which offer less favorable working conditions. We shall call this sector a leader sector (firm) or a leader for short. Residual sectors are followers. Among those residual sectors, we can distinguish leaders and followers in accordance with the wage differentials each sector is willing to pay. That is, if we have three sectors with wage rates w_1, w_2 and w_3 where $w_1 > w_2 > w_3$, sector 2 plays the role of the follower of sector 1, while sector 2 plays the role of leader of sector 3. Follower sector 2, against leader sector 1, recruits workers with relatively higher grades amongst residual applicants which the leader has left for followers to employ because those applicants are not fully suitable for employment from the leader's point of view. Sector 2 as a leader against sector 3 will again leave undesirable labor suppliers. This pattern can be viewed as continuing indefinitely, ith sector 3 acting as a leader to sector 4, and so on.

Let us imagine a labor market which consists of two sectors to simply present the basic characteristics of the model, where one of the sectors is able to attain a given level of production Q_i $(i = 1, 2)$ by varying G_i and L_i in the production function (2.1) .

The distribution function (2.5), $N \cdot \nu(G)$, is depicted in the fourth quadrant in Figure 1. The curve GN is the cumulative distribution curve from the top labor grade $G(= 1)$. Suppose firm ℓ (we denote leader by ℓ) wishes to recruit workers with grades higher than G_ℓ^{min}. In this case, the labor supply curve for firm ℓ can be depicted by curve $S_\ell S_\ell'$ in the 1st quadrant. This curve stands for equation (2.6) where G^{min} is inserted for G. Now, $G^{max}(= 1)$ and G^{min} being given for the firm ℓ, the demand curve for labor is derived from the production function (2.1) and (2.1′) by applying the condition of cost minimization. This is depicted by curve D_ℓ in the first quadrant. The intersection of the supply curve $s_\ell\, s'_\ell$ and D_ℓ gives the wage rate w_ℓ and the demand for labor L_ℓ by firm ℓ necessary to attain the given level of production Q_ℓ.

If firm ℓ were content to recruit workers with lesser grades, e.g. $[G^{min}] < G_\ell^{min}$, the curve $s_\ell\, s'_\ell$ would be less steep and stretched to the right. Hence, the required grade of workers would be less and the number of workers employd would increase. At any rate, given the production function (2.1), the grade distribution function (2.3) , and the supply probability function (2.2), the number of workers and the required grade to attain production level Q_ℓ are detemined by the procedure of cost minimization.

From (2.5) and (2.2) the number of potential applicants with grade G_ℓ^{min} and over, $L_{G_\ell}^{min}$, is given by

$$L_{G_\ell}^{min} = N_\ell \cdot G_\ell^{min} \cdot \mu = N \cdot \nu(G_\ell^{min}) \cdot \mu(w_\ell, \overline{\lambda}) \tag{2.6'}$$

which is a function of w_ℓ. Eq.(2.6′) is depicted by the curve $s_\ell s'_\ell$ in Figure 1.

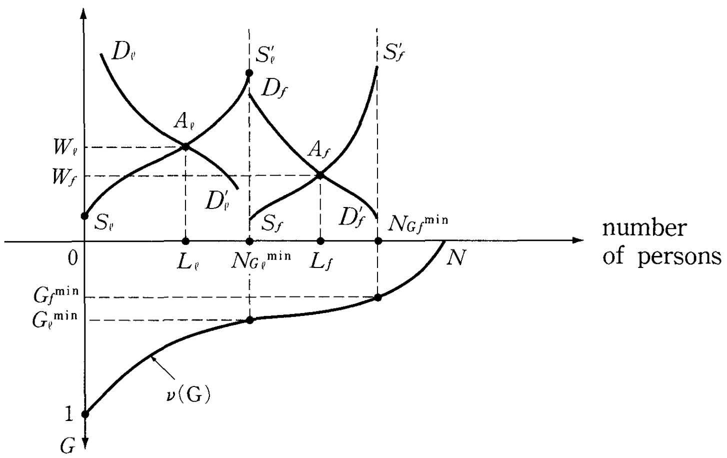


Figure 1

We have, for the leader, $G^{max} = 1$ in (2.1′). Hence, (2.1′) is written as $\overline{G}_\ell = \overline{G}_\ell(G_\ell^{min}, 1)$. Substituting this function and (2.6′) into (2.1) gives the leader's production function.

$$Q_\ell = F[N \cdot \nu(G_\ell^{min}) \cdot \mu(w_\ell^{min}, \overline{\lambda}), \overline{G}_\ell(G_\ell^{min}, 1), A_\ell], \tag{2.1''}$$

where the subscript i in (2.1) has been replaced by ℓ to denote that eq. (2.1″) refers to the leader. Defnition of cost is given by

$$C_\ell = C_0^\ell + w_\ell \cdot L_{G_\ell}^{min} = C_0^\ell + w_\ell N \cdot \nu(G_\ell^{min}) \cdot \mu(w_\ell, \overline{\lambda}) \tag{2.7}$$

where C_0^ℓ stands for capital cost which is regareded as given.

We can obtain w_ℓ and G_ℓ^{min} by minimizing C_ℓ in (2.7) under the constraint (2.1″), Q_ℓ being given:

Letting

$$\psi_\ell = C_\ell + k\{Q_\ell - F[N \cdot \nu(G_\ell^{min}) \cdot \mu(w_\ell, \overline{\lambda}), \overline{G}_\ell(G_\ell^{min}, 1), A_\ell]\} \tag{2.8}$$

where k is Lagrangian multiplyer, and C_ℓ is given by (2 - 7), we have

$$\frac{\alpha\psi_\ell}{\partial G_\ell^{min}} = \frac{\alpha\psi_\ell}{\partial w_\ell} = 0. \tag{2.9}$$

Solving (2.1″) and (2.9) simultaneously for G_ℓ^{min} and w_ℓ , we obtain,

$$G_\ell^* = G_\ell^*(\overline{\nu}_0, \overline{\lambda}, A_\ell, Q_\ell) \tag{2.10}$$

and

$$w_\ell^* = w_\ell^*(\overline{\nu}_0, \overline{\lambda}, A_\ell, Q_\ell) \tag{2.11}$$

where $\overline{\nu}_0$ is a set of parameters in the grade distribution function (2.3). Equations (2.10) and (2.11) give optimal values for G_ℓ^{min} and w_ℓ both minimizing cost G_ℓ for the given production level Q_ℓ. The solution for employment L_ℓ can be calculated by substituting (2.10) and (2.11) into (2.6) for G and w respectively. We shall call the number of workers thus obtained, and G_ℓ^* and w_ℓ^* given by (2.10) and (2.11) as the "leader solution".

3.2. Follower's Behavior

The highest grade of workers attainable to the follower is G_ℓ^{min} which is the lowest grade for the leader. Let the lowest grade of people in the group of potential applicants for the follower be G_f^{min}. The number of people with grades between G_f^{min} and G_ℓ^{min}, which we denote by $N_{G_f}^{min}$, is given by

$$N_{G_f}^{min} = N \cdot \nu(G_f^{min}) - N \cdot \nu(G_\ell^{min}), \tag{2.12}$$

which is shown by the length of $N_{G_f}^{min} \sim N_{G_\ell}^{min}$ in Figure..1. Hence, the number of suppliers to the follower $L_{G_f}^{min}$ is written as

$$L_{G_f}^{min} = N_{G_f}^{min} \cdot \mu = N[\nu(G_f^{min}) - \nu(G_\ell^{min})] \cdot \mu(w_f, \overline{\lambda}). \tag{2.13}$$

Substituting (2.13) into (2.1), we have the ptoduction function of the follower;

$$Q_f = F[N \cdot (\nu(G_f^{min}) - \nu(G_\ell^*)) \cdot \mu(w_f, \overline{\lambda}), \overline{G}_f(G_f^{min}; G_\ell^*), A_f] \tag{2.14}$$

where A_f is the set of parameters of the follower's production function, and G_ℓ^* is given by (2.10). The definition of follower's cost C_f is given by

$$C_f = C_0^f + w_f \cdot L_f = C_0^f + w_f \cdot N[\nu(G_f^{min}) - \nu(G_\ell)] \cdot \mu(w_f, \overline{\lambda}). \tag{2.15}$$

where C_0^f is capital (fixed) cost and (2.13) is substtitued for L_f .

