Title	Bayesian estimation of discrete choice models : labor supply of multiple household members
Sub Title	
Author	Matsuno, Kazuhiko
Publisher	Keio Economic Observatory (Sangyo Kenkyujo), Keio University
Publication year	1992
Jtitle	Keio Economic Observatory occasional paper. E No.13 (1992. 4)
JaLC DOI	
Abstract	The aim of this paper is to estimate models for labor supply probabilityof household members by using Bayesian procedures. Our modelsdescribe the labor supply behavior of multiple household members. Particularly,we deal with the behavior of households with one, two or threenon-principal earners (a wife and/or children). The related literaturemainly studies the labor supply of one non-principal earner. The modelsare of discrete choice or qualitative response type to yield a laborsupply probability that non-principal earners accept an employment opportunity.We apply Bayesian estimation methods to the models and carry outempirical tests of the models using Japanese data.Douglas law of labor supply shows that the labor participation rateof wives are negatively correlated with husband's income. This law hasbeen reconfirmed by empirical analyses of household-level surveys.Although the analyses are limited to a case of one non-principal earner,it may be possible to say that the labor supply of multiple non-principalearners is also negatively correlated with husband's income.The situation is, however, complicated when we consider householdswith multiple household members. In fact, it is difficult to define laboruparticipation rate at a household level when the household has multiplenon-principal earners. We may define the rate as a ratio of number ofparticipants to that of non-principal earners within a household. We needa theoretical basis for the definition. We need a model to describe thelabor supply behavior of households with multiple non-principal earners.For this purpose we provide a general discrete choice model. Three modelsfor labor supply are derived from it.In applications of Bayesian methods, a difficult problem is how toformulate prior distributions. The diffuse or uniform prior distributionis usually employed. In this study we take up Bayesian methods for estimatingone model to incorporate the information obtained in the priorestimation of other models. We also use maximum likelihood methods forcomparison.We make some experimental calculations based on the estimated parametersto clarify theoretical implications of the models. We calculateelasticities of labor supply probabilities with respect to changes ofexogenous conditions.
Notes	
Genre	Technical Report
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA10818580-00000013-0001

KEIO ECONOMIC OBSERVATORY
OCCASIONAL PAPER

April, 1992

Bayesian Estimation of Discrete Choice Models: Labor Supply of Multiple Household Members

by

Kazuhiko Matsuno

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E.No. 13

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April 1992

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*) This study was undertaken while the author was a visiting researcher at Economic Research Institute of The Economic Planning Agency.

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1. Introduction

The aim of this paper is to estimate models for labor supply probability of household members by using Bayesian procedures. Our models describe the labor supply behavior of multiple household members. Particularly, we deal with the behavior of households with one, two or three non-principal earners (a wife and/or children). The related literature mainly studies the labor supply of one non-principal earner. The models are of discrete choice or qualitative response type to yield a labor supply probability that non-principal earners accept an employment opportunity. We apply Bayesian estimation methods to the models and carry out empirical tests of the models using Japanese data.

Douglas law of labor supply shows that the labor participation rate of wives are negatively correlated with husband's income. This law has been reconfirmed by empirical analyses of household-level surveys. Although the analyses are limited to a case of one non-principal earner, it may be possible to say that the labor supply of multiple non-principal earners is also negatively correlated with husband's income.

The situation is, however, complicated when we consider households with multiple household members. In fact, it is difficult to define labor uparticipation rate at a household level when the household has multiple non-principal earners. We may define the rate as a ratio of number of participants to that of non-principal earners within a household. We need a theoretical basis for the definition. We need a model to describe the labor supply behavior of households with multiple non-principal earners. For this purpose we provide a general discrete choice model. Three models for labor supply are derived from it.

In applications of Bayesian methods, a difficult problem is how to formulate prior distributions. The diffuse or uniform prior distribution is usually employed. In this study we take up Bayesian methods for estimating one model to incorporate the information obtained in the prior estimation of other models. We also use maximum likelihood methods for comparison.

We make some experimental calculations based on the estimated parameters to clarify theoretical implications of the models. We calculate elasticities of labor supply probabilities with respect to changes of exogenous conditions.

2. Models for Labor Supply of Household Members

We present models of household labor supply as well as a model of discrete choice. The models of labor supply are derived from the discrete choice model as its applications. In the later sections we apply Bayesian procedures of estimation to the models. For Bayesian procedures we need theoretical bases to utilize prior information. And we have to explain the structure of the models in detail.

2.1 Models of household labor supply

We consider the labor supply of households, members of which consist of an employee principal earner (husband) and M potential earners (a wife and/or children of age over 16). Our model is concerned with the labor supply behavior of the potential eaners. The principal earner is already employed.

The utility indicator to express household's income-leisure preference is a function of (real) income X and leisure hours Λ,

$$u=u(X, \Lambda). \tag{1}$$

The income X is a sum of the principal earner's income I and the potential earners' possible income. The leisure hours Λ are a sum of leisure hours of potential earners'. The principal earner's income is an exogenous variable. A different utility indicator

$$u=u(I, X_1,\ldots,X_M, \Lambda_1,\ldots, \Lambda_M)$$

might have been employed, where the income X_m and leisure hours Λ_m of the m-th (m=1,...,M) potential earner's are explicitly introduced without aggregation.

We consider a case where an employment opportunity with a wage rate w and work hours h is open to the M potential earners. When the s potential earners accept the employment opportunity, we have

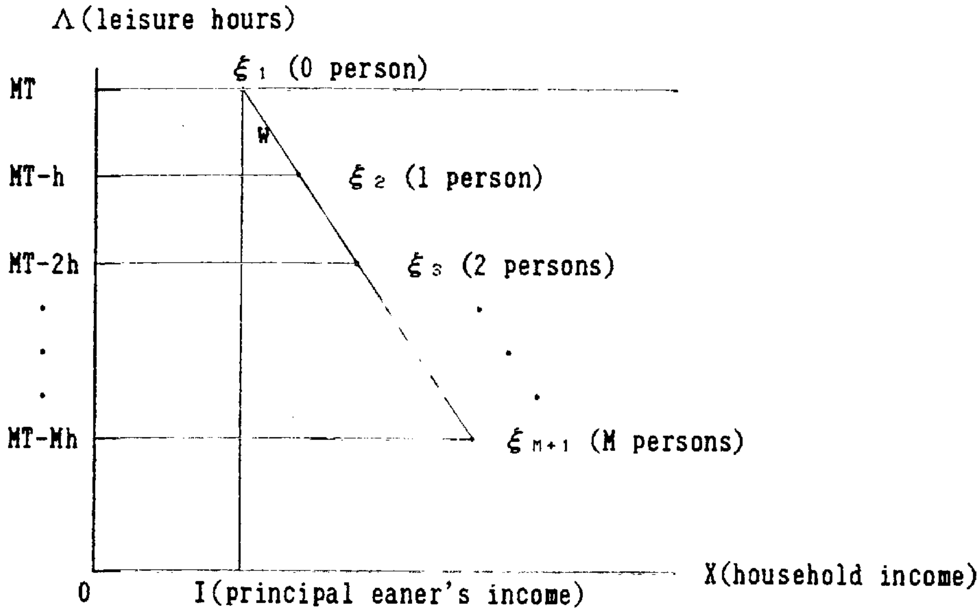
$$X=I+swh, \ \Lambda=MT-sh, \tag{2}$$

where T is total disposable hours of household members (24 hours a day, for example). The problem is to describe how many potential earners of the household accept the employment opportunity.

It should be noted that we assume the work hours are assined and fixed by demand side. Labor suppliers can not adjust the work hours at their own will. They can only accept or reject the employment opportunity with a condition of a fixed wage rate and work hours. So, the model is a discrete choice type. The framework of the model is introduced by Obi (1983), but his derivation is different from ours.

Fig. 1 is a discrete choice set of the household labor supply with M+1 elements in the income-leisure preference field. The figure illustrates the household's possible options ξ's when it has M non-principal earners and faces an employment opportunity with a wage rate w and work hours h. The option at the point ξ_1 means that no earner accepts the employment opportunity. The point ξ_2 means that one earner accepts the opportunity, etc.

Fig. 1 Discrete Choice Set for M Potential Earner Household



We consider a population of the households mentioned above. The households face the discrete choice set of Fig. 1. It may occur that choices of all the households concentrate on a single option, or that no household chooses a particular option. We consider which of the options ξ_0, ..., ξ_M are possibly chosen by the households, and what proportion of the population chooses each of the options.

We specify the utility indicator of the households as the quadratic form in X and Λ,

$$u=\frac{1}{2}\gamma_1 X^2+\gamma_2 X+\gamma_3 X\Lambda+\gamma_4\Lambda+\frac{1}{2}\gamma_5\Lambda^2 \tag{3}$$

$$\equiv\gamma_1 Z_1+\gamma_2 Z_2+\gamma_3 Z_3+\gamma_4 Z_4+\gamma_5 Z_5, \tag{4}$$

where γ's are parameters and Z's are defined by $Z_1=\frac{1}{2}X^2$, $Z_2=X$, etc. The indicator is linear in γ and Z. The parameter γ_4, a constant term of the marginal utility of leisure hours, is a stochastic variable to take account of the variation of preferences among the households. The other parameters are not stochastic. The distribution of γ_4 is represented by a density function f,

$$\gamma_4 \sim f(\gamma_4)>0, \text{ if } r<\gamma_4<R, \qquad (5)$$

$$=0, \text{ otherwise},$$

where the interval (r, R) is the range of the distribution. As the function f can take any form, we will specify, in the later sections of empirical analyses,

$$f = \text{Normal}(\mu, \sigma^2). \qquad (6)$$

The normality is, however, not yet determined. It is subject to a further empirical study.

