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Model Spaces

*Yosaku Nishiwaki**

In this paper we shall show a general framework of studying some properties between relational structures and first order languages describing them. Recently usual two-valued models have been quite naturally extended to the Boolean-valued models especially in set theory when Cohen's forcing method was studied by the Boolean-valued method. Here we want to clarify a general construction of Boolean-valued models using the analogous method of a construction of Stone-Čech compactification and ultrafilters of Boolean algebras. Properly speaking, the concept of compactifications is one of the important concepts in topology, but applying its method we can easily treat model constructions if we are given a language and a structure of the same similarity type.

In section 1 we will define a Boolean-valued structure. Then we study classical models in section 2. Further we show the relations between classical models and so called Kripke models with respect to a introduction of constants in section 3.

1.

In this section we define a Boolean-valued structure (abbreviated B-valued structure) and the concepts "true" and "false" in it. The reason why we think a B-valued structure is that truth concepts in a two-valued interpretation correspond to the ultrafilter in B-valued one and then we can investigate truth concepts algebraically.

Let ρ be an ordinal and $\tau \in \omega^{\rho}$ a similarity type. A (first order) language with equality L consists of usual means with logical

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symbols $\neg, \vee, \wedge, \rightarrow, \forall, \exists$ and equality symbol$=$, $\tau(i)$-ary predicate symbol P_i.

If L is a language of type τ, a structure for L is a relational system of type τ. And we denote this structure as

$$\mathfrak{A}=\langle A, R_i^{\mathfrak{A}}\rangle_{i<\rho}$$

Let L be a language and let $\mathfrak{A}$ be a structure for L. Given a subset C of A, we let L_C be the language whose symbols are the symbols of L together with a distinct constant symbol c_a for every $a \in C$. A structure for L_C is a relational system of the form

$$\begin{aligned}\mathfrak{A}_C&=\langle A, R_i^{\mathfrak{A}}, c_a\rangle_{i<\rho, a\in C}\\ &=\langle \mathfrak{A}, c_a\rangle_{a\in C}\end{aligned}$$

The basic model-theoretical concept of a model for a sentence may be defined by the usual method due to Tarski. Now we will generalize the notion of an ordinary two-valued structure to that of a B-valued structure by replacing the Boolean algebra $\mathbb{2}$ of two truth values with any complete Boolean algebra $\mathbb{B}$. Throughout this paper let $\mathbb{B}=\langle B, +, \cdot, -, 0, 1\rangle$ be a (complete) Boolean algebra.

By a B-valued structure we mean a system

$$\mathfrak{A}^{\mathbb{B}}=\langle A, \bar{R}_i, \bar{c}_j\rangle_{i<\rho, j<\alpha}$$

where A is a nonempty set called the universe of $\mathfrak{A}^{\mathbb{B}}$ and $\bar{R}_i$ is a n-ary function defined on A with the values in B;

$$\bar{R}_i : A^{n_i} \rightarrow B,$$

and $\bar{c}_j : c_j \rightarrow A$.

We are now going to give the definition of B-valued satisfaction. (def. 1) Let h be any infinite sequence of elements of A. We put:

(1) $\|(x_i=x_j, h)\|=\begin{cases}0 \text{ if } h_i \neq h_j,\\ 1 \text{ if } h_i=h_j.\end{cases}$

(2) $\|(P_i(x_{i_c}, \cdots, x_{i_n}), h)\|=\bar{R}_i(h_{i_c}, \cdots, h_{i_n})$

(3) $||(\neg\varphi, h)|| = {}^{-}||(\varphi, h)||$

(4) $||(\varphi \vee \psi, h)|| = ||(\varphi, h)|| + ||(\psi, h)||$

(5) $||(\exists x_i\varphi(x_i), h)|| = \sum\{||(\varphi(x_i), h')|| \mid \forall j(i \neq j \rightarrow h_i = h'_j)\}$

Remark. The condition (5) is well defined in any complete Boolean algebra, but the supremum does not always exist in any (not complete) Boolean algebra.