Let us minimize C_f in (2.15) under the constraint of (2.14) where the level of Q_f is given.

$$\psi_f = C_f + j[Q_f - F\{\cdot\}]. \tag{2.16}$$

where j is the Lagrangean multiplier.

The minimization condition is as follows.

$$\frac{\alpha\psi_f}{\partial G_f^{min}} = \frac{\alpha\psi_f}{\partial w_f} = 0. \tag{2.17}$$

Solving (2.17) for G_f^{min} and w_f ,

we have

$$G_f^* = G_f^*(\overline{\nu}_0, \overline{\lambda}_1, A_f, Q_f, G_\ell^*) \tag{2.18}$$

$$w_f^* = w_f^*(\overline{\nu}_0, \overline{\lambda}, A_f, Q_f, G_\ell^*). \tag{2.19}$$

where G_ℓ^* is already given by the leader's solution (2.10). L_f can be obtained from (2.13) by inserting (2.18) and (2.19). We shall call this employment level and (2.18) and (2.19) the "follower's solution".

3.3. Succession Equilibrium

When we have three or more firms (sectors), we can successively apply the above leader-follower relationship. We shall call the state of market shared by firms (sectors) playing the role of leaders and followers successively as succession equilibrium. Let us suppose two firms are in a state of succession equilibrium. Now, suppose relative or absoluate changes in the production level of the leader cause a "leader's solution" with a wage rate w lower than the follower's. Then, of course, the initial state of the market cannot be sustained. A new leader-follower relation has to be established. The former follower succeeds to the position of leader and the former leader now becomes a follower. However, alternative cases could be considered. If the initial leadaer expects that he will not be able to hold the position of leader without augmenting the marginal productivity of his workers and if he finds losing his leader position is not profitable, he might invest in capital to augment his workers' productivity.

3.4. Conditions for Succession-Equilibrium in Labor Market

Let us concentrate on the leader unit A and the successive follower unit B. By definition we have $w_A > w_B$ and $G_A^{min} > G_B^{min}$. We shall examine the condition that guarantees a stable structure of wage differentials. We use the term "succession eqilibrium" to characterize a labor market with stable wage differentials.

The necessary condition for succession equilibrium is that

$$w_\ell > w, \tag{2.20}$$

where w_ℓ and w stand for leader A's and follower B's wage rate respectively. Necessary and sufficient conditions read as follows. (Precise discussion is given in Obi[7])

(a) Letting A and B be the leader and follower respectively, when (2.20) holds,the leader-follower relationship is stable, if the following condition is satisfied.

(a.1) Let B be a leader instead of A, A being a follower, and compute the leader solution for B. Let the solution for the wage rate be w_ℓ . Compute the follower solution for A. Let the solution be w. Then suppose (2.20),

$$w_\ell > w$$

does not hold. This is the necessary and sufficient condition for stable succession-equilibrium.

(a.2) When the leader-follower relationship between A and B is inverted in the computation procedure (a.1), if (2.20) holds in this case as well, then the leader-follower relationship cannot be stable. Now suppose that the analytical forms of the production function and the grade-distribution function are true and the estimated parameters are correct. Further suppose that numerical values of the set of parameters and the production levels of production unit A and B are such that they generate the unstable case mentioned above. On the other hand suppose, in the real labor market, a stable wage differential between unit A and B is observed. Then it must be considered that the leader-follower relationship between A and B is sustained by fctors other than those already considered ; e.g. historical or random factors. Hence, the observed leader-follower position of A and B will be inverted whenever those factors change.

(b) Letting A and B be the leader and follower respectively, when (2.20) does not hold, the inverse leader-follower relationship is stable so long as the following condition is satisfied.

(b.1) Let B be a leader instead of A and compute the leader solution for B. Let the solution for the wage rate be denoted by w_ℓ . Compute the follower solutionfor A. Let the solution be denoted by w. Next suppose (2.20)$w_\ell > w$, does not hold. In this case, it can be said that the set of estimated parameters of the model is not correct or the model itself is at fault.

(b.2) When the leader-follower relationship between A and B is inverted in the computation procedure, if (2.20) holds, then the leader-follower relationship is stable, B and A being the leader and follower respectively. However, this case, (b.2), is substantially equivalent to case (a.1), and the indepent cases are (a.1), (a.2) and (b.1). Hence, (a.1) is the necessary and sufficient condition for the stability of successive equilibrium.

3.5. Simple Model

We shall specify the analytical form of production functions (2.1) and (2.1′) as

$$Q_i = b_i L_i^{\alpha_i} (\overline{G}_i)^{\gamma_i}, \qquad \overline{\alpha}_i > 0, \quad \gamma_i > 0 \qquad (i = 1, 2, \ldots), \tag{3.1}$$

$$\overline{G}_i = (G_{i+1} \cdot G_i)^{\frac{1}{2}}, G_{i+1} < G_i, \tag{3.1'}$$

where G_i and G_{i+1} respectively stand for the highest and the lowest values of G among the workers the ith firm employs. Let $i = \ell, f$, where ℓ and f respectively stand for leader and follower.

Simplifying the distribution function $\nu(G)$ without impairing the basic characteristics of the model,we use

$$\nu(G) = \nu_0 + \nu_1 G, \tag{3.2}$$

where ν_0 and ν_1 are parameters.

We specify the supply-probability equation (2.2) as a linear function of w

$$\mu = \lambda_0 + \lambda_1 w \tag{3.3}$$

where as shown later

$$\lambda_0 < 0, \qquad \lambda_1 > 0 \qquad and \qquad 0 \leq \mu \leq 1 \tag{3.4}$$

3.6. Numerical Experiments

3.6.1. Simulation System

We shall present a few numerical experiments to examine the workability of the successive equilibrium model. Let the number of production units (or firms) be two, unit 1 and 2. Numerical values of the parameters are assinged as follows.

$$\alpha_1 = \alpha_2 = 1, \quad \gamma_1 = 0.4, \quad \gamma_2 = 0.9, \quad b_1 = b_2 = 1,$$

$$\nu_0 = 1, \quad \nu_1 = -1, \quad \lambda_0 = -0.5, \quad \lambda_1 = 0.01, \quad N = 10,000$$

Suffix 1 and 2 stand for unit 1 and 2 respectively. The elasticity of production with respect to grades for unit 2, γ_2, is larger than that for unit 1.

The levels of production of unit 1 and 2 are experimentally given as shown in the first and second columns of Table 1a through 1g. These are exogenous variables in the simple model under consideration.

(a) In Table 1a, Q_2 is increased from 150 to 300, Q_1 being constant. In this case the computation process revealed the succession-equilibrium was stable and a stable leadaer-follower relationship holds as is shown in the table ; i.e. unit 2 and 1 are theleader and follower respectively. The wage differential w_2/w_1 increases.

(b) In the second case, Q_1 and Q_2 were increased with a common rate of growth starting from $Q_1 = Q_2 = 160$, as shown in Table 1b. The leader-follower relationship does not alter. The wage differential decreases, unlike that of case (1). It can be seen that the increase in the wage differential in case (1) stems from the growth and stagnation of production of unit 2 and 1 respectively.

