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Household's Labor Supply Function in Terms of Numerical Income-Leisure Preference Field

by

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This paper was presented in 1968 at the third Far Eastern meeting of the Econometric Society and distributed to participants: Although a portion of this paper was included in "An Analysis of Household Supply of Labor in Terms of Principal Earner's Critical Income." (1969, in Japanese, Rinkai - Kakushotoku bunpu ni yoru Kinro Kakei no Rodokyokyu no Bunseki , " Mita Gakkai Zasshi" 62 kan, 1 go), the full, original version was never published. For ease of reference, it is hereby reproduced in its entirety as KEO occasional paper E. No. 12.

"Household's Labor Supply Function in Terms of Numerical Income-Leisure Preference Field"(*)

Keiichiro Obi

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Paper for Presentation at the

Third Far Eastern Meeting of

the Econometric Society

The aim of this study is (1) to clarify the relationship between the participation rates (number of persons gainfully employed ÷ number of persons) and the labor supply schedule of individual households in terms of the income-leisure preference functions, and (2) to estimate the parameters of the preference function making use of the above-mentioned relation.

I. The Income-Leisure Preference Function of the Household

I-1. The Unit of the Labor Supply Behavior

According to P.H. Douglas' pioneering work, participation rates for men 25 through 60 years of ages have been found to be not significantly

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flexible in relation to changes in their own wage rates, while between younger and middle aged women's participation rates and the above mentioned men's wages, there exists a significant (inverse) correlation. The same has been confirmed by C.D. Long's comprehensive empirical study.

From their findings it can be seen that the individual's supply of labor is not independent but is jointly connected with those of other members of the household. It would then be appropriate to construct a collective preference function with respect to the household income and the individual's leisure. Consider a household with P persons. Let their utility indicator functions be

$$U_1(\Lambda_1, X),\ U_2(\Lambda_2, X),\ \ldots\ldots.,\ U_P(\Lambda_P, X),$$

where $\Lambda_i (i=1,2,\ldots,P)$ stands for leisure and X stands for household's income in constant prices.

The collective utility indicator function would be written as

1-1) $\omega = \omega_1[\,U_1(\Lambda_1, X),\ \ldots\ldots.,\ U_P(\Lambda_P, X)\,]$

$= \omega_2[\Lambda_1,\ \ldots\ldots.,\ \Lambda_P, X]$.

Letting T be the individual's total number of hours in the defined period (day, month etc.) and denoting quantity of labor supplied by the i individual by hi, there exists the identity,

1-2) $\Lambda_i \equiv T - hi \qquad (i = 1, 2, \ldots.., P).$

Substituting 1-2) for 1-1) we obtain

1-3) $\omega = \omega_2[\,(T - h_1),\ (T - h_2),\ \ldots..,\ (T - h_P),\ X\,]$.

Again from Douglas' and Long's finding it will be conceived that in each household there exists a person on whose income the other members' labor supply behavior depend. We call this member of the household "the principal earner". The members of the household other than principal earner, except for children, we call "the potential earners". Household income, x, can be partitioned into the principal earner's income and potential earners' income (if any). As to the mechanism of the determination of the latter, the former can be treated as an exogenous variable.

In this paper only the simplest type of the "household", consisting of a gainfully employed husband, a wife, and an unspecified number of children of ages under 15 years old is treated. We call the household of this kind "Type A" for the sake of brevity. For the household of Type A with S children we have

$$P = S + 2 \ .$$

The hours of work for the children are institutionally restricted to zero, namely

1-4) $h_t = 0 \qquad (t = P,\ P-1,\ \ldots\ldots,\ P-S+1).$

Letting the first and the second members be a husband and a wife respectively, we obtain from 1-3) and 1-4)

1-5) $\omega = \omega_3 [(T - h_1),\ (T - h_2),\ X] \ .$

It will be appropriate to regard wife's supply of labor is dependent upon husband's income. This means that husband and wife are identified as a principal earner and a potential earner respectively. Denoting husband's earning by I, we have

1-6) $W_1 h_1 \equiv I,$

where W_1, the husband's wage rate, is an exogenous variable. The hours of work of household members' gainfully employed seem to be variable, being adjusted to their wage rate. But in reality institutional factors prevent them from working hours which are far different from what is assigned by the firm. At least as a first approximation, we could make h equal to $\bar{h}$, a constant, which stands for the assigned hours of the employer. Doing so, we obtain from 1-5) and 1-6), 1-7) $\omega = \omega_3 [(T - \bar{h}_1), (T - \bar{h}_2), X]$,

$X \equiv I + wh_2,$

where I is an exogenous variable and h_2 equals $\bar{h}_2$ or zero in accordance with the wife gainfully employed or not.

Now, $T - h_1$ being an institutionally given constant, the collective utility indicator function of the household is written as

1-8) $\omega = \omega_4 [(T - h), X]$, $\quad X \equiv I + wh$

where h stands for the wife's hours of work. (We drop the subscript of h for the sake of brevity.) For households of type A, therefore, the utility indicator is fully described by a function of the wife's leisure hours T - h, and the household's income X.

I-2. The Participation Rate and the Hours of Work

In most studies of the household's supply of labor, family budget surveys have been used. In this study too, we employ the budget survey of Japan, in which the husband's and wife's income (if any) are recorded separately. As in the U.S. and other countries, the wife's working hours (or weeks) are not recorded, and the only information on labor supplied is whether the wife was gainfully employed or not. Thus the directly observed quantity of labor supplied by the wives is in terms of persons.

In order to estimate a wife's supply of labor in terms of working hour making use of data of this kind, some assumptions must be made. One of the alternative assumptions is that the wife's labor, supplied in terms of man-hours during her life time, is proportional to her participation rate defined as the number of wives gainfully employed divided by number of wives in a given group of households. The interpretation of wife's participation rate given by J. Mincer also depends on an assumption of this kind.*

Another assumption is that the labor supplied in terms of man hour is approximately proportional to the ratio of the wife's earning to her husband's full time earnings. (Rosett).