If we are given a structure $\mathfrak{A}^{B}$, in order to define a truth value for closed formulas of L we first extend L to a new language L_A by introducing the family of constants $\{c_a \mid a \in A\}$, and we define $\bar{c}_a = a$. Then above definitions become as follows:

(def. 2)

(1) $||c_i = c_j|| = \begin{cases} 0 \text{ if } \bar{c}_i \neq \bar{c}_j, \\ 1 \text{ if } \bar{c}_i = \bar{c}_j. \end{cases}$

(2) $||P_i(c_1, \cdots, c_n)|| = \bar{R}_i(\bar{c}_1, \cdots, \bar{c}_n)$

(3) $||\neg\varphi|| = {}^{-}||\varphi||$

(4) $||\varphi \vee \psi|| = ||\varphi|| + ||\psi||$

(5) $||\exists x\varphi(x)|| = \sum_{a \in A} ||\varphi(c_a)||$

(def. 3) $\mathfrak{A}^{B} \models \varphi \Longleftrightarrow ||\varphi|| = 1$

$\mathfrak{A}^{B} \models \varphi$ is read "φ is true in $\mathfrak{A}^{B}$."

There is no necessity to introduce constants for all the elements of A. But by doing so, it becomes quite simple to treat models. It is sufficient that the introduced constants are able to construct a Boolean algebra. This Boolean algebra will be one of the subalgebras of a Boolean algebra of L_A.

Theoretically we may restrict constants introductions countably many, for any Boolean algebra can be embedded in a countably generated Boolean algebra.

In a B-valued structure defined above, usual axioms of predicate calculas are also true and further,

(theorem 1)

If φ is a closed formula of L, then φ is true in every B-valued structure iff φ is logically valid.

Now let h be a complete homomorphism of $\mathbb{B}$ into $\mathbb{2}$ and F be an ultrafilter for $\mathbb{B}$. Then one of the well known results of a Boolean algebra is the following theorem:

(theorem 2)

(1) If F is an ultrafilter for $\mathbb{B}$ and

$$h(x)=\begin{cases}1 \text{ if } x\in F\\ 0 \text{ if } x\in B-F,\end{cases}$$

then h is a homomorphism from $\mathbb{B}$ to $\mathbb{2}$.

(2) If h is a homomorphism from $\mathbb{B}$ to $\mathbb{2}$ and $F=\{x\in B|h(x)=1\}$, then F is an ultrafilter for $\mathbb{B}$.

From this theorem we clearly get,

(theorem 3)

$$\begin{aligned}\mathfrak{A}^{\mathbb{B}}\models\varphi &\Longleftrightarrow h(||\varphi||)=1\\ &\Longleftrightarrow ||\varphi||\in F\end{aligned}$$

From above theorems, we can say that φ is true in $\mathfrak{A}^{\mathbb{B}}$ iff there is an ultrafilter for $\mathbb{B}$ such that $||\varphi||$ is included in it and that φ is valid iff for any ultrafilter F for $\mathbb{B}$ it includes $||\varphi||$. Thus we shall concentrate our attention upon the family of ultrafilters for $\mathbb{B}$ in the next section.

2.

From section 1 if we fix a language L_A and a structure descrived by it then a model for a sentence φ of L_A is given as follows. From all ultrafilters for $\mathbb{B}$ which include the B-value of φ, we select any one and then its ultrafilter is a model of φ.

Now we set a model family generally such as:

(def. 4) A model family for L_A is the set

$$\{\langle\mathfrak{A}^{\mathbb{B}}, F_i\rangle|i<\beta\}$$

where $\mathfrak{A}^{\mathbb{B}}$ is a B-valued structure, each F_i is an ultrafilter for $\mathbb{B}$ and $\beta=2^{|\mathbb{B}|}$.

Now we show that for each $i<\beta, M_i=\langle\mathfrak{A}^{\mathbb{B}}, F_i\rangle$ is a canonical model of φs such that $||\varphi||\in F_i$. To this proof we define the notion of consistency property.