(c) In Table 1a, Q_2, the production of unit 2 which has a larger value for γ compared to unit 1, was increased. In contrast to this, production Q_1 of unit 1 is increased, Q_2 being held constant at 150, in Table 1c. For the values of $Q_1 = 160, \cdots, 190$, unit 2 occupies the position of leader, while case (b.1) appears when Q_1 exceeds 200; that is, we do not have a consistent solution for $Q_1 \geq 200$ and $Q_2 = 150$.

(d) Next, in order to clarify the response of production unit 2 against production unit 1 with $Q_1 = 200$, we tentatively assigned Q_2 values in the range $38 \leq Q_2 \leq 750$. (See Table 1d). It was found that the leader position switches if Q_2¡47. The altered leader follower relationship is stable for $Q_1 = 200$ and $38 < Q_2 < 47$.

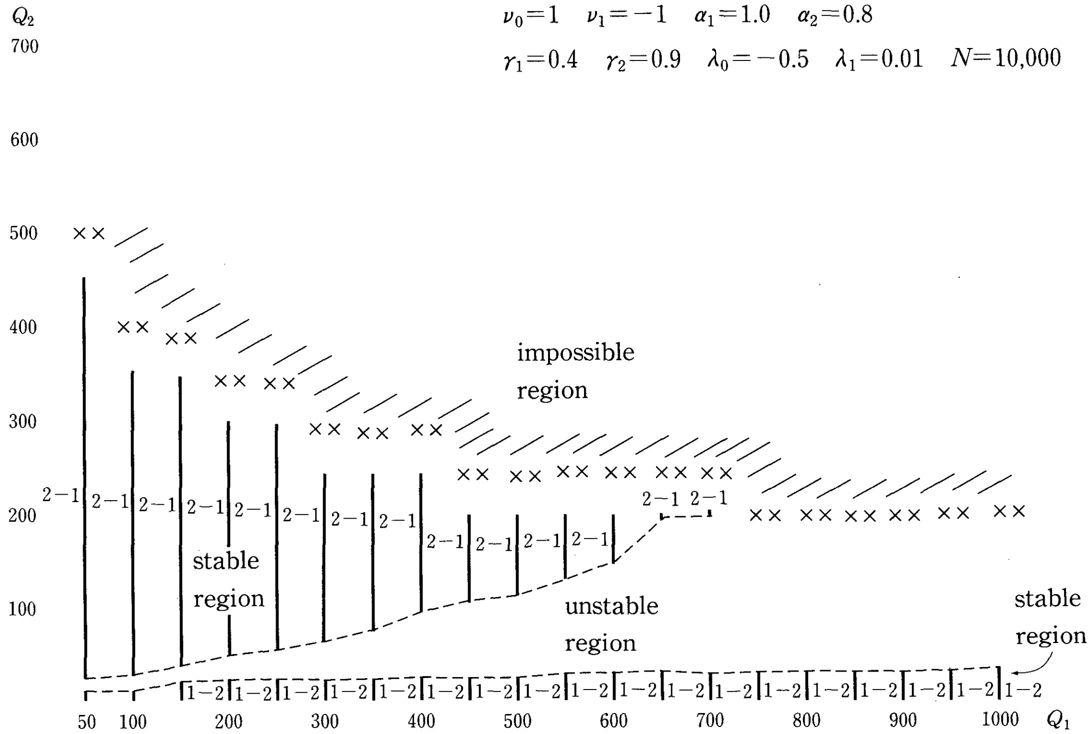
(e) *A* test analogous to (d) is shown in Table 3e. Here, Q_2 is held constant at 150, while Q_1 is varied between $63 \leq Q_1 \leq 1250$. The leader role switches when Q_1 reaches 1250.

(f) Analogous to (e), we take $Q_1 = 250$ and $38 < Q_2 < 750$. For $Q_2 > 250$, unit 2 and 1 play the leader and follower respectively. If $Q_2 \leq 54$, the relationship alters. Between $Q_2 = 54$ and $Q_2 = 250$, we do not have stable succession equilibrium (consistent solutions).

(g) Analogous to (f), we vary Q_2 between 38 and 750, Q_1 being 300. For $Q_2 > 250$, unit 2 and 1 are the leader and follower respectively. However, for $Q_2 \leq 63$ this relationship alters.

3.6.2. The Ranges of Production which Guarantee Stable Succession Equilibria

The ranges for production of sectors 1 and 2, which guarantee stable succession equilibria, are depicted in Figure 2. The thick lines and dotted lines or segments respectively stand for the ranges where succession equilibria are guaranteed and not guaranteed. Thus, it can be seen that the hatched area represents (a part of) the unstable regions.



Thick lines and segments stand for the region where stable succession equilibrium holds. Attached numbers standed for leader follower relations, e.g.2–1 states that unit 2 and 1, respectively, play the rule leader and follower.

Figure 2

	Q_1/b_1 FollowerQ	Q_2/b_2 leaderQ	leader sector	follower sector	L_1 (leader)	L_2 (follower)	G_1 (leader)	G_2 (follower)	W_1 (leader)	W_2 (follower)
	150	150	2	1	166.9	179.5	0.888	0.639	57.89	56.62
Table 1a	150	160	2	1	178.6	180.0	0.835	0.634	58.24	56.65
	150	170	2	1	190.3	180.5	0.882	0.629	58.57	56.72
	150	180	2	1	201.9	180.9	0.880	0.626	58.95	56.73
	150	190	2	1	213.8	181.4	0.877	0.621	59.26	56.80
	150	200	2	1	225.8	182.0	0.874	0.616	59.56	56.83
	150	250	2	1	273.5	183.7	0.865	0.602	60.86	56.98
	150	300	2	1	346.1	186.1	0.853	0.584	62.71	57.17
	160	160	2	1	178.6	192.4	0.885	0.630	58.24	56.98
Table 1b	170	170	2	1	190.3	205.6	0.882	0.622	58.57	57.31
	180	180	2	1	201.9	218.6	0.880	0.615	58.95	57.64
	190	190	2	1	213.8	232.0	0.877	0.607	59.26	57.99
	200	200	2	1	225.8	245.5	0.874	0.599	59.56	58.34
	250	250	2	1	285.4	313.4	0.863	0.568	61.18	60.07
	300	300	2	1	346.1	383.3	0.853	0.542	62.71	61.84
	160	150	2	1	166.9	191.9	0.888	0.635	57.89	56.91
Table 1c	170	150	2	1	166.9	204.3	0.888	0.632	57.89	57.23
	180	150	2	1	166.9	216.9	0.888	0.628	57.89	57.51
	190	150	2	1	166.9	229.4	0.888	0.625	57.89	57.80
	200	150	…	…						
	250	150	…	…						
	200	750	2	1	266.3	916.8	0.489	0.800	59.99	75.47
Table 1d	200	375	2	1	254.6	438.7	0.517	0.840	59.04	64.90
	200	250	2	1	248.4	285.4	0.582	0.863	58.55	91.18
	200	188	2	1	244.5	210.8	0.605	0.878	58.27	59.20
	200	150	…	…						
	200	50	…	…						
	200	47	1	2	217.3	74.4	0.813	0.598	56.41	56.22
	200	44	1	2	217.3	69.9	0.813	0.600	56.41	55.96
	200	42	1	2	217.3	65.8	0.813	0.602	56.41	55.79
	200	39	1	2	217.3	62.2	0.813	0.603	56.41	55.60
	200	38	1	2	217.3	59.0	0.813	0.604	56.41	55.43
	1,250	150	1	2	1,474.2	383.9	0.662	0.352	76.24	74.67
Table 1e	625	150	…	…						
			(no solution)							
	208	150	…							
	170	150	2	1	215.0	166.9	0.629	0.888	57.48	57.89
	156	150	2	1	187.2	166.9	0.636	0.888	56.71	57.89
	139	150	2	1	165.7	166.9	0.643	0.888	56.29	57.89
	125	150	2	1	148.5	166.9	0.650	0.888	55.87	57.89
	⋮		2	1						
	66	150	2	1	76.7	166.9	0.682	0.888	53.86	57.89
	63	150	2	1	72.7	166.9	0.684	0.888	53.74	57.89
	250	375	2	1	321.3	438.7	0.534	0.840	60.67	64.90
	250	250	2	1	313.4	285.4	0.568	0.863	60.07	61.18
Table 1f	250	188	…	…						
			(no solution)							
	250	58	…	…						
	250	54	1	2	273.9	89.2	0.568	0.568	57.47	57.14
	250	50	1	2	273.9	83.0	0.570	0.570	57.47	56.83
	250	47	1	2	273.9	77.6	0.572	0.572	57.47	56.57
	250	44	1	2	273.9	72.8	0.573	0.573	57.47	56.29
	250	42	1	2	273.9	68.7	0.574	0.574	57.47	56.05
	250	39	1	2	273.9	64.9	0.576	0.576	57.47	55.89
	250	33	1	2	273.9	57.7	0.577	0.577	57.71	57.48
	300	750	.							
	300	375	2	1	388.8	438.7	0.523	0.840	62.22	64.90
	300	250	2	1	379.3	285.4	0.556	0.863	61.52	61.18
Table 1g	300	188	…	…						
			(no solution)							
	300	68	…	…						
	300	63	1	2	331.0	108.5	0.782	0.542	58.52	58.25
	300	58	1	2	331.0	99.8	0.782	0.544	58.52	57.79
	300	54	1	2	331.0	92.3	0.782	0.546	58.52	57.45
	300	50	1	2	331.0	85.9	0.782	0.548	58.52	57.13
	300	47	1	2	331.0	80.3	0.782	0.550	58.52	56.85
	⋮		1	2						
	300	38	1	2	331.0	63.7	0.782	0.555	58.52	55.93

