

Title	About the applicability of the principles of logic to legal norm
Sub Title	
Author	吉野, 一(Yoshino, Hajime)
Publisher	慶應義塾大学法学研究会
Publication year	1992
Jtitle	法學研究 : 法律・政治・社会 (Journal of law, politics, and sociology). Vol.65, No.12 (1992. 12) ,p.472 (43)- 512 (3)
JaLC DOI	
Abstract	
Notes	川口實教授退職記念号
Genre	Journal Article
URL	<a href="https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AN00224504-19921228-0472">https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AN00224504-19921228-0472</a>

慶應義塾大学学術情報リポジトリ(KOARA)に掲載されているコンテンツの著作権は、それぞれの著作者、学会または出版社/発行者に帰属し、その権利は著作権法によって保護されています。引用にあたっては、著作権法を遵守してご利用ください。

The copyrights of content available on the Keio Associated Repository of Academic resources (KOARA) belong to the respective authors, academic societies, or publishers/issuers, and these rights are protected by the Japanese Copyright Act. When quoting the content, please follow the Japanese copyright act.

# ABOUT THE APPLICABILITY OF THE PRINCIPLES OF LOGIC TO LEGAL NORM

HAJIME YOSHINO

## CONTENTS

- 1 Introduction
- 2 The Starting Point of the  
Application of Logic to Legal Norms
- 3 The Concept of Truth in Logic
- 4 The Application of the Concept of Truth  
in Logic to Legal Norms
- 5 On the Logical Consequence-Relation Between Legal Norms
- 6 Legal Norms and Law Application with Respect  
to the Applicability of Logical Principles
- 7 Conclusion

## 1 Introduction

The applicability of logic to law has been debated repeatedly within the European legal theoretical world, particularly in German language community, especially the applicability of classical mathematical logic. Many scholars have argued that the classical logic is not applicable or at least not adequately applicable to law, which is a kind of norm, so they refuse the application of logic or insist that a special logic for law should

be constructed. Recently, this kind of pessimistic argument concerning the applicability of classical logic to law has been strongly emphasized by some scholars in the field of artificial intelligence of law as well [MacCarty, 1983, 1986, 1989; Bench-Capon, 1989; Herrestad, 1991; Jones, 1990; Jones & Sergot, 1992; Alchourrón & Martino, 1989]. It is, however, a pity for me that these scholars are not always acquainted with the former discussions in the German legal logical field (probably because of language barriers). Therefore, I think it is necessary to discuss the problem again on the basis of my previous works [Yoshino, 1978a, 1978b, 1981a, 1981b, 1983] to develop the resolution and to avoid further confusion of these themes.

Hans Kelsen, the worldly famous legal theoretician, not only in the German language community (but also in English as well as in Japanese), also discussed this problem, the applicability of the logical principles to legal norms [Kelsen, 1979]. He presents in his argument basic problems not only in legal logic, but also in a more general theory of legal norms.<sup>11</sup> Therefore, I would like to discuss the theme in relation to Kelsen's discussion.

He argued about this theme changing his opinion. The development of Kelsen's concept can be divided into three periods [Walter, 1980, pp. 299-314]. The first period belongs to the *Hauptprobleme der Staatsrechtslehre* (*Major Problems in the Studies of the National Law*) in 1911 [Kelsen, 1911] and *Reine Rechtslehre* (*The Pure Theory of Law*) in 1934 [Kelsen, 1934]. During this period Kelsen, like the traditional jurisprudence presupposed without hesitation that logical principles are valid in the field of law.

The second period starts with the second edition of *Reine Rechtslehre* (*The Pure Theory of Law*) in 1960 [Kelsen, 1960]. His view during this time can be expressed as follows: since legal norms are neither true nor false, but valid or invalid, the logical principles such as the Principle of the Exclusion of Contradiction and the Logical Consequence-Relation, both of which are only applicable to statements that are either true or false, can indirectly be applied to legal norms. Namely, in so far as they can be applied to the rules of law that describe legal norms and can on their part be true or false [cf. Kelsen, 1960, p. 76f, 209].

The third period is to be found in Kelsen's later works, mainly in *Law and Logic (Recht und Logik)* in 1965 [Kelsen, 1965, pp. 421-425, 495-500, 1967, p. 39f] and *The general Theory of Norms (Allgemeine Theorie der Normen)* [Kelsen, 1979] published after his death in 1979. Here, Kelsen has come to the conclusion that the two logical principles mentioned above are applicable to the relations between legal norms in neither a direct nor an indirect way, because, as he saw it, a legal norm is the meaning of acts of will [Kelsen, 1979, p. 152]. Particularly in his later years he refers more strongly to the so-called "Theory of Will" (*Willenstheorie*) of legal norms [Opalek, 1980 pp. 22, 31]. According to Kelsen, the validity and positiveness of legal norms are based on the empirical will of the creator of the legal norms; thus no logical relations between various legal norms (e. g. between general and individual norms) can be seen as the corresponding meaning of different acts of will. According to Kelsen, no parallelism exists between the validity of legal norms and the truth of statements which might justify the applicability of the logical principles [Kelsen, 1979, p. 15].

As briefly seen in the above passages, Kelsen's view on the applicability of logic to norms shows a drastic turn from the optimism of the early years to one of pessimism.

1. A norm is not a statement nor can it be described as true or false, it can only be described as valid or invalid. Logical relations, however, are only possible among statements that can be described as either true or false. There is no parallelism between the validity of norms and the truth of statements.

2. A norm is the meaning of acts of will; its validity is therefore directly bound to the will of the authorized creator of the norm sentence. It is impossible for us to discuss logical relationships between the validity of legal norms (e. g. statute as a will of a legislator and judgement as a will of a judge) . In dealing with the positive validity of legal norms the logical relations between them are beside the point.

The first reason for Kelsen's rejection is close to the conventional reason why the application of classical logic to norms is rejected by norm

-logicians (this, however, has not been explained clearly by Kelsen himself). This rejection is directed at the semantic nature of norm-sentences that are neither true nor false, as opposed to indicative statements [Weinberger, 1970, p. 189].

This reason for the rejection of the applicability of the logical principles to norms based on the "semantic nature" of norm-sentences has to be examined on logical as well as formal-semantic grounds. It is necessary in order to properly evaluate and respond to the questions raised by the drastic turn about in the light of methodology which Kelsen underwent in his last years, as well as to give a decisive answer to the confrontation regarding methodology in legal logic, terminating its controversies, and thus to establish a basis for the new development in legal logic.

The second reason depends upon whether it is appropriate to understand the legal norms as the meaning of acts of will and directly connect their validity to the will of the creator of the legal norms. We must consider what a legal norm actually is, its validity, its positiveness, etc.

These issues do not directly belong to the objects of logical consideration in the narrow sense as syntacticism; they should be given solutions through the other two departments of modern semiotics, semantics and pragmatics in the sense of property. In this sense, one can see a reasonable excuse for the past legal logic not to consider these issues. Nevertheless, when Kelsen claims that legal norm is the meaning of acts of will and that there is no applicability of logical principles among legal norms because their validity is based on the will of the creator, no longer can one evade the basic concepts of legal theories such as "legal norms" and "validity" if one is to make the applicability of logic dependent to legal norms.

Kelsen's last concept regarding the applicability of the principles of logic to legal norms provides interesting objects of discussion. The first problem cannot be resolved without considering the second. The latter is of such complexity that its solution cannot be easily developed. As a consequence, I suggest that one should approach the problem of the

ABOUT THE APPLICABILITY OF THE PRINCIPLES OF LOGIC TO LEGAL NORM

applicability of the principles of logic to legal norms in the following manner:

1. First, there is to be considered whether the logical principles are applicable to legal norms with respect to the evaluation of the truthfulness of a statement and in the face of the “semantic nature”.

2. So far as necessary for the consideration of the first point, the concept of “legal norm” and its validity will be defined. A thorough discussion of this point has to be omitted here due to space considerations so the discussion will be restricted to the formulation of arguments contrary to Kelsen.

3. If the applicability of logic to norms is theoretically proved, then the problems regarding the logical consequence-relations between legal norms (as presented by Kelsen) can be considered with the methods of logic.<sup>21</sup>

The following must be confirmed before discussing the above considerations: the problem of the applicability of the principles of logic to legal norms can be reduced to the problem of the applicability of logic itself. This is because we are dealing with the applicability of the principles and methods of logic which is the logical system itself.

Two other aspects concerning the applicability of logic to legal norms should be considered; thus, we have to ask two questions:

1. Is classical bivalent logic immediately and adequately applicable to legal norms?

2. Is it necessary to establish a special logic of norms such as “deontic logic”? If so, in what ways would it be applicable to norms?

Regarding the latter question, in forty years of research it has not been possible to go beyond the experimental stage of establishing such a system.<sup>3</sup> Provided that we could consent to the first question, one would doubtfully ask why one should undertake the extraordinary efforts to resolve the second question. In that case we should only have to deal with the application of classical logic to legal norms and thus begin to answer the first question.<sup>41</sup>

In this paper I will set forth the logical and semantic grounds for the

applicability of the system of classical logic to legal norms after indicating my standpoint against the concepts of legal norms in the form of two brief theses (chapters 2-4). I will focus on proving the validity of the logical consequence-relation among legal norms based on the grounds mentioned above, and on clarifying the meaning of the logical consequence-relation of the application process of law which has been rejected by Kelsen. I will also indicate, in a series of theses, my views towards the principal concepts of the general theories and legal methodology of legal norms, under the logical and semantic viewpoints (chapters 5-6). This report concludes with a suggestion for a new approach and the improvement of legal thinking (chapter 7).