2.2 A discrete choice model

We provide a discrete choice model in general terms and its properties summarized as theorems. For the detail, see Matsuno (1984, 1988a).

Let J be the number of options in a discrete choice set. If an economic agent chooses the j-th option ξ_j (j=1,...,J), its utility is

$$u_j=\gamma_1 Z_{1j}+\gamma_2 Z_{2j}+\gamma_3 Z_{3j}+\gamma_4 Z_{4j}+\gamma_5 Z_{5j}. \qquad (7)$$

The quadratic utility indicator (3) is a special case of (7). Although we assume five Z variables, (7) may contain more than five variables.

One of the parameters γ, γ_4 for instance, is stochastic and has a probability density function f over the range (r, R). The density f describes the difference in the households' preference. The other γ's are not stochastic.

We define a choice probability of the j-th option as

$$P_j=\Pr(u_j>u_i \mid j\neq i), \quad j=1,\ldots,J,$$

that is, a probability that the j-th option gives the maximum utilility and is chosen. Even if we observe households' labor supply behavior under a fairly controlled condition, we find some households participate in the labor force while others do not. The difference in their behavior is described by the choice probability. The labor force participation rate is an observed counterpart of the choice probability.

The "threshold" y_{ij} is defined by calculating the difference between the utilities attained by choice of options ξ_i and ξ_j,

$$u_i - u_j = (Z_{4i} - Z_{4j})(\gamma_4 - y_{ij}), \tag{8}$$

where

$$y_{ij} = \frac{\gamma_1(Z_{1i}-Z_{1j}) + \gamma_2(Z_{2i}-Z_{2j}) + \gamma_3(Z_{3i}-Z_{3j}) + \gamma_5(Z_{5i}-Z_{5j})}{Z_{4j}-Z_{4i}}, \tag{8.a}$$

$$i \neq j, i, j = 1, \ldots, J.$$

We assume, without loss of generality, an ordering of Z_{4j} variable,

$$Z_{41} > Z_{42} > \cdots > Z_{4J-1} > Z_{4J}. \tag{9}$$

From this assumption and (8), we see that if $\gamma_4 > y_{ij}$ for $i>j$, then $u_i > u_j$. There are $J(J-1)/2$ thresholds which determine ordering relations among the J utilitites. To find an ordering among the utilities, we have:

Theorem 1: The necessary and sufficient condition for all J options to be choosen, or for all J choice probabilities to be positive, is

$$R > y_{12} > y_{23} > y_{34} > \ldots > y_{J-2J-1} > y_{J-1J} > r. \tag{10}$$

Theorem 2: If all the choice probabilities are positive, they are given by

$$P_1 = \int_{y_{12}}^{R} f(\gamma_4)\, d\gamma_4,$$

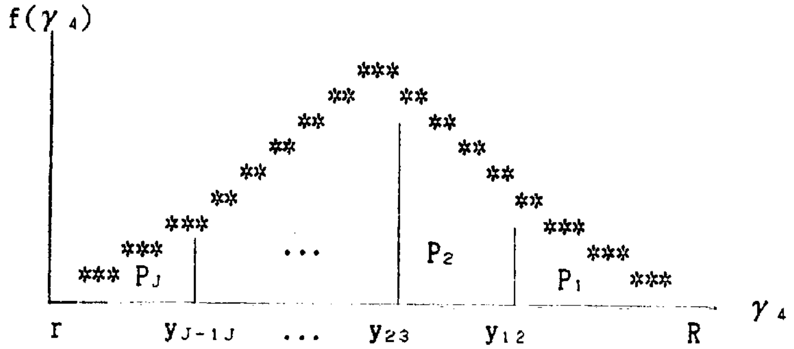
$$P_2 = \int_{y_{23}}^{y_{12}} f(\gamma_4)\, d\gamma_4, \tag{11}$$

$$\cdots\cdots$$

$$P_J = \int_{r}^{y_{J-1J}} f(\gamma_4)\, d\gamma_4.$$

Fig. 2 illustrates the meaning of the theorems. The parameter γ_4 is subject to a distribution. Some households have a large value of γ_4 and others have a small value of γ_4. Households with a larger value of γ_4, therefore with a higher preference for leisure, will choose shorter hours of work or no market work. Other households with a smaller value of γ_4 will choose longer hours of work. The distribution of choices is determined by the density function f. The households' choices are ordered and classfied into J choice groups uniquely by the thresholds satisfying the condition of Theorem 1.

Fig. 2 Distirbution of Households' Choices



If $n_1, \dots, n_J$, among n, households choose the 1-st,..., J-th option respectively, then the probalility of this event is the multinomial distribution

$$L = \frac{n!}{n_1!\ n_2!\ \cdots\ n_J!} P_1^{\,n_1}\ P_2^{\,n_2} \cdots P_J^{\,n_J}. \tag{12}$$

2.3 Model 1; a case of one potential earner

We consider an application of the discrete choice model with J=2. Economic agents in this case are households having a wife as a potential earner. The utility indicator of the households is a quadratic form (3). An employment opportunity with a wage a rate w and work hours h is open to the wife. The problem is if she accepts or rejects the opportunity.

The choice set contains two elements, no market work ξ_1 and market work ξ_2. They are represented by points

ξ_1=[I, T]=no market work,

ξ_2=[I+wh, T-h]=market work.

in the preference field of Fig. 1 with M=1.

In view of (8.a), we get a threshold

$$y_{12}=[1,\ I]\begin{bmatrix} w^2h/2 & w & w(T-h) & (h/2)-T \\ w & 0 & -1 & 0 \end{bmatrix}\gamma^{*}\equiv a_0^{*}+a_1^{*}I, \qquad (13)$$

where $\gamma^{*\prime}=[\gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_5]$. The threshold is linear in the principal earner's income I and linear in γ^{*}.

From Theorem 1, if

$$R>y_{12}>r, \qquad (14)$$

then P_1(=probability of no market work)>0 and P_2(=probability of wife's market work)>0. Provided that (14) holds, we have from Theorem 2,

$$P_1=\int_{a_0^{*}+a_1^{*}I}^{R} f(\gamma_4)d\gamma_4, \qquad (15)$$

$$P_2=1-P_1.$$

If we assume normality $N(\mu,\sigma^2)$ for the distribution of γ_4, the condition (14) always holds. Therefore, from Theorems 1 and 2, we conclude that

$$P_1=1-\Phi(a_0+a_1I),$$

$$P_2=\Phi(a_0+a_1I),$$

where the function Φ is the standard normal distribution function, and we have, for the reduced form parameters,

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}=\begin{bmatrix} a_0^{*}-\mu/\sigma \\ a_1^{*}/\sigma \end{bmatrix}=\begin{bmatrix} w^2h/2 & w & w(T-h) & -1 & (h/2)-T \\ w & 0 & -1 & 0 & 0 \end{bmatrix}\gamma, \qquad (16)$$

with $\gamma'=[\gamma_1/\sigma,\ \gamma_2/\sigma,\ \gamma_3/\sigma,\ \mu/\sigma,\ \gamma_5/\sigma]$.

In the estimation of the model, we classify the households into K income classes according to their principal earner's income I. The k-th class has the income value I_k. The probabilities P_{1k} and P_{2k} for the k-th class households are given by

$$P_{1k}=1-\Phi(a_0+a_1 I_k),$$

$$P_{2k}=\Phi(a_0+a_1 I_k).$$

If the n_{jk} households of the k-th class choose j-th (j=1,2) option, then the likelihood function for the estimation is

$$L=\prod_{k=1}^{K} \frac{n_k!}{n_{1k}!\, n_{2k}!} P_{1k}^{n_{1k}} P_{2k}^{n_{2k}} . \tag{17}$$

by independence assumption about experiments for the K classes. This is the likelihood function appearing in the usual probit analysis. The parameters a_0 and a_1 will be estimated.

Empirical studies show that a_1 is negative, or that labor participation rate of wives is negatively correlated with husband's income. We refered to this as Douglas law, see Douglas(1934).

2.4 Model 2; a case of two potential earners

We consider the labor supply behavior of two potential earners, say a wife and a child of age over 16. The utility indicator is of form (3). The wife and child can accept or reject an employment opportunity offered with a condition of a wage rate and labor hours (w, h). There are three possible options for the two suppliers, the first being that none of them will accept the opportunity, the second that one of them will accept it, and the third that two of them will accept it. These are points

ξ_1=[I, 2T]=market work of no person,

ξ_2=[I+wh, 2T-h]=market work of wife or child,

ξ_3=[I+2wh, 2T-2h]=market work of wife and child,

in Fig. 1 with M=2. The discrete choice model with J=3 applies to this case. It is noted that for the one person market work option ξ_2, the model does not tell whether it is work of wife or work of child.

Thresholds y_{12}, y_{13}, y_{23} are defined by (8.a), and two of them are

$$y_{12}=[1,\ I]\begin{bmatrix} w^2h/2 & w & w(2T-h) & (h/2)-2T \\ w & 0 & -1 & 0 \end{bmatrix}\gamma^{*}\equiv b_0^{*}+b_1^{*}I, \tag{18}$$

$$y_{23}=[1,\ I]\begin{bmatrix} 3w^2h/2 & w & w(2T-3h) & (3h/2)-2T \\ w & 0 & -1 & 0 \end{bmatrix}\gamma^{*}\equiv c_0^{*}+c_1^{*}I. \tag{19}$$

If we assume the normality of the distribution of γ_4, we have, from Theorem 1 and 2,

$$\begin{aligned} P_1&=1-\Phi(b_0+b_1I)=\text{probability of choosing } \xi_1, \\ P_2&=\Phi(b_0+b_1I)-\Phi(c_0+c_1I)=\text{probability of choosing } \xi_2, \\ P_3&=\Phi(c_0+c_1I)=\text{probability of choosing } \xi_3, \end{aligned} \tag{20}$$

where

$$\begin{aligned} b_0&=(b_0^{*}-\mu)/\sigma,\quad b_1=b_1^{*}/\sigma, \\ c_0&=(c_0^{*}-\mu)/\sigma,\quad c_1=c_1^{*}/\sigma. \end{aligned} \tag{21}$$

It is noted that we have, from (13), (18) and (19), theoretical restrictions $a_1=b_1=c_1$.