(def. 5) A consistency property for L_A is a set S of sets s such that each $s\in S$ is a set of sentences of L_A and such that all the followings hold for every $s\in S$:

(1) $\varphi\in s\Rightarrow \neg\varphi\notin s$

(2) $\varphi\wedge\phi\in s\Rightarrow s\cup\{\varphi,\phi\}\in S$

(3) $\varphi\vee\phi\in s\Rightarrow s\cup\{\varphi\}\in S$ or $s\cup\{\phi\}\in S$

(4) $\forall x\varphi(x)\in s\Rightarrow s\cup\{\varphi(c)\}\in S$ for each constant c of L_A.

(5) $\exists x\varphi(x)\in s\Rightarrow s\cup\{\varphi(c)\}\in S$ for some constant c of L_A.

(6) 1. $(c_i=c_j)\in s\Rightarrow s\cup\{(c_i=c_j)\}\in S$.

2. $\varphi(c_i), (c_i=c_j)\in s\Rightarrow s\cup\{\varphi(c_j)\}\in S$.(1)

From (def. 5) we can get the usual model existence theorem.

Extending this theorem to a B-valued structure, we can get the existence of some M_i.

(theorem 4)

For each sentence φ of L_A,

$$||\varphi||\in F_i \text{ for some } i<\beta\Longleftrightarrow\varphi\in s \text{ for some } s\in S.$$

(proof) This proof is quite the same as usual proof if we consider a non-infinitary language. And its proof consists of showing that for any $s\in S$ there is a maximal consistent $s'\in S$ and its existence is proved by Lindenbaum lemma. Then clearly from theorem 2 and theorem 3, $\mathfrak{A}^{\mathbb{B}}\models\varphi\Longleftrightarrow||\varphi||\in F\Longleftrightarrow\varphi\in s'$ for some F, s'.

But the language L_A might be uncountable. For if the universe A is not countable the set of constants $\{c_a|a\in A\}$ is not countable, too. But L_A does not contain any infinite logical conjunction and disjunction in this case. Hence the difference from the ordinary structure

$$\mathfrak{C}^{\mathbb{B}}=\langle A, \bar{R}_i\rangle_{i<\rho}$$

is only the addition of constants. So its structure become

$$\mathfrak{A}^{\mathbb{B}}=\langle A, R_i, \bar{c}_a\rangle_{i<\rho, a\in A}$$

We show that these constants can be replaced by unary predicate symbols as is done by the following construction.

Let L^+ be a language such that it is obtained from L by adding the unary predicate symbols $\{Q_\gamma | \gamma<|A|\}$. Suppose

$$\mathfrak{A}_a^{\mathbb{B}}=\langle A, R_i, Q_\gamma\rangle_{i<\rho, \gamma<|A|}$$

be a structure for L^+. Here $Q_\gamma=\{a_\gamma\}$. We associate with each formula φ $(c_{\gamma 1}, \cdots, c_{\gamma n})$ of L_A the formula φ^+ of L^+ where

$$\varphi^+=\forall x_1\cdots\forall x_n(Q_{\gamma 1}(x_1)\wedge\cdots\wedge Q_{\gamma n}(x_n)\to\varphi(x_1, \cdots, x_n))$$

Then for any $h\in A^w$,

$$||(\varphi, h)||\in F_i \text{ in } \mathfrak{A}^{\mathbb{B}}\Longleftrightarrow||(\varphi^+, h)||\in F_j \text{ in } \mathfrak{A}_a^{\mathbb{B}}$$

Therefore our proof may be reduced to the model existence theorem about the set of φ^+ type formulas of L^+. The meaning of this reduction consists upon the fact that any formula is finite and the number of occurrences of constants in a formula is also finite.

Now we shall consider a general framework of models in L_A and their B-valued structures.(2) Let $S(\mathbb{B})$=the set of all ultrafilters of $\mathbb{B}$. Then $S(\mathbb{B})\leq 2^{|\mathbb{B}|}$.