## 2 The Starting Point of the Application of Logic to Legal Norms

### 2.1 Theses on Legal Norms and Legal Norm-sentences as a Starting Point

In the following two theses, I would like to put forward my view on legal norms and the legal norm-sentences that contrasts Kelsen's:

*(T. 1) One has to start from the legal norm-sentence rather than from the legal norm.*

*(T. 2) Legal norm is the meaning of the legal norm-sentence and should not be considered as the meaning of an act of will of the creator.*<sup>5)</sup>

### 2.2 The method of Logical Formalization of Legal Norm-Sentences

It is necessary for the further discussion in this work to show the way the legal norm-sentences (the linguistic term for legal norms) should be formalized in classical mathematical logic.

Legal norm-sentences can be formalized by predicate logic of the first order (which constructs the basic part of mathematical logic). For example, there is the following legal norm-sentence:

“The murderer is sentenced to life imprisonment.”<sup>6)</sup>

Although the legal text is written in the form of an indicative sentence, the content is that of a normative sentence. Thus, the text should be rewritten, to indicate its normity, as follows:

(1) “The murderer ought to be sentenced to life imprisonment.”

In order to transfer the normative nature of the sentence to the predicate-logical formula, I would like to introduce the following symbols:

mu( . ): “. is murderer”.

sli( . ): “. is someone, who ought to be sentenced to life imprisonment”.

The legal norm-sentence is to be formalized as follows:

(1'')  $\forall P(\text{mu}(P) \rightarrow \text{sli}(P))$ .

The formula is to be read: “for every P, if P is a murderer, then P is someone who ought to be sentenced to life imprisonment”.<sup>7)</sup>

In the above, the basic scheme for the logical formalization of legal norms has been presented. For the following discussion the logical manner of formalization of legal norms is to be presented in a way to show the inner structure of legal norms in more detail. The components of a legal norm-sentence are the norm-subject and the normative modal-terms such as “obligated”, “forbidden”, etc. The following symbols will be used as a representation<sup>8)</sup>:

na( . ): . is norm-addressee

ac( . ): . is an action as norm-object

ab( ,... ): . is abstaining from ..

ob( ,... ): . is obligated to ..

fb( ,... ): . is forbidden to ..

pm( ,... ): . is permitted to ..

$\forall X( \dots )$ : universal quantifier [e. g. for all X ( ...)].



The norm-sentences can be formulated as follows:

- (2) action (ac) is obligated for all norm-addressees
- (2')  $\forall P \forall A (na(P) \wedge ac(A) \rightarrow ob(P, A))$
- (3) action (ac) is forbidden for all norm-addressees
- (3')  $\forall P \forall A (na(P) \wedge ac(A) \rightarrow fb(P, A))$
- (4) action (ac) is permitted for all norm-addressees
- (4')  $\forall P \forall A (na(P) \wedge ac(A) \rightarrow pm(P, A))$

The relations of the normative modal-terms can be defined as in (5) through (8) using predicate logic. The variable for the norm-addressees is “P”, and A 1 and A 2 are variable actions as norm-objects that are distinct.<sup>91</sup>

(5)  $\forall P \forall A 1 \forall A 2 ((ab(A 2, A 1) \rightarrow ob(P, A 2)) \leftrightarrow fb(P, A 1))$ ,  
[represents that the obligation of abstention from an action is equal to the forbiddance of that action]

(6)  $\forall P \forall A 1 \forall A 2 (ob(P, A 1) \leftrightarrow (ab(A 2, A 1) \rightarrow fb(P, A 2)))$ ,  
[that the obligation of a certain action is equal to the prohibition of the abstention from that action]

(7)  $\forall P \forall A 1 (ob(P, A 1) \rightarrow pm(P, A 1))$ ,  
[that if an action is obliged, then that action is permitted]

(8)  $\forall P \forall A 1 (pm(P, A 1) \leftrightarrow \sim fb(P, A 1))$ .  
[represents the permission of a certain action is equal to the non-forbiddance of that action]

Items (7) and (8) can logically be deducted. For example:

(9)  $\forall P \forall A 1 (ob(P, A 1) \rightarrow \sim fb(P, A 1))$   
[this expression represents that the action obligated is not forbidden]

This is logically equivalent to:

(10)  $\forall P \forall A 1 \sim (ob(P, A 1) \wedge fb(P, A 1))$ .  
[this expression represents that the obligation and the forbiddance of an action at same time is inconsistent]

The relations of the various normative terms that are defined above can be treated as extra-logical vs. logical axioms for every logical calculus (*logischen Kalkül*) of norms. They may also be treated as additional presuppositions for an already existing calculus. In the foregoing manner of formalization, these axioms are not presented as logical laws, but as laws of the linguistic usage. On the contrary, the so-called special logic of norms is concerned with systemizing these relations as logical laws, or more precisely, as norm-logical laws. For this reason it is necessary to establish a semantic foundation.

### 3 The Concept of Truth in Logic

As stated above, the scholars who reject the direct applicability of classical logic to legal norms stress the semantic nature of norms as their reason. Such a viewpoint is based on the following two assumptions and the inference resulting from them (which is famous “Dilemma of Jørgensen” [Jørgensen, 1937, pp. 288-296]).

1. Norm-sentences cannot be described as true or false.
2. The system of classical logic is based on the evaluation of sentences in respect of true or false.
3. Classical logic cannot be applied to norm-sentences.

Assumptions 1 and 2 do not require discussion; the inference is made from 1 and 2 to get 3, however, it is questionable because the terms of truth in assumptions 1 and 2 are not identical. The pessimistic view in regard to the applicability of classical logic to norm-sentences results from a misunderstanding of the concept of truth in logic and the normal epistemological truth-term[Yoshtno, 1978a, p. 142]. Here, one should first correctly present the truth-term and consider its applicability to norm-sentences.

The concept of truth in classical logic, especially in class-calculus

and in accordance with the predicate-calculus, was formally defined by Tarski.<sup>10)</sup> In the following paragraphs, the outline of Tarski's interpretation of the concept of truth will be presented in a concise manner.

The truth-valuation, i.e. the assignment of truth-values, for an atomic proposition-formula in predicate logic can be presented as described below. The following symbols will be used:

- $\Phi$ : a single term predicate
- $i$ : an interpretation-functor
- $\alpha 1, \dots, \alpha n$ : individual constant or variable

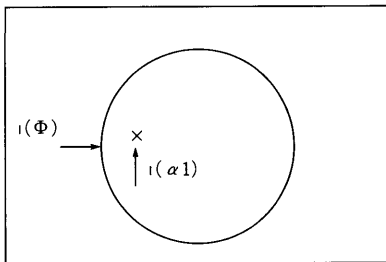
- (11)  $\Phi(\alpha 1, \dots, \alpha n)$  is true under  $i$  if  $[i(\alpha 1), \dots, i(\alpha n)] \in i(\Phi)$ , and
- (12)  $\Phi(\alpha 1, \dots, \alpha n)$  is false under  $i$  if  $[i(\alpha 1), \dots, i(\alpha n)] \notin i(\Phi)$

11 and 12 are equivalent to 11 with "if and only if, then" instead of "if, then". Accordingly, when a one term predicate applies to an individual constant or variable is part of the set which is the extension of the interpreted predicate, then the respective statement-formula is true and, if not, then it is false. For a better understanding of this principle an illustration will be given with a one-term predicate :

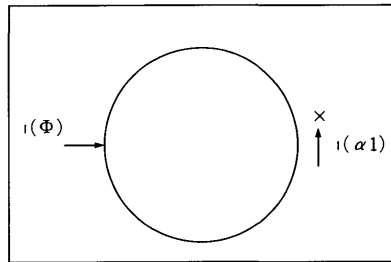
(11')

(12')

Field of Interpretation



True



False

This concept of truth is purely formal, so that this definition can be

ABOUT THE APPLICABILITY OF THE PRINCIPLES OF LOGIC TO LEGAL NORM  
 redefined in the following manner (Tarski, himself, did not do this) :

(11'') value  $(\Phi(\alpha_1, \dots, \alpha_n), i) = 1$  if  
 $[i(\alpha_1), \dots, i(\alpha_n)] \in i(\Phi)$

(12'') value  $(\Phi(\alpha_1, \dots, \alpha_n), i) = 0$  if  
 $[i(\alpha_1), \dots, i(\alpha_n)] \notin i(\Phi)$

Regarding the logical nature of the connectives (junctur) and the quantifiers (quantor) that consist of the complex statements by connecting the element statements can be defined in the following manner (where A and B are used respectively for the boolean formulae as well as i for an interpretation with the interpretation-field u) :

(13)  $\sim A$  is true under i if and only if A not true under i.

(14)  $(A \rightarrow B)$  is true under i if and only if A not true or B is true under i.

(15)  $(A \& B)$  is true under i if and only if A is true under i and B is true under i.

(16)  $(A \vee B)$  is true under i if and only if A is true under i or B is true under i.