The likelihood function for estimating the reduced form parameters b and c is

$$L=\prod_{k=1}^{K}\frac{n_k!}{n_{1k}!\ n_{2k}!\ n_{3k}!}P_{1k}^{n_{1k}}P_{2k}^{n_{2k}}P_{3k}^{n_{3k}}, \tag{22}$$

where n_{jk} is the number of households in the k-th class, choosing the j-th option and

$$P_{2k}=\Phi(b_0+b_1I_k)-\Phi(c_0+c_1I_k)$$

with similar expressions for P_{1k} and P_{3k}.

2.5 Model 3; a case of three potential earners

We consider the behavior of households with M=3 potential earners, a wife and two children of age over 16. The utility indicator takes the form (3). The households have four possible options ξ_1, ξ_2, ξ_3 and

ξ_4. The first option ξ_1 is the one that none of the three suppliers accepts the employment opportunity, the second ξ_2 is that one of them accepts it, and so on. The options corresponds to points

ξ_1=[I, 3T]=no market work,

ξ_2=[I+wh, 3T-h]=1 person market work,

ξ_3=[I+2wh, 3T-2h]=2 person market work,

ξ_4=[I+3wh, 3T-3h]=3 person market work.

in Fig 1 with M=3.

We apply the discrete choice model with J=4, of Section 2.2 to this case. Thresholds $y_{12},\dots, y_{34}$ are defined by (8.a) and are, for instance,

$$y_{12}=[1,\ I]\begin{bmatrix} w^2h/2 & w & w(3T-h) & (h/2)-3T \\ w & 0 & -1 & 0 \end{bmatrix}\gamma^{*}\equiv d_0^{*}+d_1^{*}I, \tag{23}$$

$$y_{23}=[1,\ I]\begin{bmatrix} 3w^2h/2 & w & w(3T-3h) & (3h/2)-3T \\ w & 0 & -1 & 0 \end{bmatrix}\gamma^{*}\equiv e_0^{*}+e_1^{*}I, \tag{24}$$

$$y_{34}=[1,\ I]\begin{bmatrix} 5w^2h/2 & w & w(3T-5h) & (5h/2)-3T \\ w & 0 & -1 & 0 \end{bmatrix}\gamma^{*}\equiv f_0^{*}+f_1^{*}I. \tag{25}$$

Under the normality assumption for the distribution of the stochastic parameter γ_4, Theorem 1 and 2 show that

$$\begin{aligned} P_1&=1-\Phi(d_0+d_1I)=\text{probability of choosing } \xi_1,\\ P_2&=\Phi(d_0+d_1I)-\Phi(e_0+e_1I)=\text{probability of choosing } \xi_2,\\ P_3&=\Phi(e_0+e_1I)-\Phi(f_0+f_1I)=\text{probability of choosing } \xi_3,\\ P_4&=\Phi(f_0+f_1I)=\text{probability of choosing } \xi_4, \end{aligned} \tag{26}$$

where

$$\begin{aligned} d_0&=(d_0^{*}-\mu)/\sigma, \quad d_1=d_1^{*}/\sigma,\\ e_0&=(e_0^{*}-\mu)/\sigma, \quad e_1=e_1^{*}/\sigma,\\ f_0&=(f_0^{*}-\mu)/\sigma, \quad f_1=f_1^{*}/\sigma. \end{aligned} \tag{27}$$

We see that the equality $a_1=b_1=c_1=d_1=e_1=f_1$ holds among the parameters of

Model 1, 2 and 3.

The likelihood function for the estimation in this case is obtained from the multinomial distribution, and is

$$L=\prod_{k=1}^{K}\frac{n_k!}{n_{1k}!\,n_{2k}!\,n_{3k}!\,n_{4k}!}P_{1k}^{n_{1k}}P_{2k}^{n_{2k}}P_{3k}^{n_{3k}}P_{4k}^{n_{4k}}, \tag{28}$$

where P_{jk} is the j-th choice probability of the k-th income class and n_{jk} is the number of households choosing the j-th option. The parameters d, e and f are estimated.

2.6 Theoretical restrictions

We have developed three models which have reduced form parameters with theoretical constraints

$$a_1=b_1=c_1=d_1=e_1=f_1=(w\gamma_1-\gamma_3)/\sigma. \tag{29}$$

Furthermore, we have relations for the constant terms,

$$\begin{aligned} a_0&=a_0,\\ b_0&=a_0+\delta_1,\\ c_0&=a_0+\delta_1+\delta_2,\\ d_0&=a_0+2\delta_1,\\ e_0&=a_0+2\delta_1+\delta_2,\\ f_0&=a_0+2\delta_1+2\delta_2, \end{aligned} \tag{30}$$

where

$$\begin{aligned} \delta_1&=T(w\gamma_3-\gamma_5),\\ \delta_2&=h(w^2\gamma_1-2w\gamma_3+\gamma_5). \end{aligned} \tag{31}$$

If the quadratic utility indicator (3) is a negative form, then δ_2 is negative. These constraints are tested in the later sections.

3. Data

The data used for the measurement is the 1979 cross sectional <u>Family</u>

Income and Expenditure Survey of Japan. For the earlier study using this data and particularly for the details of the data, see Matsuno(1988b). Models and estimation methods in the present analysis are different from the earlier ones.

Data 1 is a set of observations, from the survey, of 16236 households with an employee husband and one potential earner (a wife), with or without children of schooling age. Data 2 is a set of observations of 3203 households with an employee husband and two potential earners (a wife and a child of age over 16). Data 3 is a set of observations of 1211 households with an employee husband and three potential earners (a wife and two children of age over 16). Data 1 is used for the measurement of Model 1, Data 2 for Model 2 and Data 3 for Model 3.

We take the husband's income only from his employment (in the preceding year of the survey) for the observation of principal earner's income I. The households are classified into 15 income classes according to this observation. The households in the first class have I-value under 50 (×0000) yen and in the second class I lies in the interval (50, 100), etc.

Whereas FIES records information about employment status, whether an employee is a full-time worker or a part-time worker, we do not use this information. Potential earners recorded as looking for a job are regarded as having no job or choosing no market work.

4. Maximum Likelihood Estimation

We first estimate the parameters by maximum likelihood methods. The results will be used for formulating prior distributions in the later sections and for comparison with Bayesian estimates.

4.1 Likelihood function

The likelihood functions for Model 1, 2 and 3 are special cases of

$$L(\theta \mid n)=\prod_{k=1}^{K} \frac{n_k!}{n_{1k}!\ n_{2k}!\ \dots\ n_{Jk}!} P_{1k}^{n_{1k}}\ P_{2k}^{n_{2k}} \cdots P_{Jk}^{n_{Jk}}, \tag{32}$$

where

$$\begin{aligned}
P_{1k} &= 1 - \Phi(\theta_{10}+\theta_{11}I_k),\\
P_{2k} &= \Phi(\theta_{10}+\theta_{11}I_k) - \Phi(\theta_{20}+\theta_{21}I_k),\\
&\cdots\cdots\\
P_{J-1k} &= \Phi(\theta_{J-20}+\theta_{J-21}I_k)-\Phi(\theta_{J-10}+\theta_{J-11}I_k),\\
P_{Jk} &= \Phi(\theta_{J-10}+\theta_{J-11}I_k).
\end{aligned} \tag{33}$$

Model 1 has J=2 and

$\theta_{10}=a_0$, $\theta_{11}=a_1$,

Model 2 has J=3 and

$\theta_{10}=b_0$, $\theta_{11}=b_1$, $\theta_{20}=c_0$, $\theta_{21}=c_1$,

and Model 3 has J=4 and

$\theta_{10}=d_0$, $\theta_{11}=d_1$, $\theta_{20}=e_0$, $\theta_{21}=e_1$, $\theta_{30}=f_0$, $\theta_{31}=f_1$.

4.2 Estimates

The estimtes and test statistics by maximum likelihood methods are given in Table 1. The sign ^ in Table 1 indicates estimtes by maximum likelihood methods. The figures in ( ) are the standard errors of the estimates and the figures in [ ] are (asymptotic) t-values. The test statistic for the goodness of fit is Pearson's χ^2 associated with degree of freedom (d.f.) and P-value.