(def. 6) For a Boolean algebra $\mathbb{B}$ we denote by $\phi_{\mathbb{B}}$, or simply by ϕ, the function from $\mathbb{B}$ to $P(S(\mathbb{B}))$ defined by

$$\phi(a)=\{F\in S(\mathbb{B})|a\in F\}$$

The Stone topology on $S(\mathbb{B})$ is the topology determined by the subbase $\phi[\mathbb{B}]$.

(def. 7) The model space $\mathbb{M}_{\mathbb{B}}$ of $\mathbb{B}$ is the set

$$\{\langle\mathfrak{A}^{\mathbb{B}},F\rangle|F\in S(\mathbb{B})\}$$

with the Stone topology on $S(\mathbb{B})$.

Remark. In above definitions we do not define for a complete Boolean algebra. The difference of definitions will be clear immediately.

Let $\mathbb{B}(S(\mathbb{B}))$ be the set of all clopen sets of $S(\mathbb{B})$ and let $\mathbb{R}(S(\mathbb{B}))$ be the set of all regular open sets of $S(\mathbb{B})$. Then clearly $\mathbb{B}(S(\mathbb{B}))\subset\mathbb{R}(S((\mathbb{B}))$.

(lemma 1)

ψ is a Boolean algebra isomorphism from $\mathbb{B}$ onto $\mathbb{B}(S(\mathbb{B}))$.

(proof) We first show $\psi[\mathbb{B}]\subset\mathbb{B}(S(\mathbb{B}))$. From the definition of the topology of $S(\mathbb{B})$, every element of $\psi[\mathbb{B}]$ is open. Then

$$S(\mathbb{B})-\psi(a)=\psi(^{-}a)\in\mathbb{B}(S(\mathbb{B}))$$

for $a\in\mathbb{B}$, so that $\psi[\mathbb{B}]\subset\mathbb{B}(S(\mathbb{B}))$. And ψ is a Boolean algebra homomorphism. Hence ψ is a Boolean algebra embedding of $\mathbb{B}$ into $\mathbb{B}(S(\mathbb{B}))$. Now if $A\in\mathbb{B}(S(\mathbb{B}))$ then by the fact that $\psi[\mathbb{B}]$ is a base for $S(\mathbb{B})$ there is $\{a_i|i\in I\}\subset\mathbb{B}$ such that $A=\cup\{\psi(a_i)|i\in I\}$. Since A is compact there are $n<\omega$ and $i_k\in I$ for $k\leq n$ such that

$$A=\psi(a_0)\cup\cdots\cdots\cup\psi(a_n)$$

$$=\psi(a_0+\cdots\cdots+a_n)\in\psi[\mathbb{B}]$$

If X is a totally disconnected space and $p\in X$ then

$$\{a\in\mathbb{B}(X)|p\in a\}\in S(\mathbb{B}(X))$$

(def. 8)

$$h:\ X\longrightarrow S(\mathbb{B}(X))$$

$$h(p)=\{a\in\mathbb{B}(X)|p\in a\}$$

(lemma 2)

If X is a compact, totally disconnected space then h is a homeomorphism from X onto $S(\mathbb{B}(X))$.

(proof) To prove this lemma we show that (1) if X is zero-dimensional then $\bar{h}$ is a homomorphism from βX onto $S(\mathbb{B}(X))$

where $\bar{h}:\beta X\to S(\mathbb{B}(X))$ is the Stone extension of h (βX=the Stone-Čech compactification of X) and (2) if X is a compact space then X is disconnected iff X zero-dimensional. From above two statements and compactness of X, it follows that h is a homemorphism X onto $S(\mathbb{B}(X))$.

(1) Since βX is compact we have $\bar{h}[\beta X]=S(\mathbb{B}(X))$ and it is enough to prove that $\bar{h}$ is one to one. If $p, q\in\beta X$ and $p\neq q$ then there are $A, B\in Z(X)$ such that $A\in p$, $B\in q$, and $A\cap B=\phi$. ($Z(X)$=the family of all zero-sets of X) Since X is zero-dimensional there are $C, D\in\mathbb{B}(X)$ such that $A\subset C$, $B\subset D$, and $C\cap D=\phi$, and from $\bar{h}(p)=p\cap\mathbb{B}(X)$ for $q\in\beta X$, we have $\bar{h}(p)\neq\bar{h}(q)$.

