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Abstract	§ VI A Synthetic Model of Labor Supply for type A Household--Refinement of precision of the estimates of the parameters using thesynthetic model--So far, the analysis has focused on the wives' (non-principal potentialearners') acceptance and rejection of employment opportunity offered by firms .There are, however, earning opportunities without being employed by theemployers. Indeed, a number of working wives other than employees (selfemployedwives) are found in the FIES data. Hence, a comprehensive theory oflabor supply should originally be able to treat the earning behavior of selfemployed.wives. From this point of view, the theory of wives' labor supplybehavior developed so far is a first approximation in that it only describeswives' acceptance or rejection of employee status. (Hereafter, the two kinds ofworking wives are distinguished by the phrases employee wives and self employedwives). As previously, we used the simple model of labor supply theory toestimate preference parameters. However, the results in section V seem toshow that we need a more precise theory of labor supply, which should be ableto clarify the behavior of self employed as well as employee wives . Hence, amore precise model of wives' labor supply is developed in this section .
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Vol. II

Keiichiro Obi

CALAMVS GLADIO FORTIOR

KEIO ECONOMIC OBSERVATORY
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This paper is preliminary, and
the comments and discussions
are welcome.

CONTENTS

§VI A Synthetic Model of Labor Supply for type A Household

--Refinement of precision of the estimates of the parameters using the synthetic model--

So far, the analysis has focused on the wives' (non-principal potential earners') acceptance and rejection of employment opportunity offered by firms. There are, however, earning opportunities without being employed by the employers. Indeed, a number of working wives other than employees (self-employed wives) are found in the FIES data. Hence, a comprehensive theory of labor supply should originally be able to treat the earning behavior of self-employed wives. From this point of view, the theory of wives' labor supply behavior developed so far is a first approximation in that it only describes wives' acceptance or rejection of employee status. (Hereafter, the two kinds of working wives are distinguished by the phrases employee wives and self employed wives). As previously, we used the simple model of labor supply theory to estimate preference parameters. However, the results in section V seem to show that we need a more precise theory of labor supply, which should be able to clarify the behavior of self employed as well as employee wives. Hence, a more precise model of wives' labor supply is developed in this section.

[6.1.] Labor supply model of type A households constructed by taking into account wives' self-employed earning opportunities

The synthetic model of wives' labor supply for type A households should clarify the conditions by which the wife (non principal potential earner) in a given household chooses to belong to either of the following four patterns:

(1) She (or non principal potential earner) is neither an employee nor self-employed.

(2) She is not an employee but self-employed.

(3) She is an employee but is not self-employed.

(4) She is both an employee and self-employed.

Taking into account the results so far, let (1) the income leisure preference function be quadratic and (2) wives' income generating function (production function) be linear, i.e., the marginal earning rate (marginal value productivity) with respect to hours of labor is a constant. Proposition (2) is introduced for the sake of simplicity and does not impair characteristics of the model.

6.1-1. The determinants of wife's pattern of labor participation

Let us consider a group of type A households with a common level of principal earner's income, I (Fig. Ⅵ-1). Let the marginal earning rate (marginal value productivity of wife's self-employed work) be v which is assumed to be common to all the households considered. The wage rate offered by firms to the wives of the households and the assigned hours of work are denoted by w and $\bar{h}$ respectively which are also assumed to be common to all the households considered.

In Fig. Ⅵ-1 $\tan\theta_w$ and $\tan\theta_v$ stand for w and v respectively. When the wife accepts an employee opportunity, her income leisure position is given by point k. CD is the line passing through point k and parallel to aB, aB being a line of self-employed income. If the wife accepts the employee opportunity and further works as self-employed, the household income will be augmented and lie along the line k D.

Now consider a contour passing through point a. The gradient of the contour at point a, $|dx/d\Lambda|_a$, will vary among the households considered due to the difference in income-leisure preference among them.

Let us call the sub group of households i with

1) $|dx/d\Lambda|_a^i > v$

group I, and the sub group of households j with

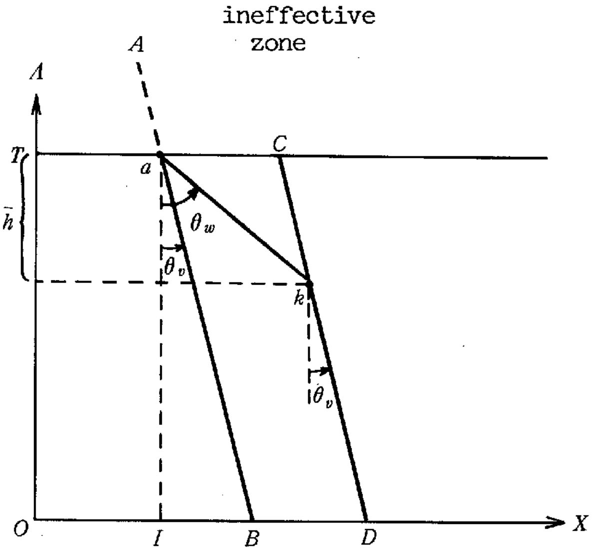
2) $|dx/d\Lambda|_a^j < v$

group II.

It is clearly seen that for any household i in group I there is no tangency point on the line aB, while there is a tangency point on the line a B for a household included in group II. Needless to say, for a household with $|dx/d\Lambda|_a = v$, the tangency point lies just on point a.

As to the households in group II, tangency points lie below point a on the line aB. On the other hand, for the households of group I, there is no tangency point between the points a and B. For those households, the tangency point will be situated at some point on the dotted line Aa which is in an ineffective zone of the indifference map.

Fig. Ⅵ-1

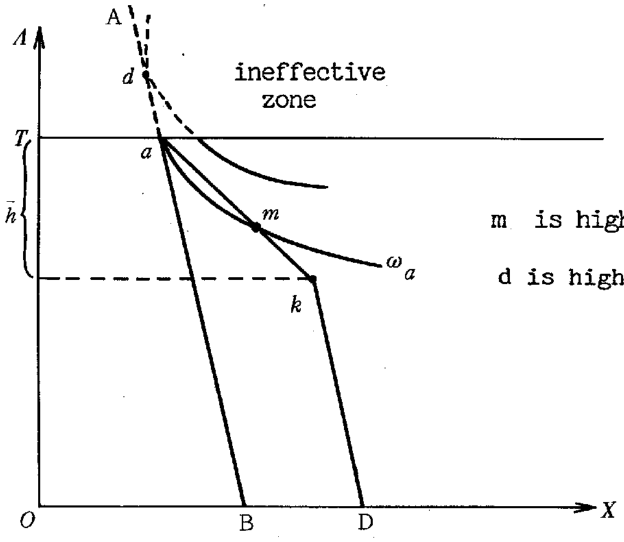


6.1.1.1 Wives' participation behavior in households in group I.

6-1.1.1.-1-

In FigⅥ-2, a contour ωa of a household in group I is depicted. The tangency point of aB and the contour is shown by point d in the ineffective zone of the indifference map. Let the intersection point of ωa and ak be m. In FigⅥ-2 point m is situated above point k on the line ak. First, we shall examine the behavior of a wife of a household with such a contour ωa as is shown in FigⅥ-2. When the wife accepts an employee opportunity, her income-leisure situation is given by point k. Her situation is shown by point a if she neither accepts the opportunity nor works to earn her self-employed income. When the wife earns both a wage and self-employed income, her situation is shown by some point between k and D on the line kD (By the definition of group I, a household wife does not choose only self employment and is therefore not situated between a and B). Among those three situations, point a is clearly the optimum because point a lies on the contour with the highest utility indicator compared to point k and any points between k and D. Hence, point a is chosen by this kind of household (wife).

Fig Ⅵ - 2
the case where a is selected



m is higher than k.

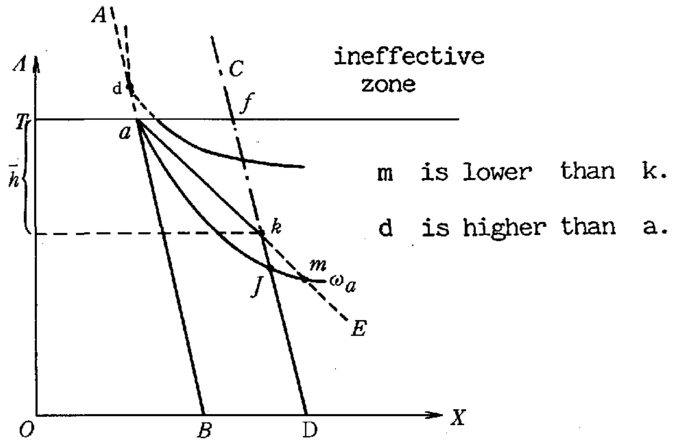
d is higher than a.

6-1.1.1. - 2 -

In Fig Ⅵ - 3 the indifference curve of a household in which the intersection point, m, of contour passing through point a, ω_a, and the line aE lies below point k. If the wife in a household of this kind of household accepted both an employee opportunity and self employed work, her income-leisure situation would be given by the tangency point of contour and line kD somewhere between k and J, as the hours of work for earning self-employed income can be adjusted as the supplier (wife) desires. It should be noted, however, that there does not exist any tangency point on the indifference curve and line between the points k and J on the line cD. If there were a tangency point, g, which is not shown in Fig Ⅵ - 3, it would be said that, when the principal earners income is Tf (in Fig Ⅵ - 3), the non principal earner's (wife's) optimal hours of work for the wage rate v ($\tan\theta_v$) is given by the ordinate difference of point f and g. If such a case occurred, it would be clear, by comparing points d and g, that the larger is the principal earner's income, the longer is the nonprincipal earner's (wife's) optimal hours of work, the nonprincipal earners wage rate v being given. This means, under the assumption of a quadratic preference function, that the locus of MHLS (see section<3.2.4>) on the X ~ Λ plane is

Fig Ⅵ-3

the case where k is selected



downward sloping. However, the downward sloping locus is evidently inconsistent with the observed facts, as has been disussed in section <3.2.5>. Hence, it was proved that, under the assumption of a quadratic preference function, there should be no tangency point between points k and J for consistency between the model and observations.

By the examination mentioned above, any points between k and J lie on the indifference curves with inferior values of the utility indicator in comparison with the indifference curve passing through point k. It is clearly seen that point k is preferable to point a. Hence, the wife of a household with such an indifference map as is shown in Fig Ⅵ-3 accepts the employee opportunity and does not earn an additional self-employed income.

6.1.1.2 Wives' participation behavior in group II households

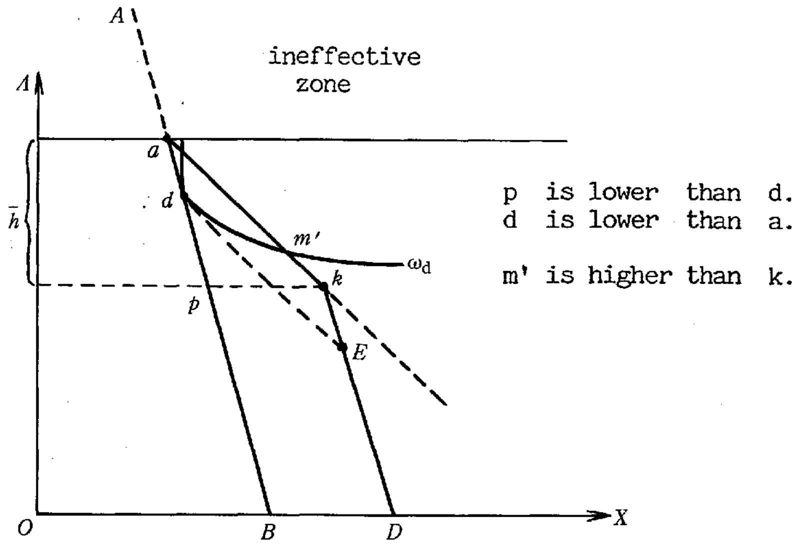
In a household in group II, there exists a tangency point of line aB and the indifference curve, d, as shown in Fig Ⅵ-4.

6-1.1.2-1- Household in which the tangency point, d, lies between points a and P.

Let the intersection of line aB and the horizontal line passing through point k be denoted by P as shown in Fig Ⅵ-4. Consider a household in which the tangency point, d, lies somewhere between points a and P. For this type of household, let the crossing point of ωa and ak be denoted by m'.

Fig Ⅵ - 4

the case where d is selected



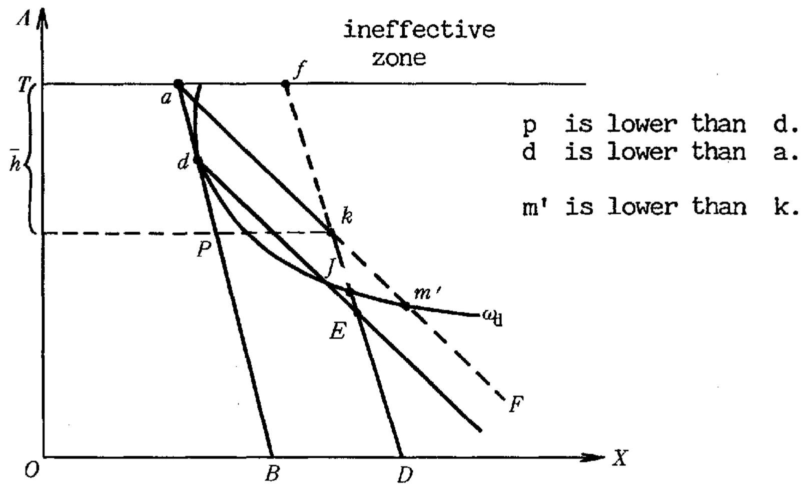
6-1.1.2-1-1- First, consider a household in which point m' lies above point k as is shown in Fig Ⅵ - 4.

The wife (non-principal potential earner) in this kind of household prefers point d, because d is situated on the indifference curve with the highest indicator among the points k, a, and all the points between k and D. Hence, she works as self-employed only and does not accept employee opportunities.

6-1.1.2-1-2- Let the extention of line ak be kF (dotted line) in Fig Ⅵ - 5.

Fig Ⅵ - 5

the case where k is selected



The intersection point of kF and contour ωd is denoted by m'. Consider a household in which point m' lies below point k as shown in FigⅥ-5.

The wife (non-principal potential earner) in this type of household will never choose any points between k and J. If she chooses those points it would mean that she is both an employee and self-employed. However, such a point would have to be a tangency point. From the previous discussion, there could not be any tangency points between k and J because of the requirement of an upward sloping MHLS locus. Hence, d is preferred to a, and any points between k and J are preferred to d. Therefore k is prefered to the points between k and J. That is, the wife will be an employee and will not work as self-employed.

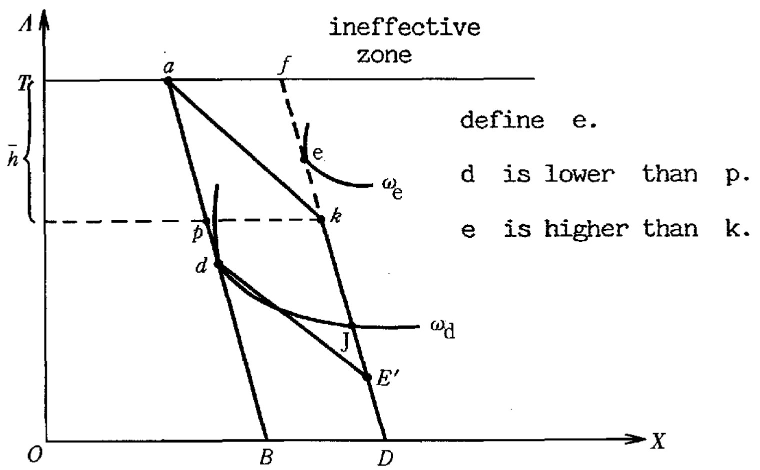
6-1.1.2-2- Household in which point d lies between points p and B.

An indifference map of this kind of household is depicted in FigⅥ-6. For this kind of household two types of households are further differentiated from each other.

6-1.1.2-2-1 In FigⅥ-6 point e is a tangency point of the indifference curve and the line fk which is the extention of line kD. Now, consider a household indifference map which has such characteristics that there exists a tangency point between the indifference curve and the line fk. For this kind of household all the points between k and D on the line k D are situated on indifference curves with smaller indicators compared to the indifference curve on which point k lies, because the gradient of contour at point k, $|dx/d\Lambda|_k$, to the vertical axis is larger than that of line k D to the

FigⅥ-6

the case where k is selected



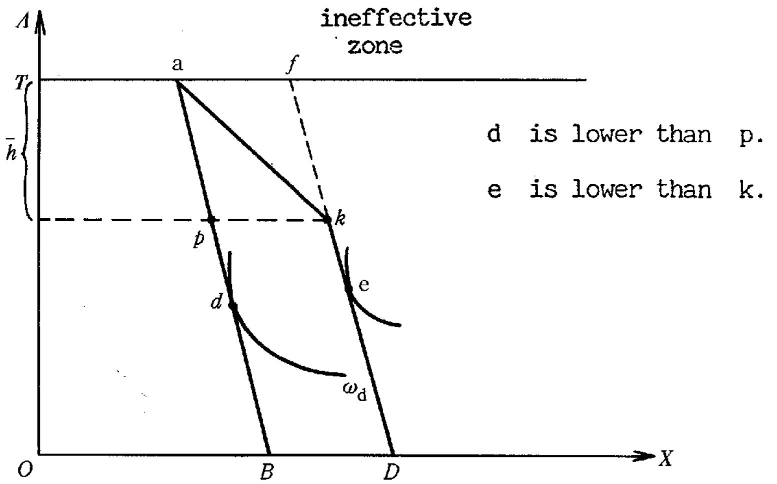
vertical axis.

Hence, among points a, d, k and all the points between k and D, d is preferred to a, all the points between k and D are preferred to d, and k is preferred to all the points between k and D; thus k is preferred. This means that the wife (non principal potential earner) of this household accepts the employee opportunity only and has no self-employed income.

6-1.1.2-2-2- Consider a household in which point e lies below point k. The indifference map of this kind of household is depicted in FigⅥ-7.

It is clearly seen that e is preferred to a, d and k. Hence, the wife (non principal potential earner) will accept the employee opportunity and at the same time she will work as self-employed.

FigⅥ-7
the case where e is selected



6.1.2 Labor participation Model of type A Households.

6.1.2.1 Summary on the patterns of wives' labor participation.

The patterns of wives' labor participatin behavior discussed in the previous section, 6.1.1 is summarized in TabⅥ-1.

Tab. Ⅵ-1

1. group Ⅰ Households with point d above a	1-1 Households with m above k	……	point a is prefered (no participation both in employee opportunity and self employed work) Fig.Ⅵ-2
	1-2 Households with m below k	……	point k is prefered (accepts employee opportunity but no self employment) Fig.Ⅵ-3
2. group Ⅱ Households with point d below a	2-1 Households with point d between a and p	……	2-1-1 households with point m' above k → poin d is prefered (earning of self-employed income only) Fig.Ⅵ-4
			2-1-2 household with point m' below k → point k is prefered (earning from employee opportunity only) Fig.Ⅵ-5
	2-2 Households with point d between p and B	……	2-2-1 households with point e above k → k is prefered (earning from being employee opportunity only) Fig.Ⅵ-6
			2-2-2 households with point e below k → e is prefered (earning from both employee opportunity and self-employed) Fig.Ⅵ-7

Now, let us denote the coordinates of point d, m, m' and e with regard to hours of work by H(d), H(m), H(m'), and H(e) respectively. The coordinates of both points k and p with respect to hours of work are $\bar{h}$ (hours of work assigned by firms). The coordinates of both points B and D with respect to hours of work are T which stands for the wife's (non principal potential earner's) total disposable time (composed of leisure and hours of work if any). Hours of work for earning self-employed income and that for income from the employee opportunity are denoted by H_{self} and H_{emp} respectively. The coordinates of point a with respect to hours of work is zero.

Making use of these notations, the conditions in TabⅥ-1 are rewritten as shown in TabⅥ-2.

