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Basic Construction of a System of Logic Based on Identity and Demonstratives

*Toshiharu Waragai**

§ 1. An Analysis of Identity

I begin this paper with a discussion about some logical nature of identity, or in other words with an analysis of the logical condition under which the verb 'is' in a sentence of the form 'A is B' is used in the sense of 'is-identical-with', where 'A' and 'B' are supposed to be general names. Let us call this 'is' of identity the verb 'is' in the role of identity.

Whenever the verb 'is' is used in the role of identity in a singular sentence 'A is B', the following conditions are evidently met:

C. 1. some object is A,
C. 2. some object is B,
C. 3. at most one object is A,
C. 4. at most one object is B,
C. 5. whatever is A is B,
C. 6. whatever is B is A.

In reverse, if there hold between the terms 'A' and 'B' the conditions C. 1.–C. 6., then the sentence constructed by means of 'A', 'B' and 'is', i. e. 'A is B' states just as much as 'A is-identical-with B'; for if the terms 'A' and 'B' meet the conditions C. 1.–C. 6., then they are clearly designative (C. 1., C. 2.) and singular (C. 3., C. 4.). From this consideration we arrive to the following equivalence:

E. 1. A is-identical-with B if and only if C. 1.–C. 6. hold.

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Taking now again into consideration that both 'A' and 'B' are singular and designative as far as they satisfy the conditions C. 1.–C. 4., we can be sure that the 'is' appearing in C. 1.–C. 6. is used in the sense of 'is-identical-with'. This fact allows us to restate E. 1. in the following way:

E. 2. A is-identical-with B if and only if

some object is-identical-with A (C. 1.)
some object is-identical-with B (C. 2.)
at most one object is-identical-with A (C. 3.)
at most one object is-identical-with B (C. 4.)
whatever is-identical-with A is-identical-with B (C. 5.)
whatever is-identical-with B is-identical-with A (C. 6.)

Since what is meant by '=' in its usual understanding is just that it is a formal substitute of the verb 'is-identical-with', we now substitute '=' in place of 'is-identical-with' in E. 2., and we obtain;

E. 3. $A=B. \equiv. (\exists x)(x=A). (\exists x)(x=B). (x, y)(x=A. y=A. \supset. x=y). (x, y)(x=B. y=B. \supset. x=y). (x)(x=A. \supset. x=B). (x)(x=B. \supset. x=A)$,

assuming here that we follow for the moment referential reading of quantifiers. Later this reading in this case will be justified. E. 3. axiomatically expresses the condition under which the verb 'is' is used in the role of identity.

§ 2. Axiom of Identity

We can take here one more step and simplify E. 3. as follows:

E. 4. $A=B. \equiv. (\exists x)(x=A). (x, y)(x=B. y=B. \supset. x=y). (x)(x=A. \supset. x=B)$

Indeed from E. 4. are the following obtainable:

Th. 1. $A=B. \supset. (\exists x)(x=B)$

Th. 2. $A=B. \supset. (x, y)(x=A. y=A. \supset. x=y)$

Proof. 1. $A=B$ (Sup.)

2. $x=A. y=A$ (Sup.)

3. $(x)(x=A. \supset. x=B)$ (1., E. 4.)
4. $x=A. \supset. x=B$ (3)
5. $y=A. \supset. y=B$ (3)
6. $x=B. y=B$ (2, 4, 5)
7. $(x, y)(x=B. y=B. \supset. x=y)$ (1, E. 4.)
8. $x=B. y=B. \supset. x=y$ (7)
9. $x=y$ (6, 8)

Th. 3. $A=B. \supset. (x)(x=B. \supset. x=A)$

Proof. 1. $A=B$ (Sup.)

2. $(x, y)(x=B. y=B. \supset. x=y)$ (1, E. 4.)
3. $x=B. A=B. \supset. x=A$ (2 ; A/y)
4. $x=B. \supset. x=A$ (3, 1)
5. $(x)(x=B. \supset. x=A)$ (4)

Th. 4. $A=B. \supset. (\exists x)(x=A). (\exists x)(x=B). (x, y)(x=B. y=B. \supset. x=y). (x, y)(x=A. y=A. \supset. x=y). (x)(x=A. \supset. x=B). (x)(x=B. \supset. x=A)$ (E. 4., Th. 1., Th. 2., Th. 3.)