For Model 1, the estimate of the coefficient a_1 is sufficiently significant. The P-value indicates satisfactory fit. The (asymptotic) variances of the estimates of a_0 and a_1 are, respectively,

0.00104, 9.3186E-09. (34)

For Model 2, the estimates of the coefficients c_1 and b_1 are both significant. The χ^2-value for the goodness of fit is not good. The fit is not good particulaly for the lower income classes. And the variances of the estimates of c_0, c_1, b_0 and b_1 are, respectively,

$$0.00591,\ 4.4721E\text{-}08,\ 0.00395,\ 2.2573E\text{-}08. \tag{35}$$

Table 1. Maximum Likelihood Estimates

Model	Estimates		χ^2	d.f.	P-value
1	$\hat{a}_0$= .090631	$\hat{a}_1$=-.0021439	21.244	13	0.073-0.055
	(.32287E-01)	(.96533E-04)			
	[2.8070]	[-22.209]			
2	$\hat{b}_0$=1.2297	$\hat{b}_1$=-.0025566	40.357	26	0.039-0.031
	(.62814E-01)	(.15024E-03)			
	[19.577]	[-17.017]			
	$\hat{c}_0$=-.43684	$\hat{c}_1$=-.0021006			
	(.76857E-01)	(.21147E-03)			
	[-5.6838]	[-9.9332]			
3	$\hat{d}_0$=2.0421	$\hat{d}_1$=-.0030841	35.417	39	0.653-0.607
	(.12381)	(.26584E-03)			
	[16.494]	[-11.601]			
	$\hat{e}_0$= .74319	$\hat{e}_1$=-.0026020			
	(.10254)	(.24886E-03)			
	[7.2479]	[-10.455]			
	$\hat{f}_0$=-.62730	$\hat{f}_1$=-.0020717			
	(.13601)	(.36375E-03)			
	[-4.6122]	[-5.6953]			

Data 3 has a smaller sample size. Therefore, the t-values for Model 3 are smaller than the former two models. But they still show significant results. The χ^2 shows good fit. And the variances of the estimates of f_0, f_1, e_0, e_1, d_0 and d_1 are, respectively, 0.01850, 1.3231E-07, 0.01051, 6.1933E-08, 0.01533, 7.0671E-08. (36)

4.3 Test(1)

We test the theoretical restrictions (29),

$a_1=b_1=c_1=d_1=e_1=f_1$.

We have estimates of b_1-c_1, d_1-e_1 and e_1-f_1 in Fig. 2. For Model 2, The P-value for testing the hypothesis; $b_1-c_1=0$, is 0.00417. The hypothesis is significant. For Model 3, the hypotheses $d_1-e_1=0$ and $e_1-f_1=0$ are not significant, if we test them separately.

Table 2. Test Statistics for Theoretical Restrictions

Estimates
(b_1-c_1)^=-.00045602 (.00022395) [-2.0363]
(d_1-e_1)^=-.00048211 (.00028558) [-1.6882]
(e_1-f_1)^=-.00053030 (.00035941) [-1.4755]

Wald's test statistic for testing the joint hypothesis; $d_1-e_1=0$ and $e_1-f_1=0$, is

$\chi^2=6.024169$, d.f.=2, P-value=0.0492. (37)

Therefore, the hypothesis is significant at 5% level.

We can get estimats of a_1, b_1, c_1, c_1, d_1, e_1 and f_1 by applying maximum likelihood method simultaneously to Model 1, 2 and 3 , and get Wald's statistic for the joint hypothesis; $a_1-b_1=0$, $b_1-c_1=0$, $c_1-d_1=0$,

$d_1-e_1=0$ and $e_1-f_1=0$. The values are

$$\chi^2=17.649877,\ d.f.=5,\ P\text{-value}=0.0034. \tag{38}$$

This means the joint hypothesis is significant.

4.4 Test(2)

From (30), we can set forth a regression equation

$$\begin{bmatrix} b_0-a_0 \\ c_0-a_0 \\ d_0-a_0 \\ e_0-a_0 \\ f_0-a_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 0 \\ 2 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} + \text{error term}. \tag{39}$$

The estimates in Table 1 are substituted as observations for the left hand side variables. Applying the least squares method, we get estimates of δ_1 and δ_2, that is,

$\delta_1\hat{}=1.0137$, $\delta_2\hat{}=-1.4011<0$.

This shows that δ_2 is estimated negative and consistent with the theoretical restriction mentioned in Section 2.6.

5. Bayesian Procedure

5.1 Specification of Prior distributions

We consider applications of Bayesian methods for estimating parameters in our models. As shown and tested above, the three models have common parameters. Therefore, estimates of one of the models can be used as prior information in estimating the other. We apply Bayesian methods in the following ways.

1) We first estimate Model 1 using Data 1 and maximum likelihood methods, and get also the variances of estimates. These statistics may

be used to specify the hyper-parameters of prior distributions in next Bayesian estimation of Model 2.

2) The information obtained in 1) is used as prior information about the parameters of Model 2. The information is combined with Data 2 by Bayesian methods to get posterior distributions of the parameters of Model 2.

3) The posterior distribution for Model 2 obtained in 2) is combined with Data 3 in the next estimation for Model 3 by Bayesian methods.

In formulating prior distributions in this study, we assume that parameters to be estimated are independently distributed. Furthermore, we assume uniform distributions for the constant terms of the thresholds and normal distribution for the slope coefficients. The hyper-parameters of normal prior distributions are specified in the way explained in 1), 2) and 3) above.

Bayesian procedures in general take account of stochastic information expressed as prior distributions into estimation. In addition, the Bayesian methods here also utilize exact information. In the calculation for getting Bayesian estimates, we impose exact theoretical restrictions, which are given in (29).

5.2 Evaluation of posterior distributions

The models to be estimated are type of qualitative response. Zellner and Rossi (1984) consider Bayesian estimtaion methods for models of this type. Features of their method are:

a) The method deals with dichotomous response models.

b) The method is based on an experiment without replications. Samples are not classified.

c) The method mainly employes normal distributions for prior distributions.

d) They obtain posterior distributions by approximating it by a normal distribution. (They also consider numerical integrations for an exact method.)

We will adopt their method with slight modifications for a) and b). And their method is now summarized.

The likelohood functions $L(\theta|O)$ for the models are of form (32). Here, "O" stands for observations. Denoting a prior distribution by $\pi(\theta)$, Bayes' formula gives the posterior distribution

$$\pi(\theta|O) \propto \pi(\theta) L(\theta|O). \tag{40}$$

This posterior distribution is maximized at the point $\theta=\theta^{\sim}$ which is defined by and obtained by solving the equation

$$\partial \log \pi(\theta|O)/\partial\theta=0. \tag{41}$$

We approximate the posterior distribution by a normal density function. Expanding the posterior distribution at the point $\theta=\theta^{\sim}$ by using Taylor's formula, we have

$$\log\pi(\theta|O) \fallingdotseq \log\pi(\theta^{\sim}|O) + (\theta-\theta^{\sim})' \partial\log\pi(\theta^{\sim}|O)/\partial\theta$$
$$+\tfrac{1}{2}(\theta-\theta^{\sim})' \left[\frac{\partial^2 \log\pi(\theta^{\sim}|O)}{\partial\theta\,\partial\theta'}\right](\theta-\theta^{\sim}). \tag{42}$$

Since the second term of the right hand side is zero in view of (41), we have

$$\log\pi(\theta|O) \fallingdotseq \log\pi(\theta^{\sim}|O) - \tfrac{1}{2}(\theta-\theta^{\sim})' H(\theta-\theta^{\sim}), \tag{43}$$

where

$$H=-\partial^2 \log\pi(\theta^{\sim}|O)/\partial\theta\,\partial\theta'. \tag{44}$$

Therefore, the approximated posterior dstribution of θ is

$$\pi(\theta|O) \fallingdotseq \text{const}\times\exp\{-\tfrac{1}{2}(\theta-\theta^{\sim})' H(\theta-\theta^{\sim})\}, \tag{45}$$

which is a normal density function with mean vector $\theta^{\sim}$ and variance-covariance matrix H^{-1}. The approximated posterior mean $\theta^{\sim}$ is used as an Bayesian estimate of θ and its variance-covariance matrix is H^{-1}.

After specifying the prior distributions, we get $\theta^{\sim}$ and H^{-1} by numerical methods. Actually, we used Newton-Raphson method to get the statistics.

6. Bayesian Estimates

We present Bayesian estimates in this section. They are summarized in Table 3.

6.1 Model 2

Prior distribution: To get a prior ditribution for Model 2, we estimate Model 1 by maximum likelihood method. The result has been given in Table 1 and is regarded as Bayesian estimates with a diffuse prior. The result of estimation means that the (posterior) distribution of the parameter a_1 is

$$a_1 \sim N(\hat{a}_1,\ \sigma_a^2)=N(-0.0021439,\ 9.3186E\text{-}09). \tag{46}$$

This can be used as a prior information for later Bayesian estimations of b_1 and c_1. For the prior distributions of b_1 and c_1, we assume that they are independently normally distributed,

$$b_1 \sim N(\bar{b}_1,\ \sigma_b^2), \tag{47}$$
$$c_1 \sim N(\bar{c}_1,\ \sigma_c^2).$$

From the restriction (29), $a_1=b_1=c_1$, we set forth numerical values

$$\bar{b}_1 = -.0021439,\ \sigma_b^2=\sigma_a^2=9.3186E\text{-}09, \tag{48.a}$$
$$\bar{c}_1 = -.0021439,\ \sigma_c^2=\sigma_a^2=9.3186E\text{-}09. \tag{48.b}$$

As for the constant terms b_0 and c_0, we do not use any information obtained at the first stage estimation of Model 1, because we have no theoretical basis for that. We assume that b_0 and c_0 are uniformly and independently distributed, that is,

$$\pi(b_0,\ c_0)=\text{const} \tag{49}$$

is the prior for b_0 and c_0, and we assume independence of all parameters.

The product of (47) and (49) with hyper-parameters (48.a,b) is the prior for b and c. The prior distribution obtained thus and the likelihood function (32) are combined to give the posterior. Its approximation is obtained as the normal density (45).