(2) If X is zero-dimensional then $\mathbb{B}(X)$ is a base for X. Then clearly X is totally disconnected. Assume X is totally disconnected. Let A, $B\in Z(X)$ with $A\cap B=\phi$. Then there is $A_{p,q}\in\mathbb{B}(X)$ such that $p\in A_{p,q}$ and $q\notin A_{p,q}$ for $p\in A$, $q\in B$. The family $\{A_{p,q}|p\in A\}$ is an open cover of the compact set A, and thus there are $n(q)<\omega$ and $p_0, \cdots, p_{n(q)}\in A$ such that

$$A\subset A_{p_0,q}\cup\cdots\cup A_{p_{n(q)},q} \qquad \text{for} \quad q\in B.$$

We set $B_q=X-(A_{p_0,q}\cup\cdots\cup A_{p_{n(q)},q})$, so that $q\in B_q\in\mathbb{B}(X)$ for $q\in B$. The family $\{B_q|q\in B\}$ is an open cover of B, and thus there are $n<\omega$ and $q_0, \cdots, q_n\in B$ such that

$$B\subset B_{q_0}\cup\cdots\cup B_{qn}.$$

We put $C=B_{q_0}\cup\cdots\cup B_{qn}$. Then $C\in\mathbb{B}(X)$, $B\subset C$ and $A\cap C=\phi$.

(def. 9) Let $\varphi:\mathbb{B}'\to\mathbb{B}$ be a Boolean algebra homomorphism. The function $S(\varphi)$ is defined on $S(\mathbb{B})$ by

$$S(\varphi)(p)=\{a\in\mathbb{B}|\varphi(a)\in p\}$$

Let $f:X\to Y$ be a continuous function between compact, totally disconnected space. $\mathbb{B}(f)$ is defined on $\mathbb{B}(Y)$ by

$$\mathbb{B}(f)(B)=f^{-1}(B)$$

Then it is clear that

$$S(\varphi):S(\mathbb{B}') \longrightarrow S(\mathbb{B})$$

$$\mathbb{B}(f):\mathbb{B}(Y) \longrightarrow \mathbb{B}(X)$$

And by lemma 1, 2, we use the notation h and ψ to denote respectively the homeomorphism and the Boolean algebra isomorphism

$$h:X \longrightarrow S(\mathbb{B}(X)) \quad \text{and} \quad \psi:\mathbb{B} \longrightarrow \mathbb{B}(S(\mathbb{B}))$$

From these definitions and lemmas, we get the following theorem.

(theorem 5)

Let $\varphi:\mathbb{B}' \to \mathbb{B}$ be a Boolean homomorphism and let $f:X \to Y$ be a continuous function between compact, totally disconnected spaces. Then

(1) φ is an embedding $\Rightarrow S(\varphi)$ is an onto function
φ is an onto function $\Rightarrow S(\varphi)$ is one to one
$\mathbb{B}'=\mathbb{B}$ and φ is the identity function $\Rightarrow S(\varphi)$ is the identity function

(2) $\mathbb{B}(f)$ is a Boolean algebra homomorphism

(3) f is one to one $\Rightarrow \mathbb{B}(f)$ is an onto function
f is an onto function $\Rightarrow \mathbb{B}(f)$ is one to one
$X=Y$ and f is the identity function $\Rightarrow \mathbb{B}(f)$ is the identity function

(4) the diagrams

$$\begin{array}{ccc} \mathbb{B}' & \xrightarrow{\varphi} & \mathbb{B} \\ \psi \downarrow & & \downarrow \psi \\ \mathbb{B}(S(\mathbb{B}')) & \xrightarrow{\mathbb{B}(S(\mathbb{B}'))} & \mathbb{B}(S(\mathbb{B})) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{f} & Y \\ h \downarrow & & \downarrow h \\ S(\mathbb{B}(X)) & \xrightarrow{S(\mathbb{B}(f))} & S(\mathbb{B}(Y)) \end{array}$$

commute.