Tab. Ⅵ-2

(1) Households with H(d)<0	(1-1) households with H(m)<$\bar{h}$	$H_{emp}=0$, $H_{self}=0$	case①	Fig.Ⅵ-2
	(1-2) households with H(m)>$\bar{h}$	$H_{emp}=\bar{h}$, $H_{self}=0$	case②	Fig.Ⅵ-3
(2) Households with H(d)>0	(2-1) households with H(d)<$\bar{h}$	(2.1.1) households with H(m')<$\bar{h}$ $H_{emp}=0$, $H_{self}>0$	case③	Fig.Ⅵ-4
		(2.1.2) households with H(m')>$\bar{h}$ $H_{emp}=\bar{h}$, $H_{self}=0$	case④	Fig.Ⅵ-5
	(2-2) households with H(d)>$\bar{h}$	(2.2.1) households with H(e)<$\bar{h}$ $H_{emp}=\bar{h}$, $H_{self}=0$	case⑤	Fig.Ⅵ-6
		(2.2.2) households with H(e)>$\bar{h}$ $H_{emp}=\bar{h}$, $H_{self}>0$	case⑥	Fig.Ⅵ-7

6.1.2.2 <u>The Relation between H(m) and H(d) for households with H(d) < 0.</u>

In order to construct a synthetic model for type A households, we shall first consider a group of households with H(d) < 0. With regard to the determinants of participation behavior of this kind of household, the position of point m in relation to the position of point d in Fig Ⅵ-2 is fundamentally important.

Let the relation of H(m) to H(d) be

1) $H(m) = \phi\ [\ H(d)\ ]$

where

2) $H(d) < 0$.

A concrete analytical form of ϕ is given in the subsequent section.

6.1.2.3 <u>The Relation between H(m') and H(d) for the households with $\bar{h}$ > H(d) > 0.</u>

For the households where H(d) > 0 holds, the position of point m' in Fig Ⅵ-4 and 5 is important. Let the relation between H(m') and H(d) be denoted by

3) $H(m') = f\ [\ H(d)\ ]$

where

4) $\bar{h} > H(d) > 0$.

An analytical form of f is given in the subsequent section.

6.1.2.4 <u>The Relation between H(d) and H(e) for the Households with H(d) > $\bar{h}$.</u>

For this kind of household the position of e is also important.

Let the relation between H(e) and H(d) be

5) $H(e) = \psi\ [\ H(d)\ ]$

where

6) $H(d) > \bar{h}$.

The analytical form of ψ is given in the subsequent section.

6.1.2.5 <u>On the graphs of functions ϕ, f and ψ.</u>

The functions ϕ, f and ψ are assumed to be monotonic and are depicted by the curves $\alpha\alpha'$, $\alpha'\beta$ and $\gamma\gamma'$ respectively in Fig Ⅵ-8 and Ⅵ-9. It should be noted that curve $\alpha\alpha'$ standing for ϕ and $\alpha'\beta$ standing for f have a point of conjunction, α', because when H(d) = 0, f[H(d)] = ϕ[H(d)] holds, as can be seen in Fig Ⅵ-3 and 4. Fig Ⅵ-8 differs from Fig Ⅵ-9 in that point α' lies above point $\bar{h}$ on the vertical axis in the former while point α' lies

Fig Ⅵ - 8

the case where the shapes of the curves are not consistent with observation

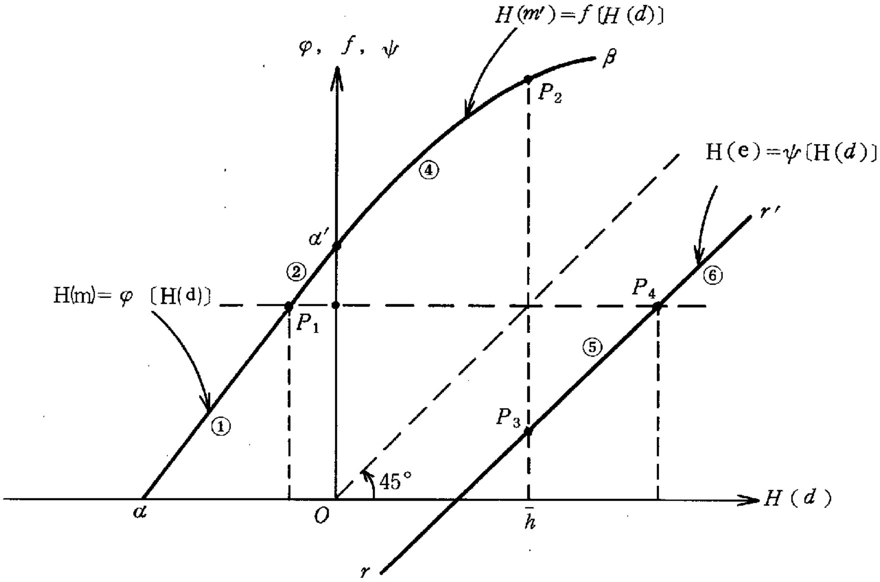
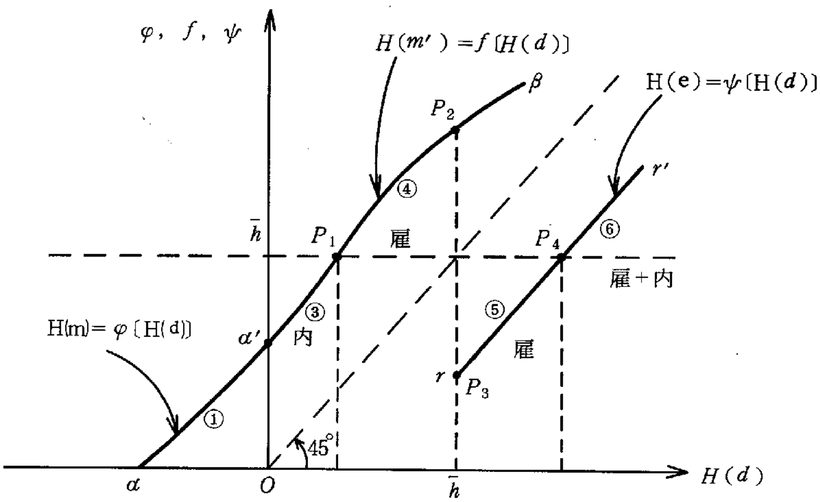


Fig Ⅵ - 9

the case where the shapes of the curves are consistent with observation



below point in the latter.

We shall begin by examining FigⅥ-8. The numbers attached to the curves correspond to those in the column of Tab.Ⅵ-2. It should be remarked that the participation pattern denoted by ③ does not occur when functions ϕ and ψ are of the shape shown in FigⅥ-8. Pattern ③ is for self-employed wives only, not wives who are employees. However according to observed facts, a pattern such as ③ does exist. Hence, since the shapes of the curves shown in FigⅥ-8 are not consistent with observation, they should be excluded. Another possible shape of the curves is shown in FigⅥ-9. In this figure it can be seen that pattern ③ exists. Although pattern ② does not appear in this figure patterns ④ and ⑤ are quite the same as ②. Hence all the participation patterns observed for type A household appear in FigⅥ-9. In this sense the shapes of functions (curves) of ϕ, f and ψ in FigⅥ-9 are consistent with observation.

Addendum

Taking into account the results in section 6.1.2.5, it can be seen that the participation patterns generated from Fig Ⅵ-3 and Ⅵ-4 are exclusive of each other. This is because we assumed the curves $\alpha\alpha'$ and $\alpha'\beta$ are upward sloping monotonic curves. This specific characteristic of the curves stems from the postulate that the preference function is approximated by a quadratic function.

Contrary to the upward sloping monotonic curves, the shape of curve $\alpha\alpha'\beta$ as shown in Fig. 10-A or 10-B might be conceivable. In Fig. 10-A , function f is not monotonic. In Fig. 10-B , function ϕ is not monotonic. In these figures, it can be seen both cases ② and ③ in Tab. Ⅵ-2 (or the cases shown in Fig. Ⅵ-3 and Ⅵ-4) can coexist. However, a quadratic preference function does not yield the curves shown in 10-A or 10-B .

Fig Ⅵ-10-A

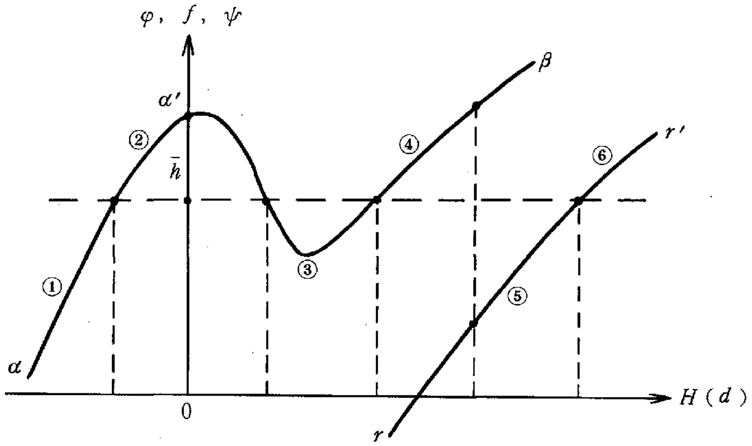
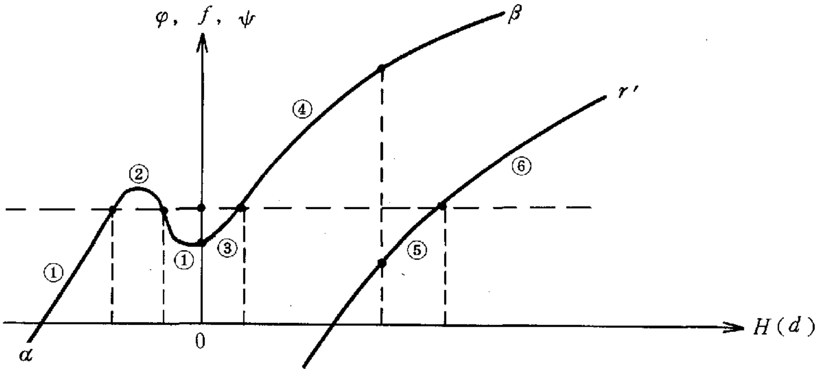


Fig Ⅵ-10-B

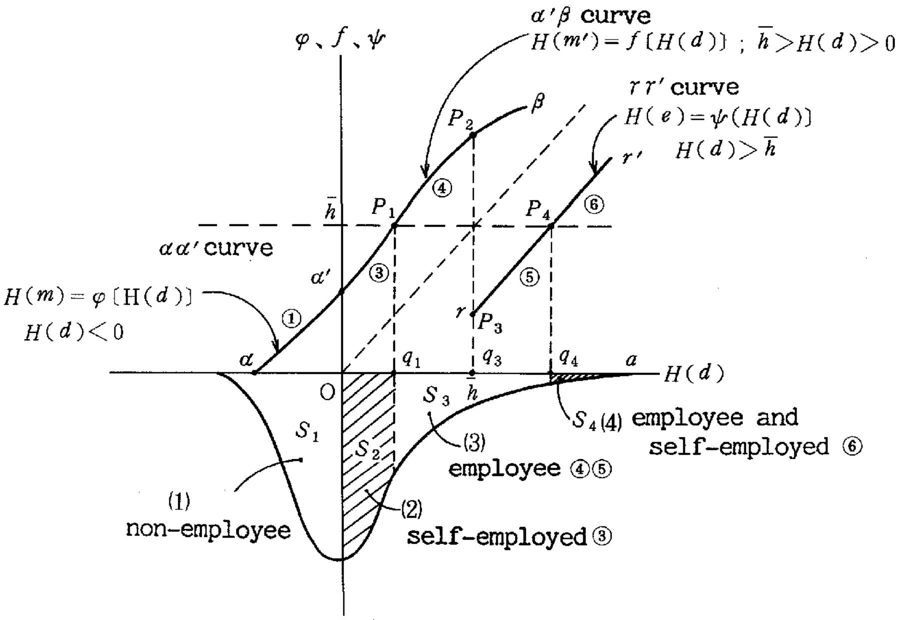


6.1.2.6 Probabilities of generating various participation patterns in type A Households.

In this section the determination of the probabilities of generating four patterns of participation in type A household will be clarified when the principal earner's (husband's) income, I, the wage rate, w, the hours of work assigned by firms, $\bar{h}$, and the earning rate of self-employed work, v, for non-principal potential earner (wife) are given. The first and second quadrants of FigⅥ-11 depict the same curves shown in FigⅥ-9. The density distribution curve of H(d) is depicted in the third and the forth quadrants. This distribution reflects the differences in magnitudes of preference parameters among households where the common values of I, w, v and $\bar{h}$ are given respectively. Taking into account the results summarized in Tab.Ⅵ-2, it will clearly be seen that area S_1 under the distribution curve, gives the probability that the wife (non-principal potential earner) is neither an employee nor self-employed. This is the probability that pattern ① in Tab.Ⅵ-2 occurs. Let us call S_1, the probability of non-participation.

Area S_2 in FigⅥ-11 gives the probability that the participation pattern ③ in Tab.Ⅵ-2 occurs. This is the probability that the wife engages in self-employed work only without accepting an employee opportunity. Let us call this

FigⅥ-11



probability the probability of self-employment participation, μ_d, where

$$\mu_d \equiv \frac{\text{number of self employed wives not accepting employee opportunity}}{\text{number of wives}}$$

Area S_3 gives the probability that either participation pattern ④ or ⑤ in Tab. Ⅵ-2 occurs. Here it should be noted ④ and ⑤ are the same pattern. Let us call this probability the probability of accepting an employee opportunity and not self-employed work, or in short, the probability of beeing an employee, μ_e (probability of employee participation), where

$$\mu_e \equiv \frac{\text{number of wives accepting employee opportunity and not self-employed work}}{\text{number of wives}} .$$

Area S_4 stands for the probability that the participation pattern ⑥ in Tab. Ⅵ-2 occurs. Let us call this probability the probability of double participation, μ_{ed}, where

$$\mu_{ed} \equiv \frac{\text{number of wives participating in both self-employed work and as an employee}}{\text{number of wives}} .$$

Of course,

$$\left(\begin{array}{l}\text{non-participation}\\ \text{probability}\end{array}\right) + \mu_d + \mu_e + \mu_{ed} = 1 .$$

Prior to drawing the curves in Fig Ⅵ-11 the values of I, w, $\bar{h}$, and v have to be given. That is, when these conditions change, the shape of all the curves change simultaneously and, in effect, the areas Si (i = 1, 2, 3, 4) or magnitude of μ_d; μ_e and μ_{ed} change. Hence analytical forms of the function ϕ, f, ψ and the size distribution function of H(d) have to be known in order to describe the changes in participation probabilities corresponding to the changes in I, w, and v. This will be discussed in the following section.

6.1.2.7 <u>Analytical Forms of Functions ϕ, f, and ψ</u>

In this section, analytical forms of ϕ, f, and ψ are determined making use of a quadratic preference function. All the available information including the plausibility of variational γ_4 model, are taken into account in the process of determining analytical forms of those functions.

6.1.2.7.1 <u>Analytical Form of ϕ</u>

6.1.2.7.-1-1. In order to obtain the concrete form of ϕ it is necessary to calculate the coordinates of point d in Fig. Ⅵ-2 or 3. The equation of line aB is given by

1) $X = I + v \cdot h$

where h and X stand for hours of work (<u>for the employee opportunity and/or self-employed work</u>) and household's income respectively. v stands for the earning rate of self-employed work.

The preference function ω is given by

2) $\omega = \frac{1}{2}\gamma_1 \cdot X^2 + \gamma_2 \cdot X + \gamma_3 \cdot X \cdot \Lambda + \gamma_4 \cdot \Lambda + \frac{1}{2}\gamma_5 \cdot \Lambda^2$

where

$\Lambda \equiv T - h.$

Under the constraint of (1), (2) is maximized with respect to h. When the value of h maximizing ω is negative, that value of h stands for H(d) in the function ϕ. This stems from the fact that the indifference maps shown in Fig. Ⅵ-2 or 3 are the maps of households with such γ_4 that places tangency point, d, on AB in the ineffective range.

Hence, we obtain

$$3)\quad H(d) = \frac{-(\gamma_1 \cdot v - \gamma_3)\ I - v\ (\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

where

$H(d) < 0.$

The value of H(d) varies among households with given I, w, $\bar{h}$ and v owing to the difference in γ_4 of each household. Hence the size distribution of γ_4 can be easily transformed to that of H(d) by using equation (3).

6.1.2.7.1-2. The equation of the indifference curve ω_a in Fig. Ⅵ-2 and 3 can be obtained as follows. By inserting the values of the ordinates of point a in

Fig. VI-2 and 3,

4) $X = I$

5) $\Lambda = T$,

into the left hand side of preference function (2), we obtain the value of indicator ω_a at point a,

$$6)\quad \omega_a = \tfrac{1}{2}\gamma_1 \cdot I^2 + \gamma_2 \cdot I + \gamma_3 \cdot I \cdot T + \gamma_4 \cdot T + \tfrac{1}{2}\gamma_5 \cdot T^2 ,$$

I and T being given. Hence, the equation of the indifference curve ω_a can be written as

$$7)\quad \omega_a = \tfrac{1}{2}\gamma_1 \cdot X^2 + \gamma_2 \cdot X + \gamma_3 \cdot X \cdot \Lambda + \gamma_4 \cdot \Lambda + \tfrac{1}{2}\gamma_5 \cdot \Lambda^2 ,$$

where ω_a is given by (6).

6.1.2.7.-1-3 Finally let us obtain the ordinate of point m in Fig. VI-2 and 3.

The equation of line ak is given by

8) $X = I + w \cdot h$.

We can solve (8) together with (7) for h. The solution is the coordinate of point m with respect to hours of work, H(m), that is,

$$9)\quad H(m) = \frac{(-\gamma_1 \cdot w + \gamma_3)\ I - w\ (\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\tfrac{1}{2}(\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5)}$$

It can be seen that the magnitude of H(m) varies among households considered owing to differences in γ_4 of each household.

6.1.2.7.-1-4 Now, we are ready to obtain a concrete form of the function ϕ. The parameter γ_4, the magnitude of which is supposed to vary among households, is included both in equations (9) and (3). Hence, by eliminating common parameter γ_4 both in (9) and (3) we obtain a relation between H(m) and H(d),

$$10)\quad H(m) = \frac{2(\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5} H(d) + \frac{2(v-w)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5}$$

---function ϕ

where $H(d) < 0$.

This is the funcution ϕ when the preference funcution is quadratic.

6.1.2.7.2 Analytical Form of funcution f

Function f stands for a relation between point m' and d in Fig.Ⅵ-4 and 5. The coordinate of point d, H(d), is previously given by (3),

$$3)\quad H(d) = \frac{-(\gamma_1\cdot v-\gamma_3)\ I - v\ (\gamma_2+\gamma_3\cdot T)+\gamma_4+\gamma_5\cdot T}{\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5}$$

However, with regard to the case shown in Fig.4 and 5, it should be noted that contrary to the previous case,

3') $H(d) > 0$.

That is, the magnitudes of parameter of preference funcution γ_4, which generates the indifference curve as shown in Fig.4 and 5 must be of a value which makes the right hand side of equation (3) positive.

6.1.2.7.2-1 We shall obtain the equation of ω_d in Fig.Ⅵ-4 and 5.

The coordinates of point d are given by

11) $X = I + v\cdot H(d)$

12) $\Lambda = T - H(d)$

where H(d) is given by (3). Inserting (11) and (12) into (2) we have

$$13)\quad \omega_d = \tfrac{1}{2}\gamma_1[I+v\cdot H(d)]^2+\gamma_2[I+v\cdot H(d)]+\gamma_3[I+v\cdot H(d)][T-H(d)] + \gamma_4[T-H(d)]+\tfrac{1}{2}\gamma_5[T-H(d)]^2 \quad .$$

Given I and v, the value of ω_d in (13) is specific to each household with specific value of γ_4.

The equation of contour ω_d in Fig.4 and 5 is given by

$$14)\quad \omega_d = \tfrac{1}{2}\gamma_1\cdot X^2+\gamma_2\cdot X+\gamma_3\cdot X\cdot\Lambda+\gamma_4\cdot\Lambda+\tfrac{1}{2}\gamma_5\cdot\Lambda^2$$

where ω_d is given by (13).

The equation of the segment ak or that of the extention of the segment is given by

15) $X = I + w\cdot h \quad ; \quad T - \Lambda = h.$

where h stands for hours of work for the employee opportunity and/or self-employed work. Hence, we can obtain the ordinate of point m' by solving (14) and (15) simultaneously with respect to h. By denoting this solution H(m') we have

$$16)\quad H(m') = \frac{-1}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5} \{ I(\gamma_1 \cdot w - \gamma_3) + (\gamma_2 + \gamma_3 \cdot T)w - \gamma_4 - \gamma_5 \cdot T \}$$
$$\pm \{ [I(\gamma_1 \cdot w - \gamma_3) + (\gamma_2 + \gamma_3 \cdot T)w - \gamma_4 - \gamma_5 \cdot T]^2 - 2(\gamma_1 \cdot w^2 - 2\gamma_3 + \gamma_5)$$
$$\times \{ \tfrac{1}{2}\gamma_1 I^2 + (\gamma_2 + \gamma_3 \cdot T)I + \gamma_4 \cdot T + \tfrac{1}{2}\gamma_5 \cdot T^2 - [\tfrac{1}{2}\gamma_1 (I + v \cdot H(d))^2$$
$$+ \gamma_2 (I + v \cdot H(d)) + \gamma_3 (I + v \cdot H(d))(T - H(d)) + \gamma_4 (T - H(d))$$
$$+ \tfrac{1}{2}\gamma_5 (T - H(d))^2] \} \}^{\frac{1}{2}} \frac{1}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5}$$

where H(d) is given by (3).