These assumptions have been necessary to interpret the supply functions, estimated by making use of data in which the wife's working hours are not surveyed, as describing the wife's supply behavior in terms of man-hours over her life time or in the particular period. If we are interested in the wife's supply behavior in terms of participation only, it would suffice to assume a specific income-leisure preference field in which each contour kinks at some point under the specified wage rate, and thus causes the wife not to work.

Yet historical data show that the direction of change in the labor supplied by individual woman in terms of working hours differs from that of the number of women in the labor force. In other words, in the course of economic development, a reduction in the length of the work day for women and, at the same time, an augmentation of their participation rate have been observed in Japan as well as in other countries.

In order to deal with those divergent movements, we have constructed a model which could describe changes in the number of women entering the labor force and changes in working hours as well. This means that we need to

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* Mincer interpreted the wives' participation rate as an indicator of the wife's labor supplied during her life time: The ratio of the wife's hours of work to her life time could be regarded as the probability of being employed at each time point. If so, the participation rate of a group of wives, which is the probability of being gainfully employed for a wife, is an indicator of the quantity of labor supplied in terms of man hours. However, in order that this interpretation be plausible, the time distribution of the wife's hours of work during her life time must be random. The adequacy of this interpretation still seems to be an open question.

inquire into the mechanism by which the number of workers are determined in terms of the income-leisure preference function.

II. A Model for Labor Force Participation in Terms of Income-Leisure Preference Functions

2-1. Optimal Hours of Work

Let us consider a household of Type A. We have here one principal earner (husband) and one potential earner (wife).

Let the income-leisure preference function of the household be, as shown by (1- δ),

2-1) $\omega = \omega(X, \Lambda)$

where,

2-2) $\Lambda = T - h,$

h being the potential earner's supply of labor.

Household income is defined by

2-3) $X \equiv I + wh$

where W, the potential earner's wage rate and I, the principal earner's income, are given.

Substituting 2-2) and 2-3) into 2-1), we can obtain the values of h maximizing ω by solving the equation,

2-4) $\frac{d\omega}{dh} = 0.$

This system is shown in Fig. 1. Leisure, Λ , and the household's total income, X, are scaled on the vertical axis and the abcissa respectively. Total disposable hours for the potential earner, T, depends on the time interval on which X, Λ, and I are defined; e.g. if these variables are defined by the time rate of 24 hours, T is 24.

Let I_1 be the value of principal earner's income, and let $\tan\theta$ be the potential earner's wage rate as given by the employer. The potential earner's optimal working hours and the household's total income are shown by the coordinate of P^*, which is the tangency point of the income-leisure contour ω, and the income line AB. Obviously, h^* is the solution of equation 2-4), and it could be written as:

2-5) $h^* = h^*(I, W, \alpha_1, \ldots\ldots, \alpha_n)$

$\alpha_i (i = 1, \ldots\ldots n)$ being parameters of the preference function 2-1). The value of h^* varies as I, W, and parameters of the preference function $\alpha_1 \ldots \alpha_n$ change. Equation (2-5), corresponding to the locus of P^* in figure 1, is the supply schedule of the potential earner. The schedule is also, because of the specification of the household, the supply schedule of household itself.

Now, if the supplier were able to determine working hours in accordance with his supply schedule, he would work exactly h^* hours under the wage rate assigned by the employer. However in reality, workers have to accept the institutionally assigned normal working hours $\bar{h}$, in order to be employed. These normal working hours ($\bar{h}$) need not be equal to the optimal hours (h^*).

2-2. The Principal Earner's Critical Income

In this section, we discuss the range of the principal earner's income over which the potential earner accepts work under the condition that both the wage rate and the working hours $\bar{w}$ and $\bar{h}$ are assigned by the employer.

Let the principal earner's income be I_2, which is higher than I_1, in Fig. 1. Suppose that the wage rate and the assigned working hour are $\tan\theta$ ($= \bar{w}$) and $T\bar{h}$ ($= \bar{h}$) respectively. If the potential earner were to accept this work, the household's position with regard to income and leisure would be shown by point H. At this point the household is obviously worse off than that point E where the potential earner does not work at all and the household's total income is equal to the principal earner's income, I_2. Hence, so long as the principal earner's income is higher than I_1, the potential earner does not accept employment in which the wage rate and the hours assigned by employer are $\tan\theta$ and $T\bar{h}$ respectively. In the same manner, it can be shown that the household is better off if it accepts work under these conditions if the principal earner's income is less than I_1. When the principal earner's income is exactly I_1, the household is indifferent to the choice between acceptance and rejection of that job.

Let us call the principal earner's income I_1 the critical level of principal earner's income with regard to that specified employment opportunity, or in short, the principal earner's critical income (PECI).

As can be seen from Fig. 1, the principal earner's critical income varies with changes in the assigned working hours $\bar{h}$, the wage rate, $\bar{w}$, and the shape of the contour of the preference map. For instance, if the assigned working hours were less than what is shown by point G, the potential earner would work because the household would be better off. Consequently, denoting the principal earner's critical income by I*, we have

2-6) $I^* = I^* (\bar{w}, \bar{h}, \alpha_1, \alpha_2, \ldots\ldots, \alpha_n).$

2-3. Size distribution of the principal earner's critical income

Let us consider a group of m households of type A in which the principal earner's income and the assigned working hours open to each household's potential earner are the same. Were we able to single out, among m households, $m'(\leq m)$ households whose preference among income and leisure are exactly the same, it is obvious from equation 2-6), that their principal earner's critical income, $I_j^*(i = 1, \ldots\ldots, m')$ must be equal (a constant I_1^*). Since it is difficult to identify households whose preference functions are exactly the same, there is a need to introduce the probability density distribution of the critical income (PECI),

2-7) $g(I^* | \bar{h}, \bar{w}, \bar{\alpha}_1, \bar{\alpha}_2, \ldots\ldots, \bar{\alpha}_n)$

where $\bar{\alpha}_i$'s$(i = 1, \ldots\ldots, n)$ are the means of n preference parameters distributed among the households considered. The functional form of the probability distribution of preference parameters α_{ij}'s$(j = 1, \ldots\ldots, m)$ among the households.