Table 3 Bayesian Estimates

Model Estimates	Standard Error	t-value	<Procedure> Estimates	Standard Error	t-value
		Model 2	<1-a>		
$\tilde{b}_0$=1.1187	(.38800E-01)	[28.832]	$\tilde{b}_1$=-.0022726	(.80250E-04)	[-28.319]
$\tilde{c}_0$=-.43489	(.41358E-01)	[-10.515]	$\tilde{c}_1$=-.0021172	(.87800E-04)	[-24.114]
		Model 2	<1-b>		
$\tilde{b}_0$=1.1060	(.38024E-01)	[29.086]	$\tilde{b}_1$=-.0022411	(.78579E-04)	[-28.521]
$\tilde{c}_0$=-.39472	(.39972E-01)	[-9.8748]	$\tilde{c}_1=\tilde{b}_1$		
		Model 3	<2-a>		
$\tilde{d}_0$=1.6799	(.56823E-01)	[29.565]	$\tilde{d}_1$=-.0022499	(.89623E-04)	[-25.104]
$\tilde{e}_0$=.57355	(.50133E-01)	[11.441]	$\tilde{e}_1$=-.0021764	(.88678E-04)	[-24.543]
$\tilde{f}_0$=-.61958	(.61540E-01)	[-10.068]	$\tilde{f}_1$=-.0021237	(.93292E-04)	[-22.764]
		Model 3	<2-b>		
$\tilde{d}_0$=1.6793	(.55234E-01)	[30.403]	$\tilde{d}_1$=-.0022507	(.87011E-04)	[-25.867]
$\tilde{e}_0$= .60113	(.50245E-01)	[11.964]	$\tilde{e}_1=\tilde{d}_1$		
$\tilde{f}_0$=-.57642	(.61524E-01)	[-9.3691]	$\tilde{f}_1=\tilde{d}_1$		
		Model 3	<3-a>		
$\tilde{d}_0$=1.7044	(.52546E-01)	[32.437]	$\tilde{d}_1$=-.0023083	(.74720E-04)	[-30.893]
$\tilde{e}_0$= .60439	(.46669E-01)	[12.951]	$\tilde{e}_1$=-.0022573	(.74170E-04)	[-30.434]
$\tilde{f}_0$=-.58445	(.59017E-01)	[-9.9030]	$\tilde{f}_1$=-.0022248	(.76814E-04)	[-28.964]
		Model 3	<3-b>		
$\tilde{d}_0$=1.7018	(.51527E-01)	[33.027]	$\tilde{d}_1$=-.0023038	(.73196E-04)	[-31.474]
$\tilde{e}_0$= .62158	(.46872E-01)	[13.261]	$\tilde{e}_1=\tilde{d}_1$		
$\tilde{f}_0$=-.55769	(.59266E-01)	[-9.4099]	$\tilde{f}_1=\tilde{d}_1$		

Estimates: The estimates are the mean vectors of the approximated posterior distribution. Their values and standard deviations are given in Table 3 <1-a>. The variances of c_0, c_1, b_0 and b_1 are, respectively,

$0.00171,\ 7.7089E-09,\ 0.00151,\ 6.4401E-09.$ (51)

Compared to the standard errors of the maximum likelihood estimates in Section 4, these values are smaller.

Testing: We test the hypothesis; $b_1-c_1=0$, which was rejected when we used maximum likelihood estimates in Section 4. Based on the Bayesian estimtes, we have the distribution of b_1-c_1,

$$b_1-c_1 \sim N(-.00015539,\ .00011542^2). \tag{52}$$

This shows that b_1-c_1 approximately lies in the 1-sigma range from zero. Therefore, the hypothesis is not significant.

Estimation with exact restriction: Then we estimate the parameters under the exact restriction $b_1=c_1$, which is shown not significant. The prior distribution for the parameters b_0, c_0 and $b_1(=c_1)$ is obtained again by (47), (48.a) and (49). The posterior distribution is now maximized under the constraint $b_1=c_1$. The estimates and standard deviations for b_0, b_1 and c_0 are given in Table 3 <1-b>. Their variances are, respectively,

$0.00160,\ 6.1747E-09,\ 0.00145.$ (53)

The variances are smaller than those given in (51). The approximated posterior distribution of b_1 under the exact restriction is

$$b_1(=c_1) \sim N(-.0022411,\ 6.1747E-09). \tag{54}$$

6.2 Model 3

6.2.1 Estimation (1)

Prior distribution: We have given the distribution (46) of a, which is obtained by maximum likelihood method using Data 1. This can be used for the prior of d_1, e_1 and f_1 when estimating Model 3, since we have restrictions $a_1=d_1=e_1=f_1$. Therefore, we assume

$$\begin{aligned} d_1 &\sim N(-.0021439,\ 9.3186E-09),\\ e_1 &\sim N(-.0021439,\ 9.3186E-09),\\ f_1 &\sim N(-.0021439,\ 9.3186E-09), \end{aligned} \tag{55}$$

for the prior of d_1, e_1 and f_1. Furthermore, we assume that the constant terms of the model are uniformly and independently distributed. We here do not take any information contained in Data 2 to formulate the prior of the parameters d, e and f.

The Bayesian estimates are calculated in the same way as in Section 6.1. They are given Table 3 <2-a>. We find that the estimates of d_1, e_1 and f_1 take close values each other. The variances of f_0, f_1, e_0, e_1, d_0, and d_1 are, respectively,

$$0.00379,\ 8.7034E-09,\ 0.00251,\ 7.8637E-09,\ 0.00323,\ 8.0323E-09. \qquad (56)$$

The variances are smaller than those of the maximum likelihood estimates.

Testing: From the posterior distribution, we have the distribution of d_1-e_1 and e_1-f_1,

$$(d_1-e_1)\sim N(-0.000073485\ 0.00012277^2), \qquad (57)$$
$$(e_1-f_1)\sim N(-0.000052744\ 0.00012665^2).$$

This shows that the differences d_1-e_1 and e_1-f_1 are not significantly different from zero.

Estimation with exact restriction: We can estimates d, e and f with exact restrictions $d_1=e_1=f_1$. The parameters to be estimated are now d_0, e_0, f_0 and d_1. The values of hyper-parameters are given in (46). The posterior distribution is maximized under the constraints $d_1=e_1=f_1$. The estimates and the standard deviations are given in Table 3 <2-b>. The variacnces of f_0, $d_1(=e_1=f_1)$, e_0 and d_0 are

$$0.00379,\ 7.5709E-09,\ 0.00252,\ 0.00305. \qquad (58)$$

It is shown that the variance of d_1 is smaller than those given in (57), but the variance of e_1 is not smaller.

6.2.2 Estimation (2)

The results of Estimation (1) utilize the information of Data 1 and 3 as well as theoretical information or restrictions. But they do not contain the information from Data 2. We consider the estimation which might incorporate the information from Data 1, 2 and 3.

Prior distribution: We have given estimates in Table 3 <1-b>, which are Bayesian estimates from Data 1 and Data 2. From those estimates or from (54), we see that

$$b_1(=c_1) \sim N(-.0022411,\ 6.1747E-09). \tag{59}$$

This can be used as the prior for d_1, e_1 and f_1 of Model 3. Furthermore, we have theoretical restrictions $b_1=c_1=d_1=e_1=f_1$. Therefore, we construct the prior for d_1, e_1 and f_1 as

$$\begin{aligned} &d_1 \sim N(-.0022411,\ 6.1747E-09), \\ &e_1 \sim N(-.0022411,\ 6.1747E-09), \\ &f_1 \sim N(-.0022411,\ 6.1747E-09). \end{aligned} \tag{60}$$

We assume that the constant terms of the model are independently and uniformly distributed. The prior distribution obtained in this way contains all the information in Data 1 and 2.

Estimates: In the same way as above, we calculate the estimates. They are given in Table 3 <3-a>. The variances of f_0, d_1, e_0, e_1, d_0 and f_1 are, respectively,

$$0.00348,\ 5.9004E-09,\ 0.00218,\ 5.5012E-09,\ 0.00276,\ 5.5830E-09, \tag{61}$$

which are smaller when compared to those given in (56).

Testing: To test the hypotheses $d_1-e_1=0$ and $e_1-f_1=0$, we get, from the posterior distribution, that

$$\begin{aligned} &d_1-e_1 \sim N(-0.000051007,\ 0.00010338^2), \\ &e_1-f_1 \sim N(-0.000032457,\ 0.00010562^2), \end{aligned} \tag{62}$$

This shows that the hypothese are not significant.

Estimation with exact restriction: Provided that the theoretical restriction $d_1=e_1=f_1$ holds, we assume, from Table 3 <1-b> or from (54),

$$d_1(=e_1=f_1) \sim N(-.0022411,\ 6.1747E-09), \tag{63}$$

for the prior of $d_1(=e_1=f_1)$. Futhermore, we assume that the constant terms d_0, e_0 and f_0 are independently and uniformly distributed.

The Bayesian estimates and their standard deviations under this prior are given in Table 3 <3-b>. The variances of f_0, $d_1(=e_1=c_1)$, e_0 and d_0 are

$$0.00351,\ 5.3576E\text{-}09,\ 0.00220,\ 0.00266. \tag{64}$$

The variance of d_1 is the smallest of all that obtained so far. The estimates contain all the observed and the theoretical information.

7. Experimental Calculations

Based on the estimated parameters, we make calculations concerning the non-principal earners' labor supply probabilities to derive theoretical implications of the models.

7.1 Labor supply spectrum

From Model 1, theoretical values of the labor supply probability or labor participation rate of households with one potential earner are calculated using the estimated parameters. In this case, the maximum likelihood estimates of Section 4 are used for the calculation. The values are illustrated in Fig. 3. It is noted that the principal earner's income I is scaled vertically in the figure. The probability P_2 at the level of I=0 is 0.536 and is 0.0101 at I=1125(×0000 yen/year). The probability is negatively correlated with or a decreasing function of the husband's income over the income range.