By examining Fig. Ⅵ-4 and 5, the algebraically larger root among the two given by (16) is adopted as the value of H(m').

6.1.2.7.2-2 Finally we shall obtain the function f.

By eliminating the common parameter γ_4 included in both (16) and (3), we have

$$17)\quad H(m') = \frac{-K - \sqrt{D}}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5} \qquad \text{---function f}$$

and,

$$K \equiv (w - v)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T) - (\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5)\, h^*_d$$
$$D \equiv (w - v)\{(w - v)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)^2 - 2(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)$$
$$\times (\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5) h^*_d + (\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5)[2\gamma_3 - \gamma_1 (w + v)](h^*_d)^2\}$$

where h^*_d is the abbreviation of H(d) given by (3). Equation (17) is the function f when the preference function ω is quadratic.

6.1.2.7.3 <u>Analytical Form of function ψ</u>

Function ψ stands for the relation between point d and e in Fig. Ⅵ-6 and Ⅵ-7.

6.1.2.7.3-1 Firstly the coordinate of H(d) is given by

$$(3)\quad H(d) = \frac{-(\gamma_1 \cdot v - \gamma_3)\, I - v\, (\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

as previously shown in 6.1.2.7.1-1. However,

$$(3')\quad H(d) > \bar{h}$$

must hold here, in order that point d lies below p in Fig. Ⅵ-6 and 7.

6.1.2.7.3-2 In the second place we shall obtain the coordinate of point e. Taking into account that the coordinates of point k is given by

(18) $X = I + w \cdot \bar{h}$

and

(19) $\Lambda = T - \bar{h}$,

the equation of line fD passing through point k is written as

(20) $X = I + (w - v)\,\bar{h} + v \cdot h_{fd}$,

where h_{fd} stands for the coordinate of hours of work on the line fD.

Under the constraint of (20), we shall obtain the value of h_{fd} maximizing ω in (2). This value of h_{fd} is H(e). Hence we have

(21) $$H(e) = \frac{-(\gamma_1 \cdot v - \gamma_3)[I + (w - v)\bar{h}] - v(\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

6.1.2.7.3-3 We are now ready to obtain the analytical form of ψ:
That is, by eliminating γ_4 in both (3) and (21), the relation between H(d) and H(e) is derived.

(22) $$H(e) = H(d) - \frac{(\gamma_1 \cdot v - \gamma_3)(w - v)\,\bar{h}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$ ---function ψ

where

$H(d) > \bar{h}$, and $w > v$,

is obtained. This is the function ψ when the preference function ω is quadratic.

6.1.3. Calculation of participation probability.

In this section the calculation of μ_e, μ_d and μ_{ed} is discussed.

6.1.3.1 The coordinates of points q_1 and q_4

It can be seen that function f contains preference parameters, γ_1, γ_2, γ_3 and γ_5, and exogenous variables, v, w, $\bar{h}$ and I, respectively; that is, f is rewritten as

23) $H(m') = f\,[H(d),\ \gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_5 \mid v,\ w,\ \bar{h},\ I\,]$

where $H(d) > 0$.

In the same fashion function ψ can be rewritten as

24) $H(e) = \psi\,[H(d),\ \gamma_1,\ \gamma_3,\ \gamma_5 \mid v,\ w,\ \bar{h}\,]$

where $H(d) > \bar{h}$.

Applying $H(m') = \bar{h}$ to the left hand side of equation (23), we have

25) $\bar{h} = f\,[H(d),\ \gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_5 \mid v,\ w,\ \bar{h},\ I\ ]$.

This equation can be solved for H(d). Let us denote the solution for H(d) by $H(d)q_1$. Hence

26) $H(d)q_1 = f^{-1}\,[\gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_5 \mid v,\ w,\ \bar{h},\ I\ ]$,

where f^{-1} stand for the inverse function of f. $H(d)q_1$ given by (26) is the coordinate of point q_1 on the H(d) axis in Fig. VI-11.

Now we shall obtain the coordinate of point q_4 in Fig.11.

Replacing H(e) on the left hand of equation (24) by $\bar{h}$ we have

27) $\bar{h} = \phi\,[H(d),\ \gamma_1,\ \gamma_3,\ \gamma_5 \mid v,\ w,\ \bar{h}\ ]$.

We can solve (27) with respect to H(d) and let us denote the solution by $H(d)q_4$. Hence we have

28) $H(d)q_4 = \phi^{-1}\,[\gamma_1,\ \gamma_3,\ \gamma_5 \mid v,\ w,\ \bar{h}\ ]$

where ϕ^{-1} is the inverse function of ϕ. Equation (28) gives the coordinate of point q_4 in Fig. VI-11. It can be seen that $H(d)q_4$ is invariant with the principal earner's income level, I, because ϕ and ϕ^{-1} does not contain I as an argument. This stems from the characteristics of quadratic function ω.

6.1.3.2 <u>Density distribution function of H(d)</u>

Finally we shall discuss the density distribution function of H(d). H(d) has been gives by (see 6.1.2.7.1-1)

$$(3)\quad H(d) = \frac{-(\gamma_1\cdot v - \gamma_3)\ I\ -\ v\ (\gamma_2 + \gamma_3\cdot T) + \gamma_4 + \gamma_5\cdot T}{\gamma_1\cdot v^2 - 2\gamma_3\cdot v + \gamma_5}$$

where the magnitude of γ_4 varies among households considered.

With respect to a household i, the value of γ_4^i is given by

$$(29)\qquad \gamma_4{}^i = \overline{\gamma_4}\cdot u_i$$

where $\overline{\gamma_4}$ is a constant which is common to all the households considered and u_i is a random variable, the distribution of which is log-normal with mean $E(u_i)$, and variance $\sigma_u{}^2$, where

$E(u_i) = 1$,

$\sigma_u{}^2$ being a constant. Let the density distribution of u_i be

$$(30)\qquad l\,(\,u \mid \sigma_u{}^2\,)$$

where the suffix i is deleted. By considering (29), (3) can be reduced to

$$(3)\quad H(d) = \frac{-(\gamma_1 \cdot v - \gamma_3)\ I\ -\ v\ (\gamma_2 + \gamma_3 \cdot T) + \overline{\gamma_4} \cdot u + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

Solving this equation with respect to u, we have

$$(31)\quad u = \frac{1}{\overline{\gamma_4}}\{(\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5)H(d) + (\gamma_1 \cdot v - \gamma_3)I + v(\gamma_2 + \gamma_3 T) - \gamma_5 T\}$$

or in short,

$$(32)\quad u = u\ (H(d),\ \gamma_1,\ \gamma_2,\ \gamma_3,\ \overline{\gamma_4},\ \gamma_5\ |\ v,\ I)$$

From (31) we have

$$(33)\quad du = \frac{1}{\overline{\gamma_4}}(\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5) \cdot dH(d)$$

From (32), (30) and (33), we have

$$(34)\quad l(u)du = l\,[\,u(H(d), \gamma_1, \gamma_2, \gamma_3, \overline{\gamma_4}, \gamma_5\ |\ v, I)\ |\ \sigma_u\,]\ \left|\frac{du}{dH(d)}\right|\ dH(d)$$

$$= l_{H(d)}(H(d)\ |\ \gamma_1, \gamma_2, \gamma_3, \overline{\gamma_4}, \gamma_5; v, I, \sigma_u)\ \left|\frac{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}{\overline{\gamma_4}}\right|\ dH(d)$$

This is the function which transforms the distribution function of u, $l(u)$, to that of H(d), $l[H(d)]$. The right hand side of equation (34) (except for dH(d)) is the density distribution function of H(d) depicted in Fig. Ⅵ-11. For the sake of brevity, let us denote the distribution function, the right hand side of (34), (except for dH(d)) by

$$(35)\quad l^*\ (H(d)\ |\ \gamma_1,\ \gamma_2,\ \gamma_3,\ \overline{\gamma_4},\ \gamma_5\ ;\ v,\ I,\ \sigma_u).$$

It can be seen from (34) that the distribution of H(d) is invariant with respect to changes in w.

6.1.3.3 Participation Probability

By using (35), μ_d shown by area S_2 in Fig. Ⅵ-11 is given by the definite integration of l^*, i, e.

$$(36)\quad \mu_d = \int_0^{H(d)q_1} l^*(H(d)\ |\ \gamma_1, \gamma_2, \gamma_3, \overline{\gamma_4}, \gamma_5; v, I, \sigma_u)\ dH(d)$$

where $H(d)q_1$ is given by (26).

In the same manner, μ_e shown by area S_3 in Fig Ⅵ-11 is given by

$$(37)\quad \mu_e = \int_{H(d)q_1}^{H(d)q_4} l^*(H(d) \mid \gamma_1, \gamma_2, \gamma_3, \overline{\gamma}_4, \gamma_5; v, I, \sigma_u)\, dH(d)$$

where $H(d)q_4$ is given by (28).

The value of μ_{ed} shown by area S_4 in Fig Ⅵ-11 is given by

$$(38)\quad \mu_{ed} = \int_{H(d)q_4}^{\infty} l^*(H(d) \mid \gamma_1, \gamma_2, \gamma_3, \overline{\gamma}_4, \gamma_5; v, I, \sigma_u)\, dH(d)$$

6.1.3.4 It can be seen from (36), (37) and (38) that the values of three kinds of participation probabilities μ_e, μ_d and μ_{ed}, are respectively determined by the values of $\{\gamma_i\}$ ($i = 1, \cdots\cdots, 5$), σ_u, v, w, and I. It should be noted that the magnitude of w affects the probabilities via limits of integration, $H(d)q_1$ and $H(d)q_4$, as well, because these are functions of w respectively.

Employing an abridged formulation, (36), (37) and (38) can be rewritten as

$$(39)\quad \mu d = \mu d\ (\{\gamma_i\},\ \sigma_u,\ v,\ w,\ I),$$

$$(40)\quad \mu e = \mu e\ (\{\gamma_i\},\ \sigma_u,\ v,\ w,\ I),$$

and

$$(41)\quad \mu ed = \mu ed\ (\{\gamma_i\},\ \sigma_u,\ v,\ w,\ I).$$

Making use of these relation we can proceed to obtain the estimates of the preference parameters, $\{\gamma_i\}$ and σ_u, of secondary order of precision. This procedure is shown in the following section.

[6.2.] Augmenting the Precision of Estimates of Preference Parameters

(6.2.1) Let us denote the values of preference parameters obtained previously in <5.2> by $\{\gamma_i^{(1)}\}$ and $\sigma_u^{(1)}$ ($i = 1, \cdots\cdots, 5$).(*) These values can be considered to be the first appoximation for the true values of preference parameters, $\{\gamma_i\}$, and σ_u. That is, the model used to estimate those parameters was a first appoximation in the sense that the model took into account the wife's choosing an employee opportunity but not self-employed work.

By inserting the observed values for w, v and I, w^0, v^0, and I^0, respectively, together with $\{\gamma_i^{(1)}\}$ and $\sigma_u^{(1)}$, into 39), 40) and 41), we have theoretical (or estimated) values for μ^e, μ^d and μ^{ed}, that is,

(*) In the previous section we used the notation σ for σ_u.

42) $\mu^{d}{}_{j}(1) \;=\mu^{d}\;(\{\gamma_{i}{}^{(1)}\},\;\sigma_{u}{}^{(1)},\;v^{0},\;w^{0},\;I_{j}{}^{0})$

43) $\mu^{e}{}_{j}(1) \;=\mu^{e}\;(\{\gamma_{i}{}^{(1)}\},\;\sigma_{u}{}^{(1)},\;v^{0},\;w^{0},\;I_{j}{}^{0})$

44) $\mu^{ed}{}_{j}(1) \;=\mu^{ed}(\{\gamma_{i}{}^{(1)}\},\;\sigma_{u}{}^{(1)},\;v^{0},\;w^{0},\;I_{j}{}^{0})$

Now, let us denote the observed values for participation porbabilities of the jth principal earner's income class by μ^{d}_{jo}, μ^{e}_{jo} and μ^{ed}_{jo}.

Let the differences between observed values and first approximation values be

(a) $u^{d}{}_{j}\equiv(\mu^{d}{}_{j0}-\mu^{d}{}_{j}(1))$, $u^{e}{}_{j}\equiv(\mu^{e}{}_{j0}-\mu^{e}{}_{j}(1))$, $u^{ed}{}_{j}\equiv(\mu^{ed}{}_{j0}-\mu^{ed}{}_{j}(1))$

where $\mu^{d}_{j}(1)$, $\mu^{e}_{j}(1)$ and $\mu^{ed}_{j}(1)$ are first approximation values.

Let

(b) $\delta(\{u^{d}{}_{j}\},\;\{u^{e}{}_{j}\},\;\{u^{ed}{}_{j}\})$

be an objective function properly defined, where { } stands for a row vector. Initial values for the preference parameters, $\{\gamma^{(1)}_{i}\}$ and $\sigma_{u}{}^{(1)}$, are allowed to vary so as to minimize δ.

Hence, we have to choose the proper functional form for the objective function δ. In relation to this, it should be noted that equations 39′) through 41′) are exact relations that do not include any shocks, or disturbances in relations. In contrast to those relations, ordinal consumption functions, for example, include shocks which, it is assumed, reflect random movements in consumer preference parameters and so on. In other words, as far as we treat shock model at least, discrepancies between observed values and theoretical (estimated) values for consumption are allowed. However, the present model for participation probability, or the system or equations 39) through 41), has been deduced by definite integration of the distribution function of H(d*), and the distribution function reflects the distribution function of the preference parameters. Also, upper and lower limits of the definite integration are not random variables. As a result, also the values for the definite integrals are not random variables. Hence, as far as the present model is concerned there is no room for allowing shocks in the equation system, 39) through 41).

Contrary to participation probability functions 39) through 41), each household's supply function with respect to wives' optimal hours of work for employment opportunities or an aggregation of them do include shocks reflecting differencesin the preference parameters among households, as consumption functions do. However, as explaned above, the probability equations are conceived as exact relations, and the differences between the theoretical values μ^d, μ^e and μ^{ed}, and observed μ_0^d, μ_0^e and μ_0^{ed}, respectively, are considered to reflect sampling or observational errors (disturbances in variables) caused from limited sample size.

Hence, denoting sampling or observational errors by additive random variable u_d, u_e and u_{ed}, we have

45) $\mu_0{}^d = \mu^d(\{\gamma_i\}, \sigma_u, v, w, I) + u_d$

46) $\mu_0{}^e = \mu^e(\{\gamma_i\}, \sigma_u, v, w, I) + u_e$

47) $\mu_0{}^{ed} = \mu^{ed}(\{\gamma_i\}, \sigma_u, v, w, I) + u_{ed}$

which constitute an error model, not a shock model.

Multiplying by n both sides of equations 45) through 47), respectively, we have

48) $n \cdot \mu_0{}^d = n \cdot \mu^d(\{\gamma_i\}, \sigma_u, v, w, I) + n \cdot u_d$

49) $n \cdot \mu_0{}^e = n \cdot \mu^e(\{\gamma_i\}, \sigma_u, v, w, I) + n \cdot u_e$

50) $n \cdot \mu_0{}^{ed} = n \cdot \mu^{ed}(\{\gamma_i\}, \sigma_u, v, w, I) + n \cdot u_{ed}$

where n stands for sample size (number of households or number of wives) for each principal earner's income class.

Rewriting 2-48), we have

51) $n \cdot \mu_0{}^d - n \cdot \mu^d(\{\gamma_i\}, \sigma_u, v, w, I) = \varepsilon_d$

52) $n \cdot \mu_0{}^e - n \cdot \mu^e(\{\gamma_i\}, \sigma_u, v, w, I) = \varepsilon_e$

53) $n \cdot \mu_0{}^{ed} - n \cdot \mu^{ed}(\{\gamma_i\}, \sigma_u, v, w, I) = \varepsilon_{ed}$

where

54-1) $\varepsilon_d \equiv n \cdot u_d$ 54-2) $\varepsilon_e \equiv n \cdot u_e$ 54-3) $\varepsilon_{ed} \equiv n \cdot u_{ed}$

ε_d, ε_e and ε_{ed} are, respectively, differences between observed and theoretical values, and they have a joint binomial distribution. If n is large enough, the joint distribution can be fully approximated by the normal distribution,

55) $N(0, 0, 0, \sigma_d, \sigma_e, \sigma_{ed}, \sigma^2{}_{d\cdot e}, \sigma^2{}_{d\cdot ed}, \sigma^2{}_{e\cdot ed})$

where, 0's, σ_d, σ_e, σ_{ed} stand for, respectively, means and standard deviations with respect to ε_d, ε_e and ε_{ed}, and $\sigma^2_{e\cdot d}$, $\sigma^2_{d\cdot ed}$ and $\sigma^2_{e\cdot ed}$ stand for their covariances.

From 45), 46) and 47), we have

$$u_d \equiv \frac{1}{n}\varepsilon_d, \qquad u_e \equiv \frac{1}{n}\varepsilon_e, \qquad u_{ed} \equiv \frac{1}{n}\varepsilon_{ed}$$

hence, u_d, u_e and u_{ed} have the joint probability distribution

$$N(0,\ 0,\ 0,\ \frac{1}{n}\sigma_d,\ \frac{1}{n}\sigma_e,\ \frac{1}{n}\sigma_{ed},\ \frac{1}{n}\sigma^2_{e\cdot d},\ \frac{1}{n}\sigma^2_{d\cdot ed},\ \frac{1}{n}\sigma^2_{e\cdot ed})$$

as an approximation.

Now, under the constraint that ε_d, ε_e and ε_{ed} have joint distribution 55) we shall obtain maximum likelihood estimates of $\{\gamma_i\}$ and σ_u. Because v, w and I, in 48) and 49), are fixed in repeated samples, equations 48) and 49) can be treated as regression equations having fixed variable on the right hand sides of the equations, although they constitute an error model.

Let n_d, n_e and n_{ed} be, respectively, numbers of persons employed by employers, self-employed and of those who participate in both opportunities. Taking into account that those variables have binomial distributions, we have

56-1) $\quad E(n_e) = n\cdot\mu_e$

56-2) $\quad E(n_d) = n\cdot\mu_d$

56-3) $\quad E(n_{ed}) = n\cdot\mu_{ed}$

57-1) $\quad var(n_e) = n\cdot\mu_e\cdot(1-\mu_e)$

57-2) $\quad var(n_d) = n\cdot\mu_d\cdot(1-\mu_d)$

57-3) $\quad var(n_{ed}) = n\cdot\mu_{ed}\cdot(1-\mu_{ed})$

57-4) $\quad cov(n_e,\ n_d) = -n\cdot\mu_e\cdot\mu_d$

57-5) $\quad cov(n_e,\ n_{ed}) = -n\cdot\mu_e\cdot\mu_{ed}$

57-6) $\quad cov(n_d,\ n_{ed}) = -n\cdot\mu_d\cdot\mu_{ed}$

where var and cov, respectively, stand for the variance and covariance of the variables in the parentheses.

Parameters $\{\gamma_i\}$ and σ_u are estimated so as to minimize

58) $\quad \delta \equiv U'\Sigma^{-1}U$

where

59) $\quad U' \equiv [u_e^1\ \ u_d^1\ \ u_{ed}^1\ \cdots\cdots\ u_e^m\ \ u_d^m\ \ u_{ed}^m]$

m standing for the number of principal earner's income classes, and Σ standing for the variance-covariance matrix with respect to u_d, u_e and u_{ed}. The population variance and covairance are estimated by the sample variance and covairance.

The estimation procedure is summarized below.