2-4. Participation functions in terms of income-leisure preference fields

For the given wage rate, $\bar{w}$, and assigned working hour, $\bar{h}$, the distribution of I^* is uniquely determined. Suppose we have K groups of households where within each group the principal earners' income, I_k, wage rate, W_k, and assigned working hours, $\bar{h}_k$, are the same. Let the number of households in each group $(k = 1, 2, \ldots\ldots, K)$ be N_k.

The characteristics of the distribution function, g, are supposed to be common to the K groups. For the kth group the density distribution of I^* and the level of principal earner's income, I_k, is shown in Fig. 2. Area B stands for the probability of $I^* > I_k$ holding for any one household. In the household where $I_k < I^*$ the potential earner is gainfully employed. For this group the number of households in which one potential earner is gainfully employed is equal to N_k multiplied by the value of the probability designated by the area B, and the probability designated by area B is the participation rate, μ_k, of the households in the kth group. That is, participation rate is shown by the equation, 2-8) $\mu_k = 1 - \int_a^{I_k} g(I^* | \bar{W}_k, \bar{h}_k, \bar{\alpha}_1, \ldots\ldots, \bar{\alpha}_n) dI^*$

where a is the lower limit of the integration.

Thus, from the participation equation (2-8), the participation rate μ_k of the kth group of households depends on the principal earner's income I_k, the wage rate $\bar{W}_k$ and assigned working hours $\bar{h}_k$ which is shown in (2-9).

2-9) $\mu_k = G(I_k, \bar{W}_k, \bar{h}_k, \bar{\alpha}_1, \bar{\alpha}_2, \ldots\ldots, \bar{\alpha}_n)$.

This function can be exactly interpreted as the supply function in terms of the participation rate. Multiplying μ_k by N_k yields the supply function in terms of persons.

If the variation in assigned working hours among job opportunities is negligible for all households, we can place h_k equal to $\bar{h}$, a constant, which is common to all opportunities. Then the equation (2-8) reduces to,

2-10) $\mu_k = 1 - \int_a^{I_k} g(I^* | \bar{W}_k, \bar{h}, \bar{\alpha}_1, \ldots\ldots, \bar{\alpha}_n)\ dI^*$.

Further, if the variance of $\bar{W}_k$ is so small that we can appropriately put $\bar{W}_k$'s equal to $\bar{w}$, a constant, we obtain

2-11) $\mu_k = 1 - \int_a^{I_k} g(I^* | \bar{w}, \bar{h}, \bar{\alpha}_1, \ldots\ldots, \bar{\alpha}_n) \cdot dI^*$.

or

2-12) $\mu_k = F(I_k, \bar{w}, \bar{h}, \bar{\alpha}_1, \bar{\alpha}_2, \ldots\ldots, \bar{\alpha}_n)$

The very core of the problem is to reduce the analytical form of the density function of principal earner's critical income, I*, in terms of income-leisure preference parameters. This is dealt with in the following.

III. Determination of Preference Parameters

Making use of (2-11) we will estimate the preference parameters, $\bar{\alpha}$'s, by means of simulation technique.

3-1 The Data

Data are from the family income and expenditure survey (FIES) conducted by the Bureau of Statistics, Office of the Prime Minister. Out of all surveyed households, wage earners' households of type A were selected for analysis.

We used selected data for the four years 1961 through 1964. Since 1962 the sample size of the survey has been doubled to about 8000 households.

The number of households of type A has been about 1800 for each year since 1962. Individual household income, expenditure and household member's status with regard to work are recorded for six months. However, all the values used in this study are expressed in monthly rates.

3-2. Preliminary observation

In order to estimate the preference parameters in the equation 2-12) by means of simulation technique, we have to have the approximations of the α_i's. It is the purpose of this section to obtain them.

In the first place, following regression equation was fitted to the data.

$$2\text{-}13)\quad h'_j = A_o + A_1 I_j + A_2 N_{cj} + V_j,$$

where $h'_j = 1$ or 0 in accordance with the jth wife was gainfully employed or not. I_j and N_{cj} stand for the principal earner's income (in constant prices) and the numbers of children under 6 years old respectively. V_j is a shock variable which follows a binomial distribution function with mean zero.

The estimated parameters of the equation 2-13) are shown in Tab. 1. Two points should be made concerning the result. (1) The estimates of the regression coefficients are statistically significant. (2) The estimates of A_o, A_1 and A_2 significantly vary year by year.

There might be two alternative interpretation with respect to the theoretical counterpart of the regression equation, 2-13). One is to identify the regression as an approximation for the function 2-12), the participation equation, and the other is to regard it as an approximation for the equation 2-5), the potential earner's supply function in terms of the hours of work. The latter interpretation seems to be better than the former, because the participation function, 2-12) was reduced by integrating the function $\mathcal{J}(I^*)$ as was shown in the equation 2-10), and the equation 2-12) thus obtained hardly be a linear function in the principal earner's income, I.

The labor supply function in terms of working hours consistent with the regression, 2-13), can be reduced from the income-leisure preference function of quardratic form. Here, we have two alternative hypotheses with respect to the quadratic preference function; that is, the classical "invariable preference function hypothesis" and the "habit formation hypothesis".

3-2-1. The invariable preference function hypothesis

By this hypothesis we mean invariable preference parameters over time. Let the preference function be

$$2\text{-}14)\quad \omega = -\frac{1}{2}\gamma_1 X^2 + (\gamma_2 + m_2 N_c + U_x)X + \gamma_3 X\Lambda + (\gamma_4 + n_2 N_c + U_\Lambda)\Lambda + \frac{1}{2}\gamma_5 \Lambda^2$$

where, γ_i's$(i = 1,2, \ldots, 5)$ are the preference constants, and m_2 and n_2 respectively stand for the shift of the intercept of marginal utility function of income and leisure, caused from the changes in the number of the children, N_c. U_x and U_Λ are the random variables which affect the difference in tastes among the households.