Th. 5. $(\exists x)(x=A). (\exists x)(x=B). (x, y)(x=B. y=B. \supset. x=y). (x, y)(x=A. y=A. \supset. x=y). (x)(x=A. \supset. x=B). (x)(x=B. \supset. x=A). \supset. A=B$ (E. 4.)

Th. 6. $A=B. \equiv. (\exists x)(x=A). (\exists x)(x=B). (x, y)(x=A. y=A. \supset. x=y). (x, y)(x=B. y=B. \supset. x=y). (x)(x=A. \supset. x=B). (x)(x=B. \supset. x=A)$ (Th. 4., Th. 5.)

Thus E. 3. is derivable from E. 4., so that E. 4. characterises the verb 'is' in the role of identity. Let us call E. 3. hereafter Axiom of Identity, for short Ax. I..[1]

§ 3. Basic Properties of '='

Let us investigate some properties of the verb 'is' in the role of identity. It is transitive and symmetrical. Indeed:

Th. 7. $A=B. B=C. \supset. A=C$

Proof. 1. $A=B$ (Sup.)

2. $B=C$ (Sup.)

	3.	$(\exists x)(x=A)$	(1, Ax. I.)
	4.	$x=A. \supset. x=B$	(1, Ax. I.)
	5.	$x=B. \supset. x=C$	(2, Ax. I.)
	6.	$x=A. \supset. x=C$	(4, 5)
	7.	$(x)(x=A. \supset. x=C)$	(6)
	8.	$(x, y)(x=C. y=C. \supset. x=y)$	(2, Ax. I.)
	9.	$A=C$	(3, 7, 8, Ax. I.)

Th. 8. $A=B. \supset. B=A$

Proof.	1.	$A=B$	(Sup.)
	2.	Th. 1.. Th. 2.. Th. 3.	(Th. 1., Th. 2., Th. 3.)
	3.	$(\exists x)(x=B). (x, y)(x=A. y=A. \supset. x=y). (x)(x=B. \supset. x=A)$	(1, 2)
	4.	$B=A$	(3, Ax. I.)

But notice that it is not reflexive. Indeed:

Th. 9. $(\exists x)(x=A). \supset. A=A$

Proof.	1.	$(\exists x)(x=A)$	(Sup.)
	2.	$x=A. A=y. \supset. x=y$	(Th. 7.)
	3.	$x=A. y=A. \supset. x=y$	(2, Th. 8.)
	4.	$(x, y)(x=A. y=A. \supset. x=y)$	(3)
	5.	$(x)(x=A. \supset. x=A)$	(Theorem.)
	6.	$A=A$	(1, 4, 5, Ax. I.)

Th. 10. $A=A. \supset. (\exists x)(x=A)$

Th. 11. $(\exists x)(x=A). \equiv. A=A$ (Th. 9., Th. 10.)

The left-hand component of Th. 11. fails if 'A' does not designate an object, so that the right-hand component of Th. 11. fails if 'A' is not singular-designative. Hence the non-reflexivity of '='. To require that '=' is reflexive is tantamount to presupposing that every name is singular-designative, which is surely a very strong ontological presupposition. This fact suggests that analysis of 'identity' necessarily leads us to the notion of 'non-reflexive identity' if we carry it out being free from any ontological presupposition.

Another type of analysis of 'non-reflexive identity' is to find in

Lejewski 1967.

§ 4. Demonstratives

I here understand by 'demonstratives' *pure* demonstratives. They are the particles that have the function of selecting an individual out of a domain of individuals. 'This' and 'that' are the best representatives in English. They are by nature functional, or functorial: they construct unshared names in concatanation with general name, e. g. 'this man', 'that book' etc.. Let 'a' be a general name, and let 'X' stand for a demonstrative. Then we are able to maintain about 'X' at least that it selects just one individual from the domain of individuals named by 'a'. This functorial character of demonstratives as *selector* or *individuator* is one of their most celebrated properties. They are nowadays classified under the heading of 'indexical expressions' whose logico-linguistic function is able to be treated only in the framework of *indexical semantics*, but their functorial function as selector or as individuator is in reality to be treated as a logically constant function which can have its own place in the framework of formal systems.