$b_1=b_2=1$ $\gamma_2=0.9$ $N=10{,}000$ $\lambda_0=-0.5$

$\alpha_1=\alpha_2=1$ $\gamma_1=0.4$ $C_0=0$ $\lambda_1=0.01$

* The solution does not exist in this range of production

4. An Alternative Simple Model

4.1. Basic Equations

4.1.1. Production function

We shall specify the analytical form of production functions (2.1) as [4]

$$Q_i = b_i L_i. \tag{3.1a}$$

Let the cost function of ith firm (or sector) be [5]

$$C_i = \psi_i(\overline{G}_i, L_i) + w_i L_i \tag{3.1'a}$$

where

$$\overline{G}_i = (G_{i+1} \cdot G_i)^{\frac{1}{2}}, G_{i+1} < G_i$$

where G_i and G_{i+1} respectively stand for the highest and the lowest values of G among the workers the ith firm employs.

4.1.2. The distribution function of grade indicator

Simplifying the distribution function $\nu(G)$ without impairing the basic characteristics of the model,we use

$$\nu(G) = \nu_0 + \nu_1 G, \tag{3.2}$$

where ν_0 and ν_1 are parameters. $\nu(G)$ is the ratio of the number of potential applicants with grade G and over to the total number of potential applicants (the number of the people of working age). The magnitueds of G's the potential supplier with the highest and lowest grade among all potential suppliers are respectively defined to equal unity and ε, ε being some small positive number. Hence, we have

$$\nu(G) = 1 \qquad \text{if} \quad G = \varepsilon, \tag{3.3a}$$

and

$$\nu(G) = \frac{1}{N} \qquad \text{if} \quad G = 1,$$

[4] With regard to this specification we can give an interpretation that in (2.1) we assume the analytical form of F as $\frac{\partial F}{\partial \overline{G}_i} = 0$. Another interpretaion would be that we assume Leontief type[3] (factor limitational) production function.

[5] ψ_i stands for an additional cost which is affected by the value of the grade G_i. This is an alternative and easier way of analysing the effect of G or the production behavior of the ith firm (so to speak G. Becker Version[1]) compared to the way in which G is included in the production function of ith firm, as is done in the previous section. By this alternative Sppcification of production function (3.1a) means that the demand curve for labor $D_i D_i'$ $(i = \ell, f)$ in Figure 1 are strait lines perpendicular to the abscissas.

where N stands for the number of the total potential suppliers. By applying (3.3a) to (3.2) we have

$$\nu_1 = -\frac{(1-\frac{1}{N})}{(1-\varepsilon)} \tag{3.4a}$$

$$\nu_0 = \frac{1+\varepsilon(1-\frac{1}{N})}{(1-\varepsilon)} \tag{3.5}$$

Hence, the distribution function (3.2) is written as

$$\nu(G) = 1 + \frac{\varepsilon(1-\frac{1}{N})}{(1-\varepsilon)} - \frac{(1-\frac{1}{N})}{(1-\varepsilon)} \cdot G \tag{3.2$'$}$$

By adopting the magnitude of ε as $\varepsilon = \frac{1}{N}$ (3.2$'$) is written as

$$\nu(G) = 1 + \frac{1}{N} - G \tag{3.2$''$}$$

If N is sufficiently a large number we have

$$\nu(G) \cong 1 - G. \tag{3.2$'''$}$$

The number of persons with G higher than G_j, $N(G \geq G_j)$, is given by

$$N(G \geq G_j) = N \cdot \nu(G_j) \tag{3.6}$$

Hence, applying (3.2$'$), we obtain

$$N(G \geq G_j) = N\left[1 + (1-\frac{1}{N})(\frac{\varepsilon}{1-\varepsilon}) - (1-\frac{1}{N})(\frac{\varepsilon}{1-\varepsilon}) \cdot G_j\right] \tag{3.7}$$

Making use of the relation $\varepsilon = 1/N$, (3.7) is written as

$$N(G \geq G_j) = N + 1 - NG_j = N(1-G_j) + 1 \tag{3.7$'$}$$

From (3.2$''$) we have, as a good approximation for (3.7$'$),

$$N(G \geq G_j) \cong N(1-G_j) \tag{3.7$''$}$$

4.1.3. Indirect observation of G_i

In the simple model, variables Q_i^t, L_i^t and w_i^t where i and t stands for the production unit and time respectively, are directly observable from the data. However, we cannot observe the magnitude of G_i directly and we must therefore indirectly measure it making use of the model itself. We shall discuss the procedure to measure G_i below.

Suppose we have data on Q_i^t, L_i^t and $w_i^t \quad (i = \ell, f)$. With respect to the parameters of the model we have $\nu_0 = 1$, $\nu_1 = -1$. Further suppose we have already estimated the parameters, λ_0 and λ_1, in the supply probability function. Labor supply curves for the leader and the follower respectivity pass through points A_ℓ and A_f in Figure.1. The values of the coordinates of those points A_ℓ and A_f are known from observed data on the wages and employment of the leader and the follower.(Production units (firms, sectors) are ordered by the observed wage rates. Hence, successively, leader-follower relationships can be identified by this ordering.) Therefore, we can obtain $N_{G_\ell}^{min}$ and $N_{G_f}^{min}$ by solving the simultaneous equations,

$$N_{G_\ell}^{min}(\lambda_0 + \lambda_1 w_\ell) = L_\ell$$

$$(N_{G_\ell}^{min} - N_{G_\ell}^{min})(\lambda_0 + \lambda_1 w_f) = L_f$$

where actual wages and employment w_ℓ, w_f, L_ℓ and L_f are directly obtained from the observed data and λ_0 and λ_1 , are supposed to be already estimated, as mentioned above.