(17)  $\forall_w A$  is true under i if and only if for all u the following holds:  
 $u \in U \Rightarrow A$  is true under  $i^u_w$

(18)  $\exists_w A$  is true under i if and only if for at least one u the following holds:

$u \in U$  and A is true under  $i^u_w$

(19)  $i^u_w$ : the interpretation, that is differed from i at least by that it assigns w to u.

$i^u_w := (i \setminus \{i(w), w\}) \cup \{u, w\}$

Based on the foregoing demonstrations, one should point out that the definition by Tarski of the truth-term of logic is constructed purely formally. In the above definition, that a certain statement is true (i. e. the fulfillment of the given propositional functions by the given circumstance)

is represented as the inclusion of the interpreted individual-constant or variable in the elements of interpreted predicate symbol, or, in other words, the inclusion of the objects specified by the individual-constant or variable in the elements of the objects set possessing the characteristics represented by the predicate symbol (see 11' and 12'). However, it is not questioned by what criteria the fulfillment must be decided. For the concept of truth in logic, the criteria, which recognizes or determines the above discussed fulfillment relations, or the relationship of an element to a set, does not matter. The truth-valuation of statements in logic does not require that the fulfillment relations in question are "determinable through observation" [Weinberger, 1973, p. 314]; it does not require that they be objectively determined. Logical calculations merely need the premises of the above relations from any standpoint - in other words, either affirmation or negation of the element relationship of a set. Under such premises is the kind of fulfillment relations concluded - the element relationship of a set concluded - calculated by logic. According to the definition by Tarski, the logical calculus needs as a presupposition nothing but the purely formal principle of bivalence, namely, that a value of two possible values is allotted to every sentence unambiguously and uniformly [comp. Yoshino, 1978a, p. 145].

#### 4 The Application of the Concept of Truth in Logic to Legal Norms

##### **4.1 The Direct Application of the Concept of Formal Truth in Logic to Legal Norms**

In order for the classical logical bivalent system to be applicable to legal norms, the classical logical concept of truth has to be applicable to legal norms. The standpoint, that denies the applicability of the classic logic to norm-sentences, as discussed above, denies the applicability in question because of the semantic nature of norm-sentences. That the norm-sentences cannot be determined by observation to be either true or false, and, that they cannot be therefore determined "in-subjectively" but

only subjectively, is true, as these people say. As it has been demonstrated in the last chapter, the concept of truth in logic has nothing to do with the issue if the above mentioned fulfillment relations, or the elements relations of the set, must be determined “inter-subjectively”, for example, through observation. Thus, the semantic nature of norm-sentences, which is compared to indicative sentences, can be said not to interfere with the truth evaluation of norm-sentences in logic, that is, the truth valuation to them.

The question whether the logical truth-terms can be applied to norms can be answered in a positive way according to the foregoing demonstration of the concept of truth in logic (that is, if the bivalence-principle can be applied in the field of norms).

The norm-sentence is to be understood as a linguistic term of an assertion about a normative state of affairs, namely, about who is obliged to do something or for whom something is forbidden, etc. In my opinion, the norm-sentence, also the legal norm-sentence, can be evaluated in a positive or negative way; specifically it can be valued as “valid” or “invalid”, or as “just” or “unjust”. The bivalence principle is definitely applicable here. Therefore, the concept of truth in classic logic should be applicable to legal norm-sentences.

Of course, one ought to keep in mind that a necessary condition for the assignment of truth-values in the logical calculus is that it is performed unambiguously and uniformly according to the bivalence-principle. The fact that normative evaluations are actually not inter-subjective, but relative, does not prevent the application of truth-values to norms. So long as it is a matter of logical validity, the preciseness and uniformity of the criterium of evaluation is presupposed within the inference that has to be examined, and this is sufficient [Yoshino, 1978a, p. 148; 1974, p. 52].

I will now explain the truth-evaluation of legal norms by example in relation to the predicate-logical formula of legal norms as mentioned above. Firstly, the assignment of truth-values to atomic sentences (of which the respective legal norm-sentence consists as a complex statement) is to be presented according to the Tarski formula.

If  $p_1$  is used for a defined person, then the truth-evaluation of the sentence “ $p_1$  is a murderer”, that is,  $\mu(p_1)$ , can be defined as follows (whereby the world of interpretation is the set of people):

(21+)  $\mu(p_1)$  is true under  $i$  if  $i(p_1) \in i(\mu)$

(21-)  $\mu(p_1)$  is false under  $i$  if  $i(p_1) \notin i(\mu)$

In this manner, the truth and falsity of the sentence “ $p_1$  is the person who ought to be sentenced to life imprisonment” is defined as follows:

(22+)  $sli(p_1)$  is true under  $i$  if  $i(p_1) \in i(sli)$

(22-)  $sli(p_1)$  is false under  $i$  if  $i(p_1) \notin i(sli)$

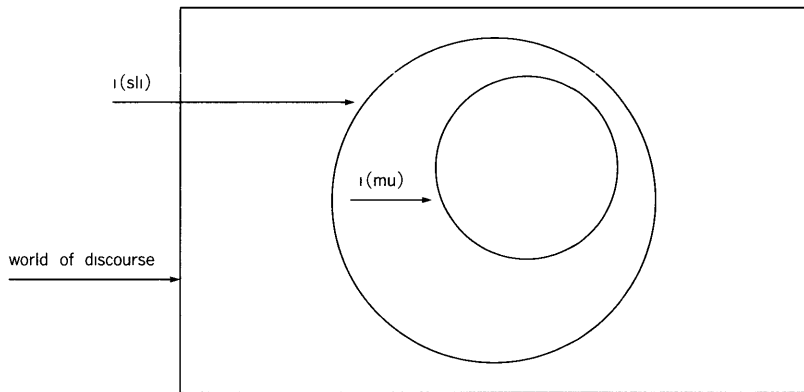
The truth and falsity of the norm-sentence as a whole sentence, consisting of these two atomic sentences, can be defined as follows:

(23+)  $\forall X(\mu(X) \rightarrow sli(X))$  is true under  $i$  if  $i(\mu) \in i(sli)$

(23-)  $\forall X(\mu(X) \rightarrow sli(X))$  is false under  $i$  if  $i(\mu) \notin i(sli)$

This connection can be illustrated as follows:

Scheme I



As far as the logical consistency of argumentation, i. e. the possibility

of calculus of consequence-relation is concerned, we can deal with logical truth in a purely formal way without interpreting it. Logical truth is purely formal; namely, “1” or “0”. Even though it may be true that the predicate that forms a norm-sentence and the one that forms an indicative sentence have different qualities. This does not interfere with the logical operation if the logical truth as such is treated as purely a formal one. Every predicate forming a statement may have its own semantic qualities and its own criteria for evaluation as long as it is evaluated unambiguously and uniformly according to these criteria wherever it occurs within the inference that has to be examined. For example, whether Monika is a cat or a woman, smart or dumb, young or old, beautiful, nice, lazy in writing, faithful, bad, guilty or unjust, these judgements can be decided according to the criterion of the respective predicate that which expresses the respective attribution in the given statement. The sentence is evaluated positively or negatively with “1” or “0” according to the respective criterion [comp. Yoshino, 1978a, p. 148]. The logical calculus only deals with this formal truth of “1” or “0”. The formulas that are deduced in a calculus can be interpreted again in daily language by reading every predicate in the same way as ordinary language is expressed in it - and this is sufficient. Thus, there occurs no problem with “mixed premises” in this way of logical treatment i.<sup>11</sup>

#### **4.2 The Application of the Normatively Interpreted Concept of Truth in Logic to Legal Norms**

It has been demonstrated in the last chapter that the concept of truth in classical logic can, due to its pure formality or lack of content, be applied to indicative sentences as well as normative sentences regardless of their “semantic” nature. In this way, the logical calculus functions without any difficulty.

There is another more extended method. It is possible to consider the “semantic” nature of sentences in the application of logic. In connection with the “semantic” nature of a given way of thinking, to which the logic is applied, it is possible to interpret the concept of truth as “indicative-



true” or “normative-true”, and to apply these truth-values to the appropriate sentences. When one speaks of the truth of an indicative sentence, that is, of the truth of a factual statement that can be established through observation of the correspondence of a sentence with the fact in some way, then it is a question of the indicative interpretation of the concept of truth. In the same way as this interpretation is permissible, the concept of truth can also be interpreted normatively. I will now further elucidate this connection.

Firstly, I will compare the structure of an indicative sentence and a norm-sentence by example. Some examples of indicative sentences are :

(24) “If water is heated up to 100 degrees Celsius under normal pressure, then it vaporizes.”

(25) “If somebody is a murderer, then he ought to be sentenced to life imprisonment.”

Both of these sentences are assertions demanding the states of affairs expressed in the sentences to be evaluated positively (i. e. to be taken as true). It is asserted in the above indicative sentence that the respective indicative state of affairs be indicatively-true ; it is asserted in the normative sentence that the respective normative state of affairs be normatively true.

If the respective truth-affirmations are presented explicitly the sentences should be expressed in the following way :

(24') “It is indicatively true that, if water is heated up to 100 degrees Celsius, under normal pressure, it vaporizes.”

(25') “It is normatively true that, if one murders, one ought to be sentenced to life imprisonment.”