Firstly, participation probabilities, μ^{d}, μ^{e} and μ^{ed}, are computed, making use of $\{\gamma_i^{(1)}\}$ and $\sigma_u^{(1)}$, by equations 45) through 47). Secondly, by using the computed participation probabilities, we have U' in 59). Inserting those values into 58), together with Σ, we have $\delta^{(1)}$, the value of δ corresponding to $\{\gamma_i^{(1)}\}$ and $\sigma_u^{(1)}$. We compute $\{\gamma_i^{(2)}\}$ and $\sigma_u^{(2)}$, respectively, using $\{\gamma_i^{(1)}\}$ and $\sigma_u^{(1)}$ so as to reduce the magnitude of δ. That is; let the shifts in $\{\gamma^{i}\}$ and σ_u be denoted by $\Delta\gamma_i (i=2, \cdots, 5)$ and $\Delta\sigma_u$ respectively. Needless to say, $\gamma_1^{(1)} \equiv -1$ and $\Delta\gamma_1 \equiv 0$. It can be seen from 42) through 44) that revised values for participation probabilities, after assigning the shifts for the parameters, $\mu^{d}(\Delta)$, $\mu^{e}(\Delta)$ and $\mu^{ed}(\Delta)$, are given by

60) $\mu^{d}{}_{j}(\Delta) = \mu^{d}(\{\gamma_i^{(1)} + \Delta\gamma_i\}, \sigma_u^{(1)} + \Delta\sigma_u, v^{o}, w^{o}, I_j{}^{o})$

61) $\mu^{e}{}_{j}(\Delta) = \mu^{e}(\{\gamma_i^{(1)} + \Delta\gamma_i\}, \sigma_u^{(1)} + \Delta\sigma_u, v^{o}, w^{o}, I_j{}^{o})$

62) $\mu^{ed}{}_{j}(\Delta) = \mu^{ed}(\{\gamma_i^{(1)} + \Delta\gamma_i\}, \sigma_u^{(1)} + \Delta\sigma_u, v^{o}, w^{o}, I_j{}^{o})$

$\delta^{(2)}$ can be computed, by employing 60) through 62), from 58).

$$\gamma_i^{(2)} = \gamma_i^{(1)} + \Delta\gamma_i \quad ; \quad i = 2, 3, 4, 5$$

and

$$\sigma_u^{(2)} = \sigma_u^{(1)} + \Delta\sigma_u$$

are revised estimates for the preference parameters.

6.2. ADDENDUM for the Computation Procedure

[6.2.1] Calculation of the abscissa of q_1 in in Fig. VI-11 for A・B Type preference function

(6.2.1.1) In equation

$$H(m') = f\ [H(d)]$$

let $H(m')=\bar{h}$, and the equation can be solved for $H(d)$. The solution is the abscissa of point q_1 .

The concrete form of function f has been given by 17) in (6.1.2.7.2). From this and $H(m')=\bar{h}$, we have

$$1)\quad \bar{h} = \frac{-K-\sqrt{D}}{\gamma_1\cdot w^2-2\gamma_3\cdot w+\gamma_5}$$

where

$$K\equiv(w-v)(\gamma_1 I+\gamma_2+\gamma_3\cdot T)-(\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5)\cdot h^*$$

$$D\equiv(w-v)\{(w-v)(\gamma_1 I+\gamma_2+\gamma_3\cdot T)^2-2(\gamma_1 I+\gamma_2+\gamma_3\cdot T)$$
$$\times(\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5)\cdot h^*+(\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5)[2\gamma_3-\gamma_1(w+v)](h^*)^2\}$$

and notation h^* is used in place of $H(d)$ for the sake of simplicity.

1) can be solved for h^*, that is, 1) can be rewritten as

$$2)\quad (\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5)(h^*)^2 - 2(\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5)\cdot\bar{h}\ h^*$$
$$+ \bar{h}\ [(\gamma_1\cdot w^2-2\gamma_3\cdot w+\gamma_5)\cdot\bar{h} + 2(w-v)(\gamma_1 I+\gamma_2+\gamma_3\cdot T)]=0$$

Among two roots of equation 2), we adopt the root h^* satisfying

$$0 < h^* < \bar{h}$$

as the plausible solution.

We rewrite 2) as

$$2')\qquad A_*\cdot(h^*)^2-2A_*\cdot\bar{h}\cdot h^*+B_*=0\quad ,$$

where

$$3-1)\quad A_*\equiv\gamma_1\cdot v^2-2\gamma_3\cdot v+\gamma_5$$

and

$$3-2)\quad B_*\equiv\bar{h}\ [(\gamma_1\cdot w^2-2\gamma_3\cdot w+\gamma_5)\cdot\bar{h} + 2(w-v)(\gamma_1 I+\gamma_2+\gamma_3\cdot T)]$$

We have the solution h^* as

$$4)\quad h^*= \frac{A_*\cdot\bar{h} \pm \sqrt{A_*^2\cdot\bar{h}^2-A_*\cdot B_*}}{A_*} = \bar{h} \pm \sqrt{\bar{h}^2-\frac{B_*}{A_*}}$$

Taking into account the requirement $0 < h^{*} < \bar{h}$, we have

$$5)\quad H(d)_{q_1} = \bar{h} - \sqrt{\bar{h}^2 - \frac{\bar{h}[(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)\bar{h} + 2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)]}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}}$$

(6.2.1.2) Calculation of abscissa for q_4 in Fig. VI-11

In equation

$$H(e) = \psi\,[H(d)]$$

we set the left hand side equal to $\bar{h}$, that is,

$$\bar{h} = \psi\,[H(d)].$$

By solving this equation for H(d), we obtain the abscissa for point q_4.

The concrete form of ψ is given by 22) in 6.1.2.7-3.

$$H(e) = H(d) - \frac{(\gamma_1 \cdot v - \gamma_3)(w-v)\cdot\bar{h}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

Applying $H(e)=\bar{h}$ for this equation, and solving it for H(d), we have

$$6)\quad H(d)_{q_4} = \bar{h} + \frac{(\gamma_1 \cdot v - \gamma_3)(w-v)\cdot\bar{h}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} = \bar{h}\,[1 + \frac{(\gamma_1 \cdot v - \gamma_3)(w-v)}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}]$$

where $H(d)_{q_4}$ stands for the abscissa of point q_4.

(6.2.1.3) Calculation of abscissa of Point a in Fig. VI-11

In equation 3) in (6.1.2.7),

$$H(d) = \frac{-(\gamma_1 \cdot v - \gamma_3)I - v(\gamma_2 + \gamma_3 \cdot T) + \gamma_4 + \gamma_5 \cdot T}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

we set $\gamma_4 = 0$. Hence, we have

$$7)\quad H(d)_{max} = \frac{\gamma_5 \cdot T - (\gamma_1 \cdot v - \gamma_3)I - v(\gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

where $H(d)_{max}$ stands for the value of H(d) for the household with the largest value of H(d) among the households considered. Accordingly, $H(d)_{max}$ represents the abscissa of point a in Fig. VI-11.

[6.2.2] Some other constraints for the Parameters to be Estimated

From the generalized labor supply model for type A households, in which self-employment opportunities are taken into account as well as market employment opportunities, we can derive some additional theoretical restrictions for the parameters to be estimated. They are :

(1) The derivative of function ϕ must be positive

This constraint can be stated, by using 10) in (6.1.2.7.4), as

1) $$\frac{2(\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5} > 0.$$

Hence we have

1′) $(\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5)(\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5) > 0$ ---constraint [1]

(2) $0 < \phi\,[H(d)=0] = f\,[H(d)=0] < \bar{h}$ must hold

This restriction means that point α', in Fig.Ⅵ-11, must lie between 0 and $\bar{h}$. By applying

$$H(d) = 0$$

to equation 10) in (6.1.2.7.1-4) we have

2) $$\phi\,[H(d)=0] = \frac{2(v-w)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5}$$

which stands for the ordinate of point α'on curve $\alpha\alpha'$, or function ϕ.

By applying $H(d)=0$, or $h^*=0$, to equation 17) in §6-1.2.2, we have,

$$f\,[H(d)=0] = \frac{-K' - \sqrt{D'}}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5}$$

where,

$$K' \equiv (w-v)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)$$

$$D' \equiv (w-v)^2 \cdot (\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)^2 = (K')^2$$

Hence, we have

3) $$f\,[H(d)=0] = \frac{-2K'}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5} = \frac{2(v-w)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5}$$

which stands for the ordinate of α' on curve $\alpha'\beta$, or function f.

Comparing 2) and 3), curve $\alpha\alpha'$ and $\alpha'\beta$ in Fig.Ⅵ-8, 9 and 10, respectively, join each other at point α'.

From 2) and 3), the constraint

$$0 < \phi\ [H(d)=0] = f\ [H(d)=0] < \bar{h}$$

can be written as

$$4)\qquad 0 < \frac{2(v-w)(\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot T)}{\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5} < \bar{h}$$

From the first and second terms in this inequality, we have

$$5)\qquad 0 < \frac{-2(w-v)(\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot T)}{\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5}$$

From the previous restriction

$$w > v, \qquad \text{---restriction [2]-0}$$

hence, we obtain

$$5.1)\qquad (\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot T)(\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5) < 0.$$

On the other hand, from the second and third terms in the inequality 4), we have

$$\frac{-2(w-v)(\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot T)}{\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5} < \bar{h}$$

or,

$$\frac{(w-v)(\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot T)}{\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5} > \frac{-\bar{h}}{2}$$

Hence, according to if the denominater in the left hand side of the last inequality is positive

$$a)\qquad \gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5 > 0$$

we have the constraint,

$$5.2)\qquad (w-v)(\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot T) > -\frac{\bar{h}}{2}(\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5),$$

and if,

$$b)\qquad \gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5 < 0$$

we have the constraint,

$$5.3)\qquad (w-v)(\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot T) < -\frac{\bar{h}}{2}(\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5).$$

The discussion below equation 5.1) can be alternat ely restated as follows: firstly, in equation 5.1), we have

6.1) $\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T > 0$, ---restriction [2]-1

because the left hand side of the inequality stands for the marginal utility of household income when its non-principal potential earner does not work at all. Hence, from 5.1) we have

6.2) $\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5 < 0$ ---restriction [2]-2

Taking into account these, we examine the inequality

$$\frac{2(v-w)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5} < \bar{h}$$

in 4). This can be rewritten as

$$\frac{(w-v)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5} > \frac{-\bar{h}}{2}$$

From 6.2), we can see the left hand side of this inequality is negative. Hence, we have

6.3) $(w-v)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T) < -\frac{\bar{h}}{2}(\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5)$.
---restriction [2]-3

From 6.2) and 1), we have

$\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5 < 0$ ---restriction [1]'

which is an alternative to restriction [1].

(3) <u>Constraint that the inequality $0 < H(d)_{q_1} < \bar{h}$ must hold.</u>

The abscissa of point q_1 is given by equation 5) in (6.2.1.1); that is,

$$H(d)q_1 = \bar{h} - \sqrt{(\bar{h})^2 - \frac{\bar{h}[(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)\bar{h} + 2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)]}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}}$$

In order that the root is positive and that

$$0 < H(d)_{q_1} < \bar{h}$$

holds, the following must be true

7) $-(\bar{h})^2 < \frac{\bar{h}[(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)\bar{h} + 2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)]}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} < 0$.
---constraint [3]

Now, from requirement 6.2) and 1′)

1″) $$\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5 < 0$$

should have been satisfied. Hence, from 7) we have

7′) $$-(\bar{h})^2 \cdot (\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5) > -\bar{h}\,[(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)\bar{h} + 2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)] > 0$$

---constraint [3]'

which is an alternative presentation for the requirement that inequality $0 < H(d)_{q_1} < \bar{h}$ holds.

(4) <u>Constraint that $\bar{h} < H(d)_{q_4} <$ a must hold</u>

From Fig. Ⅵ-11, it can be seen that point q_4 must lie between points $\bar{h}$ and a on the horizontal axis.

The abscissa of point q_4, $H(d)_{q_4}$, is given by equation (6) in (6.2.1.2), that is,

$$H(d)_{q_4} = \bar{h} + \frac{(\gamma_1 \cdot v - \gamma_3)(w-v)\,\bar{h}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

Hence, in order that the part of the constraint, $\bar{h} < H(d)_{q_4}$ be satisfied,

8) $$\bar{h} < \bar{h} + \frac{(\gamma_1 \cdot v - \gamma_3)(w-v)\,\bar{h}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$ ---constraint [4]-1

must hold.

The abscissa of point a in Fig. Ⅵ-11 can be written as

$$H(d)_{max} = \frac{\gamma_5 \cdot T - (\gamma_1 \cdot v - \gamma_3)I - v(\gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5},$$

which is shown in 7) in (6.2.1.3). Hence the part of constraint that $H(d)_{q_4} <$ a can be written as

9) $$\bar{h} + \frac{(w-v)(\gamma_1 \cdot v - \gamma_3)\bar{h}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} < \frac{\gamma_5 \cdot T - (\gamma_1 \cdot v - \gamma_3)I - v(\gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

---constraint [4]-2

From 8) it can be seen that

8′) $$\frac{(\gamma_1 \cdot v - \gamma_3)(w-v)\,\bar{h}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5} > 0,$$ ---constraint [4]-1'

where $w - v > 0$,

must hold. From 6.2), at the same time,

$$\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5 < 0$$

must hold, so that, 8′) implies

$$10) \qquad \gamma_1 \cdot v - \gamma_3 < 0 \quad . \qquad \text{---constraint [4]-2'}$$

Requirement 10) can be fullfiled whenever $\gamma_3 > 0$. Hence, constraints [4]-1′ and [4]-2′ are needed in order that

$$\bar{h} < H(d)_{q_4} < a$$

be true.

[6.3] <u>Improving exactness of estimated parameters by using employee-self employed model (A synthetic model for Type A household)</u>

(6.3.1) Search for refined estimates

This is the preliminary section for obtaining refined estimates of the preference parameters. To improve the precision of estimated parameters, we employ the synthetic model described in the previous section. In the synthetic model, a new kind of exogenous variable, v, standing for the earning rate (or marginal productivity) of self employed workers, was introduced. However, v cannot be directly observed because the Family Income and Expenditure Survey in Japan does not cover hours of work. Hence, we are compelled to estimate v as a parameter, the value of which is assumed to vary from year to year.

a) Postulates for estimating plausible values of v

The following define what constitutes a plausible value for v.

(1) If we have obtained a set of reasonably good estimates of parameters making use of the general model, we should be able to compute theoretical values for the self-employed participation rates μ_d and μ_{ed} (participation ratio for those both employed and self-employed) as well as the employee participation rate μ_e, which should reasonably fit the observables μ_d^o, μ_{ed}^o and μ_e^o.

(2) Before computing theoretical values μ_d, μ_e and μ_{ed}, we have to have numerical values for principal earners' incomes I_i of the group of households, with both w and v assumed to be common to all groups of households. I_i and w are directly observable but v is not, as mentioned above. Hence, owing

to (1), if we try a tentative value for v and if the value is a fairly good approximation to the true value of v, the computed theoretical values μ_d , μ_{ed} and μ_e will reasonably fit the observed values.

In addition to the above assumptions, the following restrictions must be satisfied by v together with observed values for I_i and w.

1. The slope of the curve of the function ϕ should be positive.

$$2(\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5) / (\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5) > 0$$

2. $0 < \phi[H(d)=0] = f[H(d)=0] < \bar{h}$

(We need this in order to attain consistency between empirical observations and the model.)

3. $0 < q_1 < \bar{h}$; where,

$$q_1 = \bar{h} - \sqrt{\bar{h}^2 - \frac{\bar{h}\{(\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5)\bar{h} + 2(w-v)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)\}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}}$$

4. $\bar{h} < q_4 < a$; where,

$$q_4 = \bar{h} + \frac{(\gamma_1 \cdot v - \gamma_3)(w-v)\,\bar{h}}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

$$a = \frac{\gamma_5 \cdot T - (\gamma_1 \cdot v - \gamma_3)I - v(\gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5}$$

(These restrictions stem from $\phi[H(d)=\bar{h}] < \bar{h}$ and $\phi[H(d)=a] > \bar{h}$

5. $\bar{h} < f(\bar{h})$
6. $\phi(\bar{h}) < \bar{h}$
7. $\bar{h} < \phi(a)$

Condition 1 yields

$$(\gamma_1 \cdot v^2 - 2\gamma_3 \cdot v + \gamma_5)(\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5) > 0.$$

Condition 2 can be rewritten as

$$0 < \frac{2(w-v)(\gamma_1 \cdot I + \gamma_2 + \gamma_3 \cdot T)}{\gamma_1 \cdot w^2 - 2\gamma_3 \cdot w + \gamma_5} < \bar{h}.$$

Hence, we have

$$\frac{(w-v)(\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot T)}{\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5} > \frac{\bar{h}}{2}$$

where $\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5 < 0$,

or

$$(w-v)(\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot T) < \frac{\bar{h}\,(\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5)}{2}$$

With respect to condition 4, in order that the value in the root be positive and $0 < q_1 < \bar{h}$

$$-(\bar{h})^2 < -\frac{\bar{h}\,\{(\gamma_1 \cdot w^2-2\gamma_3 \cdot w+\gamma_5)\bar{h}+2(w-v)(\gamma_1 \cdot I+\gamma_2+\gamma_3 \cdot I)\}}{\gamma_1 \cdot v^2-2\gamma_3 \cdot v+\gamma_5} < 0$$

must hold.

Condition 5 can be rewritten as follows.

$$\bar{h} < \bar{h} + \frac{(w-v)(\gamma_1 \cdot v-\gamma_3)\,\bar{h}}{\gamma_1 \cdot v^2-2\gamma_3 \cdot v+\gamma_5} < \frac{\gamma_5 \cdot T-(\gamma_1 \cdot v-\gamma_3)I-v(\gamma_2+\gamma_3 \cdot T)}{\gamma_1 \cdot v^2-2\gamma_3 \cdot v+\gamma_5}$$

(b) Search for plausible values of v.

To compute the theoretical values of participation rates, we first need numerical values for the preference parameters γ_i ($i=1, \cdots\cdots, 5$) and σ_u. As a first approximation, values of γ_i and σ_u obtained in the previous section,

$\gamma_1=-1.0$ $\gamma_2=100.003$ $\gamma_3=0.0797$,

$\gamma_4=6000.4$ $\gamma_5=-1399.3$, and $\sigma_u=0.3839$, were used.

Those values are first approximations because they are obtained making use of the employment opportunity model where the nonprincipal earner's income from self-employed work was ignored.

Secondly, we need observed value for I_i. We used three levels of I, $I_1=12.5018$, $I_2=35.0735$, and $I_3=57.7704$ for 1964. Finally, concerning the observed value for w, we have $w \cdot \bar{h}=15.8$ for 1964, where $\bar{h}$ is estimated as $\bar{h}=0.3501$, and hence $w=45.13$.

Inserting the values above, together with tentative values for v, into restrictions 1 through 5, we checked to see these restrictions were satisfied. Tentative values of v ranged from 1 to 90 so as to include the observed value for w, 45.13.

The intervals of the tentative values for v are 5. It was found that the values of v satisfying the restrictions were 40 and 45 as shown in Tab. Ⅵ-1.(*)

Next, we refined the process of the search, that is; (1) various sets of values for γ_i and σ_u are tentatively adopted, (numerical values of which were tentatively given in the vicinities of the values cited on the previous page) (2) the values of I_i are adopted from the complete range of 19 classes of principal earners' incomes in 1964, (3) the intervals of the tentative values for v were narrowed down to 1, and (4) the range of tentative values of v was also narrowed down to 35 through 50 instead of 1 through 90.

The results satisfying the conditions are shown in Tab. Ⅵ-2, where U stands for Theil's U with respect to the fit of theoretical values to the observations, and Φ stands for the value of the objective function(**) which, we expect, is to be minimized for the best set of parameters. It can be shown from the table that the values of v satisfying the restrictions and with smaller Φ are 45 and 46. The sets of parameters, together with v's, which were found to satisfy restrictions 1 through 7 are summarized in the table below.

γ_2	100	150	200	
γ_3	0	10	20	30
γ_4	6000	7000	8000	9000
γ_5	-400	-1400	-2400	-3400
σ_u	0.188	0.227	0.268	(+)
v	45	46	(***)	

(*) Checking whether restrictions were fulfilled or not was carried out using values of I mentioned in the text. Values of $\bar{h}$ and Λ used for the check were as follows.

$\Lambda_{max}=1.0$ $\quad\Lambda_{min}=0.25$

$\bar{h}_{min}=0.25$ $\quad\bar{h}_{max}=0.50$

(**) We use the objective function,

$$\Phi\equiv\Sigma\{(\frac{\mu_d{}^0-\hat{\mu}_d}{\hat{\mu}_d})^2+(\frac{\mu_e{}^0-\hat{\mu}_e}{\hat{\mu}_e})^2+(\frac{\mu_{ed}{}^0-\hat{\mu}_{ed}}{\hat{\mu}_{ed}})^2\}\cdot n\ .$$

Hence we are minimizing χ^2.

Where n stands for the number of households.

(***) For computation, the value of $\bar{h}$ was assumed to be 1/3.

(+) The values for σ_u are computed using given values of w, $\bar{h}$, γ_2, γ_3 and γ_5.