Applying $N_{G_\ell}^{min}$ and $N_{G_f}^{min}$ thus obtained to the left hand side of the grade distribution function $(\nu(G_i) = 1 - G_i)$ in [3.2"'], we can calculate the numerical values for G_ℓ^{min} and G_f^{min}. These are the "indirectly observed" values for G_ℓ^{min} and G_f^{min}.

4.1.4. Equation of Supply-probability

We specify the supply-probability equation (2.2) as a linear function of w

$$\mu = \lambda_0 + \lambda_1 w \tag{3.8}$$

where as shown later

$$\lambda_0 < 0, \qquad \lambda_1 > 0 \qquad and \qquad 0 \leq \mu \leq 1 \tag{3.9}$$

are postulated. [6] In order to make our model simple without impairing its basic characteristics, we use a linear function as a supply probability function. This simplification means that we implicitly employ a rectangular distribution for the minimum supply price of labor, $\underline{w}$. In equation (3.8), we have $w = \lambda_0/\lambda_1$ when $\mu = 0$, hence,

$$\mu = 0 \qquad \text{if} \quad w \leq -\frac{\lambda_0}{\lambda_1} \tag{3.10}$$

$$\mu = 1 \qquad \text{if} \quad w \geq -\frac{1-\lambda_0}{\lambda_1}$$

and for the range of w,

$$-\frac{\lambda_0}{\lambda_1} < w < \frac{1-\lambda_0}{\lambda},$$

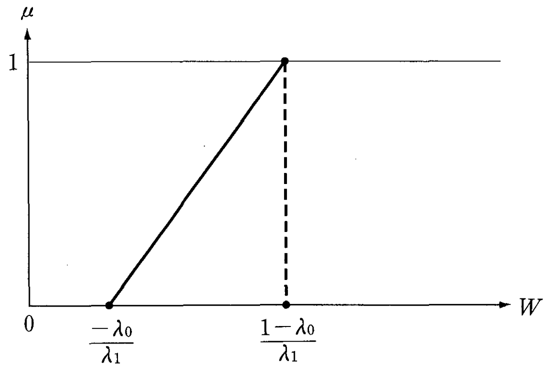


Figure 3

(3.8) holds. The supply probability curve with the characteristics (3.8) and (3.10) is depicted in Figure 3.

The numerical value of $-\lambda_0/\lambda_1$, stands for the minimal value of the range of distributed values of $\underline{w}$. This minimal values of $\underline{w}$ must be positive, and

$$-\frac{\lambda_0}{\lambda_1} > 0 \tag{3.11}$$

must hold. On account of the nature of the distribution function, λ_1 must be positive. Hence, from (3.11) we have

$$\lambda_0 < 0 \tag{3.12}$$

4.2. Behavior of the Leader in the alternative Simple Model

4.2.1. Basic Equations

Let the leader's production function be (3.1a), and we have

$$Q_\ell = b_\ell L_\ell \tag{L.1}$$

where the suffix i in (3.1a) is replaced by ℓ to show that the equation is that of the leader.

THe cost function in (4.1.1) can be written as

$$C_\ell = \psi_\ell(\overline{G}_\ell, L_\ell) + w_\ell L_\ell \tag{L.2}$$

where [7]

[6] If the true shape of μ function is linear as shown in (3.8) $\lambda_0 < 0$ must be held in the estimated relation as well. However, if the true function is non-linear, linear supply probability function, $\lambda_0 + \lambda_1 w$, originally is an aproximation. Hence, a constant term in an estimated linear supply probability function could be negative.

[7] Here after, for the sake of brevity, we assume $\overline{G}_\ell \simeq G_\ell$ where G_ℓ stands for G_ℓ^{min}. By this approximation the basic characteristtics of the model will not be impaired.

$$\overline{G}_\ell = (G_\ell^{max} \cdot G_\ell^{min})^{\frac{1}{2}} \tag{L.2'}$$

The number of suppliers to the leader is given by (letting $\overline{N}$ be population of working age)

$$L_\ell^s = \overline{N}[\nu(G_\ell) - \nu(G_\ell^{max})] \cdot \mu(w_\ell)$$

or

$$L_\ell^s = \overline{N}[\nu(G_\ell) - \nu(G_\ell^{max})] \cdot (\lambda_0 + \lambda_1 w_\ell + \lambda_2 A) \tag{L.3}$$

where A stands for the effect of changing $\overline{x}_g$ in sec. 2.3. $(L.3)$ corresponds to (2.6). This equation states that effective suppliers to the leader must be the ones with at least grade G_ℓ.

When the value of Q_ℓ is given $(L.3)$ can be written as (taking into account $(L.1)$,)

$$\frac{1}{b_\ell} Q_\ell = \overline{N}[\nu(G_\ell) - \nu(G_\ell^{max})] \cdot \mu(w_\ell) \tag{L.3'}$$

We minimize C_ℓ in $(L.2)$ under the constraint $(L.3')$. That is, defining F as

$$F = \psi_\ell(\overline{G}_\ell, L_\ell) + w_\ell L_\ell + \Lambda \left[\frac{1}{b_\ell} Q_\ell - \overline{N}[\nu(G_\ell) - \nu(G_\ell^{max})] \cdot \mu(w_\ell)\right] \tag{L.4}$$

where Λ is a Lagrangian multiplier,

$$\frac{\partial F}{\partial G_\ell} = \frac{\partial F}{\partial w_\ell} = 0 \tag{L.5}$$

has to hold if C_ℓ is minimized. Hence, from $\frac{\partial F}{\partial G_\ell} = 0$, we have

$$\frac{\partial \psi_\ell}{\partial G_\ell} + \Lambda \left[-\overline{N} \frac{d\nu(G_\ell)}{dG_\ell} \cdot \mu(w_\ell)\right] = 0 \tag{L.6}$$

From $\frac{\partial F}{\partial w_\ell}$, we have

$$L_\ell - \Lambda \overline{N}[\nu(G_\ell) - \nu(G_\ell^{max})] \cdot \frac{d\mu}{dw_\ell} = 0 \tag{L.7}$$

Taking into account (3.8) we have $\frac{d\mu}{dw_\ell} = \lambda_1$, Hence, from $(L.6)$ and $(L.7)$ we get the minimization condition

$$\frac{-\frac{\partial \psi_\ell}{\partial G_\ell}}{\mu(w_\ell)} = \frac{\frac{1}{b_\ell} Q_\ell}{(G_\ell^{max} - G_\ell)\lambda_1}, \tag{L.8}$$

where G_ℓ^{max} is given. We can solve $(L.8)$ and $(L.3')$ simultaneously for G_ℓ and w_ℓ. From $(L.8)$ and $(L.3')$

$$-\frac{\partial \psi_\ell}{\partial G_\ell} = \frac{\overline{N}}{\lambda_1} \cdot [\mu(w_\ell)]^2 \tag{L.9}$$

To begin with the simplest case we assume $\dfrac{\partial \psi_\ell}{\partial G_\ell}$ to be a linear function, that is,

$$\frac{\partial \psi_\ell}{\partial G_\ell} = \delta_0 + \delta_2 L_\ell \tag{L.10}$$

where δ_0, δ_1 and δ_2 are the paramenters, and,

$$\frac{\partial \psi_\ell}{\partial G_\ell} < 0 \tag{L.11}$$

must hold.