As terms for normative values, the following are conceivable : validity/invalidity, justice/injustice, etc. If we accept the term of validity, we can represent (25') as follows :

(25'. 1) "It is normatively valid that, one ought to be sentenced to life imprisonment if one is a murderer."

(25'. 2) "It is normatively invalid that one ought to be sentenced to life imprisonment, if one is a murderer."

In sentences (24) and (25), (24') and (25') as well as (25'. 1) a parallel structure of truth-evaluation and state of affairs can be found. The normative concept of validity is parallel to the indicative concept of truth, and the normative state of affairs is parallel to the indicative state of affairs. I propose the following thesis :

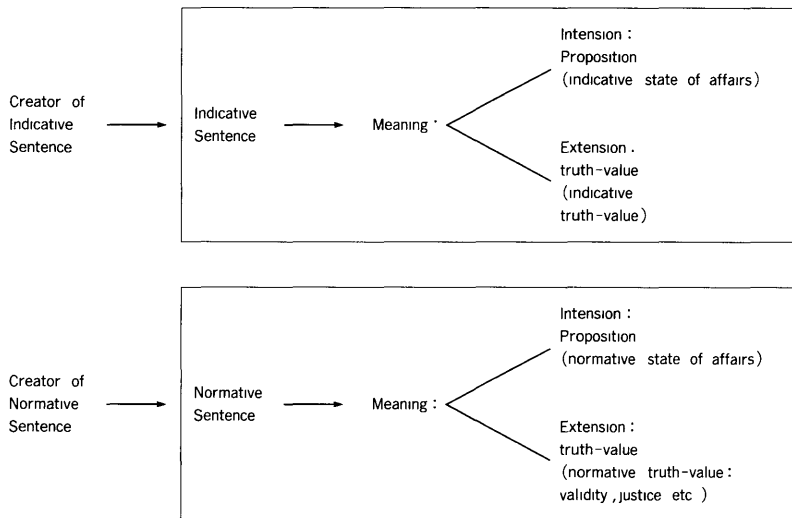
*(T. 3) The concept of normative validity can be regarded as a concept of truth, that is, a concept of normative truth.*

At this point, I would like to make a few comments about my foregoing conception of legal norms and about the normative validity as an antithesis to Kelsen. As mentioned above, Kelsen rejects the parallelism between the truth of statements and the validity of norms mainly for the following reason : according to Kelsen, the truth of a statement is the nature of a statement, whereas the validity of a norm is the meaning of the act of will. Its specific "notional" existence ; a false statement is still a statement, but if a norm is not valid, this means that this norm does not exist [Kelsen, 1979, p. 167].

I do not agree with Kelsen's concept concerning the rejection of the parallelism between norms and statements. In my opinion, it appears to be on Kelsen's misconception of the terms "statement" and "meaning". When he said that truth was the nature of a statement, he must have taken the word "statement" for an indicative statement, whereas when he said that validity was not the nature of norms but its existence, he must have understood the word "norm" to be the norm as meaning, not as the norm-sentence of which the norm is the meaning. He should have started with norm-sentences and compared indicative statements with them.

The starting point of theory of meaning that forms the basis of my foregoing criticism of Kelsen and my following discussions are based on Carnap's semantical theory [Carnap, 1947, p. 23] and can be illustrated:

Scheme II



When Kelsen says that the truth of a statement is its very nature, then the statements, according to the above mentioned context, would have to be indicative sentences, since otherwise truth could not be their nature ; for the extension of the meaning is truth. Because Kelsen understood by norm the meaning, he should have proceeded from norm-sentences and not from the act of will. If one starts with norm-sentences one can speak of a (normative) truth of norm-sentences.

As illustrated in the above scheme, one can understand the meaning of sentences in two ways (according to Carnap's semantics) : the intention of sentences is the proposition and their extension is the truth-value [Carnap, 1947, p. 25]. According to this definition of the meaning of Sentences and the illustrations above, I will propose further theses :

(T. 4) *The intention of norm-sentences is their proposition, namely their*

*normative state of affairs.*

(T. 5) *The extension of norm-sentences is their truth-value, namely their normative truth-value.*

I will now formally semantically found my conception of the applicability of the normatively interpreted concept of truth to legal norm-sentences. The first to be presented will be the definition of the indicative and then of the normative truth. The definition of the indicative truth in an atomic indicative sentence can be put in the following manner (the sign  $\pi_i$  stands for the set of indicative predicates) :

- (26+)  $\Phi(\alpha_1, \dots, \alpha_n)$  is indicatively true under  $i$  if  
 $[i(\alpha_1), \dots, i(\alpha_n)] \in i(\Phi) \ \& \ \Phi \in \pi_i$
- (26-)  $\Phi(\alpha_1, \dots, \alpha_n)$  is indicatively false under  $i$  if  
 $[i(\alpha_1), \dots, i(\alpha_n)] \notin i(\Phi) \ \& \ \Phi \in \pi_i$

In talking about an indicative sentence that could be labeled true or false by means of observation of the correspondence in some way, we are always dealing with this concept of truth. It is often regarded identical with the logical truth ; however, to be precise, it is not a logical truth in the actual sense but an indicative interpretation of the truth. If one can see this relationship, then one has no problems with the above mentioned "Dilemma of Jørgensen". The pessimistic opinion in regard to the applicability of classical mathematical logic to norms is due to the wrong identification of the indicative truth with the purely logical truth.

The truth can also be interpreted normatively. The definition of the normative truth in an atomic normative sentence will be formulated as follows (the sign  $\pi_n$  stands for the set of normative predicates) :

- (27+)  $\Phi(\alpha_1, \dots, \alpha_n)$  is normatively true under  $i$  if  
 $[i(\alpha_1), \dots, i(\alpha_n)] \in i(\Phi) \ \& \ \Phi \in \pi_n$
- (27-)  $\Phi(\alpha_1, \dots, \alpha_n)$  is normatively false under  $i$  if  
 $[i(\alpha_1), \dots, i(\alpha_n)] \in i(\Phi) \ \& \ \Phi \in \pi_n$

In talking about a normative sentence, i. e. about the norm itself, then the normative concept of truth is the central issue. The normative concept of truth, such as “normative-true” or “normative-false” can, according to the context, be determined as valid/invalid, just/unjust, etc.

One point that should be highlighted is that the difference between indicative-true and normative-true arises out of the pragmatic circumstances of predicates, which appears in the verification or assessment of one elementary sentence or of elementary sentences that are contained in a complex sentence.

In this logical discussion, we can expect that it is presupposed that the two kinds of sentences, the indicative and normative sentences can be distinguished from each other by some point of view. The semantic difference between indicative and normative sentences is commonly accepted. To precisely define the difference is an interesting but difficult problem, and there is not yet a generally accepted opinion regarding this issue. My method consists in founding the difference between the dissimilarity of interpretations of the concept of truth and explaining the semantic and pragmatic disparity of the predicates. To precisely define the difference between the indicative and normative predicates is a task that does not belong to the field of logic, but to the pragmatic field.<sup>12)</sup> Logic itself does not require that the criteria of this disparity be settled, but this disparity can provide us with a starting point. Is it not commonly accepted that the indicative predicates are different from the normative predicates?

Regarding the assessment of norm-sentences, in contrast to the indicative truth, “normative-true” and “normative-false” will probably neither be confirmed nor contradicted through the observation of the correspondence between the sentences and the objects they represent. But one can assess a norm in some manner, to some criterion whatsoever, as positive or negative, i. e. as valid/invalid, true/false, just/unjust. Here, the principle of bivalence (as states above) can be utilized. Therefore it is possible to speak of a normative truth, and to formulate and analyze the legal norms and the normative inferences logically through this normative

interpretation of the concept of truth.

The indicative truth of the indicative sentence (24) as a whole can be defined as follows where :

wa( . ) : . is water that, heated up to over 100 degrees celsius under normal pressure,

va( . ) : . vaporizes :

(24')  $\forall X(\text{wa}(X) \rightarrow \text{va}(X))$

(Df. 24' +)  $\forall X(\text{wa}(X) \rightarrow \text{va}(X))$  is indicatively true under i if and only if  $i(\text{wa}) \subset i(\text{va}) \ \& \ (\text{wa} \in \text{pi} \ \& \ \text{va} \in \text{pi})$

(Df. 24' -)  $\forall X(\text{wa}(X) \rightarrow \text{va}(X))$  is indicatively false under i if and only if  $i(\text{wa}) \not\subset i(\text{va}) \ \& \ (\text{wa} \in \text{pi} \ \& \ \text{va} \in \text{pi})$

The normative truth of the sentence (25) as a whole can be defined as follows :

mu( . ) : . is a murderer

sli( . ) : . is someone who ought to be sentenced to life imprisonment.