Tab. Ⅵ-1.

v	I	q_1	q_4	a	$\varphi(-\bar{h})$	$\varphi(0)=f(0)$	$f(\bar{h})$	$\varphi(\bar{h})$	$\varphi(a)$
	I_{min}	−0.780	0.433	1.515	1.482	1.787	2.066	0.268	1.433
10	I_{med}	−0.602	0.433	1.364	1.021	1.327	1.598	0.268	1.281
	I_{max}	−0.384	0.433	1.213	0.563	0.868	1.126	0.268	1.130
	I_{min}	−0.235	0.419	1.747	0.301	0.770	1.168	0.281	1.678
30	I_{md}	−0.138		1.453	0.103	0.571	0.954		1.383
	I_{max}	−0.0173		1.159	−0.0948	0.374	0.730		1.090
	I_{min}	−0.0952	0.397	1.697	−0.0195	0.515	0.952	0.303	1.650
35	I_{med}	−0.0206		1.396	−0.152	0.383	0.800		1.349
	I_{max}	0.0730		1.096	−0.285	0.250	0.638		1.049
	I_{min}	0.0554	0.374	1.631	−0.350	0.261	0.725	0.326	1.607
40	I_{med}	0.105		1.330	−0.418	0.194	0.637		1.306
	I_{max}	0.168		1.030	−0.484	0.127	0.540		1.006
	I_{min}	0.306	0.351	1.556	−0.691	0.00661	0.396	0.350	1.555
45	I_{med}	0.314		1.259	−0.693	0.00491	0.387		1.259
	I_{max}	0.323		0.964	−0.695	0.00321	0.377		0.964
	I_{min}	$\sqrt{\ }$ 内が負	0.328	1.479	−1.042	−0.248	$\sqrt{\ }$	0.372	1.500
50	I_{med}	$\sqrt{\ }$		1.189	−0.978	−0.184	$\sqrt{\ }$		1.211
	I_{max}	$\sqrt{\ }$		0.901	−0.915	−0.120	$\sqrt{\ }$		0.923
	* $\bar{h}$ = 0.3501								
	I_{min} = 12.5018, I_{med} = 35.0735, I_{max} = 57.5504								

Tab. VI -2

$Case$	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
1	100	0	6000	− 400	.498	43	.556	.664	.470	.881	.372	.305	.362 6				
						44	.440	.664	.468	.829	.416	.317	.300 6				
						45	.316	.664	.467	.771	.478	.332	.267 6				
						46	.239	.666	.466	.706	.571	.348	.310 6				
						47	.181	.673	.464	.626	.747	.364	.863 6				
2	100	0	6000	−1400	.389	41	.425	.668	.475	.816	.206	.298	.287 5			○	
						42	.550	.667	.474	.760	.225	.311	.244 5			○	
						43	.470	.667	.472	.701	.254	.326	.216 5			○	
						44	.374	.668	.471	.640	.297	.343	.203 5			○	
						45	.304	.669	.470	.577	.362	.361	.213 5				
						46	.249	.673	.468	.509	.466	.380	.304 5				
						47	.198	.680	.467	.431	.675	.399	.125 6				
3	100	0	7000	− 400	.498	43	.532	.705	.480	.922	.511	.330	.339 6				
						44	.311	.705	.478	.883	.550	.313	.436 6				
						45	.201	.706	.477	.839	.602	.301	715 6				
						46	.145	.708	.476	.788	.680	.292	.349 6			○	
						47	.105	.712	.475	.725	.818	.288	.117 7			○	
4	100	0	7000	−1400	.389	41	.590	.717	.486	.888	.361	.383	.597 6				
						42	.430	.717	.485	.847	.386	.358	.504 6				
						43	.286	.717	.485	.801	.420	.336	.432 6				
						44	.207	.717	.483	.754	.464	.318	.390 6				
						45	.158	.719	.482	.703	.525	.304	.385 6				
						46	.127	.721	.482	.648	.616	.294	.464 6			○	
						47	.097	.725	.481	.582	.780	.289	.127 7			○	
5	100	0	8000	−2400	.321	39	.608	.739	.495	.895	.392	.633	.288 6				
						40	.438	.740	.494	.860	.407	.602	.250 6				
						41	.280	.740	.494	.823	.428	.57	.225 6				

$Case$	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^e$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$
						42	.194	.740	.493	.784	.454	.539	.148 7				
						43	.146	.741	.493	.743	.487	.508	.130 7				
						44	.115	.742	.492	.700	.531	.477	.120 7				
						45	.093	.743	.492	.656	.590	.448	.117 7				
						46	.076	.744	.491	.610	.675	.420	.143 7				
						47	.059	.747	.491	.557	.820	.394	.390 7				
6	100	0	9000	−3400	.273	37	.620	.751	.499	.903	.448	.828	.960 6				
						38	.449	.75	.499	.873	.458	.807	.804 6				
						39	.281	.752	.499	.842	.471	.785	.697 6				
						40	.189	.752	.499	.809	.487	.761	.614 6				
						41	.138	.753	.498	.774	.508	.736	.572 6				
						42	.106	.753	.498	.739	.534	.710	.102 7				
						43	.085	.753	.498	.702	.567	.682	.101 7				
						44	.070	.754	.498	.665	.609	.654	.112 7				
						45	.059	.755	.497	.627	.664	.626	.148 7				
						46	.048	.756	.497	.588	.740	.596	.256 7				
						47	.037	.757	.497	.545	.863	.567	.388 7				
7	150	0	6000	− 400	.272	45	.315	.447	.441	.751	.211	.646	.769 3	○		○	
						46	.528	.438	.437	.597	.245	.668	.201 3	○		○	
						47	.495	.444	.433	.416	.457	.687	.141 3	○			
8	150	0	6000	−1400	.236	43	.039	.205	.428	.763	.385	.711	.142 5				
						44	.006	.178	.424	.628	.349	.728	.458 5				
						45	.002	.159	.419	.496	.306	.743	.130 3	○			
						46	.024	.151	.414	.363	.267	.756	.515 2	◎		○	
						47	.105	.174	.409	.212	.377	.768	.420 2	◎	○		
9	150	0	7000	− 400	.272	45	448	.637	.468	.831	.245	.427	.318 4			○	
						46	.269	.634	.465	.703	.338	.462	.140 4				

$Case$	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^{ed}$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$
						47	.199	.638	.462	.552	.569	.495	.171 4				
10	150	0	7000	−1400	.236	43	.050	.561	.463	.835	.254	.502	.827 4			○	
						44	.420	.552	.459	.703	.224	.536	.525 3	○		○	
						45	.539	.545	.456	.569	.196	.567	.196 3	○		◎	
						46	.485	.543	.452	.433	.212	.595	.104 3	○		○	
						47	.406	.553	.448	.282	.426	.621	.179 3	○			
11	150	0	7000	−2400	.209	42	.014	.388	.449	.698	.422	.621	.195 4				
						43	.000	.369	.444	.573	.391	.647	.259 3	○			
						44	.015	.352	.440	.453	.351	.670	.907 2	◎			
						45	.059	.341	.436	.339	.298	.690	.416 2	◎		○	
						46	.137	.340	.431	.225	.240	.709	.232 2	◎		○	
						47	.303	.366	.427	.108	.333	.725	.319 2	◎	◎		
12	150	0	8000	− 400	.272	45	.216	.712	.484	.898	.432	.287	.362 5				○
						46	.109	.711	.482	.808	.521	.283	.205 5				○
						47	.076	.714	.480	.695	.711	.293	.293 5			◎	○
13	150	0	8000	−1400	.237	43	.415	.684	.483	.906	.179	.283	.182 5			◎	○
						44	.405	.681	.481	.803	.197	.299	.506 4			○	○
						45	.247	.679	.478	.693	.241	.325	.289 4				
						46	.188	.680	.476	.578	.337	.357	.204 4				
						47	.148	.686	.473	.451	.573	.391	.369 4			○	
14	150	0	8000	−2400	.209	42	.145	.625	.476	.787	.277	.358	.237 4			○	
						43	.509	.619	.474	.665	.250	.396	.760 3	○		○	
						44	.528	.615	.471	.545	.217	.433	.408 3	○		○	
						45	.454	.613	.468	.428	.185	.471	.248 3	○		◎	
						46	.391	.614	.465	.313	.200	.505	.183 3	○		○	
						47	.320	.626	.461	.197	.424	.538	.401 3	○	◎		
15	150	0	8000	−3400	.188	41	.002	.505	465	.643	.446	.497	.786 3	○			

Case	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
						42	007	.4 3	.461	.526	.421	.533	.197 3	○			
						43	.048	.482	.458	.416	.389	.566	.876 2	◎			
						44	.118	.474	.454	.313	.346	.596	.472 2	◎			
						45	.213	.470	.450	.215	.289	.623	.289 2	◎	○	○	
						46	.336	.474	.446	.124	.219	.647	.210 2	◎	◎	○	
						47	.499	.498	.442	.081	.310	.669	.453 2	◎	◎		
16	150	0	9000	− 400	.272	45	.100	.745	.494	.944	.638	.513	.633 6				
						46	.041	.744	.492	.887	.702	.461	.378 6				
						47	.025	.746	.491	.813	.830	.411	.548 6				
17	150	0	9000	−1400	.237	43	.621	.735	.49	.953	.371	.491	.267 6				
						44	.186	.733	.49	.886	.405	.436	.137 6				
						45	.098	.733	.49	.810	.460	.385	.811 5				
						46	.070	.734	.4	.725	.551	.342	.551 5				
						47	.052	.737	.48	.627	.736	.309	.878 5				
18	150	0	9000	−2400	.209	42	.580	.712	.492	.877	.178	.374	.430 5			◎	
						43	.358	.710	.490	.783	.189	.330	.244 5			◎	
						44	.223	.709	.488	.686	.213	.299	.147 5			○	○
						45	.170	.710	.487	.587	.264	.284	.928 4			○	○
						46	.140	.711	.485	.488	.365	.286	.665 4				○
						47	.113	.717	.483	.385	.602	.302	.112 5				
19	150	0	9000	−3400	.188	41	.275	.666	.387	.748	.282	.282	.551 4			○	○
						42	.557	.663	.484	.636	.284	.284	.314 4			○	○
						43	.499	.661	.482	.526	.203	.303	.192 4			○	
						44	.416	.659	.480	.422	.203	.332	.121 4			○	
						45	.358	.660	.478	..323	.176	.367	.798 3	○		◎	
						46	.310	.664	.475	.231	.206	.403	.598 3	○	○	○	
						47	.253	.675	.472	.159	.445	.440	.113 4		◎		

Case	r_2	r_3	r_4	r_5	σ	v	R^2 μ^d	R^2 μ^e	R^2 μ^{ed}	TU μ^d	TU μ^e	TU μ^{ed}	ϕ		ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
20	200	0	7000	− 400	.190	46	.029	.254	.385	.661	.346	.815	.658	3	○			
						47	.005	.254	.377	.431	.412	.825	.141	3	○			
21	200	0	8000	− 400	.188	46	.404	.354	.432	.681	.230	.704	.369	3	○		○	
						47	.543	.350	.426	.431	.369	.727	.774	2	◎			
22	200	0	8000	−1400	.171	45	.020	.007	.412	.654	.390	.768	.604	3	○			
						46	.002	.000	.404	.466	.329	.785	.120	3	○			
						47	.003	.002	.397	.268	.342	.799	.481	2	◎	○		
23	200	0	8000	−2400	.156	44	.154	.622	.386	.680	.526	.817	.335	4				
						45	.132	.643	.377	.552	.485	.828	.734	3	○			
						46	.116	.649	.368	.433	.431	.837	.264	3	○			
						47	.093	.624	.360	.315	.403	.845	.127	3	○			
24	200	0	9000	− 400	.188	46	.309	.618	.465	.762	.233	.497	.135	4			○	
						47	.215	.618	.460	.547	.463	.543	.518	3	○			
25	200	0	9000	−1400	.171	45	.397	.489	.453	.695	.266	.606	.425	3	○		○	
						46	.547	.475	.447	.493	.212	.642	.894	2	◎		○	
						47	.507	.481	.441	.277	.324	.673	.455	2	◎	○		
26	200	0	9000	−2400	.156	44	.012	.209	.436	.648	.446	698	.562	3	○			
						45	.000	.172	.429	.473	.394	724	.115	3	○			
						46	.008	.152	.422	.312	.318	746	.411	2	◎			
						47	.048	.175	.415	.155	.289	765	.249	2	◎	◎		
27	200	0	9000	−3400	.144	43	.138	.198	.414	.656	.566	.769	.251	4				
						44	.114	.280	.406	.530	.533	.786	.525	3	○			
						45	.099	.334	.397	.421	.488	.800	.200	3	○			
						46	.084	.345	.389	.327	.419	.813	.907	2	◎			
						47	.060	.255	.381	.252	.349	.824	.519	2	◎	○		
28	100	10	6000	− 400	.627	42	.365	.561	.456	.825	.242	.414	.855	5			○	
						43	.485	.559	.454	.767	.270	.432	.128	6			○	

Case	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
						44	.489	.557	.452	.705	.311	.449	.823 6				
						45	.450	.558	.450	.639	.374	.465	.273 6				
						46	402	.562	.448	.567	.474	.481	.299 6				
						47	.344	.574	.446	.479	.677	.496	.410 7				
29	100	10	7000	− 400	.627	42	.506	.624	.465	.868	.315	.317	.578 6				
						43	.503	.622	.464	.818	.349	.329	.920 6				
						44	.424	.622	.462	.764	.395	.342	.170 6				
						45	.356	.623	.460	.707	.458	.357	.295 6				
						46	.301	.626	.459	.644	.554	.372	.125 7				
						47	.247	.635	.457	.566	.735	.387	.579 6				
30	100	10	7000	−1400	.478	40	.456	.667	.478	.850	.223	.294	.229 6			○	○
						41	.546	.666	.477	.801	.240	.294	.158 6			○	○
						42	.475	.665	.476	.750	.264	.297	.117 6			○	○
						43	.390	.665	.474	.696	.297	.305	.933 5			○	
						44	.326	.666	.473	.641	.343	.315	.903 5				
						45	.278	.668	.471	.584	.409	.327	.125 6				
						46	.238	.671	.470	.523	.511	.341	.316 6				
						47	.195	.679	.468	.451	.708	.356	.165 7				
31	100	10	8000	400	.627	42	.552	.664	473	.900	.406	.300	.128 6				
						43	.445	.663	.471	.858	.440	.295	.103 7				○
						44	.341	.663	.470	.813	.483	.294	.890 6				○
						45	.272	.664	.468	.764	.542	.296	.158 7				○
						46	.224	.665	.467	.709	.629	.301	.193 6				
						47	.179	.673	.465	.641	786	.308	.406 6				
32	100	10	8000	−1400	.478	40	.575	.704	.486	.897	.338	.395	.392 6				
						41	.488	.704	.485	.858	.359	.370	.303 6				
						42	.355	.703	.484	.816	.386	.349	.262 6				

Case	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^e$	$TU\mu^e$	$TU\mu^{ed}$
						43	.270	.704	.483	.772	.420	.330	.282 6				
						44	.217	.704	.481	.726	.466	.315	.820 6				
						45	.180	.706	.480	.677	.528	.304	.115 7				
						46	.151	.708	.479	.624	.618	.297	.695 6				○
						47	.121	.714	.478	.562	.782	.293	.319 7				○
33	100	10	9000	−2400	.390	38	.596	.734	.495	.915	.386	.643	.143 7				
						39	.500	.734	.495	.984	.400	.614	.105 7				
						40	.335	.734	.494	.850	416	.585	.816 6				
						41	.237	.734	.494	.814	.438	.555	.661 6				
						42	.181	.735	.493	.776	.465	.526	.568 6				
						43	.145	.735	.492	.737	.499	.497	.582 6				
						44	.121	.736	.492	.697	.543	.469	.724 6				
						45	.102	.737	.491	.655	.601	.442	.174 7				
						46	.086	.739	.490	.611	.684	.416	.136 7				
						47	.069	.742	.489	.560	.826	.392	.496 7				
34	150	1	6000	− 400	.334	44	.100	.388	.440	.767	.245	.645	.113 4				
						45	.341	.375	.43	.634	.227	.665	.280 3				
						46	.479	.369	.431	.496	.251	.682	.112 3				
						47	.520	.381	.427	.335	.453	.698	.126 3				
35	150	10	7000	− 400	.334	44	.530	.580	.462	.829	.190	.475	.302 4				
						45	.454	.574	.459	.712	.217	.505	.124 4			○	
						46	364	.572	.456	.588	.300	.532	.712 3	○			
						47	.299	.579	.453	.444	.534	.558	.109 4				
36	150	10	7000	−1400	.288	43	.175	.504	.457	.734	.270	.550	.898 3	○		○	
						44	.427	.494	.453	.613	.238	.577	.323 3	○		○	
						45	.521	.488	.450	.493	.209	.602	.157 3	○		○	
						46	.524	.487	.446	.370	.221	.624	.959 2	◎		○	

Case	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
						47	.479	.501	.442	.232	.428	.645	.183 3	○	○		
37	150	10	7000	−2400	.254	41	.022	.369	.450	.7511	.434	.611	.975 5				
						42	.001	.349	.446	.6355	.409	.635	.526 3	○			
						43	.005	.332	.442	.5222	.378	.656	.168 3	○			
						44	.029	.318	.438	.415	.339	.676	.750 2	◎			
						45	.072	.310	.434	.312	.289	.694	.392 2	◎		○	
						46	.142	.312	.430	.209	.240	.710	.242 2	◎	○	○	
						47	.292	.340	.425	.105	.353	.724	.405 2	◎	◎		
38	150	10	8000	− 400	.334	44	.460	.669	478	.882	.269	.307	.240 5			○	
						45	.253	.666	475	.790	.325	.330	.123 5				
						46	.188	.666	472	.688	.425	.356	.771 4				
						47	.149	.671	470	.567	.641	.385	.116 5				
39	150	10	8000	−1400	.288	43	.558	.640	.476	.808	.179	.350	.610 4			◎	
						44	.421	.636	.473	.703	.181	.381	.326 4			◎	
						45	.330	.634	.470	.595	.209	.412	.193 4			○	
						46	.277	.635	.468	.485	.294	.444	.134 4			○	
						47	.229	.643	.465	.363	.535	.474	.232 4				
40	150	10	8000	−2400	.254	41	.034	.588	.474	.820	.316	.397	.612 5				
						42	.267	.580	.471	.707	.293	.432	.139 4			○	
						43	.497	.574	.468	.595	.264	.465	.713 3	○		○	
						44	.537	.569	.465	.485	.229	.497	.412 3	○		○	
						45	.510	.566	.462	.379	.195	.527	.255 3	○		◎	
						46	.467	.569	.458	.273	.206	.554	.185 3	○	○	○	
						47	.400	.584	.455	.164	.422	.580	.378 3	○	◎		
41	150	10	8000	−3400	.227	40	.007	.481	.465	.713	.452	.503	.462 4				
						41	.001	.467	.462	.602	.432	.535	.456 3	○			
						42	.022	.455	.458	.496	.407	.565	.192 3	○			

$Case$	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
						43	.068	.444	.455	.396	.374	.592	.101 3	○			
						44	.131	.437	.451	.302	.332	.617	.591 2	◎			
						45	.211	.434	.447	.212	.277	.639	.374 2	◎	○	○	
						46	.316	.439	.443	.127	.218	.659	.275 2	◎	◎	○	
						47	.472	.466	.439	.081	334	.678	.602 2	◎	◎		
42	150		9000	− 400	.334	44	.298	.715	.488	.923	.426	.344	.285 6				
						45	.138	.713	.486	.855	.480	.314	.147 6				
						46	.098	.714	.484	.777	.569	.294	.885 5				○
						47	.074	.717	.482	.681	.746	.286	.122 6				○
43	150	10	9000	−1400	.288	43	.412	.704	.488	.872	.249	.316	.999 5		○		
						44	.224	.702	.486	.790	.285	.295	.548 5		○	○	
						45	.164	.702	.484	.703	.345	.287	.315 5			○	
						46	.133	.703	.482	.611	.446	.291	.206 5			○	
						47	.108	.708	.480	.507	.662	.304	.305 5				
44	150	10	9000	−2400	.254	41	.271	.681	.488	.886	.189	.299	.105 6			◎	○
						42	.554	.678	.486	.793	.181	.288	.250 5			◎	○
						43	.388	.676	.484	.696	.175	.290	.143 5			◎	○
						44	.297	.675	.482	.599	.182	.304	.850 4			◎	
						45	.249	.675	.480	.503	.216	.327	.525 4			○	
						46	.215	.677	.477	.408	.309	.355	.366 4				
						47	.178	.686	.475	.309	.553	.385	.586 4				
45	150	10	9000	−3400	,227	40	.082	.636	.484	.792	.322	.295	.107 5				○
						41	.363	.631	.482	.685	.304	.316	.466 4				
						42	.539	.627	.479	.580	.281	344	.274 4			○	
						43	.529	.624	.477	.480	.251	.375	.167 4			○	
						44	.482	.622	.474	.383	.216	.409	.104 4			○	
						45	.437	.622	.471	.290	.184	.442	.675 3	○	○	◎	