From 2-14), marginal utility functions with respect to income and leisure,

2-15) $\frac{\partial \omega}{\partial X} = \gamma_1 X + \gamma_2 + m_2 N_c + \gamma_3 \Lambda + U_x$

2-16) $\frac{\partial \omega}{\partial \Lambda} = \gamma_5 \Lambda + \gamma_4 + n_2 N_c + \gamma_3 X + U_\Lambda$,

are obtained.

Applying the condition of maximizing the ω, 2-4), to 2-14) and by using 2-2) and 2-3), we get the labor supply function in terms of the working hours for the jth household,

$$\text{2-18)} \quad h_j = \frac{(\gamma_5 - \gamma_3 W)T + (\gamma_4 - \gamma_2 W)}{\Omega} + \frac{\gamma_3 - \gamma_1 W}{\Omega} I_j + \frac{n_2 - m_2 W}{\Omega} N_{cj} + \frac{U_{\Lambda j} - W U_{xj}}{\Omega}$$

where,

2-19) $\Omega \equiv \gamma_1 W^2 - 2\gamma_3 W + \gamma_5$.

To begin with the simpler case, we put the $U_{\Lambda j}$ equale to zero. This means that we presume the differences in taste among the households can be fully described by the random shift in the intercept of the marginal utility curve of income. Furthermore, let U_{xj} be the random variable which follows a binomial distribution with mean zero. Hence, h_j can also be regarded as the random variable following binomial distribution (with mean,

$$\frac{(\gamma_5 - \gamma_3 W) T + \gamma_4 - \gamma_2 W}{\Omega} + \frac{\gamma_3 - \gamma_1 W}{\Omega} I_j + \frac{n_2 - m_2 W}{\Omega} N_{cj} \text{).}$$

Let the values taken by h_j be $\bar{h}$ and 0*. By deviding both sides of the

* This means that the shifts of contours of the indefference maps among the households are such that the tangency points of the contour and the income line in each household are restricted to the two points whose vertical coordinates are zero and $\bar{h}$.

equation 2-18) by $\bar{h}$ we have

$$\text{2-20)}\quad h_j' = \frac{(\gamma_5-\gamma_3)NT + (\gamma_4-\gamma_2W)}{\Omega\bar{h}} + \frac{\gamma_3-\gamma_1W}{\Omega\bar{h}} \cdot I_j + \frac{n_2-m_2W}{\Omega\bar{h}} \cdot N_{cj} + \frac{WU_{xj}}{\Omega\bar{h}},$$

where,

$$\text{2-21)}\quad h_j' \equiv \frac{h_j}{\bar{h}} = 1 \text{ or } 0.$$

Comparing 2-20) with 2-13) we have

$$\text{2-22)}\quad (\gamma_5-\gamma_3W)T + (\gamma_4-\gamma_2W) = A_o\Omega\bar{h}$$

$$\text{2-23)}\quad \gamma_3 - \gamma_1W = A_1\Omega\bar{h}$$

$$\text{2-24)}\quad n_2 - m_2W = A_2\Omega\bar{h}.$$

Inserting 2-19) into these relations, we obtain

$$\text{2-25)}\quad \bar{h}W^2A_o\gamma_1 - W(2hA_o- T)\gamma_3 + (hA_o- T)\gamma_5 - \gamma_4 + W\gamma_2 = 0$$

$$\text{2-26)}\quad W(\bar{h}WA_1+1)\gamma_1 - (2\bar{h}WA_1+1)\gamma_3 + \bar{h}A_1\gamma_5 = 0$$

$$\text{2-27)}\quad \bar{h}W^2A_2\gamma_1 - (2\bar{h}WA_2)\gamma_3 + \bar{h}A_2\gamma_5 - n_2 + Wm_2 = 0.$$

As far as "the invariable preference hypothesis" is concerned the parameters of the preference function ω, $\gamma_P(P = 1, \ldots, 5)$, m_2 and n_2 are assumed to be constant over time. In the above equations, 2-25) through 2-27), A_o, A_1 and A_2 change year by year as shown in Tab. 1, and so does W, the wive's average wage rate.(*) Institutionally determined hours of work, $\bar{h}$, will be admittedly considered to be constant over time. Taking into account of prevailing 8 hours of work a day, fairly good approximation of $\bar{h}$ will be $\frac{1}{3}$, as we put T, total desposable time, equal to unity.

It will, therefore, be possible to determine γ_1, γ_5, γ_3, γ_2 and γ_4 from

(*) See Tab. 2.

2-25) making use of the observations on the A_0 and W for four years, 1961 through 1964, because one of the parameters can be taken as unity on account of the normilization rule. In the same manner equation 2-26) also determines r_1, r_3 and r_5 (one of which is normalized) making use of the data for two years at least.

The same for the equation 2-27). So long as the above hypothesis be plausible, three sets of the parameters (r_1, r_3 and r_5) independently determined from 2-25) through 2-27) have to coincide with each other. This provides a test for our hypothesis. We calculated the parameters making use of 2-27) which are shown in Tab. 3. As far as the hypothesis be sustainable, these estimates must fulfill 2-26) as well. By inserting the estimates of r_1, r_3, r_5 and W in Table 3, we calculated the value of A_1 for the year 1961. The value of A_1 thus obtained was - 0.87, which is far different from its actual value in 1961, -0.0005039. Thus, the invariable preference hypothesis turned out to be falsified as long as the form of ω, (2-14) is concerned.

3-2-2. Habit formation hypothesis

The hypothesis of invariable preference field seems to be dubious so far. As long as the quadratic preference function, 2-14), which will have general applicability in the sense that various ordinary functions can be reduced to a quadratic form by the Taylor expansion, is concerned, discrepancy between the directly estimated value of A_1 and that of indirectly calculated must be accounted for by introducing the alternative hypothesis.

In fact, it is shown that the above mentioned discrepancy is explainable if we take into account the possible shifts in the intercept of the marginal utility curve of income and/or leisure.