(25')  $\forall X(\text{mu}(X) \rightarrow \text{sli}(X))$

(Df. 25' +)  $\forall X(\text{mu}(X) \rightarrow \text{sli}(X))$  is normatively true under i if  $i(\text{mu}) \subset i(\text{sli}) \ \& \ (\text{mu} \in \text{pn} \ \& \ \text{sli} \in \text{pn})$

(Df. 25' -)  $\forall X(\text{mu}(X) \rightarrow \text{sli}(X))$  is normatively false under i if and only if  $i(\text{mu}) \not\subset i(\text{sli}) \ \& \ (\text{mu} \in \text{pn} \ \& \ \text{sli} \in \text{pn})$

If we take as the normative truth-value the normative value of validity, the definitions will be as follows (where png stands for the set of the normative predicates that are to be assessed with respect to the normative validity)<sup>13)</sup> :

(Df. 25.1' +)  $\forall X(\text{mu}(X) \rightarrow \text{sli}(X))$  is normatively valid under i if  $i(\text{mu}) \subset i(\text{sli}) \ \& \ (\text{mu} \in \text{png} \ \& \ \text{sli} \in \text{png})$

(Df. 25.1' -)  $\forall X(\text{mu}(X) \rightarrow \text{sli}(X))$  is normatively invalid under i if  $i(\text{mu}) \not\subset i(\text{sli}) \ \& \ (\text{mu} \in \text{png} \ \& \ \text{sli} \in \text{png})$

On grounds of the presentation above, one could draw the conclusion that the logical concept of truth in an extended way is also applicable to norms.

Here one probably needs to point out the following: when such a normative interpretation of the concept of truth takes place — although this is not always necessary for the logical calculus, but useful for some practical purposes such as the foundation of the validity of legal norms, for example — then this concept must be used consistently and continuously within the inference of which the classical-logical consequence-relation is to be examined.<sup>14)</sup> Regarding this point I can add the following:

1. So long as it is a question of a logical relation between norms, that is, a consequence-relation or a contradiction, we are always dealing with the same truth-concept, the normative truth, provided that the logical truth is normatively interpreted.

2. Regarding the first part of the norm-sentence that expresses the legal requirement, its “semantic” nature is no longer purely indicative since the truth of the antecedent ought to be assessed under the very criterion of truth-evaluation of the norm-sentence as a whole, and also of the consequence. For example, if a man is convicted of murder, this does not mean a pure description of facts but exactly the classification of this man to the group of those who ought to be imprisoned; something that can only be obtained by a legal value judgement.

In the above, the applicability of the concept of truth of classical logic was established in the formal concept of truth as well as in the normatively interpreted concept of truth on semantic grounds. As a consequence, one should not maintain any more that classical logic cannot be applied to legal norms because of the “semantic” nature of legal norm-sentences as opposed to indicative statements. The system of classical predicate logic can directly be applied to legal norm-sentences of which the meaning is legal norm. This is the case for the principle of logical consequence relation as well as for the principle of the exclusion of contradiction.

## 5 On the Logical Consequence-Relation Between Legal Norms

### 5.1 The Applicability of Logical Consequence-Relation Between Legal Norms and the Normative Validity of Legal Norms

Here I would like to discuss the logical consequence-relation between legal norm-sentences. In order to answer the question whether the logical consequence-relation is applicable within the framework of legal norm-sentences, one must precisely understand the term “logical consequence-relation”. Since the applicability of the concept of truth in logic has been proven in the last chapter, it is only necessary here to define the logical consequence-relation between legal norm-sentences by means of the concept of truth. Here I am using the normatively interpreted truth-concept that I defined in the last chapter. The definition of the logical consequence-relation of legal norms can be presented as follows:

$N_1$  : a set of legal norm-sentences

$N_2$  : a legal norm-sentence, not identical with  $N_1$

$\Rightarrow$  : from . it follows logically . .

(Df. 2)  $N_1 \Rightarrow N_2$  , if and only if for all interpretations of  $\varphi$  the following is valid: if  $N_1$  is normative-true, then  $N_2$  is normative-true.

In the foregoing definition of the logical consequence-relation between legal norm-sentences, “normative-true” can be replaced by “normative-valid” since the normative-valid concept can be thought of as a part of the concept of normative truth. From the definition one can conclude that a norm-sentence is normatively valid when it logically follows from the set of norm-sentences that are established as normatively valid. By means of the logical inference, the normative validity of norm-sentences can be transferred to the norm-sentence which is logically deduced from it. Thus, the logical consequence-relation plays an essential part in the justification of legal norms to be set up. In any case, the demonstration in chapter 4 of the applicability of the logical truth-



concept to legal norms and the definition in this chapter of the logical consequence-relation between legal norm-sentences have proven that the logical consequence-relation is applicable within the framework of legal norms.

## 5.2 On Judicial Syllogism as Justification Process

In the process of law-applications, the inference relations are conceived mainly in two dimensions: the phase at which individual legal decisions are acquired — decision or discovery process — and the phase at which the acquired decisions are justified — justification process. Since both are inferential processes, logic commonly plays a part in both aspects. The logical consequence-relation plays a crucial part, however, in the second process. Here the validity of each individual concrete legal norm-sentence as the acquired legal decision is justified, as the norm-sentence is logically deduced from other legal norm-sentences whose validity is already recognized together with the factual statements about describing a given concrete case. The essence of justification can be said to be a logical demonstration. The logical structure of the justification inferences in law application has been known as syllogism. This kind of syllogism is called judge syllogism (*richterlicher Syllogismus*) [Roedig, 1973, p. 163] or judicial syllogism (*Justizsyllogismus*); it has been also called normative syllogism or practical syllogism [Stammler, 1911, p. 656], because of its normative character.

It is essentially correct that the logical structure of the justification process has been explained as a syllogism. However, it has yet been explained why the decision can be justified, in other words, why is the decision to be regarded as valid when it is logically deduced? In my opinion, the validity of the decision is due because a decision, as a conclusion, inherits the validity of law as the major premise through logical deduction. The validity of legal norm-sentences, as demonstrated above, is to be regarded as a logical truth value of them and so the conclusion is valid when it is a logical consequence deduced from a valid premise as it is true when it is a logical consequence from a true premise.

In this sense the logical consequence-relation in law-applications plays an important role in the justification of individual legal decisions.

It is also to be noted that a legal decision is seldom actually deduced directly from only the statute and facts. The statute is abstract; individual legal norm-sentences, which represent the legal decision for individual cases, are concrete; and a concrete norm-sentence cannot be logically derived directly from an abstract norm-sentence. This does not mean, however, that logical consequence-relation does not play a part in law-applications. Between the statute and the concrete fact of the case, further presuppositions are added as the result of the law-text interpretation for the subsumption of the facts of the case under the law.

This is the “making concrete” (Konkretisierung) of the normative meaning of the statute [Roedig, 1973, p. 173]. This can be presented in several stages. Globally speaking it can be divided into two stages :

1. the “making concrete” of statute for its general application to various possible situations that is given by text of commentary or in published opinions of the court ; and
2. the further substantialization of the substantialization above for the special application of law to a concrete case that is sometimes stated additionally in the judgement deposition.

If these presuppositions are demonstrated clearly and added to the justification process, then the decision can be proved as a logical deduction from all these presuppositions of which the validity has already been established, and the validity of this individual legal norm is justified [Yoshino, 1978b, p. 282]. This justification process can be seen as a modified judicial syllogism when the statute together with its assumptions of substantialization is accepted as an actual law<sup>15)</sup> and these, in total, are understood to be a major premise.

In the following, the logical structure of the justification process in law-application is presented schematically [Yoshino, 1978b, p. 282]<sup>16)</sup> whereby the following terms are represented by the following symbols :

lr( . ) : “ . fulfills statute legal requirement”

If(.): “. legal effect is generated”

lr1(.): “. fulfills concrete legal requirement”

lr2(.): “. fulfills second-stage concrete legal requirement”

Scheme III

- |  |  |
|--|--|
| (a) Statute  | $\forall P(lr(P) \rightarrow lf(P))$   |
| (b) Published opinions of the court<br>or text of the commentary             | $\forall P(lr1(P) \rightarrow lr(P))$  |
| (c) Additional interpretative sentence<br>made by judge in the concrete case | $\forall P(lr2(P) \rightarrow lr1(P))$ |
| (d) Fact   | $lr2(p1)$                              |
- 
- |               |          |
|---------------|----------|
| (e) Judgement | $lf(p1)$ |
|---------------|----------|

In this manner the logical consequence-relation is valid for the justification process in law-application. The logical deduction of (e) from premise (a) to (d) can be proved as follows :

- |  |             |
|--|-------------|
| 1 . $\forall P(lr(P) \rightarrow lf(P))$   |             |
| 2 . $\forall P(lr1(P) \rightarrow lr(P))$  |             |
| 3 . $\forall P(lr2(P) \rightarrow lr1(P))$ |             |
| 4 . $lr2(p1)$                              |             |
|  | $lf(p1)$    |
| <hr/>                                      |             |
| 5 . $lr(p1) \rightarrow lf(p1)$            | 1, U. I.    |
| 6 . $lr1(p1) \rightarrow lr(p1)$           | 2, U. I.    |
| 7 . $lr2(p1) \rightarrow lr1(p1)$          | 3, U. I.    |
| 8 . $lr1(p1)$                              | 4, 7, M. P. |
| 9 . $lr(p1)$                               | 8, 6, M. P. |
| 10. $lf(p1)$                               | 9, 5, M. P. |

### 5.3 About Kelsen's Formulation of the Applicability of Logical Consequence-Relation to Legal Norms

I will now express my view according to the foregoing scheme concerning Kelsen's opinion about the applicability of logical consequence-relation to legal norms. Kelsen mentions his doubts about the normative syllogism mainly for the reason that both premises here are of a different nature ; that is, the major premise statute, is a norm and the minor premise fact, is a proposition [Kelsen, 1979, p. 185]. Kelsen's further discussion deals with the possibility to deduce individual norms from general norms and also with the possibility to deduce less general norms from more abstract norms. Kelsen denies the possibility to logically deduce individual norms from general norms. The reason for this rejection is that what matters to these two norms is the meaning of the act of will, and the will of the creator of law in general norms does not include the will of the judge in individual norms [Kelsen, 1979, p. 185]. However, he considers the deduction of less general norms from more abstract norms possible insofar as the less general concepts of these norms are included in the more general concepts of this norm [Kelsen, 1979, p. 201].