Collecting above results, we can say about the model family for L_A that if we are given some $\mathbb{B}$, the family of truth sets of each model is $\mathbb{R}(S(\mathbb{B}))$. And this is a kind of geometrization of models.

Further a relation of two model spaces will be shown as such;

$$\begin{array}{ccc} \mathfrak{A} & \cdots\cdots\cdots\rightarrow & \mathfrak{A} \\ \downarrow & & \downarrow \\ \mathfrak{A}^{\mathbb{B}} & \xrightarrow{\quad\varphi\quad} & \mathfrak{A}^{\mathbb{B}'} \\ \downarrow & \mathbb{B}(S(\varphi)) & \downarrow \\ \mathbb{M}_{\mathbb{B}} & \longrightarrow & \mathbb{M}_{\mathbb{B}'} \end{array}$$

3.

In section 2, in order to speak of the algebraic relation of all the models of classical logic we did not alter the language L_A and the structure $\mathfrak{A}^{\mathbb{B}}$. Only the kinds of ultrafilters were the subject of our inquiry. But in this section conversely we fix an ultrafilter and consider many languages.

Let L be a language with any constants and let $\mathfrak{A}$ be a structure of the same similarity type. We shall consider the method of the introduction of new constants to L. Then there may be many methods, say, the method of introducing new constants one by one to L cumulatively. Then the sequence

$$L, L_1, \cdots\cdots, L_i, \cdots\cdots$$

where $L_i = L_{i-1} \cup \{c_i\}$ is obtained. Of course, it is not necessary that the introduction is done by a totally ordering. Further there might be the process of erasing the constants. Hence there are very many methods of getting new languages by the introductions, the erasures or their complex. But here we consider on the process of the introduction of constants. By this assumption we can say that at least the process of introductions is a partial order. Various algebraic structures of constants introductions can be considered, for example,

1. a poset with L, L_A be zero and unit
2. a lattice „
3. a Boolean algebra „

Before considering the subject, we notice some remarks.

* By the introduction of constants only the language may be changed but the structure is invariant through it.

* The order of a introduction is quite free and there is no relation with our real methods of it.

* The family of sets of new constants are given a priori and we can use them with no restriction.

Although there are quite many introduction methods, in this paper we restrict them to the following sets:

$$C^*=\{C(B) \mid B \subseteq A \wedge |B|=\omega\}$$

where $C(B)$ is the set of constants of all elements of B. This restriction is only by the results of non-classical models already constructed, so there is no logical necessity of such a restriction. Now we define the Kripke-type models with our languages. First we fix A the universe of $\mathfrak{A}^{\mathbb{B}}$ and an ultrafilter F. Then let set the following sequence:

$$\begin{array}{ccccccccc}
\cdots\cdots, & L_{B_1}, & L_{B_2}, & \cdots\cdots, & L_{B_n}, & \cdots\cdots, & L_A \\
 & | & | & & | & & | \\
\cdots\cdots, & <\mathfrak{A}^{\mathbb{B}_1}, F^1>, & <\mathfrak{A}^{\mathbb{B}_2}, F^2>, & \cdots\cdots, & <\mathfrak{A}^{\mathbb{B}_n}, F^n>, & \cdots\cdots, & <\mathfrak{A}^{\mathbb{B}}, F>
\end{array}$$

$$(n<\omega)$$

Now we shall characterize this sequence. For that purpose we consider the factors of the sequence respectively as follows:

α $B_1, B_2, B_3, \cdots\cdots, B_n, \cdots\cdots$

β $\mathbb{B}_1, \mathbb{B}_2, \mathbb{B}_3, \cdots\cdots, \mathbb{B}_n, \cdots\cdots$

γ $F^1, F^2, F^3, \cdots\cdots, F^n, \cdots\cdots$ $(n<\omega)$

1. the sequence α

From the definition of C^*, for any B_n all the elements of B_n have their names and $C(B_n)$ is the set of them. A relation R of C^* is defined by many ways. For example, R can be defined by the set inclusion relation $\subseteq$ of the set

$$\{B_n \mid C(B_n) \in C^*\}$$

Then in this case the relation R is a partial ordering relation and at least it is transitive and reflexive.