Let us suppose that the magnitudes of the intercepts of the marginal utility curves of income and leisure shift over time on account of the habit formation effect (on the preference among income and leisure). There will be correlation between the intercepts of the marginal utility curves and the scale of the income earned in the past for the households. The income earned in the past will correlate with the present position of the income (except for the household whose principal earner is unemployed). Let the relation between the present level of the principal earner's income and the magnitudes of the intercepts of income and leisure be

$$2\text{-}28)\quad r_2 = r_2^{\circ} + m_1 I$$

$$2\text{-}29)\quad r_4 = r_4^{\circ} + n_1 I$$

Inserting these relations to 2-18), we obtain

$$2\text{-}30)\quad h_j = \frac{r_3 - (r_1 + m_1)W + n_1}{\bar{h}\Omega} I_j + \frac{n_2 - Wm_2}{\bar{h}\Omega} N_{cj} + \frac{NT(r_5 - \gamma_3 W) + r_4^{\circ} + r_2^{\circ} W}{\bar{h}\Omega} + \frac{U_I - WU_X}{\bar{h}\Omega}.$$

By comparing this relation with the regression 2-13), we have

$$2\text{-}31)\quad (\bar{h}W^2 A_o) r_1 - W(2\bar{h}A_o - T) r_3 + (\bar{h}A_o - T) r_5 - r_4^{\circ} + W r_2^{\circ} = 0$$

$$2\text{-}32)\quad W(\bar{h}WA_1 + 1) r_1 - (2\bar{h}WA_1 + 1) r_3 + \bar{h}A_1 r_5 + m_1 W - n_1 = 0$$

$$2\text{-}33)\quad (hW^2 A_2) r_1 - (2\bar{h}WA_2) r_3 + \bar{h}A_2 r_5 - n_2 + Wm_2 = 0$$

These relations are reduced to 2-25) through 2-27) when the shift coefficients are deleted. It can be seen from 2-33) that the equation 2-27) is invariable under the introduction of shift factors. We are, then, able to utilize the estimates r_1, r_3 and r_5 again in Tab. 3. On the other hand m_1

and n_1 can not be determined directly making use of 2-32), for these vary over time. However, this time, the discrepancy between the calculated and the observed value of A_1 which was found in the previous section is accounted for by the existence of shift coefficients, m_1 and n_1 in the equation 2-32).

To determine m_1 and n_1 we use simulation technique in the following sections, where the values of r_1, r_3 and r_5 already obtained are used as a first approximation.

3-3. Numerical determination of Preference Parameters by means of a S imulation technique

3-3-1. Derivation of the Principal earner's critical income in terms of Preference Parameters

In this section we convert the equation of the principal earner's critical income 2-6), into a concrete analytical form making use of the preference function

$$3\text{-}1)\quad \omega = \frac{1}{2} r_1 X^2 + (r_2 + m_2 N_c)X + r_3 X\Lambda + (r_4 + n_2 N_c)\Lambda + \frac{1}{2} r_5 \Lambda^2,$$

where,

$$3\text{-}2)\quad r_2 = r_2^* U_x + m_1 I \quad ; \quad E(U_x) = 1,$$

and

$$3\text{-}3)\quad r_4 = r_4^* + n_1 I.$$

Here, r_1, r_3 and r_5 are the preference constants and r_2^*, m_1 , r_4^* and n_1 are the parameters.

The relation 3-2) means that the magnitude of intercept of the marginal utility curve of the household income, r_2, depends on the level of the principal earner's income, I, and the random variable with the mean unity, U_x, which is supposed to affect the cross sectional difference in the "taste" among households. The relation 3-3) means that the intercept of the marginal utility curve of leisure varies in accordance with the cross sectional difference in the principal earner's income among households.

In the first place we will obtain an equation of the contour passing through the point A in Fig. 1. At the point A we have

3-4) $\Lambda = TN$

3-5) $X = I$

by applying $h = 0$ to 2-2) and 2-3). Inserting 3-4) and 3-5) into 3-1) we get

$$3\text{-}6)\quad \omega_o = \frac{1}{2} \cdot r_1 I^2 + (r_2 + m_2 N_c)I + r_3 I \cdot TN + (r_4 + n_2 N_c)T + \frac{1}{2} r_5 (T\ )^2$$

To obtain the equation of the contour passing through the point C in Fig. 1, we put h equal to $\bar{h}$ in 2-2) and 2-3), and insert these relations to 3-1); that is, we have

$$3\text{-}7)\quad \omega_o' = \frac{1}{2} \cdot r_1 (I+w\bar{h})^2 + (r_2+m_2N_c)(I+w\bar{h}) + r_3(I+w\bar{h})(T-\bar{h}) + (r_4+n_2N_c)(T-\bar{h}) + \frac{1}{2} r_5 (T-\bar{h})^2$$

By the postulate that the I in 3-6) and 3-7) be the principal earner's critical income, I^*, we have

3-8) $\omega_o = \omega_o'$.

From this, we get

$$3\text{-}9)\quad I^* = \frac{(r_4-r_2W) + (n_2-m_2W)N_c - r_3(T-\bar{h})W + r_5(T-\frac{1}{2}\bar{h}) - \frac{1}{2} r_1 W^2 \bar{h}}{r_1 W - r_3} .$$

Inserting r_4 and r_2 in 3-2) and 3-3) respectively into this equation we have

$$3\text{-}10)\quad I^* = \{r_4^* - (r_2^* U_x)W + (n_1 - m_1 W)I + (n_2 - m_2)N_c - r_3 W(T - \bar{h}) + r_5(T - \tfrac{1}{2}\bar{h}) - \tfrac{1}{2} r_1 W^2 \bar{h}\} / (r_1 W - r_3).$$

This equation corresponding to 2-6) is the principal earner's critical income of the household whose intercept of the marginal utility curve of income is equal to $r_2^* U_x + m_1 I$ (I being the present level of principal earner's income).

It is seen from the equation 3-10) that I^* is obtained by the linear transformation of U_x, the random variable, for a given set of values of w, $\bar{h}$, N_c, r_4^*, r_2^*, r_3, r_5, n_1 and m_1.