Inserting $(L.10)$ into $(L.9)$ and making use of $(L.3)$, we have

$$\delta_0 + \delta_2 L_\ell = \frac{-[\frac{1}{b}Q_\ell]^2}{\overline{N}(G_\ell^{max} - G_\ell)^2 \lambda_1} \tag{L.12}$$

From this we have

$$G_\ell = G_\ell^{max} \pm \sqrt{\frac{-(\frac{1}{b}Q_\ell)^2}{\overline{N}\lambda_1(\delta_0 + \delta_2 L_\ell)}} \tag{L.13}$$

Taking into account $G_\ell < G_\ell^{max}$ we adopt

$$G_\ell = G_\ell^{max} - \sqrt{\frac{-(\frac{1}{b}Q_\ell)^2}{\overline{N}\lambda_1(\delta_0 + \delta_2 L_\ell)}} \tag{L.13'}$$

where

$$\delta_0 + \delta_2 L_\ell < 0.$$

$(L.13')$ is the "leader solution" of the grade variable.

Taking into account $\nu(G_\ell) - \nu(G_\ell^{max}) = G_\ell^{max} - G_\ell$, $(L.3)$ can be written as,

$$L_\ell = \overline{N}(G_\ell^{max} - G_\ell)(\lambda_0 + \lambda_1 w_\ell + \lambda_2 A) \tag{L.3''}$$

where A stands for the effect of changing $\overline{x}_g$ in sec. 2.3.

From $(L.3'')$

$$w_\ell = \frac{L_\ell}{\lambda_1 \overline{N}(G_\ell^{max} - G_\ell)} - \frac{\lambda_0}{\lambda_1} - \frac{\lambda_2}{\lambda_1}A \tag{L.14}$$

Substituting G_ℓ in $(L.14)$ by $(L.13')$ we obtain

$$w_\ell = \frac{1}{\lambda_1} \cdot \frac{L_\ell}{\overline{N} \cdot \sqrt{\dfrac{-L_\ell^2}{N\lambda_1(\delta_0 + \delta_2 L_\ell)}}} - \frac{\lambda_0}{\lambda_1} - \frac{\lambda_2}{\lambda_1}A \tag{L.15}$$

or

$$w_\ell = \frac{1}{\lambda_1} \cdot \frac{\dfrac{1}{b_\ell}Q_\ell}{\overline{N} \cdot \sqrt{\dfrac{-(\dfrac{1}{b_\ell}Q_\ell)^2}{N\lambda_1(\delta_0 + \delta_2 \dfrac{1}{b_\ell}Q_\ell)}}} - \frac{\lambda_0}{\lambda_1} - \frac{\lambda_2}{\lambda_1}A \tag{L.15'}$$

This is the "leader solution" of the wage variable .

4.3. Behavior of the Follower in the Simple Model

Let the production function and the cost function of the follower be respectively,

$$Q_f = b_f L_f \tag{F.1}$$

and

$$C_f = \psi_f(\overline{G}_f, L_f) + w_f L_f \tag{F.2}$$

where [8]

$$\overline{G}_f = (G_f^{max} \cdot G_f^{min})^{\frac{1}{2}} \tag{F.2'}$$

The number of suppliers to the follower is given by

$$L_f^s = \overline{N}[\nu(G_f) - \nu(G_f^{max})] \cdot \mu(w_f) \tag{F.3}$$

G_f stands for the minimum value of the grade indicator of labor the follower can accept. G_f^{max} is the maximum value of the grade indicator which the follower can attain. When Q_f is given $(F.3)$ can be written as

[8] The same assumption as that made in footnote 6 is made, that is $\overline{G}_f \simeq G_f$, where G_f stands for G_f^{min} .

$$\frac{1}{b_f}Q_f = \overline{N}[\nu(G_f) - \nu(G_f^{max})] \cdot \mu(w_f) \tag{F.3'}$$

We minimize C_f in $(F.2)$ with constraint $(F.3')$. The values of w_f and G_f minimizing C_f can be given by,

$$\frac{\partial F}{\partial G_f} = \frac{\partial F}{\partial w_f} = 0, \tag{F.4}$$

where

$$F = \psi_f(\overline{G}_f, L_f) + w_f L_f + \Lambda\left[\frac{1}{b_f}Q_f - \overline{N}[\nu(G_f) - \nu(G_f^{max})] \cdot \mu(w_f)\right] \tag{F.5}$$

From $(F.4)$ we get

$$\frac{-\dfrac{\partial \psi_f}{\partial G_f}}{\mu(w_f)} = \frac{\dfrac{1}{b_f}Q_f}{(G_f^{max} - G_f)\lambda_1} \tag{F.6}$$

Making use of $(F.6)$ and $(F.3')$ we have

$$-\frac{\partial \psi_f}{\partial G_f} = \frac{\overline{N}}{\lambda_1} \cdot [\mu(w_f)]^2 \tag{F.7}$$

We assume $\frac{\partial \psi_f}{\partial G_f}$ to be linear, that is

$$\frac{\partial \psi_f}{\partial G_f} = \varepsilon_0 + \varepsilon_2 L_f \tag{F.8}$$

Using $(F.3)$

$$\mu(w_f) = \frac{L_f}{\overline{N}[\nu(G_f) - \nu(G_f^{max})]} \tag{F.9}$$

where

$$\nu(G_f) = 1 - G_f \tag{F.10}$$

and

$$\nu(G_f^{max}) = 1 - G_\ell \tag{F.11}$$

because $G_f^{max} = G_\ell$.

Hence

$$(\nu(G_f) - \nu(G_f^{max})) = (1 - G_f) - (1 - G_\ell) = G_\ell - G_f \tag{F.12}$$

Taking into account $(F.8)$, $(F.9)$ and $(F.12)$, $(F.7)$ is rewritten as

$$\varepsilon_0 + \varepsilon_2 L_f = -\frac{\overline{N}}{\lambda_1}\left[\frac{L_f^2}{\overline{N}^2(G_\ell - G_f)^2}\right] \tag{F.13}$$

where G_ℓ is given by the solution for the leader. Again we suppose $\varepsilon_1 = 0$ for brevity. Hence $(F.14)$ can be written as

$$\varepsilon_0 + \varepsilon_2 L_f = \frac{-(\frac{1}{b_f}Q_f)^2}{\overline{N}(G_\ell - G_f)^2\lambda_1} \tag{F.14}$$

From this we have

$$G_f = G_\ell \pm \sqrt{\frac{-(\frac{1}{b_f}Q_f)^2}{\overline{N}\lambda_1(\varepsilon_0 + \varepsilon_2\frac{1}{b_f}Q_f)}} \tag{F.15}$$

Because $G_f < G_\ell$ we adopt

$$G_f = G_\ell - \sqrt{\frac{-(\frac{1}{b_f}Q_f)^2}{\overline{N}\lambda_1(\varepsilon_0 + \varepsilon_2\frac{1}{b_f}Q_f)}} \tag{F.16}.$$

where

$$\varepsilon_0 + \varepsilon_2 L_f < 0.$$

$(F.16)$ corresponds to $(L.13')$ in the leader's case, and gives the "follower's solution" of the grade variable. From $(F.3)$ and $(F.12)$, we have

$$L_f = \overline{N}(G_f^{max} - G_f)(\lambda_0 + \lambda_1 w_f + \lambda_2 A) \tag{F.17}$$

By solving $(F.17)$ we get

$$w_f = \frac{L_f}{\lambda_1\overline{N}(G_f^{max} - G_f)} - \frac{\lambda_0}{\lambda_1} - \frac{\lambda_2}{\lambda_1}A \tag{F.18}$$

Substition of G_f in $(F.18)$ by $(F.15)$ gives

$$w_f = \frac{1}{\lambda_1} \cdot \frac{L_f}{\overline{N} \cdot \sqrt{\dfrac{-L_f^2}{\overline{N}\lambda_1(\varepsilon_0 + \varepsilon_2 L_f)}}} - \frac{\lambda_0}{\lambda_1} - \frac{\lambda_2}{\lambda_1} A \tag{F.19}$$

or

$$w_f = \frac{1}{\lambda_1} \cdot \frac{\dfrac{1}{b_f} Q_f}{\overline{N} \cdot \sqrt{\dfrac{-(\frac{1}{b_f} Q_f)^2}{\overline{N}\lambda_1(\varepsilon_0 + \varepsilon_2 \frac{1}{b_f} Q_f)}}} - \frac{\lambda_0}{\lambda_1} - \frac{\lambda_2}{\lambda_1} A \tag{F.19'}$$

This is the "follower solution" of the wage variable.

