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| Abstract | Father Bochenski, while in a conversation in this summer, mentioned of the unclearness of the Lukasiewicz rejection-symbol as to its logical status : whether it is a functor, and if so to what category it should belong. I have been feeling obliged to answer this question, for in my above mentioned work, I made essential use of the rejection-symbol as proposition forming functor. |
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# A remark on the semantic category of the Łukasiewicz rejection-symbol: Addendum to "The notion of Rejection and a Proof of Ł-Completeness of the Two-Valued Logic"[1]

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Father Bocheński, while in a conversation in this summer, mentioned of the unclearness of the Łukasiewicz rejection-symbol as to its logical status: whether it is a functor, and if so to what category it should belong. I have been feeling obliged to answer this question, for in my above mentioned work, I made essential use of the rejection-symbol *as proposition-forming functor*.

In [1], p. 143, I enlarged at first the notion of deductive systems by supplying them with rejection rules and rejected axioms, while on the other side the notion of well-formed formulae was enlarged as to make the notion of proof in the enlarged systems resemble the normal one. The enlarged notion of wffs was given as follows:

$$1 \quad \alpha, \beta \in \mathrm{L} \rightarrow \vdash \alpha,\ \dashv \alpha,\ \alpha \vdash \beta \in \mathrm{Lr}$$

where L is the set of all the wffs in a deductive system and Lr is

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1) Owing to my carelessness, there are some misprints in my [1]. The title should be '...Ł-Completeness...' in place of '...Ł-Decidability...'. On page 140, $\mathrm{r}_2^-$ should be as follows:

$$\mathrm{r}_2^- : \dashv \phi(\mathrm{p}/\phi_1) \rightarrow \dashv \phi(\mathrm{p}),$$

and on p. 142 two rules are given without index. They should be indexed as '$\mathrm{r}_1^+$' and '$\mathrm{r}_2^+$'.

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the set of all the wffs in the correspodning enlarged system·

With this enlargement of the notion of wffs, there soon arises quite an interesting problem with regard to the semantic categories of the symbols '$\vdash$', '$\dashv$', '$\vdash^*$'. (I shall use $\vdash^*$ for $\vdash$ in a wff of the form '$\alpha\vdash\beta$' in order to distinguish it from the morphollogically same symbol $\vdash$ in a wff of the form '$\vdash\alpha$'.)

According to the theory of semantic category originally proposed by S. Leśniewski and developed by K. Ajdukiewicz, every meaningful expression should be in possession of some definite semantic category. Now that what Leśniewski admitted as fundamental ones were the following two,

2 the category of proposition (p)
3 the category of name (n),

if we should like to treat the symbols '$\vdash$', '$\dashv$', '$\vdash^*$' as proposition-forming functors, an easy calculation will show that their corresponding categories are respectively 'p/p', 'p/p', 'p/p, p'. But this category assignment is an undesirable one, for it forces us to admit the following symbol-connexi;

$$\vdash\dashv\alpha,\ \vdash\vdash\alpha,\ \vdash\alpha\dashv\dashv\alpha \text{ etc.}$$

which are, however, patently meaningless. This seems to show the impossibility of category assignment to them within the theory of semantic category of Leśniewski-Ajdukiewicz, which in turn should mean that it is impossible to treat them as functors.

But when we take into consideration the fact that we make essential use of them in rejective proofs, it seems necessary to assign them some appropriate categories and treat them as functors of proposition-forming kind.

For this purpose, I propose to introduce one new category which is propositional and by one degree higher than the category 'p'. The newly introduced category will be denoted as 'z'.

Now let us assume that the wffs in Lr belong to the category 'z' and not to 'p', while the wffs in L belong to the category 'p'. An easy calculation will show that the categories of '⊢', '⊣', '⊢*' are 'z/p', 'z/p', 'z/p, p'. The difficulty with the symbol-connexi which are patently meaningless disappears, for such connexi are grammatically impossible.

* * *

I think that the new category of propositional kind is of some philosophical importance, especially with respect to the problem of 'truth-talk'. Indeed, the admittance of the category 'z' suggests to us the existence of another kind of propositions which differ from the propositions of the category 'p'. In principle, we should admit as many cotegories of proposition as the number of different types of propositions.

In normal discourse, we distinguish at least two basic types of propositions:

4 propositions without assertive power,

5 propositions with assertive power,

though they may take the same morphological figure. As Frege often mentioned, a proposition can once be used without assertive power and can once be used with assertive power, e. g. take a proposition 'she loves me', and consider the following two propositions containing this proposition:

6 if she loves me, I would be happy,

and

7 Oh, she loves me!

In 7, the proposition in question is used without assertive power, and in 8, in an appropriate situation it is used with assertive power, though they take the same morphological figure. Now it is desirable to devise some method for distinguishing their semantico-pragmatic status. To the former proposition, we can assign e. g. the category 'p', and to the latter the category 'z'. Further, if we take into consideration what Strawson maintains in his 'On Referring' (*Mind*, 1950), it seems that we need at least three kinds of propositional categories

8 the category of propositions as pure gramatical symbol-connexi (s)
9 the category of propositions used without assertive-power (p)
10 the category of propositions used with assertive power (z).

's', 'p', 'z' can be tentatively called 'sentence-category', 'proposition-category' and 'statement-category'.

Now let us confine ourselves to the categories 'p' and 'z', and consider the meaning of the functors of the category 'z/p'. This is a functor which is essentially *intercategorial*. Such a functor functions as type-elevating one, i. e. it functions on a proposition without assertive power and produces a statement, as is mirrored in its category structure.

I hold that the phrase 'it is true that....' is of the category 'z/p', if it is used properly.

I have outlined an idea of application of categories to the problem of truth-talk. I shall discuss it in detail in the paper which is now in preparation.

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