2. the sequence β

This is the sequence of Boolean algebras. What properties are there between them?

(1) For any $n<\omega$, $\mathbb{B}_n$ is a subalgebra of $\mathbb{B}$. It is clear from the fact that $B_n \subseteq B$ and $\mathbb{B}_n$ is a Boolean algebra.

(2) For any $n<\omega$, $\mathbb{B}_n$ is a countably generated Boolean algebra. Because $C(B_n)$ is countable and its elements are generated from $C(B_n)$.

(3) If we start our study using only the symbols of L_A, then we can think that $\mathbb{B}$ is a free Boolean algebra of B-values of sentences of L_A and each $\mathbb{B}_n$ is also a free Boolean algebra of B-values of sentences of L_{B_n}.

3. the sequence γ

This is the sequence of ultrafilters F^n which are included in F of $\mathbb{B}$.

(1) Each F^n is an ultrafilter of $\mathbb{B}_n$ and $F^n \subseteq F$ for any $n<\omega$.

We will be able to understand more clearly if γ is considered such that every F^n is much the same as F, but F^n has one more property, i.e., every element of F^n have a name of itself. Therefore we can write

$$F^n = \{a \in F \mid c_a \in C(B_n)\} \qquad \text{for any } n<\omega.$$

(2) From 1 we get easily

$$B_i \subseteq B_j \Longrightarrow F^i \subseteq F^j \qquad \text{for any } i, j.$$

We did not prove above properties, but their precise proofs are not so difficult. They are well known properties of Boolean algebras without exception.

Here we assume above properties for the next definition.

(def. 10)

1. $L^* = \bigcup_{n<\omega} L_{B_n}$
2. $G = \{<\mathfrak{A}^{B_n}, F^n> | n<\omega\}$
3. $R = \{\ll \mathfrak{A}^{B_i}, F^i>, <\mathfrak{A}^{B_j}, F^j \gg | B_i \subseteq B_j\}$
4. $P: G \longrightarrow C^*$

   $\Gamma R \varDelta \longrightarrow P(\Gamma) \subseteq P(\varDelta)$
5. $||\varphi|| \in F^n \Longleftrightarrow \Gamma_n \models \varphi$
6. $K = <G, R, \models, P>$

(def. 11) Definition of Kripke models(8).

By an intuitionistic model we mean an ordered quadruple $K' = <G', R', \models', P'>$ where

1. G' is any set.
2. R' is a transitive relation.
3. P' is a function from G' to C^* satisfying the condition

$$\Gamma R' \varDelta \longrightarrow P'(\Gamma) \subseteq P'(\varDelta)$$

4. $\models'$ is a relation between members of G' and sentences of L^* satisfying, for each $\Gamma \in G'$,

   a. $\Gamma \models' \varphi, \Gamma R' \varDelta \Longleftrightarrow \varDelta \models' \varphi$, for atomic

   b. $\Gamma \models' (\varphi \land \psi) \Longleftrightarrow \Gamma \models' \varphi$ and $\Gamma \models' \psi$

   c. $\Gamma \models' (\varphi \lor \psi) \Longleftrightarrow \Gamma \models' \varphi$ or $\Gamma \models' \psi$

   d. $\Gamma \models' \neg \varphi \Longleftrightarrow$ for each $\varDelta \in G'$ such that $\Gamma R' \varDelta$, not $\varDelta \models' \varphi$

   e. $\Gamma \models' (\varphi \rightarrow \psi) \Longleftrightarrow$ for each $\varDelta \in G'$ such that $\Gamma R' \varDelta$, if $\varDelta \models' \varphi$ then $\varDelta \models' \psi$

   f. $\Gamma \models' \forall x \varphi(x) \Longleftrightarrow$ for each $\varDelta \in G'$ such that $\Gamma R' \varDelta$, $\varDelta \models' \varphi(c)$ for each $c \in P'(\varDelta)$

g. $\Gamma \models' \exists x\varphi(x) \Longleftrightarrow \Gamma \models' \varphi(c)$ for some $c \in P'(\Gamma)$

From these definitions we have the next result.