Let the (density) distribution function of U_x, $f(u_x)$, be a normal distribution in logarithmic scale with the mean and the variance 0 and σ^2 respectively. From the equation 3-10) we have

$$3\text{-}11)\quad U_x = \{ - I^*(r_1 W - r_3) + r_4^* + (n_1 - m_1 W)I + (n_2 - m_2)N_c - r_3 W(T - \bar{h}) + r_5(T - \tfrac{1}{2}\bar{h}) - \tfrac{1}{2} r_1 W^2 \bar{h}\} / r_2^* W$$
$$= K(I^* | I, W, \bar{h}, N_c).$$

Inserting the function K into the distribution function f, the probability density function with respect to U_x, is expressed in I^* that is,

$$3\text{-}12)\quad f[K(I^* | I, W, \bar{h}, N_c)\sigma^2], \text{ or } g_f(I^* | I, W, \bar{h}, N_c).$$

Hence we can get the distribution function of I^*,

$$3\text{-}13)\quad \int_0^{u_x} f(U_x)dU_x = \int_a^I g_f(I^* | I, W, \bar{h}, N_c) \left| \frac{dK}{dI^*} \right| dI^*,$$

where, α is a lower limit of the integration.

Suppose the kth group of households in which I and N_c are approximately the same among the households, that is, $I_j \simeq I_k$, $N_{cj} \simeq N_{ck}$ (where j stands for the jth household in the kth group of households). Furthermore, let $\bar{w}$ and $\bar{h}$ be approximately the same for all the groups of households considered. Applying 3-13) to 2-11) we have the theoretical values of μ for the kth group, μ_k^*. Let the observed value of the participation rate of the kth group be μ_k^o, and define d by

$$3\text{-}14) \quad d = \sum_k (\mu_k^o - \mu_k^*)^2 ,$$

where d depends on the values of preference constants and the shift parameters for the given set of values of I_k and N_{ck}. That is,

$$3\text{-}15) \quad d = d\,[\, I_k,\, N_{ck} \mid r_1,\, r_3,\, r_5,\, r_2^*,\quad r_4^*,\, n_1,\, m_1,\, n_2,\, m_2, \sigma^2]$$

d's in the equation 3-15) were computed for the various set of values of the parameters, r_2^*, r_4^*, σ^2, n_1 and m_1. The ranges of the values assigned to the parameters are shown in Tab. 4.(*)

The values assigned to r_3, r_5, m_2 and n_2 are those listed in Tab. 3. The value of $\bar{h}$ is fixed at 1/3. Doing so, 7700 (=5 x 11 x 5 x 4 x 7) sets of the parameters were obtained for each year, 1961 through 1964. Out of 30800(= 7700 x 4) sets, 81 sets of the parameters for which $d < 0.1$ was held were selected. Of these 81 sets of the parameters, the best and the second best sets for each year are shown in Tab. 5., where the parameters are normalized such that $r_1 \equiv -1$.

It can be seen that the parameters except for m_1 and n_1 are fairly stable during the four years, m_1 and n_1 are the parameters which stand for the differences in the intercepts of the marginal utility curve of income and leisure respectively between the groups of households. It is observed that out of eight m_1's six are positive. Positive values of m_1 means that the intercept of the marginal utility curve of income in the higher (principal earner's) income group is larger than that of the lower income group. Accordingly positive m_1 and n_1 is consistent with the habit formation hypothesis.(**) However, higher precision in the parameters is required to conclude on this point.(***)

(*) The wider ranges were tried in the preliminary computations. Out of these results, the ranges of parameters which made μ negative were deleted. Further, in order to make the ranges narrower, we made use of 2-32) for m_1 and n_1. In the same manner 2-31) were used for r_4^* and r_2^*.

(**) As to the habit formation hypothesis applied to the consumer behavior, see (Houthacker and Taylor) and (Tsujimura and Sato) in the references.

(***) Especially, the technique which enables d not to attain its local minimum is needed.

Conclusions

As far as the present quadratic preference function and the data employed in this study are concerned the following conclusions were obtained.

1) The invariable preference field-hypothesis is not consistent with the observed data.
2) A difference seems to exist in the intercept of the marginal utility curve of income among groups of households (classified by principal earner's income). The same can be said with respect to the intercept of the marginal utility curve of leisure. By examining the sign of m_1 and n_1, it could be said that the results were favorable to the habit formation hypothesis.
3) In order to get the exact numerical labor supply function in terms of working hours, a technique which enables to obtain more precise estimates for the parameters are needed. Consequently, the above conclusions are of an intermediate nature.

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Tab. 1

	1961	1962	1963	1964
A_2	-0.07102 (5.2)	-0.06204 (5.5)	-0.08407 (7.8)	-0.09344 (8.0)
A_1	-0.0005039 (5.4)	-0.0007444 (8.7)	-0.0009271 (10.4)	-0.0004438 (6.7)
A_o	0.2576 (9.7)	0.3406 (14.5)	0.4155 (16.6)	0.3239 (14.9)
$\bar{R}$	0.2501	0.2443	0.2965	0.2481
d.f.	806	1556	1643	1534

The Values in the parentheses are t-statistics.

Tab. 2 The values* wh

1961	12.8**
1962	13.9
1963	14.7
1964	15.8

* in constant prices;
1961 = 100

** in thousand yen

Tab. 3

r_1	1.000
r_3	59.5185
r_5	3229.33
n_2	-38.5583
m_2	-0.9225

Tab. 4

	Upper limit	Lower limit	Interval	Numbers of the intervals
r_2^*	2	-30	70	5
r_4^*	5151	-20	500	11
σ	0.05	0.13	0.02	5
m_1	10	-2	4.4	4
n_1	500	-100	100	7

Tab. 5

	1961		1962		1963		1964	
r_2^*	55	55	55	55	55	55	55	55
r_4^*	3150	3150	3150	3150	2650	2650	3150	3150
σ	0.130	0.090	0.130	0.110	0.130	0.110	0.110	0.130
m_1	1.2	-1.0	1.2	-1.0	10.0	5.6	5.6	3.4
n_1	100.0	0.0	100.0	0.0	500	300	300	200
d	0.034	0.026	0.044	0.039	0.039	0.036	0.029	0.027