Regarding the first point of the problem of normative syllogism suggested by Kelsen, it could belong to the problem of "mixed premises" which has been also brought up by some legal logicians and norm-logicians. The problem was also discussed in connection with the formalization of legal norms, according to which the antecedent as a legal requirement would be an indicative statement, whereas the second part as a legal effect would be a norm-sentence. This is the problem of "mixed-premises".<sup>171</sup> As I have already pointed out in the previous chapter, one can ignore this problem in terms of the logical consequence-relation if one proceeds with the concept of formal truth (4.1). The Problem of "mixed premises" disappears here. If, however, one applies the concept of normatively interpreted truth in order to demonstrate the normative nature of the norm-sentence explicitly, then "mixed premises" may occur; but we have to consider here whether in judicial syllogism the fact, more precisely, the sentence that expresses that the fact in a concrete case fulfills the

legal requirement that was “made concrete”, is a purely indicative sentence. This sentence is not a purely indicative sentence, and normative truth-terms ought to be assigned to it. For, as I have already mentioned, the judgment that the fact fulfills the legal requirement should be made in connection with the judgment that the relevant legal effect is to be assigned to the fact of the case. Logically, in the judicial syllogism, a real problem does not occur with “mixed premises”.

The second problem suggested by Kelsen — deducing individual norms from general norms and deducing less general norms from highly abstract norms — can be explained as a problem of justification in the logical structure in law-application, or, more precisely, from the standpoints of “exemplification” and “particularization”[Roedig 1973, p. 173]. The former is an issue of “exemplification” and the latter, of “particularization”. From a logical point of view, the second aspect cannot be regarded as logical deduction; to derive a less general norm-sentence from a more abstract norm-sentence, concreteness, which is not included in the premise, must be added, for logic does not allow that something not included in the premise be deduced in the conclusion. The less general norm-sentence can only be deduced if “particularization” is performed, that is, if one is to add the additional presuppositions that present more precisely the induction of less general concepts of less general norms in more general concepts of more abstract norms. (It should be noted that Kelsen himself assumes that the less general concept of the former norm is included in the more general concept of the latter norm.) Consequently, it is a matter of establishing the additional presuppositions that are to be mentioned in the points (b) and (c) of the foregoing scheme III and thus a matter of their acceptability; not of the immediate logical deductibility of less general norms from more abstract norms.

Regarding the aspect of exemplification, that is the deduction of individual norms from general norms, from the logical point of view there has to exist logical deduction because of the logical principle of “universal exemplification”. However, matters become complicated when Kelsen says that the general norm as the meaning of the act of will of the

legislator cannot imply the individual norm as the meaning of the act of will of the judge. We cannot argue directly and logically against this concept of Kelsen because we are dealing with the concept of legal norms and that of their practical validity; to answer those questions is not a logical assignment. How should we deal with Kelsen's discussion? There is no doubt that one cannot accept a logical consequence-relation between the actual act of will of the legislator and the judge. How should we counter-argue this view of Kelsen? In what respect does the logical relation play its part?

In his later years, Kelsen associated the concept of legal norms with the practical act of will, defining it as the meaning of the act of will. He based the practical validity of legal norms on the experiential act of will by the authorized norm creator. One can perceive symptoms of inclination to empiricism in this act, which had been brought about by Kelsen's stay in the United States. It is not, however, appropriate to consider legal norms the meaning of the act of will, as I have already expounded upon this.

If one does not start with the meaning of the act of will of the creator of the norm-sentence, but with the meaning of the norm-sentence that is established by the creator of norm — in short, not with acts of will but with norm-sentences —; then one can deal with the logical relation between two legal norm-sentences that are established by different creators of legal norm. This is the case, because once the meaning of legal norm-sentences is given, it is not directly tied to the will of the creator of the legal norm anymore. Once the sentence is declared a law, the meaning of the general legal norm-sentence as law is not identical with the meaning of the act of will actually presented by the actual legislator.

Legal norm as the meaning of the legal norm-sentence, however, cannot exist as an entity. In each case it is dependent on the person who interprets the respective norm-sentence. As Kelsen puts it: "Without an imperator, there is no imperative" [Kelsen, 1979, pp. 23, 187]. Rather I would say: "Without an interpreter, there is no interpretation, i. e. no legal norm as the meaning of the legal norm-sentence." The world of

norm as the meaning of law only occurs in the conception of the judge in his respective interpretation of the statute. The judge as an individual creator of law ought to start with the statute by also comparing with the actual will of the legislator and by keeping in mind the social circumstances to which laws are applied. He also ought to consider the effect of further decisions according to his decision, evaluate them and establish the normative meaning of the law for the concrete case by considering it in this way. During this thinking, he can have an idea of positive legal norms as a meaning that is not identical with the actual meaning of the original legislator. Then he has to reconcile his decision with the normative world presented by the actual legislator. In his mind the judge has to reconcile his decision with normative world of legal norm imaged by him. The logical consequence-relation is valid for this relationship in the judge thought. I am going to describe this relationship more figuratively in the following chapter.

It has been clarified in the last chapter that logical principles can be directly applied to legal norm-sentences; between the legal norm-sentences, which represent legal norms, a logical consequence-relation is found. Is this also true for the legal norms as the meaning of legal norm-sentences? To those, logical principles apply indirectly, through legal norm-sentences. Since legal norms as meaning appears merely in the conceptual world of the interpreter, the relation between legal norms — e.g. the relation between abstract, general legal norms (the meaning of statute) and individual, concrete legal norms (the meaning of decision) — is to be only discussed in the conceptual world of the judge as the same interpreter. Logical consequence-relation applies as a result of the interpretation, between the legal norms that appeared in the conceptual world, through legal norm-sentences. This, however, does not mean that logical principles merely apply only within each subjectivity. In order to establish communication, to lead logical debates, and to have a consistent theory, logical relations are required to be valid in the world of meaning which is symbolized by different subjectivities, as long as there is a demand for theoretical communication among different subjectivities, transmitting

ABOUT THE APPLICABILITY OF THE PRINCIPLES OF LOGIC TO LEGAL NORM  
the symbolized world of meaning to others in the form of sentences,  
through language. Logic is exactly the premise of theoretical communica-  
tion.

## 6 Legal Norms and Law Application with Respect to the Applicability of Logical Principles

According to the above frame work I would like to explain my  
version of legal norms and law application concerning mainly the applica-  
bility of logic in the following theses by likening law application to the  
painting of a picture.

1. Statute is a general and abstract legal norm-sentence that has been  
linguistically represented. It is a linguistically expressed, abstract, printed  
picture of legal norms, comparable to a children's painting book (see (a)  
in the scheme III).

2. Jurists, judges or legal scholars paint their pictures of legal norms in  
the frame of this printed picture so as to substantiate the normative  
content of general legal norm-sentences. The difference between judge  
and legal scholars lies in that judge is legally authorized to paint the  
picture by other legal norms, whereas a legal scholars is not. The legal  
norms are still the meaning of the painted picture. (see (b) in the scheme).

3. The judge, if necessary, paints the details of the picture that corre-  
sponds to the concrete case in order to make the legal norm more  
concrete. This is a more substantial picture; the legal norms are still the  
meaning of this picture (see (c) in the scheme).

4. The judge, in a concrete case, decides upon the facts and embeds  
them in the picture (see (d) and (e) in the scheme).

5. At each stage, the legal norm is the meaning of the legal norm-



sentence, that is, of the linguistically painted picture. The picture expresses legal norms as normative states of affairs.

6. The picture is painted by the creator of legal norms ; once painted, however, it can have an independent meaning. The meaning is dependent on the person who sees the picture, i.e. the interpreter.

7. Jurists paint their picture of legal norms without looking at legal norms themselves as objects of painting.

8. The fact that jurists paint the picture of legal norms does not mean that legal norms themselves actually exist beforehand as objects of painting.

9. The legal norm-sentences, i.e. the statutes, cases and commentaries etc., as linguistic expressions, do exist. Legal norms as their meaning, however, do not exist as realities in themselves. It is just as the meaning of the picture, that is, the world in a picture, does not necessarily exist, even if the picture itself exists.

10. The positive existence of legal norms itself is a presupposition for jurists and their work; however, it is merely a fiction of the jurists.

11. The logical principles are valid in the world of this painted picture, that is, between legal norm-sentences, All the drawing lines must be logically and correctly connected with each other so that in the picture the drawing-lines on each of the stages ((a)-(e)) or between the stages do not contradict, that they are consistent and especially that the judgment is consistently drawn in this picture. In this sense, logical principles such as the consequence-relation and the principle of the exclusion of contradictions are valid between legal norm-sentences of which legal norms are the meaning.