For Model 2, the theoretical restriction $b_1-c_1=0$ is not significant. Under this restriction, the condition

$$\infty>y_{12}>y_{23}>-\infty \tag{65}$$

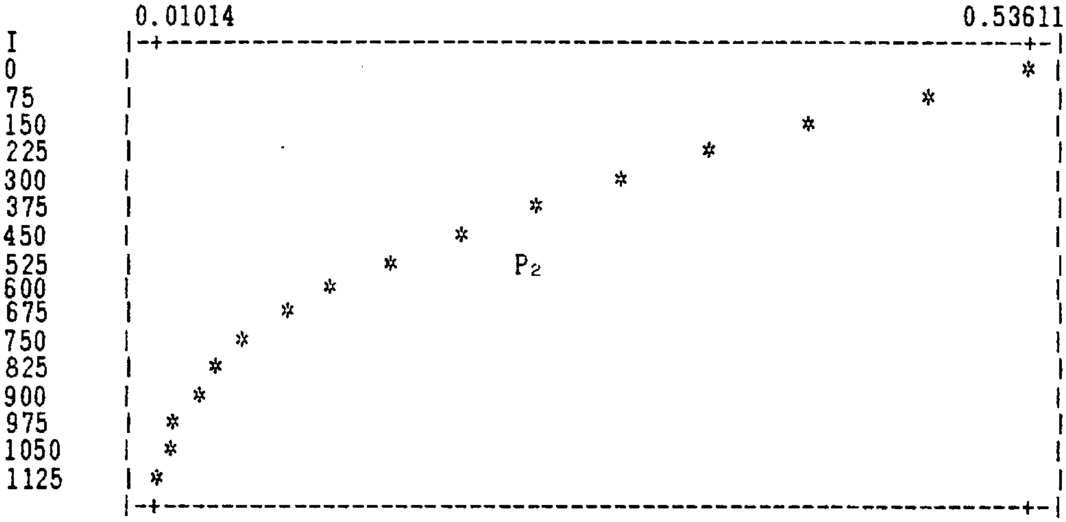
is satisfied at every income level. Since (65) is the necessary and sufficient condition (Theorem 1) for $P_1>0$, $P_2>0$ and $P_3>0$, we may conclude that the three options of Model 2 are chosen with positive probabilities at every income level. If we adopt the estimated result $b_1<c_1$ in Table 3

<1-a>, we have

$$\infty > y_{23} > y_{13} > y_{12} > -\infty \tag{66}$$

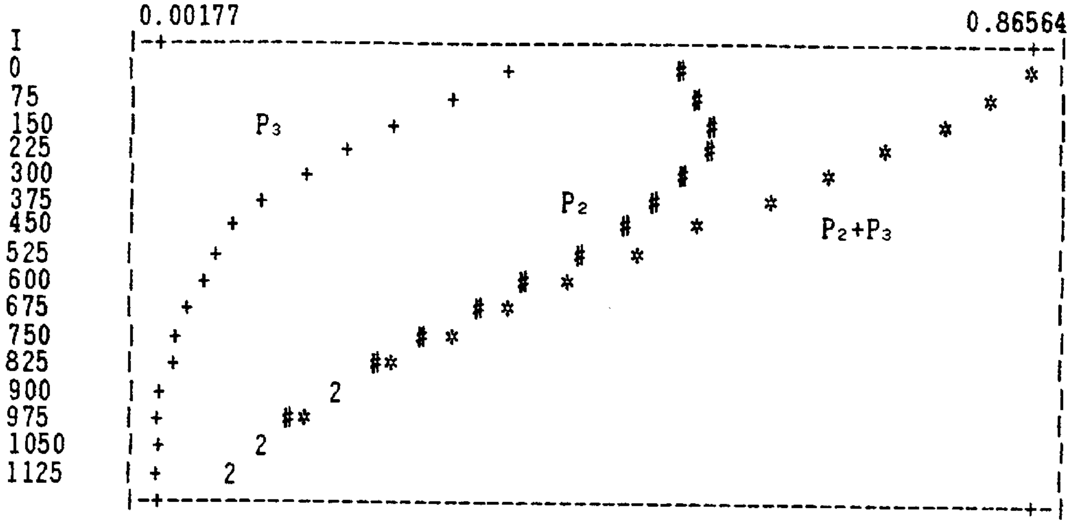
at income levels above I=(1.1187+.43489)/(-.0021172+.0022726)=9997. For households having income higher than 9997, there will be no possibility of choosing one-person employment and there will remain two possibilities of no-person employment and two-person employment. However, our data do not include households with husband's income higher than 9997. So, for the data year, we do not have the possibility that any of the options is not identically chosen.

Fig. 3 Labor Supply Probability of One Potential Earner



From Model 2, the three probabilities P_1, P_2 and P_3 are calculated with the estimates of Table 3 <1-b>. They are given in Fig. 4. The probability P_3 takes small values over the range, and the probability P_2 takes large values. The probability P_2+P_3 of one or two person employment is quite large. P_2+P_3 and P_3 are found to be decreasing functions of income I exept for the lower income classes, where P_2 is an increasing function of I. If income I reaches at the average level, P_2 is a decreasing function of I and takes small value.

Fig. 4 Labor Supply Probabilities of Two Potential Earners



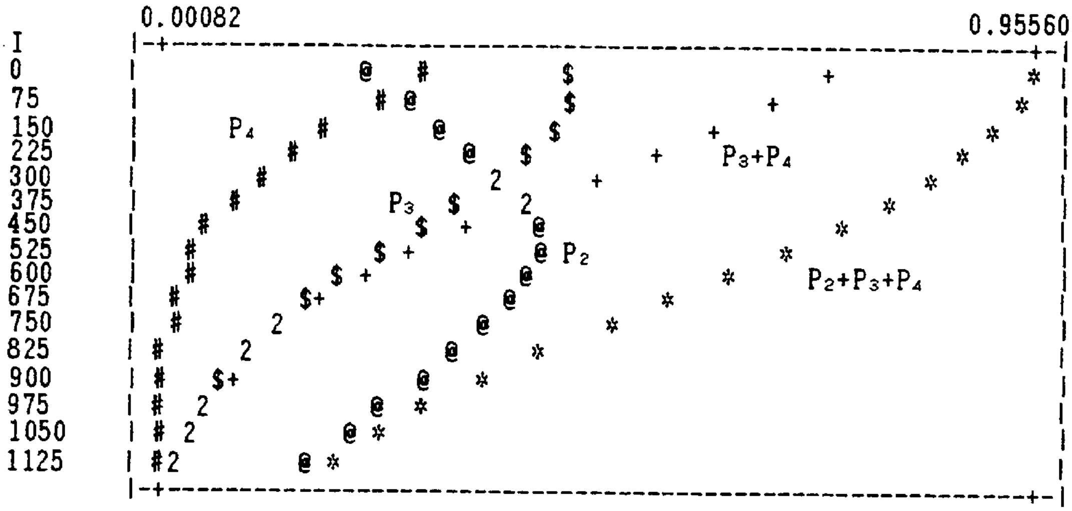
Based on the Bayesian estimates of Model 3 in Table 3 <1-a>, the hypothesis $d_1=e_1=f_1$ is not significant. Therefore, we see that the inequality

$$\infty > y_{12} > y_{23} > y_{34} > -\infty \tag{67}$$

holds at any income class. In view of Theorem 1, the four probabilities P_1, P_2, P_3 and P_4 are all positive at any income level. We see no income range where any of them becomes zero. If we adopt the estimates by the maximum likelihood estimation, then the inequality $y_{12}<y_{23}$ holds for income level above 2584. The probability P_3 is zero for the classes with income beyond this level. In addition, the inequality $y_{13}<y_{24}$ holds for households with income above 2655. The probability P_2 is zero and the probabilities P_1 and P_3 are positive for households with income beyond this level. But our data do not include observations of households with income of this high level. We may conclude that the four probabilities are positive for households within the normal income range.

The theoretical values of the four probabilitites of Model 3 and their combinations are calculated and illustrated in Fig. 5, using estimated parameters of Table 3 <3-b>. It is almost certain that at least one non-potential earner of lower income households paticipates in the labor force. We find that the probability of two-person participation is possibly as large as 50% and that the probability of three-person participation is small. It is noted that the combinations of probabilities, $P_4+P_3+P_2$, P_4+P_3 and P_4, are decreasing functions of income I, and that there is an income range where the probabilities P_3 and P_2 are incresing functions of I. But, if income I becomes large enough, then P_3 and P_2 become decreasing functions of I.

Fig. 5 Labor Supply Probabilities of Three Potential Earners



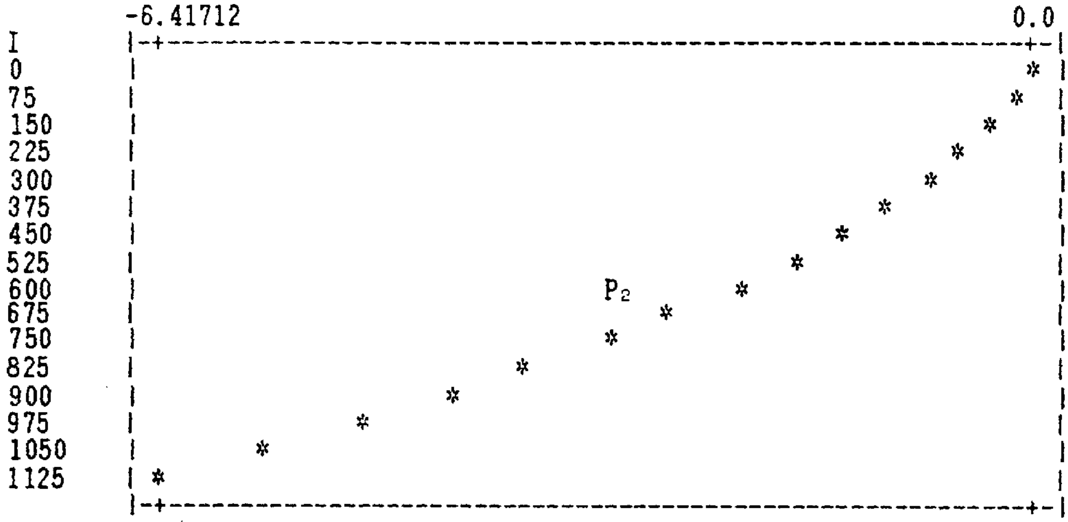
7.2 Income elasticity

We calculate the elasticity of probabilities with restpect to the change of the principal earner's income I. For Model 1, the formula is

$$(\partial P_2/\partial I)(I/P_2)=\phi(a_0+a_1 I)a_1 I/P_2<0 \tag{68}$$

The maximum likelihood estimates are used for the calculation. The calculated elasticity is given in Fig. 6. Households with higher income response greatly when the principal earner's income becomes large. The observed (weighted) average of the principal earner's income is 332.58. The elasticity at this income level is -0.87829. Therefore, 1% increase of I yields 0.88% reduction of the labor supply probability.