In must be noticed that the demonstratives constitute a *proper* part of the expressions of the semantic category 'n/n' because of their specific logico-linguistic character. Let 'F' be a usual adjective. Using 'Xa' properly, we select an object out of the domain of a's, which we in principle cannot do by using 'Fa'. And the proper use of 'Xa' usually presupposes the existence of a, while the mere use of 'Fa' does not. In order to mirror this fact in a logical language, let us accept a special category 'd' of the demonstratives constituting a proper part of n/n. From this consideration we see that 1) every expression of d is of n/n, but 2) the expressions of n/n are in general *not* substitutable for the variables of d. Let us express this relation between d and n/n as $d \subset n/n$.

I should here mention another type of coping with demonstratives.

Prof. C. Lejewski proposed in his "Proper names" to treat them as a kind of proper name of special character:

> Demonstrative pronouns can be used as unshared names of any object. They are introduced into language axiomatically, as it were... The expression 'that rose' can be regarded as a conjunction or product of two names. (pp. 251–2)

This is in spite of its artificiality one of the most proper ways in treating demonstrative pronouns as well as pure demonstratives in the framework of Leśniewski's Ontology$_L$. The logical relations subsisting between my treatise and his treatise will be discussed in another work in preparation.

Let 'X', 'Y',... be the variables of the semantic category '*d*' whose logical behaviour is ruled by the following axioms:

Ax. D. 1. $a=a. \supset . a=Xa$[2)]

D. $a\varepsilon b. \equiv . (\exists X)(a=Xb)$

Ax. D. 2. $(\exists x)(x\varepsilon a). \supset . Xa=Xa$

Ax. D. 3. $(X, Y)(Xa=Ya). \supset . a=a$

I will explain their intuitive sense. As will be clear in § 6, '$a=a$' states just 'a is an individual'. Now Ax. D. 1. reads as follows:

Ax. D. 1* if a is an individual, then it is unindividuatable.

Before explaining the intuitive sense of D., I have to remark on the reading of quantifiers which bind the expressions of demonstratives. In this case, I do not follow the referential reading, but I simply read them as 'for some particular' and 'for every particular'. This reading will be justified in § 6. By D. the symbol 'ε' is introduced. Its reading is simply 'is' or 'is-a'. The whole has the following reading:

D.* a is(-a) b if and only if a is-identical-with a certain particular b.

An example: Socrates is(-a) man, since Socrates is-identical-with a certain particular man. It is clear that the reading of '$(\exists X)(\exists x)(x=Xa)$' is 'some individual is a certain particular a', or simply 'a exists'. Now Ax. D. 2. reads as follows:

Ax. 2* if a exists, then every particular a is an individual.

An example: man exists, so *this* man is an individual.

Ax. 3* if no individuation affects a, then a is an individual.

Let us enumerate some theses obtainable on this base.

Th. 12. $a=b. \supset. a\varepsilon b. b\varepsilon a$

Proof.	1.	$a=b$	(Sup.)
	2.	$b=a$	(1, Th. 8.)
	3.	$b=b$	(2, Th. 9.)
	4.	$b=Xb$	(3, Ax. D. 1.)
	5.	$a=Xb$	(1, 4, Th. 7.)
	6.	$(\exists X)(a=Xb)$	(5)
	7.	$a\varepsilon b$	(6, D.)
	8.	$b\varepsilon a$	(cf. 1-7)
	9.	$a\varepsilon b. b\varepsilon a$	(7, 8)

Th. 13. $a\varepsilon b. b\varepsilon a. \supset. a=b$

Proof.	1.	$a\varepsilon b$	(Sup.)
	2.	$b\varepsilon a$	(Sup.)
	3.	$(\exists X)(a=Xb)$	(1, D.)
	4.	$(\exists Y)(b=Yb)$	(2, D.)
	5.	$b=Y^*a$	(4)
	6.	$Y^*a=b$	(5, Th. 8.)
	7.	$b=b$	(6, Th. 9.)
	8.	$a=X^*b$	(3)
	9.	$b=X^*b$	(7, Ax. D. 1.)
	10.	$X^*b=b$	(9, Th. 8.)
	11.	$a=b$	(8, 10, Th. 7.)