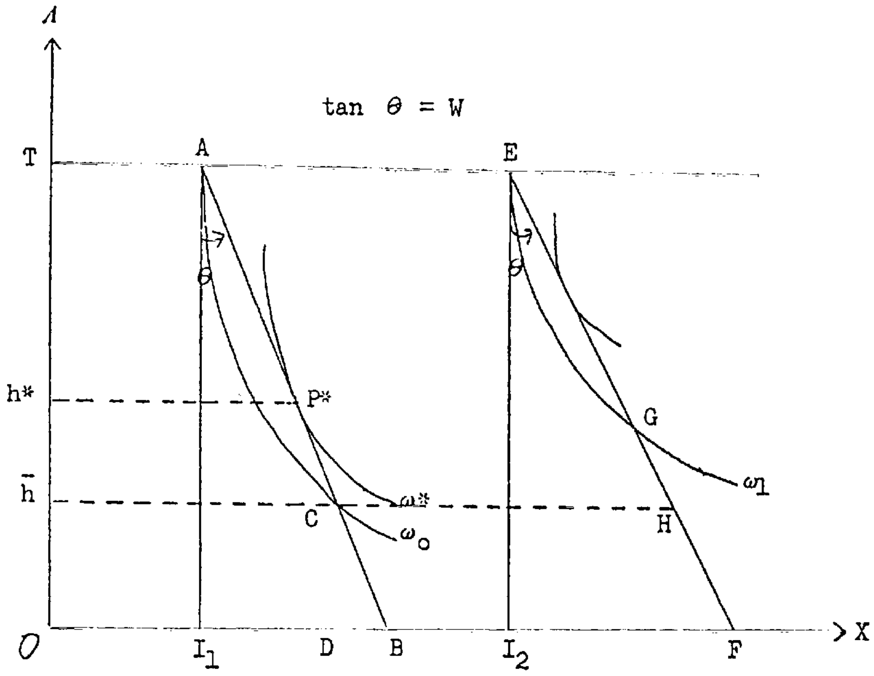


Fig. 1

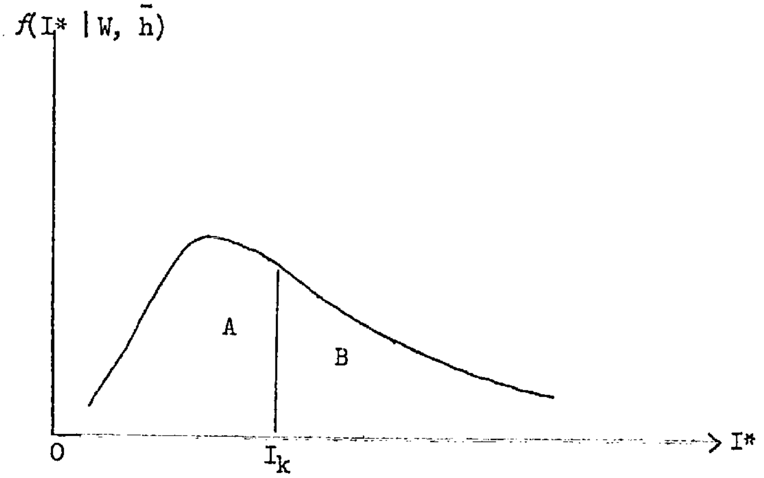
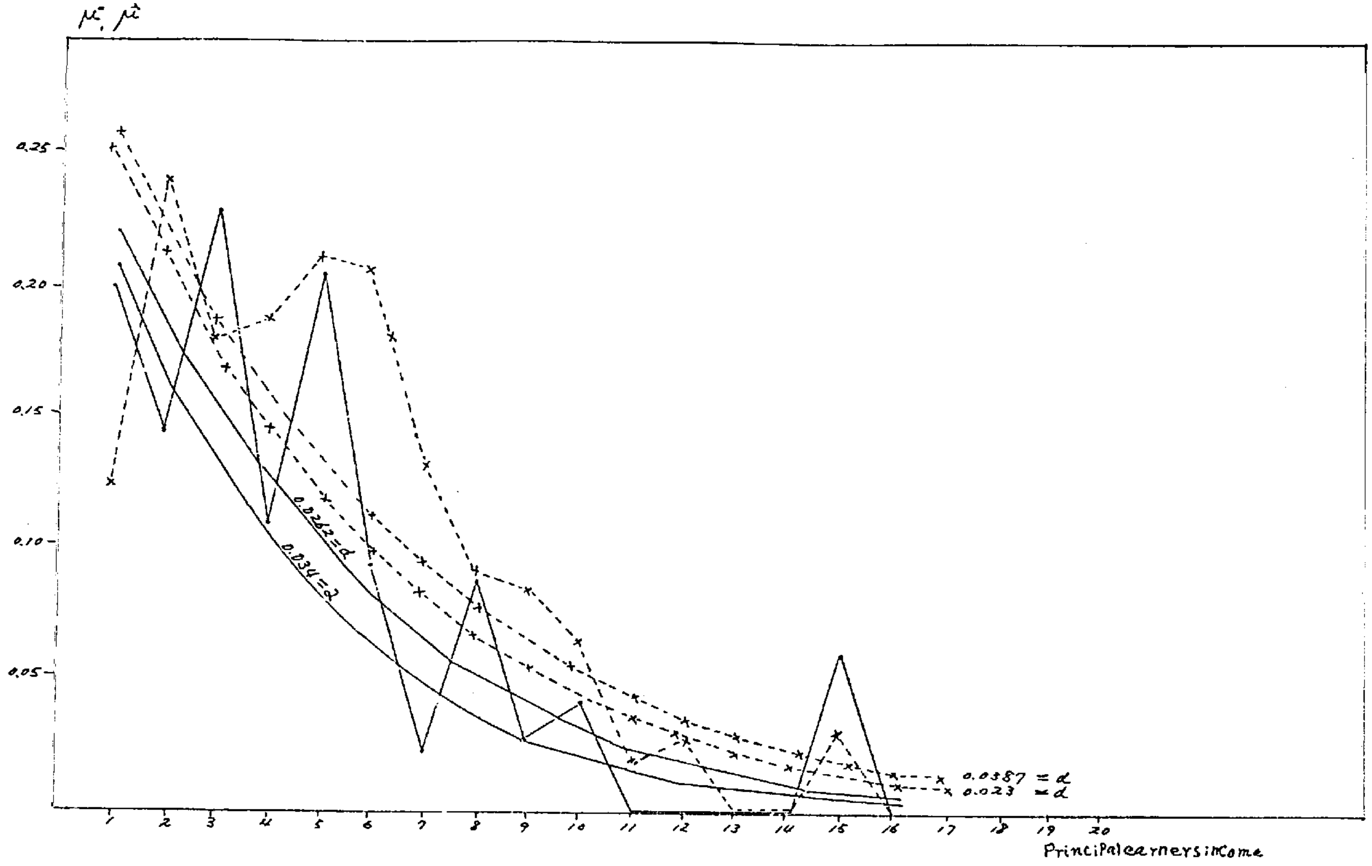