12. The normative world of positive legal norm — which does not exist in reality, but is nevertheless assumed by jurists and unconsciously exist as a fiction — ought to have no contradictions and be consistent. In that respect, the logical principles would also have to be valid in this fictitious world. As far as this world of meaning is symbolized in the substantiated legal norm sentences by judge who interpretes the statute, the logical consistency of the world could be tested.

## 7 Conclusion

Finally, I would like to stress that I have more sympathy for the earlier work of Kelsen, where I find the intention and undertaking to thoroughly analyze the world of legal norms structurally, especially from the logical point of view and to develop law into a precise science. In his earlier years, however, Kelsen unfortunately had no precise concept and the firm method of mathematical logic available for law and legal science.

Now, we can be sure about the applicability of classical mathematical logic to the field of law, and the precise method of its application is available to us. We can even pursue and realize the intentions and undertakings of Kelsen by means of this application so that legal science will become a precise science and legal thinking can be improved. On the way, one will be able to expose the falsity of the substantiation of legal norms themselves which can be observed in Kelsen as well as in many other jurists.<sup>18)</sup>

If one consistently starts from and stands on legal norm-sentences to analyze their logical structure thoroughly, then one could develop a genuine legal science — science of law — which should be called “Logical Jurisprudence”.<sup>19)</sup>

## References

- Alchourrón & Martino. 1989. A Sketch of Logic without Truth. *The Second International Conference on Artificial Intelligence and Law* New York (ACM) 1989, 165-179.
- Alexy. 1978. *Theorie der juristischen Argumentation*.
- Bench-Capon. 1989. Deep Models, Normative Reasoning and Legal Expert Systems. *Proceedings of the Second International Conference on AI and Law*: 37-45.
- Carnap. 1947. *Meaning and Necessity*.
- Hinst. 1974. Wahrheit und Bedeutung. Vorschlaege zu einem fundamental-semantischen Aufbau von Wissenschaftssprachen. (Unpublished).
- Herrestad. 1991. Norms and Formalization. *The Third International Conference on Artificial Intelligence and Law*: 175-194.
- Jones. 1990. Deontic Logic and Legal Knowledge Representation., *Ratio Juris*: Vol. 3. No. 2.
- Jones & Sergot. 1992. Deontic Logic in the Representation of Law: Towards a Methodology. *Artificial Intelligence and Law*: Vol 1. No. 1.
- Jørgensen. 1937. Imperatives and Logic, *Erkenntnis* 7, 1937/38, 288-296.
- Kalinowski. 1973. *Einfuehrung in die Normenlogik*.
- Kaufmann, A. 1972. Gesetz und Recht. Kaufmann, *Rechtsphilosophie in Wandel*: 135-171.
- Kelsen, Hans. 1923. *Hauptprobleme der Staatsrechtslehre*. 2nd ed. Tuebingen.
- . 1934. *Reine Rechtslehre*.
- . 1960. *Reine Rechtslehre* 2nd ed.
- . 1965. Recht und Logik. *Forum* 12: 421-425.
- . 1967. Recht und Logik. *Forum* 14: 495-500.
- . 1979. *Allgemeine Theorie der Normen*.
- Klug. 1966. *Juristische Logik*.
- Kutscherea & Breitkopf. 1971. *Einfuehrung in die moderne Logik*.
- Kutscherea. 1973. *Einfuehrung in die Logik der Normen, Werte und Entscheidungen*.
- McCarty, L.T. 1983. Permissions and Obligations. *Proceedings of the Eighth International Joint Conference on Artificial Intelligence*: 287-294.
- . 1986. Permissions and Obligations - an Informal Introduction. Martino A. A. & Socci N. F. (eds.), *Automated Analysis of Legal Texts*.
- . 1989. A Language for Legal Discourse. I. Basic Features. *Proceed-*

- ings of the Second International Conference on Artificial Intelligence and Law*: 180-189.
- Opalek. 1980. *Ueberlegungen zu Hans Kelsens "Allgemeiner Theorie der Normen"*.
- Engisch. 1963. *Logische Studien zur Gesetzanwendung*, 3rd ed.
- Roedig. 1972. Ueber die Notwendigkeit einer besonderen Logik der Normen. Albert et al (eds.), *Rechtstheorie als Grundlagenwissenschaft der Rechtswissenschaft, Jahrbuch fuer Rechtssoziologie und Rechtstheorie*: vol 2. 163-185.
- . 1973. *Die Theorie des gerichtlichen Erkenntnisverfahrens. Die Grundlinien des zivil-, straf- und verwaltungsgerichtlichen Prozesses*.
- Stammler. 1911. *Theorie der Rechtswissenschaft*, 2nd ed 1923.
- Stegmueller. 1957. *Das Wahrheitsproblem und die Idee der Semantik*.
- Taira, Ryo & Yoshino, Hajime. Meaning, Issues and Methods of Legal Logics. [Japan. Houronrigaku noigi oyobi kadai to hoho]. *Keio Law Journal*: vol 45 78ff. (Keio University).
- Tarski. 1935. Der Wahrheitsbegriff in den Formalisierten Sprachen, *Studia Philosophica Commentarii Philosophicae Polonorum*: I. 261-405.
- . 1944. The semantic conception of truth and the foundation of semantics. *Journal of Philosophy and Phenomenological Research*: 4, 341-375.
- . 1952. Semantics and philosophy of language. Urbana: III 13-47.
- Trapp. Zur rationalen Rekonstruktion des richterlichen Urteils. Roch-Rottleuthner (ed.), *Juristische Argumentationstheorie*: I, 3ff.
- Wagner & Haag. 1970. *Die moderne Logik in der Rechtswissenschaft*.
- Walter. 1980. Das Problem des Verhaeltnisses von Recht und Logik in der Reinen Rechtslehre. *Rechtstheorie* 11: 299-314.
- Weinberger, Ch. & Ota. 1979. Logik Semantik, Hermeneutik. Logik der deskriptiven Sprache, Logik der praeskriptiven Sprache.
- Weinberger, Ota. 1970. *Rechtslogik*.
- . 1973. Bemerkungen zu J. Roedigs "Kritik des normlogischen Schliessens". *Theory and Decision* III: 314
- . 1979. Kann man das normenlogische Folgerungssystem philosophisch begruenden? Ueberlegungen zu den Grundlagen des Juristischen Folgerns. *ARSP*: LXV/2. 177-183.
- von Wright. 1951. Deontic logic. *Mind* 60, 58-74.
- . 1964. A new system of deontic logic. *Danish Yearbook of Philosophy*: I, 173-182.
- Yoshino, Hajime. 1974. Justice and logic. The role of deductive methods in

- reasoning about justice [Japn.] *Justice, the Annual of Legal Philosophy* 1974: 38-68.
- . 1978a. Ueber die Notwendigkeit einer besonderen Nomenlogik als Methode der juristischen Logik. Klug, Ramm, Rittner, Schmiedel (eds), *Gesetzgebungstheorie, Juristische Logik, Zivil und Prozessrecht*. Gedächtnisschrift fuer Juergen Roedig. (Springer).
- . 1978b. Zu Ansätzen der juristischen Logik, in: Tammelo-Schriener, *Strukturierungen und Entscheidungen im Rechtsdenken*: 277-287.
- . 1980. The Model of Logical Structure of the Justification in Judicial Decision [Japn.] *The Meiji Gakuin Law Review*: 26, 83ff.
- . 1981a. Die logische Struktur der Rechtsnorm bezueglichder logischen Formalisierungsweise fuer die Rechtsnorm:  $Ax(Moe(x) \rightarrow St(x))$ . ARSP (Archiv fuer Rechts- u. Sozial Philosophie) Supplementa Bd. 1. 3: 235-243. 235ff.
- . 1981b. Die Logische Struktur der Argumentation bei der Juristischen Entscheidung. Aarnio, Niiniluoto, Uusitalo(Hrsg.), *Methodologie und Erkenntnistheorie der juristischen Argumentation*: 235ff.
- . 1983. Legal Logic - the Direct Application of Mathematical Logic to Legal Logic [Japn.]. *Legal Theory: Modern Legal Philosophy 1*: 197ff.
- . 1986. *Foundation of Legal Expert System* [Japn.] Tokyo (Gyosei).
- . 1987b. Yoshino, Hajime et. al., Legal Expert System LES-2. Wada (ed.), *Logic Programming*, 86: 34ff.
- . 1987c. Logical Structure of Law and the Possibility of Computer Aided Legal Reasoning. *Archiv fuer Recht- und Sozial Philosophie (ARSP)*, Nr. 30: 185-202.
- . 1990a; 1991a; 1992a. *Study on Claryfing of Structure of Legal Knowledge and Construction of Legal Knowledge Base* [Japn.]. Tokyo (Kikai System Shinko Kyokai) (1): 1990; (2): 1991; (3): 1992.
- . 1990b; 1991b; 1992b. *Study on Development and Prove of Inference Edgine of Legal Expert System* [Japn.]. Tokyo (Kikai System Shinko Kyokai) (1): 1990; (2): 1991; (3): 1992.
- . 1991b. Foundation of Systematization of Legal Analogy [Japn.] *Proceedings of Anual Meeting of Japan Society of Artificial Intelligence 1991*: 219ff.