Fig. 6 The Income Elasticity of Labor Supply Probability;
A Case of One Potntial Earner



For Model 2, the formulae for calculating the income elasticity are

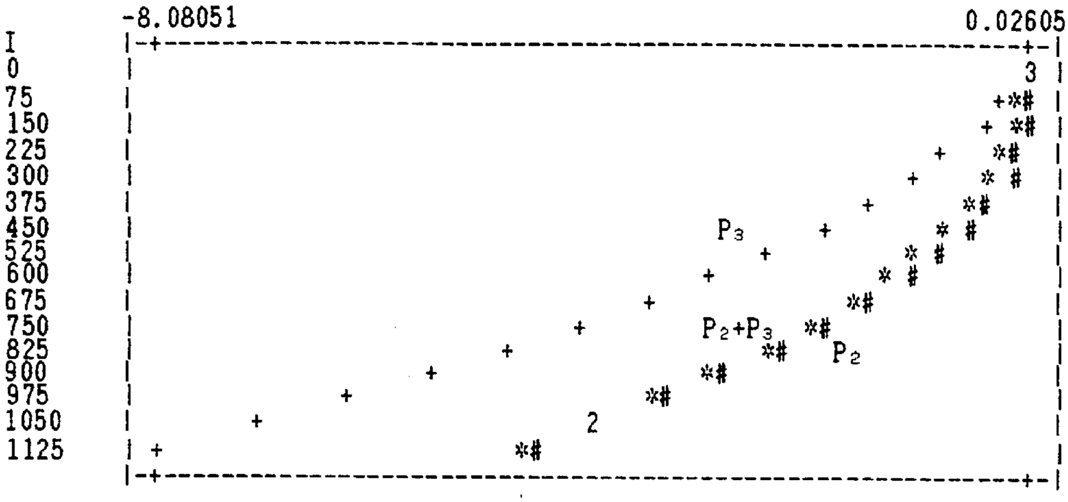
$$(\partial P_2/\partial I)(I/P_2)=\{\phi(b_0+b_1 I)b_1-\phi(c_0+c_1 I)c_1\}I/P_2=? \qquad (69)$$

$$(\partial P_3/\partial I)(I/P_3)=\phi(c_0+c_1 I)c_1 I/P_2<0$$

$$\{\partial(P_2+P_3)/\partial I\}\{I/(P_2+P_3)\}=\phi(b_0+b_1 I)b_1 I/(P_2+P_3)<0$$

The estimates in Table 3 <1-b) are used for the calculations. The calculated elasticities are illustrated in Fig. 7. We see that the elasticity of P_2 is negative over the range except for lower income classes and that it takes large negative values at higher income classes. The observed average income is 384.03. The elasticities of P_2+P_3, P_3 and P_2 at this income level are respectively -0.5582, -1.4917 and -0.3596. The probability of two-person participation responses greatly for households with two potential eaners. That is, this probability decreases most when the income increases.

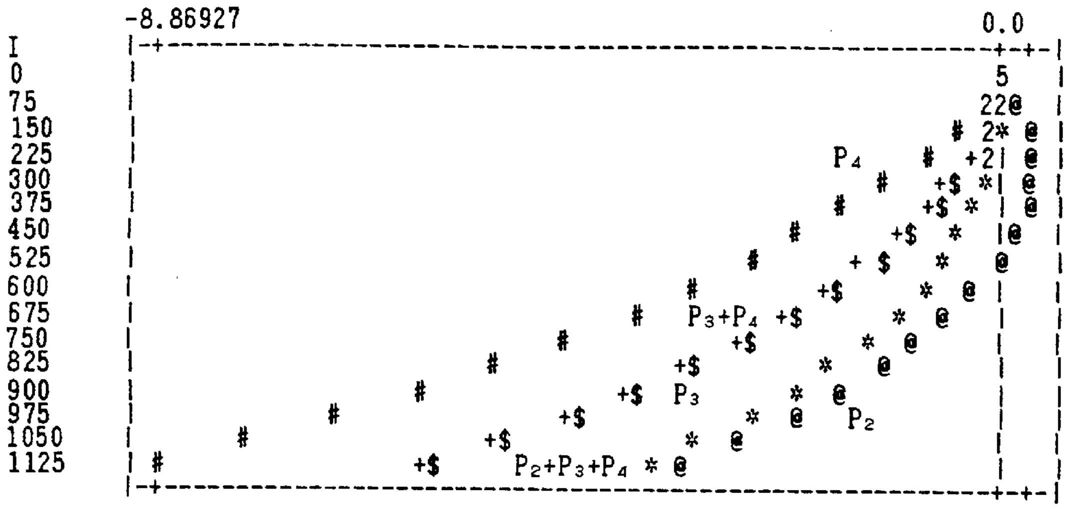
Fig. 7 The Income Elasticities of Labor Supply Probability; A Case of Two Potential Earners



For Model 3, we have similar formulae to calculate the income elasticities. The calculated values by using the parameters in Table 3 <3-b> are illustrated in Fig. 8. The elasticities of P_2 for income classes up to 525 are positive and their average is positive. That of P_3 is positive for the two lowest income classes. The observed average income is 401.08. The elasticities of $P_2+P_3+P_4$, P_3+P_4, P_4, P_2 and P_3 at this level are, respectively, -0.3485, -0.92388, -1.77698, +0.19910, and -0.73461. This shows that P_4 is most negatively elastic. If the income gets large, then P_4 decreases substantially and P_2 substitutes it.

Fig. 8 The Income Elasticites of Labor Supply Probability;

A Case of Three Potential Earners



7.3 Composition of household members

The number of labor force paticipants can be calculated since the models describe the labor supply in terms of "man". Let r_k be the number of households with one potentail earner, s_k be that of households with two potential earners and t_k be that of households with three potential earners in the k-th income class. Then the number of labor force participants S_k in the k-th class is

$$
\begin{aligned}
S_k = & r_k\Phi(a_0+a_1I_k) \\
& +s_k\{\Phi(b_0+b_1I_k)-\Phi(c_0+c_1I_k)\}+2s_k\Phi(c_0+c_1I_k) \\
& +t_k\{\Phi(d_0+d_1I_k)-\Phi(e_0+e_1I_k)\}+2t_k\{\Phi(e_0+e_1I_k)-\Phi(f_0+f_1I_k)\} \\
& +3t_k\Phi(f_0+f_1I_k)
\end{aligned}
\tag{70}
$$

The calculation is based on the estimates of the parameter a by maximum likelihood method, and those of b and c given in Table 3 <1-b>, and those of d, e, and f given in Table 3 <3-b>. A histogram of distribution of S_k is plotted with a sign '*' in Fig. 9. The observed number of households is 20650 and the observed number of potentila earners is 26275. The calculated total number of participants is 8239. The participation rate is 31.357%.

Fig. 9 The Effect of Change of Household Composition

```
        2.5(persons)                                         1396
k       |-+----------------------------------------------------+-|
1       | 2                                                      |
2       |    2                                                   |
3       |           + *                                          |
4       |                       +   *                            |
5       |                                      +    *            |
6       |                                           +    *       |
7       |                                           +      *     |
8       |                              +    *                    |
9       |                    +   *                               |
10      |              +   *                                     |
11      |              +   *                                     |
12      |       +*                                               |
13      |  +*                                                    |
14      |  2                                                     |
15      | 2                                                      |
(class) |-+----------------------------------------------------+-|
```

Then, we consider the effect that the change of household composition may cause to the labor supply. For argument purpose, let us assume that the size of a household gets smaller. For instance, let us assume that a half of households with two potential earners become households with one potential earner, and that a half of the households with three potential earners become those with two potential earners. The situation is described by letting r_k become $r_k+(s_k/2)$, s_k become $s_k+(t_k/2)$ and t_k become $(t_k/2)$. Other things like a distribution of principal earners' income are kept the same.

Making use of (70) and the same parameters set as above, we can get the number of household members who choose to be employed after the change of household composition. The number S_k is plotted in Fig. 9 with a sign "+". The number of labor force paticipants now is 7158 or a decrease by 1081. The number of households stays the same and is 20650. The total number of potential earners is now 24068. Therefore, the labor force participation rate is 29.74%, or 1.62 point decrease.