Fig. 2

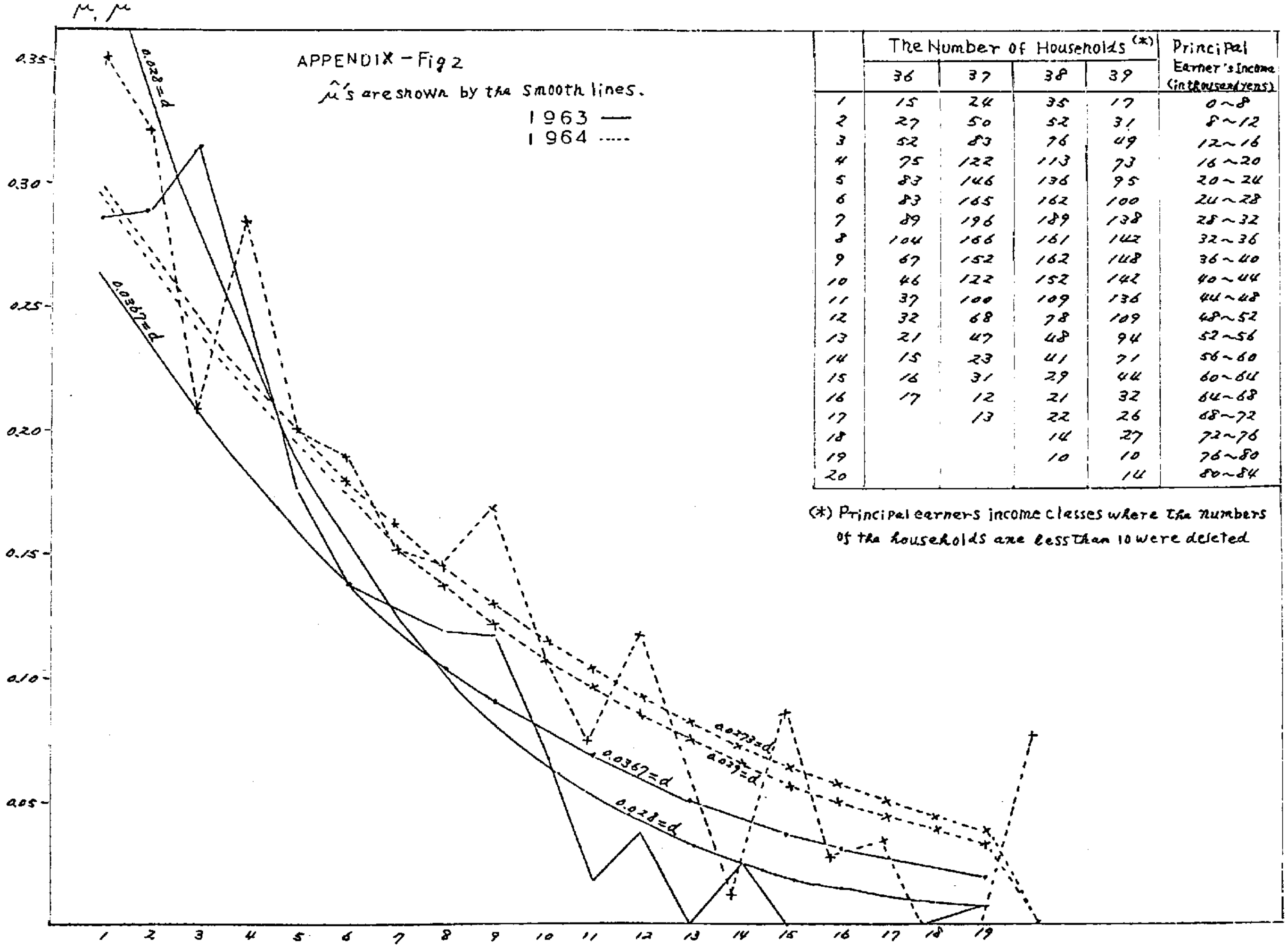
APPENDIX— Fig 1

$\hat{\mu}$'s are shown by the smooth lines. 1961 —— 1962 ----



APPENDIX − Fig 2

$\hat{\mu}$'s are shown by the smooth lines.



	The Number of Households (*)				Principal Earner's Income (in thousand yens)
	36	37	38	39	
1	15	24	35	17	0～8
2	27	50	52	31	8～12
3	52	83	76	49	12～16
4	75	122	113	73	16～20
5	83	146	136	95	20～24
6	83	165	162	100	24～28
7	89	196	189	138	28～32
8	104	166	161	142	32～36
9	67	152	162	148	36～40
10	46	122	152	142	40～44
11	37	100	109	136	44～48
12	32	68	78	109	48～52
13	21	47	48	94	52～56
14	15	23	41	71	56～60
15	16	31	29	44	60～64
16	17	12	21	32	64～68
17		13	22	26	68～72
18			14	27	72～76
19			10	10	76～80
20				14	80～84

(*) Principal earners income classes where the numbers of the households are less than 10 were deleted