5. Application of the alternative Simple Model to Japanese data

The result of application of the model in the previous section to Japanese data is shown in this section.

It can be seen the wage of financial sector is always at the top of the wage differential among the sectors(industries) during the observational period, 1970 through 1991. Hence financial sector can be identified as "leader sector". The other sectors are aggregated and can be identified as "follower sector" as is shown by Figure 4. Hence, application of the model of leader behavior and follower behavior is straightforward . [9]

The observed yearly values of w_i, L_i and $Q_i (i = \ell, f)$ were obtained from SNA data arranged by Economic Planning Agency. Observational period is 1970 through 1991. These are shown in Table 3.

The estimated values of the parameters λ_0, λ_1 are shown in Table.2. [10]

Indirectly observed values obtained by using the estimation method in 4.1.3 function are shown in Table 3. Estimated values of δ_0 and δ_1 are also shown in Table.2. The estimated and observed values for $w_i, G_i \quad (i = \ell, f)$ are shown in Table.3. The estimated and observed values for w_i and $G_i \quad (i = \ell, f)$ are shown in Figure 4 and Figure 5.

Table 2: Estimated Parameters of Structural Equations

λ_0	1.515465
λ_1	0.5524484E-06
λ_2	-0.1202143E-01
δ_0	-0.7462759E+10
δ_2	-2385271.
ε_0	-0.9816546E+10
ε_2	564116.1

[9] In case where the order of wage differential(i.e, top. second, third etc.) change during observational period, the application of the models of leader and follower was discussed in (3.4).

[10] In this section μ is specified as $\mu = \lambda_0' + \lambda_1 w + \lambda_2 A$ where A affects the changes in $\overline{x}_g$ in (2.3). However, values of A to be observed were substitued by time trend, that is, $\mu = \lambda_0' + \lambda_1 w + at$ where a stands for the coefficient of the time trend term. λ_0' could be negative as shown in footnote footnote 6

6. Conclusion

From these results obtained in Figure 4, we can conclude that the "alternative simple model of the continually heterogeneous labor market" seems to be applicable to the Japanese data.

This model would also be applicable to the growth mechanism of developing economies as was shown elsewhere[8].

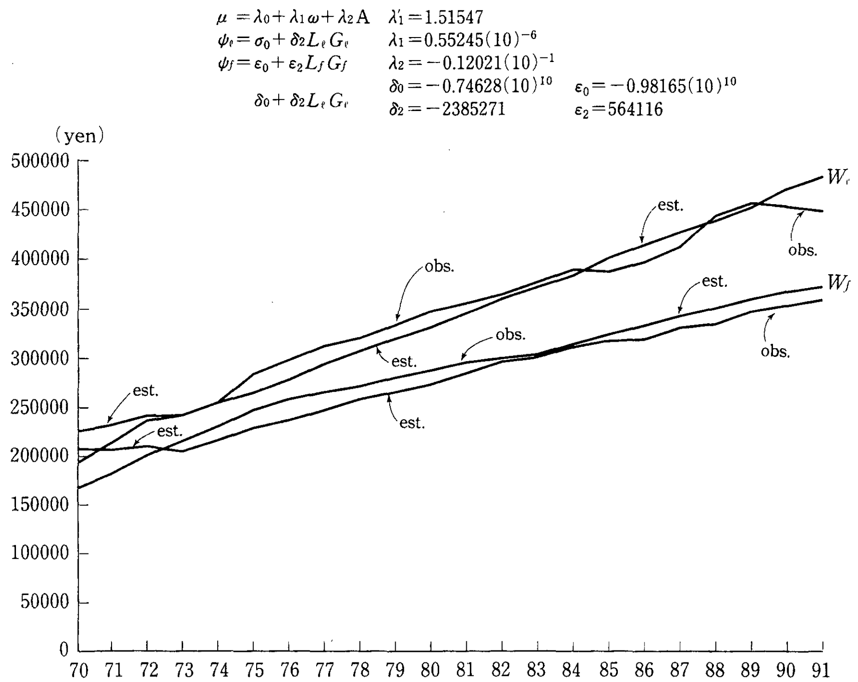


Figure 4

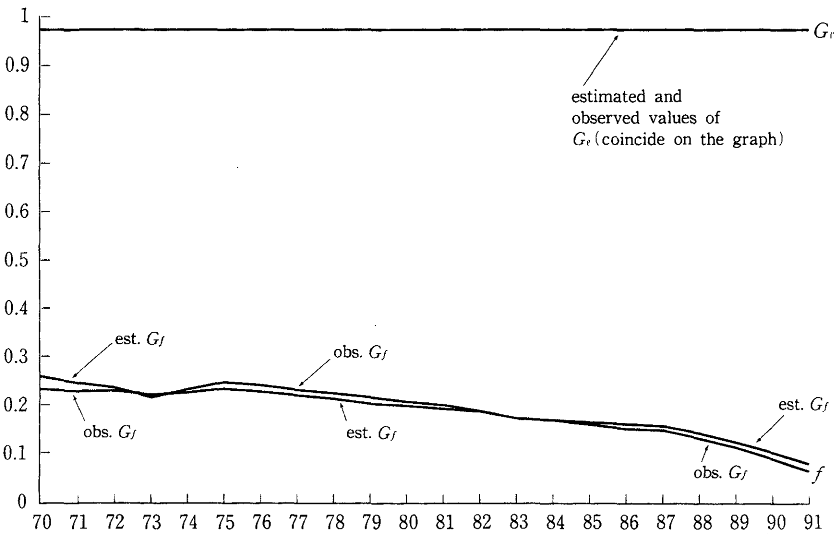


Figure 5

Table 3: Observations and Simulated Values of Endogenous Variables

year	W_ℓ	W_f	G_ℓ	G_f
70 obs.	190723.9	164117.4	0.9686200	0.2366719
70 sml.	225210.9	207227.4	0.9693688	0.2595301
71 obs.	212242.0	181815.5	0.9670115	0.2333205
71 sml.	231979.4	206573.3	0.9674667	0.2467064
72 obs.	236075.2	202183.1	0.9672889	0.2357605
72 sml.	240207.1	211359.1	0.9673843	0.2406927
73 obs.	242492.2	216001.8	0.9666206	0.2261282
73 sml.	242717.1	205364.5	0.9666260	0.2203420
74 obs.	254782.4	229392.6	0.9660790	0.2373796
74 sml.	253821.1	216853.4	0.9660554	0.2305865
75 obs.	283595.2	247586.6	0.9658823	0.2491550
75 sml.	265557.8	228423.5	0.9654353	0.2384550
76 obs.	298899.2	259103.2	0.9651493	0.2452453
76 sml.	278351.2	237753.7	0.9646257	0.2331408
77 obs.	312024.5	266191.5	0.9636751	0.2351443
77 sml.	293480.8	248250.0	0.9631802	0.2247163
78 obs.	319961.3	271786.4	0.9630249	0.2269524
78 sml.	307068.5	258810.4	0.9626725	0.2192794
79 obs.	334005.4	280536.4	0.9620401	0.2178648
79 sml.	319750.1	266674.2	0.9616375	0.2094702
80 obs.	345523.0	288243.2	0.9610689	0.2122245
80 sml.	330872.6	273689.6	0.9606411	0.2032560
81 obs.	354781.7	296014.9	0.9595347	0.2042027
81 sml.	345882.0	284714.5	0.9592632	0.1971854
82 obs.	364612.8	298871.4	0.9587308	0.1937988
82 sml.	359798.1	295412.0	0.9585801	0.1915384
83 obs.	376508.9	305068.8	0.9576265	0.1786572
83 sml.	372576.0	302153.6	0.9574993	0.1766975
84 obs.	387559.1	313688.6	0.9573984	0.1760640
84 sml.	384304.5	311829.1	0.9572917	0.1747734
85 obs.	387313.4	316914.0	0.9570192	0.1671898
85 sml.	396973.9	322461.5	0.9573409	0.1711148
86 obs.	394973.4	319244.1	0.9556640	0.1560513
86 sml.	410530.0	332169.2	0.9562019	0.1651734
87 obs.	408629.6	330245.8	0.9545292	0.1518785
87 sml.	423995.9	342070.6	0.9550777	0.1603886
88 obs.	440171.0	334388.6	0.9552247	0.1347124
88 sml.	436453.7	349725.5	0.9550931	0.1452633
89 obs.	453168.6	344986.5	0.9541766	0.1166702
89 sml.	450787.6	357213.9	0.9540898	0.1253832
90 obs.	452937.7	351832.4	0.9515034	0.0936549
90 sml.	467234.0	365684.3	0.9520572	0.1045376
91 obs.	448169.7	358395.9	0.9505394	0.0684420
91 sml.	481306.3	373074.6	0.9518565	0.0811564