(theorem 6)

K is an intuitionistic model.

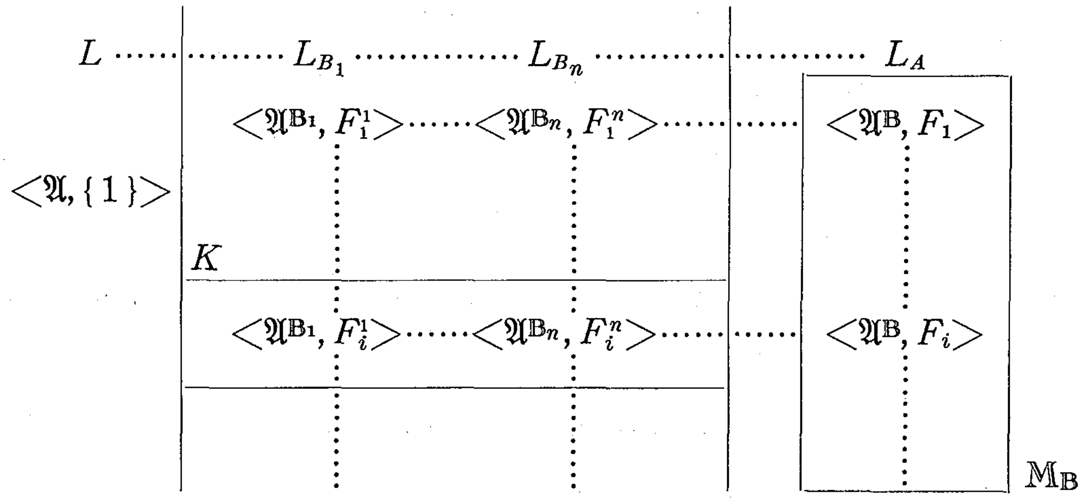
(proof) If K satisfies 1, 2, 3 and 4 in (def. 11), then the theorem is proved. Hence we must show $\Gamma_n \models \varphi \Rightarrow \Gamma_n \models' \varphi$ for $n<\omega$. To prove this let use the properties of the sequence γ. From the property of ultrafilter F, a, b and c are satisfied. In order to prove d$\sim$g, it is sufficient to use (2) of the sequence γ.

It will be better to say that K is a Boolean intuitionistic model.

4.

In the preceding sections we investigated separately the structures of classical models and intuitionistic models by the idea of constants introductions. If we unify the classical model space $\mathbb{M}_\mathbb{B}$ of L_A and the intutionistic model K, then we get the following diagram:

(unified model space)



Each column means a classical model space $\mathbb{M}_{\mathbb{B}_i}$ and each row corresponds to an intuitionistic model K_i.

NOTES

(1) To satisfy (6) we must add the following axioms to def. 2.

1. $||c=c||=1$
2. $||c_1=c_2||=||c_2=c_1||$
3. $||c_1=c_2||\cdot||c_2=c_3||\leq||c_1=c_3||$
4. For each n-ary predicate symbol P,

$$||c_1=c_1'||\cdots||c_n=c_n'||\cdot||P(c_1, \cdots, c_n)||\leq||P(c_1', \cdots, c_n')||$$

(2) Following contents of section 2 are written more precisely in [1], [2].

(3) We took this definition from [3].

REFERENCES

1. Comfort, W. W. & Negerepontis, S., *The Theory of Ultrafilters*, Springer, 1974.
2. Chandler, R. E., *Hausdorff Compactifications*, M. Dekker, 1976.
3. Fitting, M., "Model Existence Theorems for Modal and Intuitionistic Logics", *J. S. L.*, 1973, 38, 613–627.