Th. 14. $a=b. \equiv. a\varepsilon b. b\varepsilon a$ (Th. 12., Th. 13.)

Th. 14. is accepted in Ontology$_L$ as the definition for introducing '=' by means of 'ε'.

Th. 15. $a=a. \equiv. a\varepsilon a$ (Th. 14.)

§ 5. Basic properties of 'ε'

Now I pass on to proving one of the main result of this paper: the symbol 'ε' introduced by means of D. satisfies the sole axiom of Ontology$_L$. This fact says that Ax. D. 1.–Ax. D. 3. are sufficient as a formal characterisation of selector or individuator, and that these functorial components have a logically well founded right to find a fixed place in a syntactically well regulated formal system.

Th. 16. $a\varepsilon b. \supset . a\varepsilon a$

Proof.	1.	$a\varepsilon b$	(Sup.)
	2.	$(\exists X)(a=Xb)$	(1, D.)
	3.	$a=X^*b$	(2)
	4.	$X^*b=a$	(3, Th. 8.)
	5.	$a=a$	(4, Th. 9.)
	6.	$a=Xa$	(5, Ax. D. 1.)
	7.	$(\exists X)(a=Xa)$	(6)
	8.	$a\varepsilon a$	

Th. 17. $a\varepsilon b. \supset . (\exists x)(x\varepsilon a)$

Th. 18. $a\varepsilon b. \supset . (x, y)(x\varepsilon a. y\varepsilon a. \supset . x\varepsilon y)$

Proof.	1.	$a\varepsilon b$	(Sup.)
	2.	$x\varepsilon a$	(Sup.)
	3.	$y\varepsilon a$	(Sup.)
	4.	$(\exists X)(a=Xb)$	(1, D.)
	5.	$(\exists Y)(x=Ya)$	(2, D.)
	6.	$(\exists Z)(y=Za)$	(3, D.)
	7.	$a=X^*b$	(4)
	8.	$x=Y^*a$	(5)
	9.	$y=Z^*a$	(6)
	10.	$a=a$	(7, Th. 8., Th. 9.)
	11.	$a=Y^*a$	(10, Ax. D. 1.)
	12.	$Y^*a=a$	(11, Th. 8.)
	13.	$x=a$	(8, 12, Th. 7.)

14.	$y=a$	(9, 10 ; cf. 10–13)
15.	$x=y$	(13, 14, Th. 8., Th. 7.)
16.	$x\varepsilon y$	(15, Th. 14.)

Th. 19. $a\varepsilon b. \supset. (x)(x\varepsilon a. \supset. x\varepsilon b)$

Proof.

1.	$a\varepsilon b$	(Sup.)
2.	$x\varepsilon a$	(Sup.)
3.	$a\varepsilon a$	(1, Th. 16.)
4.	$a=a$	(3, Th. 15.)
5.	$(\exists X)(x=Xa)$	(2, D.)
6.	$x=X^*a$	(5)
7.	$a=X^*a$	(4, Ax. D. 1.)
8.	$x=a$	(6, 7, Th. 7., Th. 8.)
9.	$(\exists X)(a=Xb)$	(1, D.)
10.	$a=X^{**}b$	(9)
11.	$x=X^{**}b$	(8, 10, Th. 7.)
12.	$(\exists X)(x=Xb)$	(11)
13.	$x\varepsilon b$	(12, D.)

Th. 20. $a\varepsilon b. \supset. (\exists x)(x\varepsilon a). (x, y)(x\varepsilon a. y\varepsilon a. \supset. x\varepsilon y). (x)(x\varepsilon a. \supset. x\varepsilon b)$

Proof. Obvious from Th. 17, Th. 18, Th. 19.

Notice that one need make use of Ax. D. 1. in proving Th. 20., while the remaining two axioms are not used.

Th. 21. $(\exists x)(x\varepsilon a). (x, y)(x\varepsilon a. y\varepsilon a. \supset. x\varepsilon y). (x)(x\varepsilon a. \supset. x\varepsilon b). \supset. a\varepsilon b$

Proof.