Notes

\* The author dedicates this paper to Professor Minoru Kawaguchi commemorating the anniversary of his retirement from the Faculty of Law at Keio University. He has guided and helped me during and after my study under the late of Professor Teruo Minemura. I am deeply grateful to Professor Kawaguchi for all that he has done for me.

This paper is a refinement of my previous theories which originally appeared in German (Hajime Yoshino, "Zur Anwendbarkeit der Regeln der Logik auf Rechtsnormen" in: *Die Reine Rechtslehre in wissenschaftlicher Diskussion*, Wien, 1982, S. 142ff., my paper for the International Symposium commemorating the 100th anniversary of Hans Kelsen's birth). I have done this refining work during my visiting study in August, 1992 at the School of Law and the College of Computer Science at Northeastern University, Boston, MA (USA). The study was supported by the International Joint Research Project Funding by the Japanese Ministry of Culture, Education and Science.

I'd like to thank the Ministry for the funding and the faculty at Northeastern University for providing the good study environment and help. The following people at Northeastern have been especially helpful: Professor Donald Berman, my host at the School of Law, Professor Carole Hafner, my host at the College of Computer Science and my research student Philip Johnson, undergraduate of the College of Computer Science.

- 1) I distinguish between "legal norm-sentence" and "legal norm" in following chapters of this paper. In this chapter, however, only the term "legal norm" will be used, according to the general usage, which does not distinguish between them. In chapter 2 and after, also, the original expression will be respected when referring to the theories by Kelsen et al.
- 2) Regarding the problem of the applicability of the principle of the exclusion of contradiction, my original contribution to the Kelsen Symposium included the discussion of this matter. I want to omit this out of reasons of space here and publish the discussion on another occasion.
- 3) The special logic of norms is called "deontic logic" or "norm-logic". They are understood to be a part of the so-called philosophical logic that many different authors have attempted to develop in all directions beyond classical logic. Regarding norm-logic, von Wright has had considerable influence especially with: *Deontic Logic*, *Mind* 60, 1951, 58-74 (the first system by

Wrights); and also: A New System of Deontic Logic, Danish Yearbook of Philosophy, I, 1964, 173-182 (the second system by Wrights). Today there is a great amount of literature available in this field. About various attempts of norm-logic see Kalinowski, Einführung in die Normenlogik (translated by Klein), 1973. In the field of logic of law, Ota Weinberger tried to establish a special logic for norms. Compare O. Weinberger, 1970; and Ch. Weinberger and O. Weinberger, Logik, 1979. None of these attempts have yet been successful in finding a reliable system useful for juridical purposes. About my criticism of some of these systems of this special logic see: Yoshino, 1978a, mainly p. 151ff.

- 4) I am of the opinion that classical bivalent logic can be applied directly and productively to legal norms and that a special logic for norms is therefore in principle not necessary. About this comment see Rödig, 1972, pp. 163-185. I have also discussed in detail the problem of applicability of classical logic and the futility of a special normlogic as a method of judicial logic elsewhere. Compare: Yoshino, 1978a. There after this work was criticized by Weinberger [1979], so this problem has become a real problem of juridical logic again. In order to prevent misunderstandings and doubts in regard to the application of classical logic to legal norms it is necessary to fully solve this problem.
- 5) Kelsen, as mentioned later, takes norm as the meaning of an act of will of the creator of norms. He also distinguishes between legal norm (Rechtsnorm) and rules of law (Rechtssatz). The latter, also called "statements about legal norms" by Kelsen, he regards as the description of legal norms. (Kelsen, 1960, 73; 1979, 123). I do not assume this distinction here. Here the term "norm-sentence" is taken as the sentence, the linguistic term, of norm.
- 6) cf. e.g. S211 StGB of the Federal Republic of Germany.
- 7) In the presented formula the normative moment of legal norm is expressed in the predicate. It is dealt with here as quality. For this predicate-logical manner of formalization of legal norms compare Klug, 1966, 55ff, 178. Yoshino, 1978a, 145. Some Logicians of Law have criticized this formalization. (Weinberger, 1979, 177ff; Wagner-Haag, 1970, 81ff.) I will discuss these criticisms later.
- 8) Following formalization of the normative modal-terms I partly took from Roedig (comp. Roedig, 1972, 180f, newprint in: Schriften zur juristischen Logik, 180f). Here I have reconstructed legal norms from the following three elements:
  1. norm-addressee (with a one-term predicate such as "na(.)")
  2. norm-object (also with a one-term predicate such as "ac(.)")

3. normative modal-condition of norm-subject in relation to norm-object (with two-term predicates such as “ob(. . .)”).

- 9) In the systems of normlogic it is usual to express the default of an action “A” through the application of a negator with “ $\sim A$ ”. This is problematic since with the negation of action “A” the component of the multitude “A” is understood, namely not only for the default of “A” but also for everything outside of “A”. In the normalization of default of the predicate I am using here “ab(. . .)”. Comp. Yoshino, 1978a, 151, note 41; also Roedig, 1972, 174ff and 180f. The relations between the normative modal-terms that are presented predicate-logically here correspond to those that attempt to formalize normlogic in a special normlogical way (comp. e.g. Kutschera, 1973, 21ff).
- 10) About this semantic foundation see also Yoshino, 1978a, 144-147. For Tarski’s formal semantics, see two of his works: Tarski, 1935, 261-405; newprint in: Berka-Kreiser (ed.), *Logik-Texte*, 1971, 447-559; Tarski, 1944, 341-375, newprint in: Linsky (ed.), *Semantics and the Philosophy of Language*, Urbana (III), 1952, 13-47. For the system of this semantics, see e.g.: Kutschera-Breitkopf, 1971, 89-90, for a detailed description, see e.g.: Stegmüller, 1957. In this paper, the formulization of the definition of the meta-linguistical truth-term (interpretation-semantics) by Tarski is to a great extent based on the description by Kutschera (Kutschera-Breitkopf, 89) and Hinst (Hinst, 1974).
- 11) Wagner and Haag object against the predicate-logical formalization of legal norm by Klug “ $x (Ve(x) \rightarrow So(x))$ ” the antecedent  $Ve(x)$  (in the words of Klug: “x is a behavior of the manner A”) taking up truth-terms, but the second part  $So(x)$  (in words: “x is a behavior that ought to be”) none and likewise the whole formula (Wagner-Haag, 1970, 81f). Weinberger in his criticism of my formalization (comp. formula (I’) in the presented paper) stresses again the difference of the semantic natures of the first and the second part of the formula (Weinberger, 1979, 178f). On grounds of the explication above of the logical-truth-term it has become clear that there is no problem in the logical truth-valuation of the first part as well as the second part and also of the whole formula of this formalization.
- 12) The purpose of the discussion at hand is to present the possibility of normative interpretation of the truth-term in relation to normative predicates and to the applicability of the normatively interpreted truth-term that is interpreted in this manner. In order to do this, however, it is not necessary to decide under what criteria this interpretation of the truth-term or its assignment has to be performed. We can also assume that the indicative predicate can be distinguished from each other by some criterion—this is usually



presumed.

- 13) It is hard to say what is the content of the term of validity, in other words, under what criterion is the given legal norm to be considered valid. This question is neither a syntactical nor a semantic, but a pragmatic problem. Therefore it is not necessary for the present logical and formal-semantic examination to answer the question above.
- 14) However, I am not excluding the possibility that the indicatively-interpreted truth-term and the normatively-interpreted truth-term can at the same time be applied to the inference of which the logical consequence-relation is to be examined. I want to mention the examination of this problem and its formal semantic foundation of another occasion.
- 15) Arthur Kaufmann distinguishes between “law” and “statute” in assuming that “law” is not a positive law but a living, actually “concrete” right for the concrete case. Here I see the term of “law” in somehow the same way as Kaufmann (except for his ontological view). Comp. Kaufmann, 1972, 135-171, 159f and 163f.
- 16) This scheme or a similar one was established not only by me but also by Alexy as well as Koch almost at the same time. Comp. Alexy, 1978, 279. Koch and Rottleuthner also lead a DFG project “Juristische Argumentationstheorie”; in this framework a similar scheme was developed: Trapp, Zur rationalen Rekonstruktion des richterlichen Urteils, in: Koch-Rottleuthner (ed.), Juristische Argumentationstheorie, Vol. 1, 3ff. Further analysis of juridical decisions, ARSP, supplement 14, 1980, 181-212. About my more detailed opinion and my further developed logical model of the justification process of law-application, also including the “Konkretisierung” of law-inference, comp. Yoshino, 1980, 83ff.
- 17) Comp. FN 11.
- 18) In the history of legal theory there has been a plethora of incorrect thought that considers the legal norm as real when in fact it does not exist at all. This kind of thinking can be called, in my opinion, “the substantiation of the legal norm” because it attempts to make the ideal concept, which does not exist, into something real.
- 19) I have coined “Logical Jurisprudence”, the development of which, is presently one of my most important tasks. I have endeavoured to do this in connection with the research on the development of the Legal Expert System (LES). Logical Jurisprudence can also provide the basis of the development of LES and the research on LES can likewise stimulate Logical Jurisprudence; both compliment and enhance each other [Yoshino 1978a-b, 1980, 1981b, 1983,

ABOUT THE APPLICABILITY OF THE PRINCIPLES OF LOGIC TO LEGAL NORM  
1986, 1987b-c, 1990a-b, 1991a-b, 1992a-b].