Case	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ		ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
						46	.391	.627	.469	.202	.204	.474	.495	3	○	○	○	
						47	.327	.641	.466	.132	.433	.504	.901	3	○	◎		
46	200	10	7000	− 400	.229	45	.033	.046	.399	.734	.372	.789	.150	4				
						46	.008	.075	.392	.550	.333	.801	.233	3	○			
						47	.000	.061	.384	.348	.397	.812	.873	2	◎			
47	200	10	8000	− 400	.229	45	.136	.341	.436	.763	.266	.682	.923	3	○		○	
						46	.411	.320	.431	.568	.236	.706	.150	3	○		○	
						47	.523	.325	.424	.351	.374	,726	.577	2	◎			
48	200	10	8000	−1400	.220	44	.027	.059	.422	.737	.418	.740	.168	4				
						45	.003	.034	.415	.565	.371	.758	.241	3	○			
						46	.001	.023	.408	.401	.313	.773	.733	2	◎			
						47	.019	.035	.401	.227	.344	.787	.405	2	◎	○		
49	200	10	9000	− 400	.229	45	.533	.578	.463	.818	.185	.510	.186	4			◎	
						46	.366	.570	.458	.643	.221	.550	.439	3	○		○	
						47	.294	.573	.453	.447	.443	,585	.281	3	○			
50	200	10	9000	−1400	.207	44	.118	. 67	.454	.777	.313	.592	.117	4				
						45	.442	.449	449	.599	.265	.626	.196	3	○		○	
						46	.533	.438	.444	.424	.216	.655	.603	2	◎			
						47	.532	.449	.438	.234	.340	.681	.458	2	◎			
51	200	10	9000	−2400	.189	43	.021	.250	.442	.741	.459	.670	.207	4				
						44	.0002	.215	.436	.576	.422	.696	.248	3	○			
						45	.006	.188	.430	.424	.370	.719	.774	2	◎			
						46	.025	.177	.423	.280	297	.739	.332	2	◎	○	○	
						47	.087	.203	.417	.140	.301	.756	.251	2	◎	◎		
52	200	10	9000	−3400	.174	42	.128	.005	.425	.728	.563	.736	.101	5				
						43	.096	.030	.418	.593	.538	.755	.776	3	○			
						44	.078	.064	.411	.476	.504	.771	.250	3	○			

Case	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
						45	.065	.092	.404	.373	.456	.786	.111 3	○			
						46	.052	.094	.397	.285	.387	.798	.564 2	◎	○		
						47	.030	.040	.390	.222	.334	.809	.400 2	◎	○		
53	150	20	7000	−1400	.343	42	.043	.453	.455	.773	.310	.561	.243 4				
						43	.203	.441	.451	.660	.283	.585	.819 3	○		○	
						44	.364	.431	.447	.549	.251	.607	.396 3	○		○	
						45	.461	.425	.444	.439	.222	.628	.213 3	○		○	
						46	.509	.426	.430	.325	.232	.646	.132 3	○		○	
						47	.517	.444	.436	.197	432	.664	.226 3	○	◎		
54	150	20	8000	−1400	.343	42	.352	.594	.472	.829	.215	.396	.163 5			○	
						43	.535	.588	.469	.727	.199	.426	.790 4			◎	
						44	.476	.588	.466	.625	.188	.455	.415 4			◎	
						45	.417	.580	.463	.522	.201	.483	.231 4			○	
						46	.370	.582	.460	.416	.274	.509	.144 4			○	
						47	.316	.593	.457	.299	.511	.534	.208 4		○		
55	150	20	8000	−2400	.300	41	.085	.540	,468	.755	.328	.457	.404 4				
						42	.289	.532	.465	.648	.304	.487	.192 4				
						43	.450	.524	.462	. 45	.274	.515	.105 4			○	
						44	.515	.519	.459	.443	.239	.542	.603 3	○		○	
						45	.528	.517	.456	.344	.206	.566	.361 3	○		○	
						46	.514	.520	.452	.244	.214	.589	.245 3	○	○	○	
						47	.467	.538	.449	.140	.424	.610	.424 3	○	◎		
56	150	20	8000	−3400	.268	40	.001	.440	.462	.675	.443	537	.136 4				
						41	.006	.426	.458	.572	.423	.564	.505 3	○			
						42	.031	.414	.455	.474	.397	.589	.261 3	○			
						43	.071	.404	.451	.382	.364	.612	148 3	○			
						44	.121	.397	.447	.294	.323	.633	.888 2	◎	○		

$Case$	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ		ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
						45	.186	.395	.444	.209	.271	.652	.560	2	◎	○	○	
						46	.278	.401	.440	130	.222	.670	402	2	◎	◎	○	
						47	.431	.430	436	.083	.353	.686	.812	2	◎	◎		
57	150	20	9000	−1400	.343	42	.569	.667	.483	.877	.183	.292	.202	6			◎	○
						43	.404	.664	.481	.792	.196	.298	.976	5			◎	○
						44	.296	.662	.478	.705	.224	.313	.493	5			○	
						45	.245	.661	.476	615	.278	.344	.260	5			○	
						46	.211	.663	.474	.521	.380	.358	.153	5				
						47	.177	.670	.471	.416	.610	.384	.188	5				
58	150	20	9000	−2400	.300	41	.438	.644	.482	.818	.220	.303	.533	5			○	
						42	.535	.640	.480	.723	.204	.322	.278	5			○	
						43	.444	.636	.478	.628	.188	.346	.152	5			◎	
						44	.378	.634	.475	.535	.180	.373	.851	4			◎	
						46	.333	.634	.473	.442	.199	401	.497	4			◎	
						46	296	.637	.470	.350	.280	.430	.321	4			○	
						47	.251	.649	.467	.254	.523	.458	.448	4		○		
59	150	20	9000	−3400	.268	40	.133	.598	.479	.740	.337	.349	.109	5				
						41	.366	.592	.477	.639	.318	.379	.594	4				
						42	.505	.587	.474	.541	.293	.409	.341	4			○	
						43	.535	.583	.471	.446	.263	.440	.201	4			○	
						44	.523	.581	.468	.356	.227	.469	.121	4			○	
						45	.497	.581	.465	.268	.193	.498	.755	3	○	○	◎	
						46	.461	.586	.463	183	.207	.524	.524	3	○	◎	○	
						47	.398	.603	.459	.113	.429	.549	.856	3	○	◎		
60	200	20	8000	−1400	.245	43	.046	.102	.429	.817	.439	.716	.182	5				
						44	.006	.071	.422	.656	.404	.734	.547	3	○			
						45	.000	.050	.416	.504	.358	.751	.144	3	○			

Case	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
						46	.005	.042	.410	.357	.302	.765	.545 2	◎			
						47	.033	.061	.414	.201	.350	.778	.381 2	◎	○		
61	200	20	9000	−1400	.245	43	.006	.443	.456	.853	.348	.582	.105 5				
						44	.220	.423	.451	.691	.312	.613	.463 3	○			
						45	.416	.407	.4 5	.533	.265	641	.131 3	○		○	
						46	.500	.400	.440	.376	.222	.666	.501 2	◎		○	
						47	.536	.414	.435	.204	.353	.688	.494 2	◎	○		
62	200	20	9000	−2400	.223	43	.003	.238	.441	.672	.442	.673	.637 3	○			
						44	.003	.209	.435	.525	.404	.696	.159 3	○			
						45	.016	.189	.429	.389	.353	.716	.610 2	◎			
						46	.040	.182	.424	.259	.285	.734	.295 2	◎	○	○	
						47	.113	.211	.418	.131	.315	.750	.267 . 2	◎	◎		
63	150	30	8000	−1400	.402	41	.090	.541	.467	.858	.260	.436	.140 6			○	
						42	.370	.532	.464	.761	.241	.463	.532 5			○	
						43	.504	.525	.461	.662	.221	.489	.223 5				
						44	.511	.519	.458	.564	.204	.514	1.000 4				
						45	.488	.516	.455	.466	.206	.537	.477 4				
						46	.456	.518	.451	.365	.266	.559	.250 4				
						47	.407	.533	.448	.251	.497	.580	.259 4				
64	150	30	8000	−2400	.350	40	.011	.498	.466	.806	.358	.478	.263 5				
						41	.103	.487	.463	.704	.338	.505	.965 4				
						42	.248	.477	.460	.604	.314	.530	.467 4				
						43	.369	.469	.456	.507	.284	.554	.238 4			○	
						44	.447	.463	.453	.412	.250	.577	.126 4			○	
						45	.494	.461	.450	.319	.217	.597	.694 3	○		○	
						46	.518	.465	.446	.224	.225	.617	.420 3	○	○	○	
						47	.511	.486	.443	.123	.427	.634	.563 3	○	◎		

Case	r_2	r_3	r_4	r_5	σ	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$
	150	30	9000	−1400	.402	41	.277	.622	.478	.894	.191	.319	.536 5			◎	
						42	.540	.617	.475	.810	.186	.339	.541 6			◎	
						43	.456	.613	.473	.722	.186	.362	.211 6			◎	
						44	.385	.610	.470	.634	.200	.386	.905 5			○	
						45	.339	.608	.467	.544	.242	.412	.408 5			○	
						46	.303	.611	.465	.450	.338	.437	.202 5				
						47	.260	.622	.462	.346	.573	.461	.182 5				
66	150	30	9000	−2400	.350	40	.140	.605	.479	.853	.264	.336	.270 6			○	
						41	.450	.599	.476	.760	.247	.360	.117 6			○	
						42	.531	.594	.474	.667	.228	.386	.544 5			○	
						43	.497	.590	.471	.576	.207	.413	.262 5			○	
						44	.455	.587	.468	.486	.191	.440	.133 5			◎	
						45	.418	.586	.465	.396	.196	.466	.706 4			◎	
						46	.381	.590	.463	.307	.265	.491	.406 4			○	
						47	.330	.604	.460	.213	.503	.415	.455 4		○		
67	200	30	9000	−1400	.285	43	.050	.396	.451	.775	.348	.604	.139 4				
						44	.227	.377	.446	.627	.312	.631	.305 3	○			
						45	.358	.362	.441	.484	.267	.655	.110 3	○		○	
						46	.442	.357	.436	.342	.229	.676	.485 2	◎		○	
						47	.516	.375	.431	.184	.366	.695	.552 2	◎	◎		
68	200	30	9000	−2400	.258	42	.015	.247	.444	.764	.457	.654	.265 4				
						43	.000	.218	.439	.621	.429	.677	.369 3	○			
						44	.007	.194	.434	.488	.391	.698	.124 3	○			
						45	.021	.178	.428	.364	.341	.716	.540 2	◎			
						46	.046	.175	.423	.244	.278	.732	.282 2	◎	○	○	
						47	.121	.206	.417	.127	.330	.747	.295 2	◎	◎		

Figures of parameters under which bars are attached give relatively small values for Φ (the smaller Φ, the more favorable). With regard to γ_3, $\overline{\gamma_4}$ and γ_5, the values 0, 9000 and -3400 respectively are terminal values of the ranges for the parameters. Among those ranges, tentative values of parameters were given to compute theoretical values for μ_e , μ_{ed}, μ_d . By employing those theoretical values and the corresponding observed values, numerical values for the objective function Φ were computed. Examining values for Φ, it was found, with respect to $\overline{\gamma_4}$ and γ_5, terminal values 9000 and -3400, respectively, yielded relatively smaller values of Φ.

Hence, we extended the range of trial values of the parameters $\overline{\gamma_4}$ and γ_5. The range for γ_3 was not extended because the analyses in the previous sections show that positive values for γ_3 give relatively favorable results. Thus we extended as shown below ranges for the values of $\overline{\gamma_4}$ and γ_5 tentatively given to compute (new) theoretical values for μ^e, μ^{ed} and μ^d.

γ_2	150			
γ_3	0	10	20	
$\overline{\gamma_4}$	9000	10000	11000	12000
γ_5	-3400	-4400	-5400	-6400

The tentative value for $\bar{h}$ is fixed at 1/3. The computations were conducted for the year 1964. Among the sets of parameters tried (See Tab. Ⅵ-3) the follwing sets were adopted.

Tab. Ⅵ-3

Case	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ		ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$	判定
1	150	0	9000	−3400	.188	44	.422	.208	.332	.121	4			○		
						45	323	.176	.367	.798	3	○		◎		
						46	.231	.206	.403	.598	3	○	○	○		
2	150	0	9000	−3400	.227	44	.442	.202	.430	.826	2	◎		○		
						45	.345	.178	.463	.503	2	◎		◎		
						46	.248	.213	.494	.401	2	◎	○	○		
3	150	0	9000	−3400	.268	44	.470	.210	.516	.697	2	◎		○		
						45	.377	.194	.541	.379	2	◎		◎		
						46	.280	.234	.564	.248	2	◎	○	○		
4	150	0	9000	−4400	.188	44	.231	.321	.540	.317	2	◎	○			
						45	.151	.263	.569	.221	2	◎	◎	○		
						46	.087	.198	.596	.182	2	◎	◎	◎		
5	150	0	9000	−4400	227	44	.293	301	.602	.294	2	◎	○			
						45	.213	.250	.623	.200	2	◎	○	○		
						46	.137	210	.643	.156	2	◎	◎	○		
6	150	0	9000	−4400	③											
7	150	0	9000	−5400	①											
8	150	0	9000	−5400	②											
9	150	0	9000	−5400	③											
10	150	0	9000	−6400	①											
11	150	0	9000	−6400	②											
12	150	0	9000	−6400	③											
13	150	0	10000	−3400	.188	44	.594	.249	.411	.652	5			○		
						45	.508	.308	.362	.403	5					
						46	.425	.415	.323	.275	5					
						44	.570	.198	.267	.862	3	○		◎	○	

Case	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu^e$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$	判定
14	150	0	10000	−3400	.227	45	.483	.251	.271	.621 3	○		○	○	
						46	395	.356	.286	.548 3	○			○	
15	150	0	10000	−3400	.268	.44	.562	.185	.322	.159 3	○		◎		
						.45	.475	.230	.350	.994 2	◎		○		
						.46	386	.330	.378	.811 2	◎				
16	150	0	10000	−4400	.188	.44	.334	.185	.295	.630 3	○		◎		
						.45	.252	.170	.322	.442 3	○	○	◎		
						.46	.178	.233	.353	.360 3	○	◎	○		
17	150	0	10000	−4400	.227	.44	.359	.191	.395	.465 2	◎		◎		
						.45	.276	.175	.426	.311 2	◎	○	◎		
						.46	.193	.224	.456	.284 2	◎	◎	○		
18	150	0	10000	−4400	③										
19	150	0	10000	−5400	①										
20	150	0	10000	−5400	②										
21	150	0	10000	−5400	③										
22	150	0	10000	−6400	①										
23					②										
24					③										
25	150	0	11000	−3400	.188	44	.755	.523	.738	.653 6					
						45	.691	.579	.692	.404 6					
						46	.624	.664	.642	.138 7					
26	150	0	11000	−3400	.227	.44	.703	.390	.470	.133 5					
						.45	.632	.454	.423	.953 4					
						.46	.558	.553	.379	.792 4					
27	150	0	11000	−3400	.268	.44	.667	.307	.275	.711 3	○			○	
						.45	.592	.373	.260	.519 3	○			○	
	150	0	11000	−3400	.268	46	.514	.479	.255	.489 3	○			○	

Case	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu^{d}$	$TU\ \mu^{ed}$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\ \mu^{d}$	$TU\ \mu^{e}$	$TU\mu^{ed}$	判定
28	150	0	11000	−4400	.188	44	.506	.259	.431	.210 5					
						45	.431	.323	.382	.142 5					
						46	.360	.434	.340	.106 5					
29	150	0	11000	−4400	.227	44	.483	.201	.261	.391 3	○		○	○	
						45	.40	.259	.260	.300 3	○		○	○	
						46	.329	.368	.270	.291 3	○			○	
30	150	0	11000	−4400	.268	44	.479	.188	.310	.828 2	◎		◎		
						45	.402	.236	.335	.557 2	◎		○		
						46	.323	.388	.362	.525 2	◎				
31	150	0	11000	−5400	.188	44	.268	.171	.271	.379 3	○	○	◎	○	☆
						45	.199	.171	.289	.280 3	○	◎	◎	○	☆
						46	.142	.245	.313	.246 3	○	◎	○		
32	150	0	11000	−5400	②										
33					③										
34	150	0	11000	−6400	①										
35					②										
36					③										
37	150	0	12000	−3400	.188	44	.873	.762	.913	.263 7					
						45	.833	.795	.892	.241 7					
						46	.789	.844	.867	.256 7					
38	150	0	12000	−3400	.227	44	.813	.607	.725	.223 6					
						45	.761	.656	.685	.152 6					
						46	.706	.729	.643	.117 6					
39	150	0	12000	−3400	.268	44	.763	.481	.476	.425 4					
						45	.704	.541	.434	.328 4					
	150	0	12000	−3400	.268	46	.640	.630	.394	.313 4					
40	150	0	12000	−4400	.188	44	.678	.517	.735	.750 6					

$Case$	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ		ϕ	$TU\ \mu^d$	$TU\ \mu^e$		判定
						45	.617	.577	.692	.469	6					
						46	.555	.666	.646	.327	6					
41	150	0	12000	−4400	.227	44	.620	.382	.460	.448	4					
						45	.554	.449	.416	.343	4					
						46	.486	.552	.375	.314	4					
42	150	0	12000	−4400	.268	44	.584	.300	.264	.320	3	○			○	
						45	.515	.368	.252	.250	3	○			○	
						46	.443	.476	.248	.261	3	○			○	
43	150	0	12000	−5400	.188	44	.433	.271	.448	.884	4			○		
						45	.367	.339	.400	.632	4					
						46	.306	.453	.357	.511	4					
44	150	0	12000	−5400	.227	44	.412	.208	.257	.210	3	○		○	○	
						45	.343	.269	.252	.170	3	○		○	○	
						46	.276	.380	.258	.182	3	○	○		○	
45	150	0	12000	−5400	③											
46	150	0	12000	−6400	.188	44	.217	.164	.260	.254	3	○	○	◎	○	☆
						45	.159	.180	.267	.196	3	○	◎	◎	○	☆
						46	.118	.271	.283	.184	3	○	◎	○	○	☆
47	150	0	12000	−6400	②											
48					③											
49	150	10	9000	−3400	.188	44	.370	.227	.328	.127	6			○		
						45	.281	.193	.357	.678	5		○	◎		
						46	.207	.206	.391	.380	5		○	○		
50	150	10	9000	−3400	.227	44	.383	.216	.409	.104	4			○		
50	150	10	9000	−3400	.227	45	.290	.184	.442	.675	3	○	○	◎		
						46	.202	.204	.474	.495	3	○	○	○		
51	150	10	9000	−3400	.268	44	.406	.215	.489	.801	2	◎		○		

Case	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\ \mu^d$	$TU\ \mu^e$	$TU\mu^{ed}$	判定
						45	.315	.189	.516	.511 2	◎		◎		
						46	.222	.216	.541	.413 2	◎	○	○		
52	150	10	9000	−4400	.188	44	.183	.333	.515	.606 3	○	◎			
						45	.110	.273	.547	.382 3	○	◎	○		
						46	.074	.202	.576	.262 3	○	◎	○		
53	150	10	9000	−4400	.227	44	.238	.309	.576	. 2	◎	○			
						45	.161	.254	.600	. 2	◎	◎	○		
						46	.096	.202	.621	. 2	◎	◎	○		
54	150	10	9000	−4400	③										
55	150	10	9000	−5400	①										
56					②										
57					③										
58	150	10	9000	−6400	①										
59					②										
60					③										
61	150	10	10000	−3400	.188	44	.541	.242	.456	.394 6			○		
						45	.460	.296	.403	.570 6			○		
						46	387	.399	.358	.362 6					
62	150	10	10000	−3400	.227	44	.517	.194	.293	.270 5			◎	○	
						45	.433	240	.287	.168 5			○	○	
						46	.352	.341	.293	.114 5				○	
63	150	10	10000	−3400	.268	44	.506	.177	.310	.638 3	○		◎		
						45	.421	.215	.335	.451 3	○		○		
	150	10	10000	− 3400	.268	46	.336	.311	.362	.385 3	○				
64	150	10	10000	−4400	.188	44	.297	.204	.301	.314 5		○	○		
						45	.224	.182	.319	.187 5		○	◎		
						46	.169	.221	.346	.118 5		◎	○		