1.	$(\exists x)(x\varepsilon a)$	(Sup.)
2.	$(x, y)(x\varepsilon a. y\varepsilon a. \supset. x\varepsilon y)$	(Sup.)
3.	$(x)(x\varepsilon a. \supset. x\varepsilon b)$	(Sup.)
4.	$(x, y)(x\varepsilon a. y\varepsilon a. \supset. y\varepsilon x)$	(2)
5.	$(x, y)(x\varepsilon a. y\varepsilon a. \supset. x=y)$	(2, 4, Th. 15.)
6.	$(x, y)((\exists X)(x=Xa). (\exists Y)$ $(y=Ya). \supset. x=y)$	(5, D.)
7.	$(x, y, X, Y)(x=Xa. y=Ya.$ $\supset. x=y)$	(6)
8.	$Xa=Xa. Ya=Ya. \supset. Xa=Ya$	(7)

9.	Xa=Xa	(1, Ax. D. 2.)
10.	Ya=Ya	(1, Ax. D. 2.)
11.	Xa=Ya	(8, 9, 10)
12.	(X, Y)(Xa=Ya)	(11)
13.	a=a	(12, Ax. D. 3.)
14.	aεa	(13, Th. 15.)
15.	aεa. ⊃. aεb	(3)
16.	aεb	(14, 15)

Th. 22. aεb. ≡. (∃x)(xεa). (x, y)(xεa. yεa. ⊃. xεy). (x)(xεa. ⊃. xεa)

Proof. Obvious from Th. 20, Th. 21.

Th. 22. shows that the symbol 'ε' introduced by D. satisfies the sole axiom of Ontology$_L$. Notice that in proving Th. 21. we made use of Ax. D. 2. and Ax. D. 3., while Ax. D. 1. was not needed.

§ 6. System *LID*

I will construct in this chapter a logical system which I call 'a system of logic based on identity and demonstratives', for short *LID*, and prove a metatheorem concerning it.

1. Construction: *LID* has as its basic semantic categories '*n*', '*s*' and '*d*', which stand for 'general name', 'sentence' and 'demonstrative' respectively. As for the compound semantic categories, we stipulate that they are composed only of '*n*' and '*s*'. The relation $d \subset n/n$ is accepted.

Let us construct *LID* in the following successive way. At first, we construct its beginning status *B*. *B* has:

1. variables of *s*; p, q, . . .
2. variables of *n*; a, b, c, . . . , x, y, z, A, B, C, . . .
3. constant of $s/n, n$; =
4. variables of *d*; X, Y, Z, . . .

and as the rules of inference;

R. 1. modus ponens

R. 2. substitution

R. 3. quantification

R. 4. propositional definition

R. 5. propositional extension,

with four axioms and one definition;

Ax. I., Ax. D. 1., D., Ax. D. 2., Ax. D. 3.,

where the definition D. is garanteed by R. 4.. Now we extend *B* by adding stepwise the theses to it until we reach Th. 22. Now at this stage we add the following rule of inference stated in terms of 'ε' to *LID*:

R. 6. nominal definition.

The status thus obtained constitutes the basic status of *LID* from which *LID* will be developed. Let us call this status *S*.

2. Metatheorem: At *S* we single out Th. 22., accepting R. 1.–R. 6. as the rules of inference. Let us call the status thus obtained S_0. Its only thesis is:

Ax. 0. (=Th. 22.) $(a, b)(a\varepsilon b. \equiv. (\exists x)(x\varepsilon a). (x, y)(x\varepsilon a. y\varepsilon a. \supset. x\varepsilon y).$
$(x)(x\varepsilon a. \supset. x\varepsilon b))$

The system developed on the basis of S_0 is evidently Ontology$_L$ without the rule of *nominal extensionality*[3]. Hence the following metatheorem:

Theorem: *LID* is an extension of Ontology$_L$ (without the rule of nominal extensionality).

After having reached the correspondent of Th. 22., we could supply *LID* (at *S*) with the rule of nominal extensionality as well as the rule of nominal definition. Let us call it *LID**. Just as the addition of the rule of nominal extensionality radically changes the logical relations holding between theses in Ontology$_L$[4], its addition to *LID* directly affects the logical relation holding between the axioms of *LID* and the axiom of Ontology$_L$. Indeed, Ax. D. 1. and D. suffice to prove the axiom of Ontology$_L$ in *LID**. I do not go further on this point here, and leave it to another occasion. For the time being, let us confine our study to *LID*.