Unit of W_ℓ,W_f: yen per month (constant price of 1985)

Table 4: Gross Domestic Product by kind of Economic Activity

year	XTI	XAF	XFI	XRE	PXTI85	PXAF85	PXFI85	PXRE85
70	70387.9	4488.0	3120.5	5899.0	46.0	46.8	64.4	39.4
71	77039.0	4273.8	3766.4	6972.9	47.7	46.8	62.3	43.0
72	88648.7	5049.9	4550.5	8135.8	49.7	48.7	54.7	46.5
73	108763.0	6675.1	5560.9	9853.5	56.5	61.2	66.0	50.0
74	127727.1	7505.9	7001.1	10944.5	67.5	69.5	97.3	53.1
75	138707.7	8141.1	7795.8	12138.0	71.6	75.6	92.2	57.8
76	156191.4	8870.0	8348.7	14208.2	77.5	86.9	95.1	64.0
77	172864.1	9401.6	9050.5	16663.5	82.5	94.5	89.7	70.2
78	190517.4	9440.6	10294.0	19036.6	86.6	94.7	88.2	76.3
79	207251.8	9623.0	11413.0	20965.4	88.0	95.2	95.1	79.6
80	224266.2	8847.2	12440.4	22654.3	91.3	96.9	104.4	82.2
81	239883.1	9075.4	12307.3	24402.3	94.2	98.2	101.0	86.2
82	252930.0	9238.4	13990.5	25675.4	96.4	95.3	113.6	89.8
83	264260.5	9516.4	15370.2	27409.2	97.4	96.5	111.0	93.6
84	281948.5	9956.9	15843.5	29802.4	98.9	97.2	102.0	96.7
85	301175.2	10213.7	16971.9	32358.5	100.0	100.0	100.0	100.0
86	313154.4	9974.9	17714.3	34729.0	102.4	99.7	94.2	103.0
87	328761.1	9767.5	19228.1	37734.4	101.7	94.6	91.8	106.7
88	351749.3	9753.8	21015.0	40653.1	101.9	97.6	91.5	109.3
89	379150.4	10131.8	23436.1	43569.0	103.4	98.1	92.0	112.1
90	407334.4	10552.6	23021.5	46507.8	105.0	101.7	92.4	116.2
91	431061.2	10442.3	22896.0	49098.7	107.1	109.8	92.6	120.6

Notes
Unit: Billion Yen
Periodicity: Calendar Yearly Data
Source: Annual Report on National Accounts
XTI:Gross Domestic Product,Producers'Values-Industries
XAF:Gross Domestic Product by Industry-Agriculture,Forestry and Fishery
XFI:Gross Domestic Product by Industry-Finance and Insurance
XRE:Gross Domestic Product by Industry-Real Estate
PXTI85: Gross Domestic Product,Producers'Values-Industries (Deflator)
PXAF85: Gross Domestic Product by Industry-Agriculture,Forestry and Fishery (Deflator)
PXFI85: Gross Domestic Product by Industry-Finance and Insurance (Deflator)
PXRE85: Gross Domestic Product by Industry-Real Estate (Deflator)

Table 5: Number of Employed Persons by kind of Economic Activity / Population 15 Years Old and Over-Total / Cash Earnings

year	EWTI	EWAF	EWFI	EWRE	PT	WSMR1	WSMFI	WSMRE
70	5052.9	1073.6	131.6	35.0	7886	75494	84958	98167
71	5081.4	995.8	139.9	39.6	7979	86726	98056	112486
72	5097.7	947.8	142.8	39.0	8070	100485	114372	128158
73	5205.6	903.3	145.8	43.2	8239	122041	134753	144619
74	5166.2	880.9	149.2	44.8	8341	154840	172061	171702
75	5139.8	861.8	153.4	45.9	8443	177272	206658	191010
76	5177.3	844.3	158.3	47.4	8540	200805	238408	209067
77	5236.7	836.6	164.6	51.2	8631	219608	266636	227793
78	5279.5	826.3	166.4	54.0	8726	235367	287393	245327
79	5331.3	797.6	171.0	57.6	8824	246872	302055	269788
80	5358.6	756.6	176.9	60.1	8932	263166	323773	291001
81	5393.2	732.9	182.4	64.9	9017	278846	346036	300952
82	5435.8	718.9	186.7	66.7	9117	288112	366614	309144
83	5521.1	695.4	192.5	70.0	9232	297137	384973	316523
84	5538.7	668.9	195.0	71.1	9347	310238	402622	330292
85	5578.3	659.8	194.7	73.1	9465	316914	407887	332516
86	5626.4	643.3	201.6	75.9	9587	326906	424815	350368
87	5676.3	633.4	208.7	78.6	9720	335860	435997	361355
88	5774.5	622.3	209.7	79.8	9849	340742	474191	381113
89	5899.0	613.4	217.0	81.5	9974	356716	485684	423026
90	6028.9	605.7	228.9	85.6	10090	369424	488887	440013
91	6157.8	587.1	230.3	87.8	10199	383842	490963	451207

Notes on EWTI, EWAF, EWFI, EWRE
Source: Annual Report on National Accounts Unit: 10000 Persons
EWTI:Employed by kind of Economic Activity-Industries
EWAF:Employed by kind of Economic Activity-Agricultrue, Forestry and Fishery
EWFI:Employed by kind of Economic Activity-Finance and Insurance
EWRE:Employed by kind of Economic Activity-Real Estate
Notes on PT
Source: Monthly Reprot on the Labour Force Survey Unit: 10000 Persons
PT: Population 15 Years Old and Over-Total
Notes on WSMR1, WSMFI, WSMRE
Source: Monthly Labour Survey Unit: yen Periodicity: Monthly Data
WSMR1: Ave.Monthly Cash Earnings of Regular Workers(incl.Bonus) -All Industries
WSMFI: Ave.Monthly Cash Earnings of Regular Workers(incl.Bonus) -Finance and Insurance
WSMRE: Ave.Monthly Cash Earnings of Regular Workers (incl.Bonus)-Real Estate
$W_\ell = (WSMFI \cdot EWFI + WSMRE \cdot EWRE)/(EWFI + EWRE)$
$W_f = [WSMR1 \cdot EWTI - W_\ell \cdot (EWFI + EWRE)]/(EWTI - EWFI - EWRE)$

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