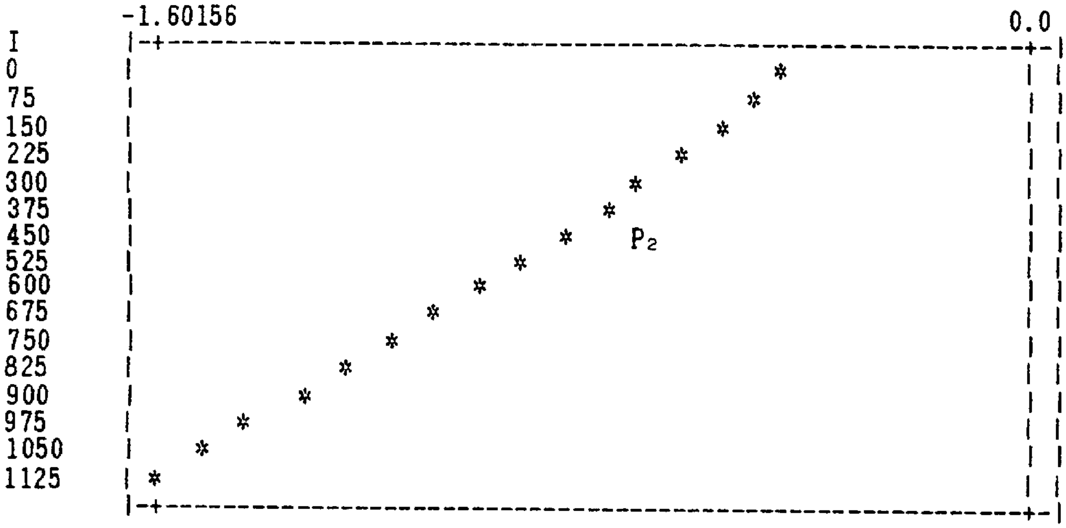
7.4 Work hours

Applying the least squares method to the regression equation (39) with the Bayesian estimates for the left hand side variables of it, we obtain the estimates of δ_1 and δ_2,

$$\tilde{\delta}_1 = .85323,\quad \tilde{\delta}_2 = -1.2039. \tag{71}$$

With these estimates, we can evaluate the effect of the change in working hours to the labor supply probabilities .

Fig. 10 The Work Hours Elasticity of Labor Supply Probability: A Case of One Potential Earner



For Model 1, we calculate the elasticity of the probability P_2 with respect to work hour change. We have

$$(\partial P_2/\partial h)(h/P_2) = \phi(a_0 + a_1 I)\delta_2/2P_2$$

$$= -.60195\,\phi(.0906319 - .0021439 I_k)/P_2 < 0 \tag{72}$$

This shows that if work hours get shorter then the labor supply probability of one potential earner household gets larger. The calculated values are given in Fig. 10. The elasiticity is -0.74148 at the average income level. The 1% decrease in work hours causes 0.74% increase of P_2.

For Model 2, we have

$$(\partial P_3/\partial h)(h/P_3)=\phi(c_0+c_1 I)\delta_2 3/2P_3<0$$

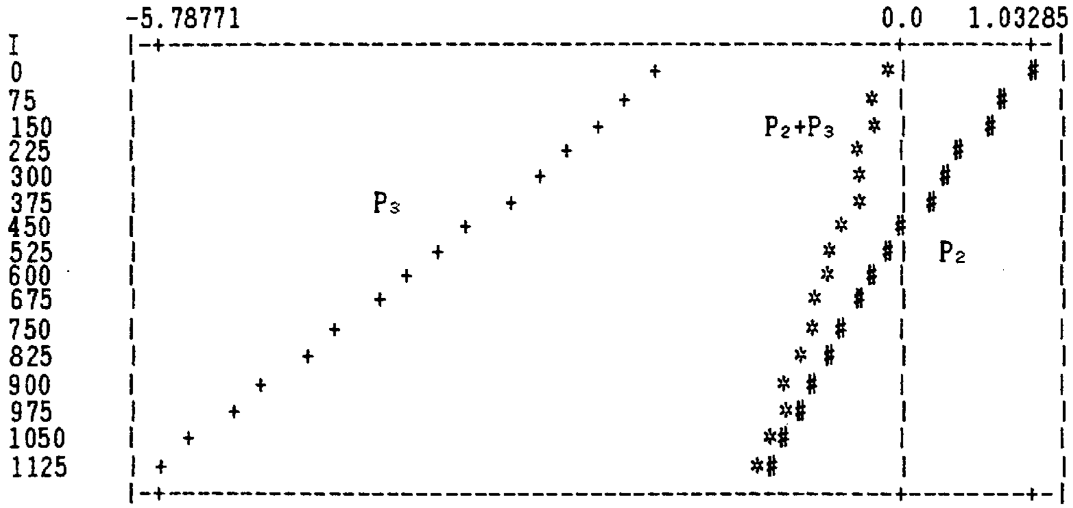
$$\{\partial(P_2+P_3)/\partial h\}\{h/(P_2+P_3)\}=\{\phi(b_0+b_1 I)\delta_2\}/2(P_2+P_3)<0$$

$$(\partial P_2/\partial h)(h/P_2)=\{\phi(b_0+b_1 I)\delta_2-3\phi(c_0+c_1 I)\delta_2\}/2P_2=? \quad (73)$$

for calculating the elasticities. The sign of the third elasticity is not uniquely determined. These values are given in Fig. 11. The elasticities at the average income level are -3.13001,-0.39038 and +0.19219. That of P_2 is positive at low and mid income classes. That of P_3 is negatively large, since the change in work hours effects two-hold to the two potential earners.

Fig. 11 The Work Hours Elasticity of Labor Supply Probabilities;

A Case of Two Potential Earners

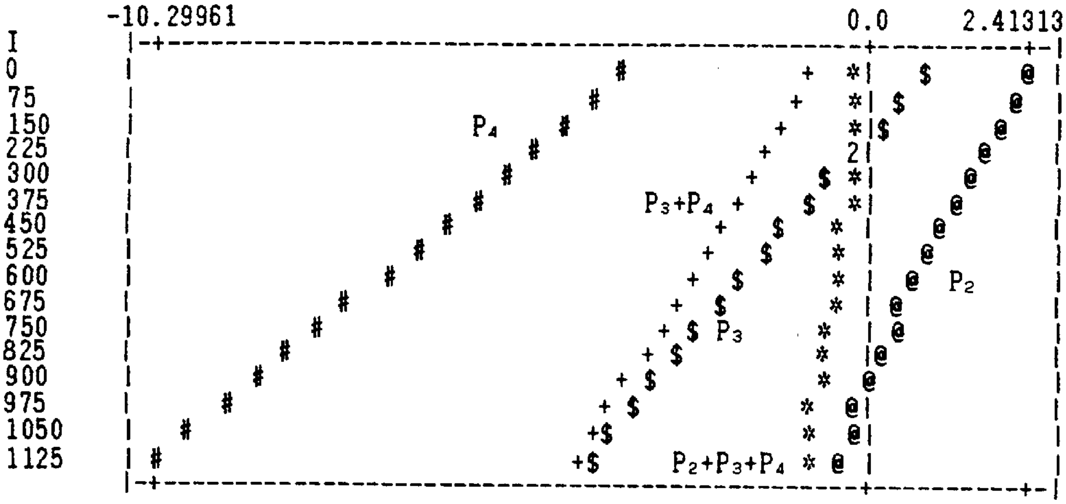


For Model 3, we have similar formulae, which provide the elasticities given in Fig. 12. At the lower income classes, the elasticities of P_2 and

P_3 are positive. The probabilities $P_2+P_3+P_4$, P_3+P_4 and P_4 have negative elasticities. The elasticities of $P_2+P_3+P_4$, P_3+P_4, P_4, P_2 and P_3 at the average income level are -0.22703, -1.80559, -5.78813, 1.27534 and -0.92202. P_4 is most elastic for the change in work hours, since the change of work hours affects three-hold to the three suppliers. It is noted that P_2 has a positive elasticity over the fairly wide range.

Fig. 12 The Work Hours Elasticity of Labor Supply Probabilities;

A Case of Three Potential Earners



We have a formula (70) to calculate S_k. We then calculate the total number of labor participants S by summing S_k. Multiplying S by work hours h, we have the total amount of labor sypply $\varkappa=Sh$ in terms of man-hours. By (70), we calculate the elasticity of total amount of labor supply with respect to work hours change. The reduction of work hours may have two effects to $\varkappa$. The one is, of course, a negative effect to h and the other may be a positive effect to the labor supply probabilities. Overall effect is calculated using the Bayesian estimates and is

$$(\partial \varkappa/\partial h)(h/\varkappa)=.21412. \tag{74}$$

That is, 1% decrease in work hours causes 0.214% decrease of the total labor supply.

We have indirectly estimated the parameter δ_2 to find the effect of work hour reduction by making use of the three models. It is not possible to estimate δ_2 only by using one of the models and cross section data. Generally a set of cross section data over some time periods is necessary to estimate δ_2.

7.5 Wage rate

We can not deal with effects of wage rate change without estimating the parameters γ in the utility function or without estimating marginal utility of income. For this matter too, we have to have sets of cross section data over many time points.

8. Concluding remark

We have estimated Model 1, Model 2 and Model 3 by maximum likelihood methods and tested the theoretical restrictions based on maximum likelihood estimates. The theoretical restrictions turned out to be significant. When we uesed Bayesian estimates to test the restrictions, they are not significant, although we did not use exact Bayesian methods of hypothesis testing with Bayes factors.

When classical statistical methods were used, we tested the theoretical restrictions without using prior information. On the other hand, when we used Bayesian methods, we utilized prior information about the parameters and got different conclusion. For instance, in estimating b_1 and c_1 and testing the equality, we utilized prior information about b_1 and c_1, which was obatained from prior estimation of a_1 with a larger set of samples. This might be a cause of the different conclusion.

By building the three models within a systematic or autonomous framework, we derived the theoretical restrictions for the parameters. This was the basis for using Bayesian procedure for formulating prior distributions. Or else, the prior distribution had no basis to be incorporated in Bayesian estimates. It should be noted that autonomous model building clarifies relations among parameters in a model and relations among parameters in different models. This kind of information is hoped to provide theoretical bases for using prior information by Bayesian procedures.

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