3. On quantifier-reading in *LID*: Notice that thanks to Th. 15. and Theorem, what 'a=a' states is just 'a is an object'[5]. In *LID* the following are theses:

E. 3. 1. $(\exists x)(x=A). \equiv. (\exists x)(x=x. x=A)$

E. 3. 2. $(x, y)(x=B. y=B. \supset. x=y). \equiv. (x, y)(x=x. y=y. x=B. y=B. \supset. x=y)$

E. 3. 3. $(x)(x=A. \supset. x=B). \equiv. (x)(x=x. x=A. \supset. x=B)$,

thanks to:

Th. 9* $x=A. \supset. x=x$,

which is a variant of Th. 9.. Clearly the quantifiers in E. 3. 1.–E. 3. 3. may be interpreted to run over the domain of individuals. Thus the referential reading in §1 is justified.

I follow here *provisionally* what I called in Waragai 1980 categorial reading of quantifiers as the official version in *LID* with the following categorial conventions as to names and demonstratives:

C(*n*): a name indefinitely names the object which falls under the extension of the name, and if '='-relation holds of the name, it designates an individual.

C(*d*): a demonstrative selects just one individual out of the extension of the name to which it is concatanated.

That we may supply *LID* with C(*d*) as the categorial convention of demonstratives is a matter to be proved. I skip the proof of it in this paper.

The general form of quantifier reading in *LID* is as follows:

CRQ $(\alpha)(\ldots\alpha\ldots)$ for every expression 'α' of the categorial convention $C:\ldots\alpha\ldots$

In general, we supply with an appropriate categorial convention to the expressions whose semantic function we need to state explicitely. Otherwise, we do not state it, and follow the substitutional reading of quantifiers. Thus the quantification over the demonstratives is justified, and we are able to quantify every expression appearing in *LID*.

§ 7. Problems untouched

There are some problems left untouched in this paper. I list up three of them which seem to be interesting.

1. A very natural problem that follows Theorem in § 6 is whether *LID* is a *proper* extension of Ontology$_L$. The answer to this question seems to be positive, since the demonstratives constitute a *proper* part of the expressions of the semantic category 'n/n'. Adjectives are of the category 'n/n'. But what are bound by 'X', 'Y' in the axioms are demonstratives, for which the adjectives are not substitutable. The axioms of *LID* characterise the logical role of a proper part of the expressions of the semantic category 'n/n', so that *LID* is able to treat a proper part of the expressions of the semantic category 'n/n', separating it from the other parts of the expressions of the same semantic category, which Ontology$_L$ cannot do. A positive answer to this problem will be given with a proof in a paper in preparation.

2. The second point to be considered is the relation between the treatment of demonstratives in *LID* and that proposed by Prof. C. Lejewski. I point out a logical fact concerning this point. The following holds: under some condition, what can be expressed by means of a demonstrative can be expressed without it.

3. What is interesting is how the indefinite and definite articles are incorporated into *LID*. By some appropriate way, they are incorporatable into *LID* as the expressions of the semantic category 'd'. As a byproduct of this survey, the following theorem was obtained: if we restrict the property-expressions to non-empty ones, the quantifiers are eliminable.

Remarks

1) Notice the resemblence of Ax. I. to the axiom of Ontology$_L$, i. e. Th. 22.. Ax. I. can be shortened to '$(a, b): a=b. \equiv .(\exists x). a=x. x=b$' which is the sole

axiom of Lejewski 1967. Notice that this again resembles to the shortest axiom of Ontology$_L$, i. e. '(a, b) : aεb. ≡. (∃x). aεx. xεb'. Through the analysis of ontological commitment free identity, I reached Ax. I. without realising its equivalence to Lejewski's axiom.

2) That $d \subset n/n$ is here of essential importance, since this prohibits such a substitution as '*⟨x⟩/X', where the functor '*' is introduced by the definition 'aε*⟨b⟩(c). ≡. aεb. aεc'. If this substitution were to be allowed, then Ax. D. 1. leads to a contradicition.

3) On this rule and R. 1.–R. 6., cf. Lejewski 1958, 1967 as well as Słupecki 1955.

4) Cf. Sobocinski 1934.

5) For a philosophical discussion on this point, cf. Waragai 1980.

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