Case	*r*	r_3	r_4	r_5	σ	v	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	判定
65	150	10	10000	−4400	.227	44	.312	.200	.370	.396 3	○		○		
						45	.232	.175	.405	.276 3	○	○	◎		
						46	.159	.214	.436	225 3	○	◎	○		
66	150	10	.0000	−4400	③										
67	150	10	10000	−5400	.188	44	.133	.304	.462	.337 3	○	◎			
						45	.080	.245	.494	227 3	○	◎	○		
						46	.084	.186	.524	.169 3	○	◎	◎		
68	150	10	10000	−5400	②										
69	150	10	10000	−5400	③										
70	150	10	10000	−6400	①										
71	150	10	10000	−6400	②										
72	150	10	10000	−6400	③										
73	150	10	11000	−3400	.188	44	706	.498	.764	.207 7					
						45	.640	.556	.718	.211 7					
						46	.575	.647	.668	.246 7					
74	150	10	11000	−3400	.227	44	.655	.370	.513	.714 6					
						45	.584	.434	.464	.412 6					
						46	.512	.536	.417	.258 6					
75	150	10	11000	−3400	.268	44	.618	.288	.314	.654 4					
						45	.544	.354	.291	.458 4				○	
						46	.468	.461	.277	.372 4				○	
76	150	10	11000	−4400	.168	44	.467	.252	.476	.213 6			○		
	150	10	11000	−4400	.188	45	.397	.312	.424	.971 6					
						46	.335	.422	.378	.571 6					
77	150	10	11000	−4400	.227	44	.441	.196	.291	.677 4			◎	○	
						45	.367	.248	.280	.461 4			○	○	
						46	.297	.355	.281	.352 4		○		○	

Case	r_2	r_3	r_4	r_5	a	v	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^e$	$TU\mu^e$	$TU\mu^{ed}$	判定
78	150	10	11000	−4400	.268	44	.434	.178	.298	.241 3	○		◎	○	
						45	.358	.220	.320	.181 3	○		○		
						46	.282	.321	.345	.173 3	○	○			
79	150	10	11000	−5400	.188	44	.241	.185	.287	.110 5		○	◎	○	△
						45	.181	.178	.294	.709 4		◎	◎	○	△
						46	.142	.242	.311	.492 4		◎	○		
80	150	10	11000	−5400	.227	44	.257	.186	.345	.189 3	○	○	◎		
						45	.188	.172	.374	.140 3	○	◎	◎		
						46	.127	.228	.403	.125 3	○	◎	○		
81	150	10	11000	−5400	③										
82	150	10	11000	−6400	①										
83					②										
84					③										
85	150	10	12000	−3400	.188	44	.836	.740	.923	.377 7					
						45	791	.777	.902	.421 7					
						46	.743	.832	.878	.327 7					
86	150	10	12000	−3400	.227	44	.774	.585	.752	.102 7					
						45	.719	.637	.712	.657 6					
						46	.662	.714	.670	.102 7					
87	150	10	12000	−3400	.268	44	.722	.460	.519	.679 5					
						45	.661	.521	.475	.456 5					
	150	10	12000	−3400	.268	46	.596	.614	.433	.346 5					
88	150	10	12000	−4400	.188	44	.636	.497	.761	.443 6					
						45	.575	.559	.718	.418 6					
						46	.516	.662	.672	.185 7					
89	150	10	12000	−4400	.227	44	.580	.366	.504	.115 6					
						45	.515	.433	.458	.743 5					

Case	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$	判定
						46	450	.538	.414	.526 5					
90	150	10	12000	−4400	.268	44	.543	.284	.301	.184 4			○		
						45	.475	.351	.281	.140 4				○	
						46	.405	.461	.269	.128 4				○	
91	150	10	12000	−5400	.188	44	.404	.264	.492	.372 6			○		
						45	.343	.330	.442	.228 6					
						46	.290	.443	.396	.146 6		○			
92	150	10	12000	−5400	.227	44	.378	.201	.289	.238 4			○	○	
						45	.313	.259	.275	.174 4			○	○	
						46	.251	.370	.271	.146 4		○		○	
93	150	10	12000	−5400	③										
94	150	10	12000	−6400	.188	44	.197	.173	.284	.486 4		◎	◎	○	△
						45	.149	.182	.280	.334 4		◎	◎	○	△
						46	.129	.266	.288	.250 4		◎	○	○	△
95	150	10	12000	−6400	②										
96					③										
97	150	20	99000	−3400	.188	44	.334	.252	.330	.550 6			○		
						45	.258	.214	.354	.255 6		○	○		
						46	206	.213	.385	.517 6		○	○		
98	150	20	9000	−3400	.227	44	.339	.235	.395	.848 5			○		
	150	20	99000	−3400	.227	45	.254	.198	.427	.442 5		○	◎		
						46	.181	.203	.460	.243 5		◎	○		
99	150	20	99000	−3400	.268	44	.356	.227	.469	.121 4			○		
						45	.268	.193	.498	.755 3	○	○	◎		
						46	.183	.207	.524	.524 3	○	◎	○		
100	150	20	9000	−4400	.188	44	.155	.347	.496	.581 5		◎			
						45	.096	.288	.529	.297 5		◎	○		

Case	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	判定
						46	.094	.214	.560	.159 5		◎	○		
101	150	20	9000	− 400	.227	44	.196	.319	.556	.562 3	○	◎			
						45	.124	.262	.581	.350 3	○	◎	○		
						46	.077	.204	.605	.238 3		◎	○		
102	150	20	9000	−4400	③										
103	150	20	9000	−5400	①										
104					②										
105					③										
106	150	20	9000	−6400	①										
107					②										
108					③										
109	150	20	10000	−3400	.188	44	.497	.241	.492	.175 7			○		
						45	.424	.287	.436	.206 7			○		
						46	.362	.386	.387	.252 7					
110	150	20	10000	−3400	.227	44	.474	.198	.321	.246 6			◎		
						45	.395	.234	.306	.131 6			○		
						46	.324	.329	.305	.905 6					
111	150	20	10000	−3400	.268	44	.461	.179	.309	.192 5			◎		
						45	.379	.208	.329	.115 5			○		
	150	20	10000	−3400	.268	46	.301	.298	.354	.752 4			○		
						44	.274	.225	.312	.332 6		○	○		
						45	.213	.198	.323	.169 6		○	◎		
						46	.179	.224	.345	.131 7		◎	○		
113	150	20	100000	−4400	.227	44	.278	.214	.362	.141 5		○	○		
						45	.205	.184	.392	.842 4		○	◎		
						46	.146	.242	.422	.536 4		◎	○		
114	150	20	10000	−4400	③										

Case	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\ \mu^d$	$TU\ \mu^e$	$TU\mu^{ed}$	判定
115	150	20	10000	−5400	.188	44	.111	.317	.444	.160 5		◎			
						45	.077	.258	.477	.919 4		◎	○		
						46	.105	.195	.509	.552 4		◎	◎		
116	150	20	10000	−5400	②										
117					③										
118	150	20	10000	−6400	①										
119					②										
120					③										
121	150	20	1100	−3400	.188	44	.658	.473	.784	178 7					
						45	.593	.544	.738	230 7					
						46	532	.629	.689	173 7					
122	150	20	11000	−3400	.227	44	.611	.353	.548	.191 6					
						45	.542	.417	.497	.135 7					
						46	.475	.521	.448	.113 7					
123	150	20	11000	−3400	268	44	.576	.275	.349	.297 6			○		
						45	.503	.338	.320	.167 6					
						46	.431	.446	.301	.102 6					
124	150	20	11000	−4400	.188	44	.435	.249	.511	.321 6			○		
	150	20	11000	−4400	.188	45	.372	.304	.457	.329 6					
						46	.321	411	.408	.879 6					
125	150	20	11000	−4400	.227	44	.408	.198	.320	.353 6			◎		
						45	.339	.243	.302	.201 6			○		
						46	.278	.345	.295	.117 6		○		○	
126	150	20	11000	−4400	.268	44	.397	.177	.297	.358 4			◎	○	
						45	.323	.212	.315	.240 4			○		
						46	.254	.309	.337	.181 4		○			
127	150	20	11000	−5400	.188	44	.227	.204	.306	.855 6		○	○		

Case	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\mu^d$	$TU\ \mu^e$	$TU\mu^{ed}$	判定
						45	.179	.190	.305	.447 6		◎	◎		
						46	.160	142	.316	.249 6		◎	○		
128	150	20	11000	−5400	.227	44	.229	.197	.336	.382 4		○	◎		
						45	.166	.177	.362	.250 4		◎	◎		
						46	.121	.223	.390	.178 4		◎	○		
129	150	20	11000	−5400	③										
130	150	20	11000	−6400	①										
131					②										
132					③										
133	150	20	12000	−3400	.188	44	.796	.716	.930	.382 7					
						45	.748	.758	.910	.546 7					
						46	.698	.818	.886	.229 7					
134	150	20	12000	−3400	.227	44	.735	.563	.772	167 7					
						45	.678	.618	.733	.279 7					
						46	.621	.700	.691	.585 7					
135	150	20	12000	−3400	.268	44	.684	.441	.553	.431 6					
						45	.622	.504	.508	.259 6					
	150	20	12000	−3400	.268	46	.558	.600	.465	.139 7					
136	150	20	12000	−4400	.188	44	.597	.476	.781	.195 7					
						45	.538	541	.738	.384 7					
						46	.482	.638	.692	.115 7					
137	150	20	12000	−4400	.227	44	.545	.352	.539	.809 6					
						45	.492	.419	.491	.464 6					
						46	,421	.526	.446	.322 6					
138	150	20	12000	−4400	.268	44	.509	.272	.336	.371 5			○		
						45	.442	.338	.310	.240 5					
						46	376	.448	.292	.170 5				○	

Case	r_2	r_3	r_4	r_5	σ	v	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	判定
139	150	20	1200	−5400	.188	44	.382	.260	.527	.116 7					
						45	.328	.322	486	.641 6					
						46	.285	.434	.427	.486 6		○			
140	150	20	12000	−5400	.227	44	.353	.200	.320	.629 5			○		
						45	.292	.253	.300	392 5		○	○		
						46	.239	.361	.289	.263 5		○		○	
141	150	20	12000	−5400	③										
142	150	20	12000	−6400	.188	44	.190	.188	.310	198 6		◎	◎		
						45	.154	.190	.298	115 6		◎	◎	○	△
						46	.148	.264	.299	709 5		◎	○	○	△
143	150	20	12000	−6400	②										
144	150	20	12000	−6400	③										

Case		γ_2	γ_3	$\overline{\gamma_4}$	γ_5	σ_u	v			Φ		
①	31	150	0	11000	-5400	0.188	44	45		.379	.280	
②	46	150	0	12000	-6400	0.188	44	45	46	.254	.196	.184
③	79	150	10	11000	-5400	0.188	44	45		.110	.709	
④	94	150	10	12000	-6400	0.188	44	45	46	.486	.334	.250
⑤	142	150	20	12000	-6400	0.188		45	46	.115	.709	

The observed and theoretical values of participation rates computed by employing the above parameters are shown in Fig. Ⅵ-13.

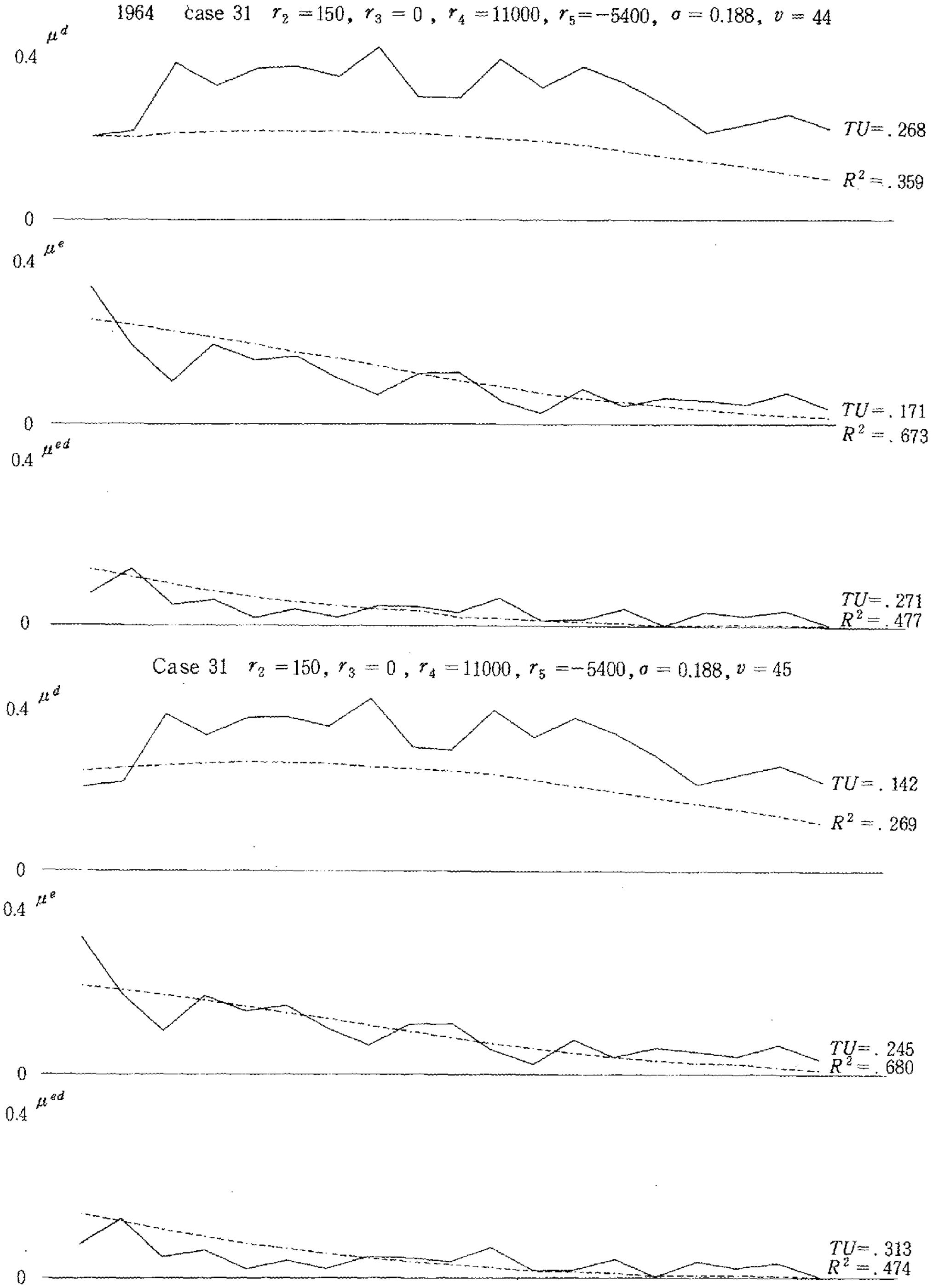
The next step of the analysis is to search for the value of v for the years 1961 through 1963. We used numerical values of the preference parameters shown as ①~⑤ in the above table. The range of trial values of v was $30 \leqq v \leqq 50$ and intervals of the v values were set at 1. From Tab. Ⅵ-4 it was found that the ranges of v satisfying restrictions 1 through 7 were as follows.

Parameter set	Ranges		
①	$36 \leqq v_{63} \leqq 44$	$34 \leqq v_{62} \leqq 41$	$30 \leqq v_{61} \leqq 38$
②	$34 \leqq v_{63} \leqq 44$	$33 \leqq v_{62} \leqq 41$	$30 \leqq v_{61} \leqq 38$
③	$35 \leqq v_{63} \leqq 44$	$33 \leqq v_{62} \leqq 41$	$30 \leqq v_{61} \leqq 38$
④	$34 \leqq v_{63} \leqq 44$	$32 \leqq v_{62} \leqq 41$	$30 \leqq v_{61} \leqq 38$
⑤(*)	$34 \leqq v_{63} \leqq 44$	$32 \leqq v_{62} \leqq 41$	$30 \leqq v_{61} \leqq 38$

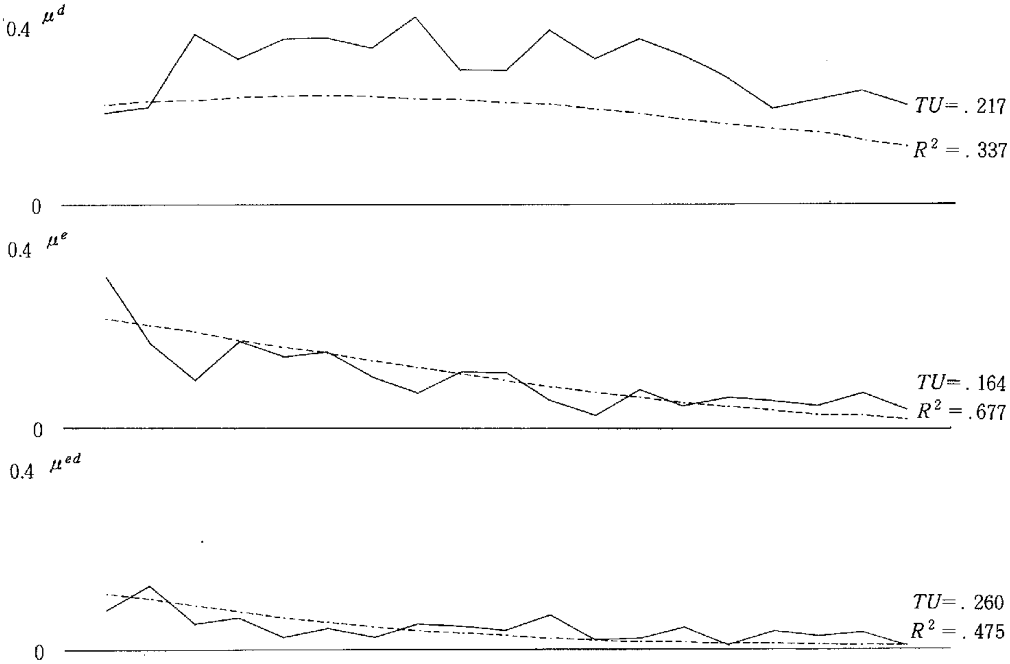
(*) ①~⑤ correspond to those in the above table.

Fig. VI-13 (a)

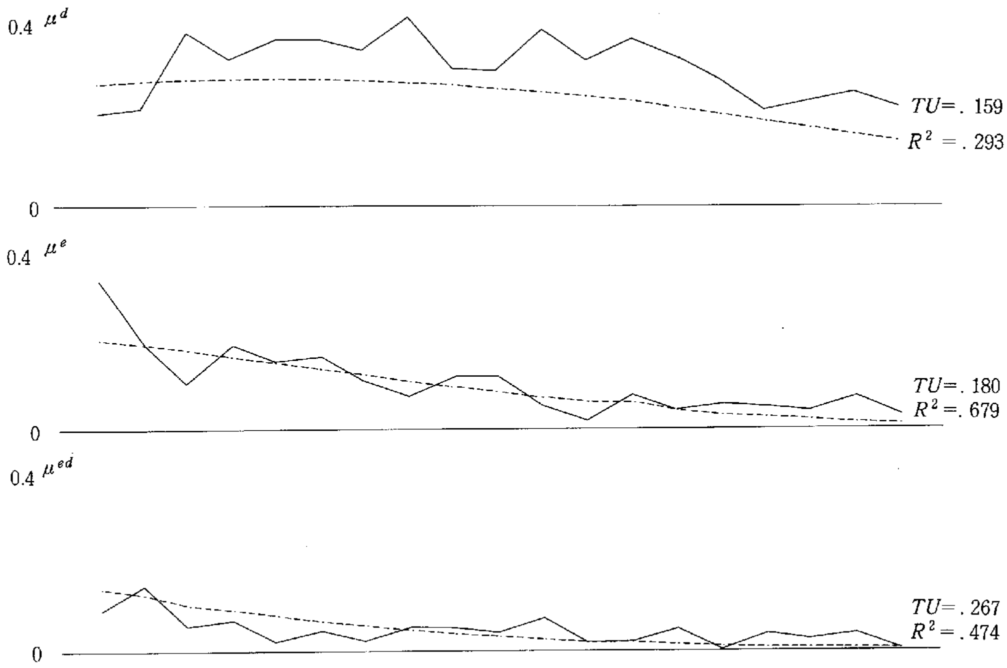
VI—13 (#79—01—23)



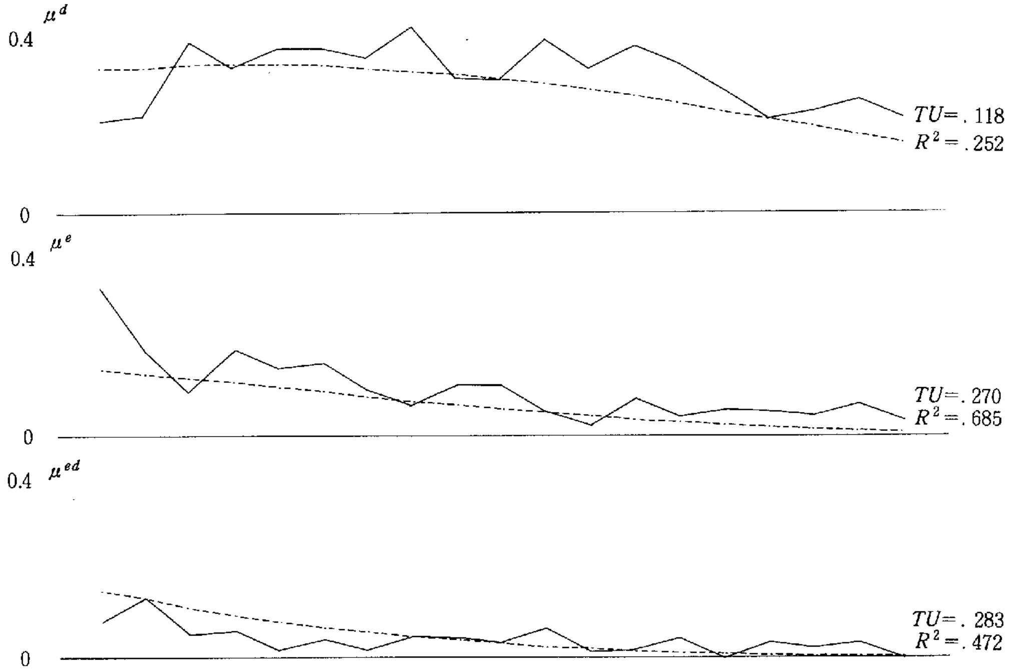
Case 46 $r_2 = 150$, $r_3 = 0$, $r_4 = 12000$, $r_5 = -6400$, $\sigma = 0.188$, $v = 44$



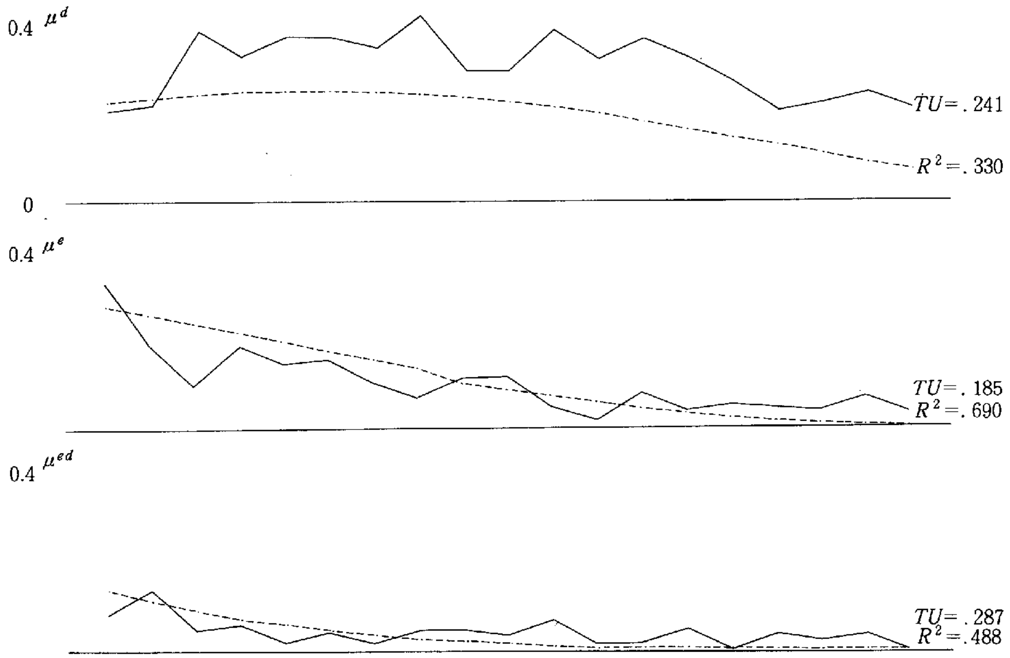
Case 46 $r_2 = 150$, $r_3 = 0$, $r_4 = 12000$, $r_5 = -6400$, $\sigma = 0.188$, $v = 45$



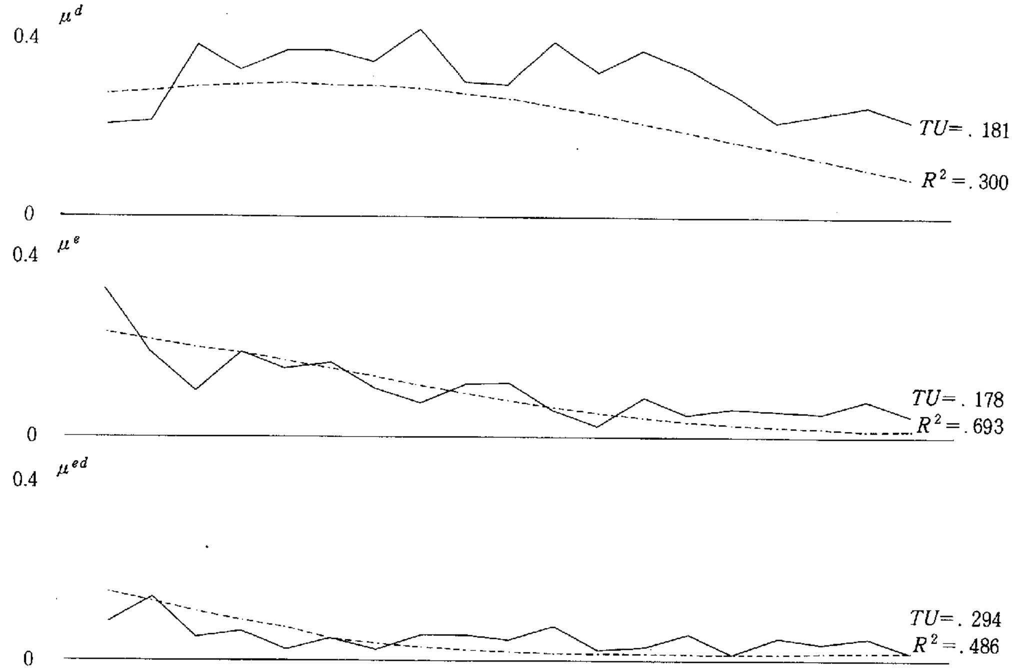
Case 46 $r_2 = 150$, $r_3 = 0$, $r_4 = 12000$, $r_5 = -6400$, $\sigma = 0.188$, $v = 46$



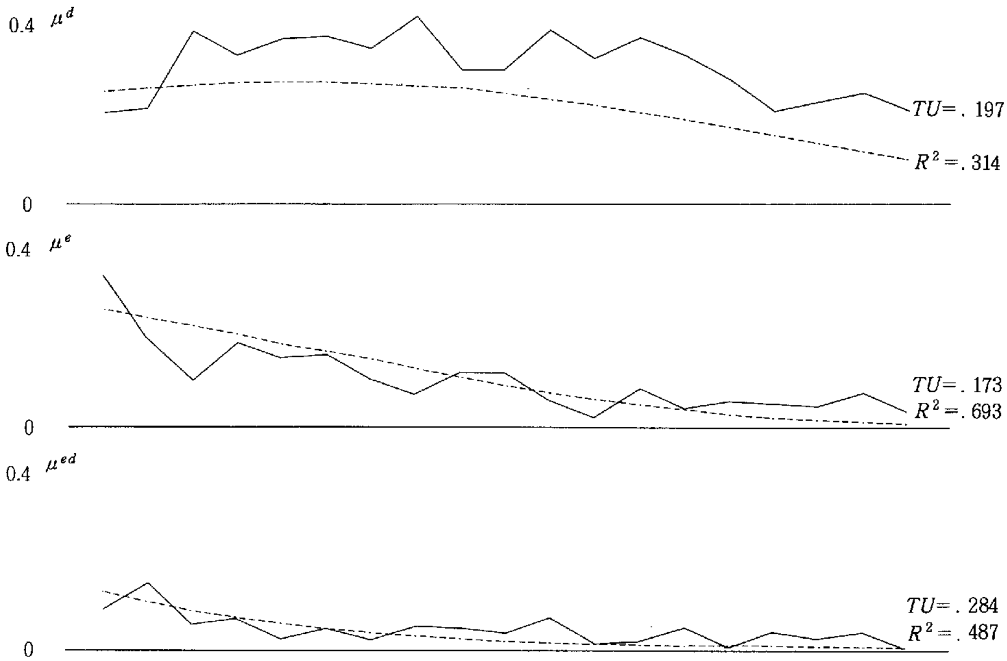
Case 79 $r_2 = 150$, $r_3 = 10$, $r_4 = 11000$, $r_5 = -5400$, $\sigma = 0.188$, $v = 44$



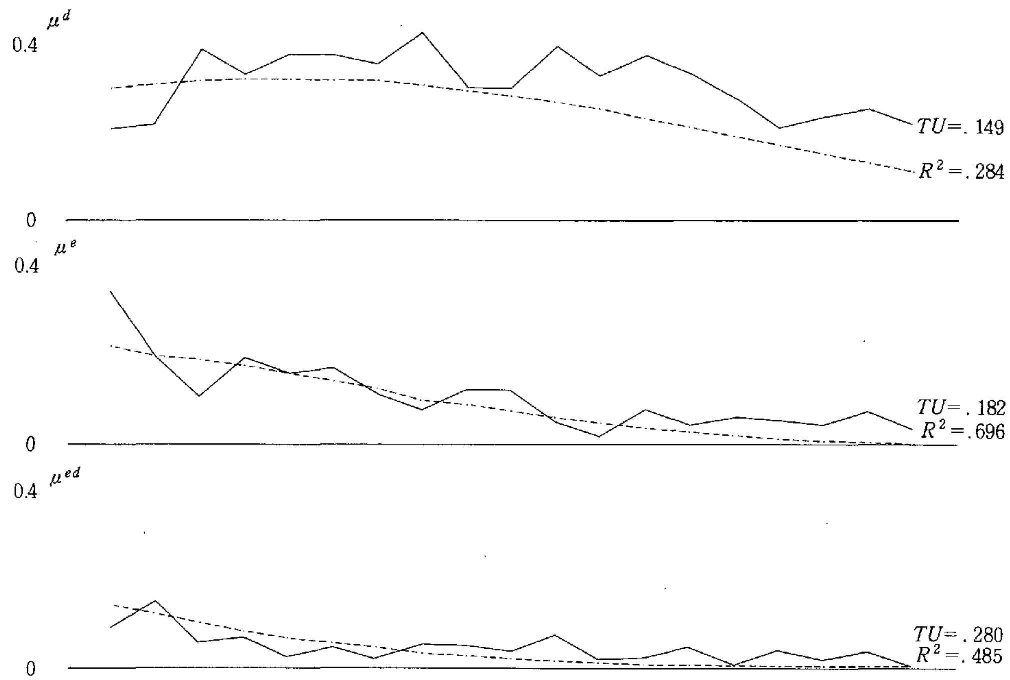
Case 79 $r_2 = 150$, $r_3 = 10$, $r_4 = 11000$, $r_5 = -5400$, $\sigma = 0.188$, $v = 45$



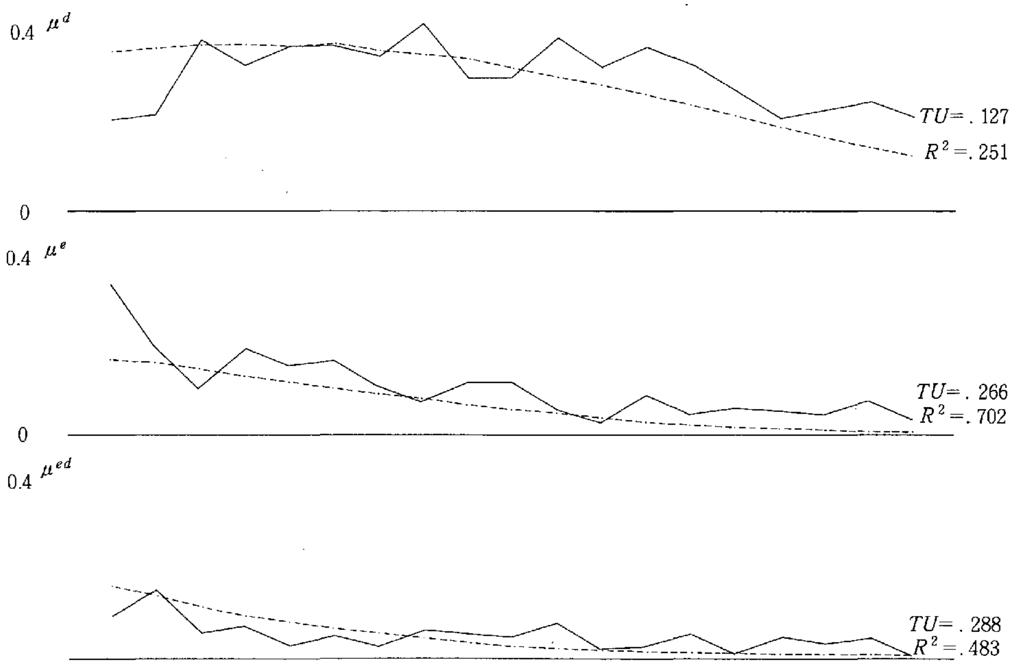
Case 94 $r_2 = 150$, $r_3 = 10$, $r_4 = 12000$, $r_5 = -6400$, $\sigma = 0.188$, $v = 44$



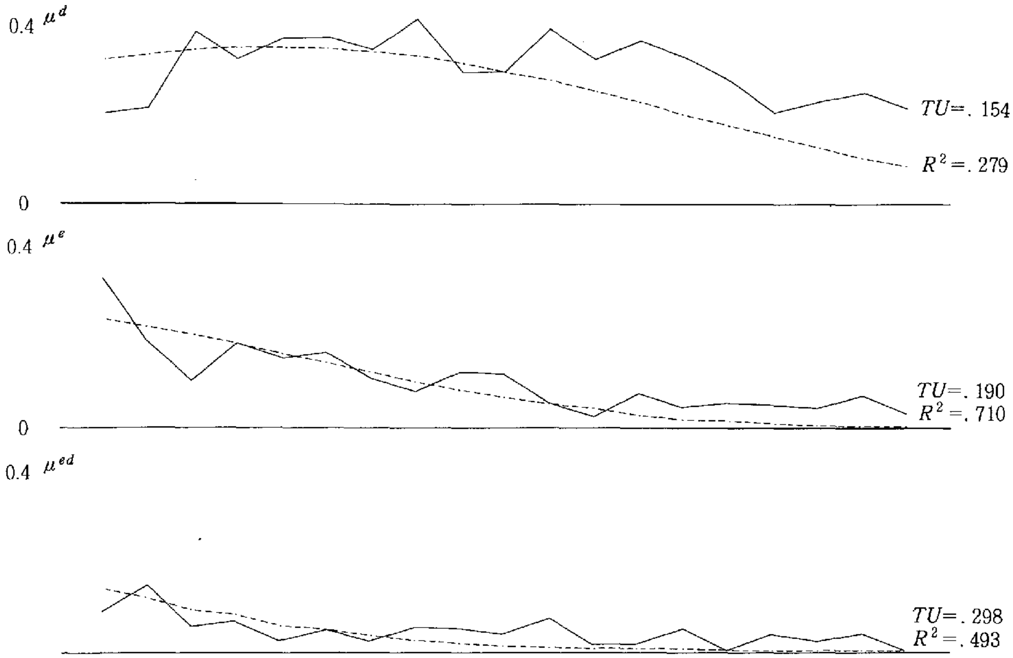
Case 94 $r_2=150$, $r_3=10$, $r_4=12000$, $r_5=-6400$, $\sigma=0.188$, $v=45$



Case 94 $r_2=150$, $r_3=10$, $r_4=12000$, $r_5=-6400$, $\sigma=0.188$, $v=46$



Case 142 $r_2 = 150$, $r_3 = 20$, $r_4 = 12000$, $r_5 = -6400$, $\sigma = 0.188$, $v = 45$



Case 142 $r_2 = 150$, $r_3 = 20$, $r_4 = 12000$, $r_5 = -6400$, $\sigma = 0.188$, $v = 46$

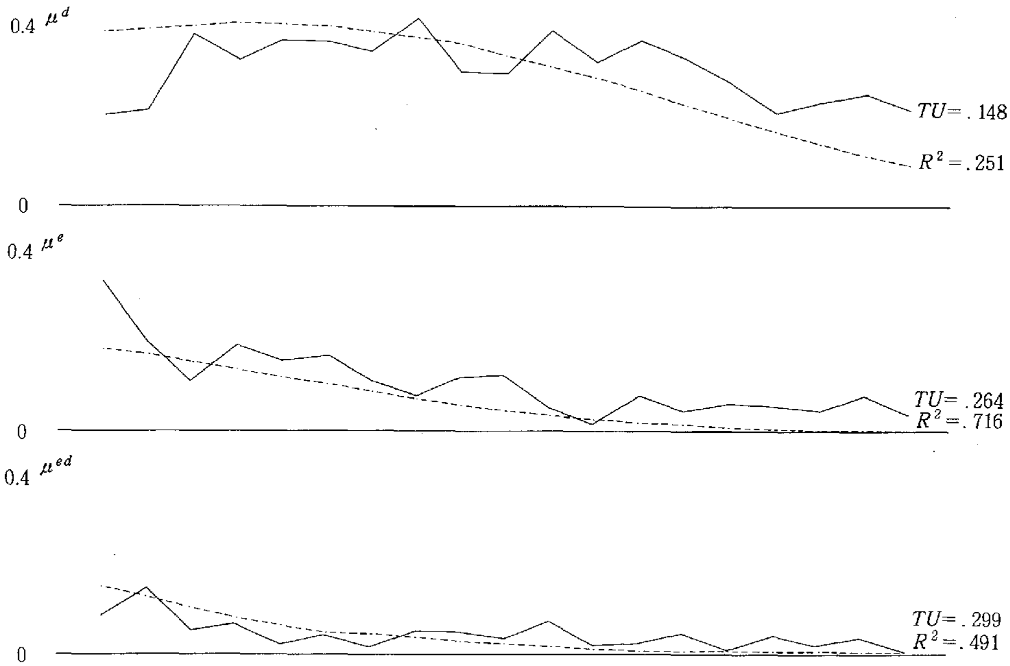


Fig. Ⅵ-13 (b)

plots of the estimated non-participation ratios of the four selected cases and the actual non-participation ratio

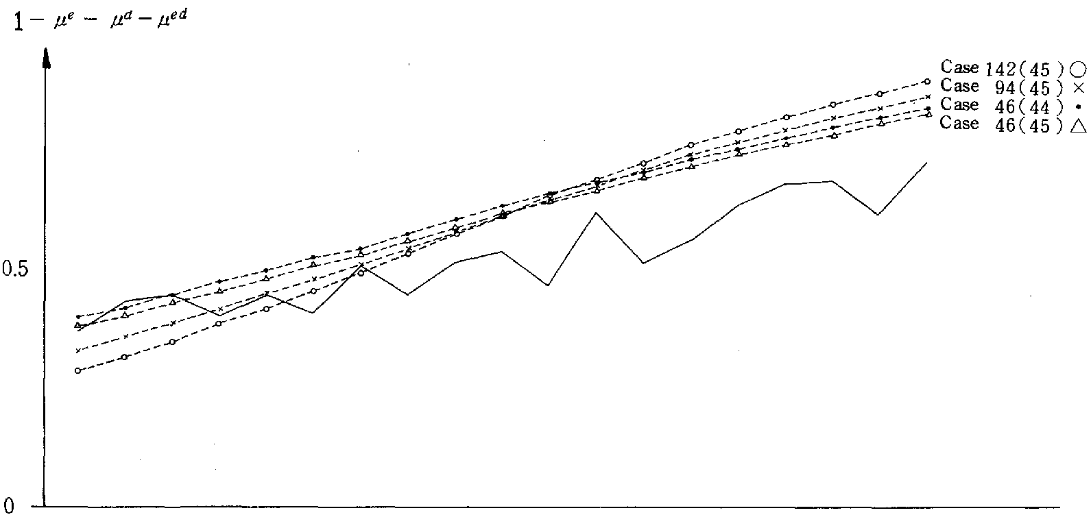
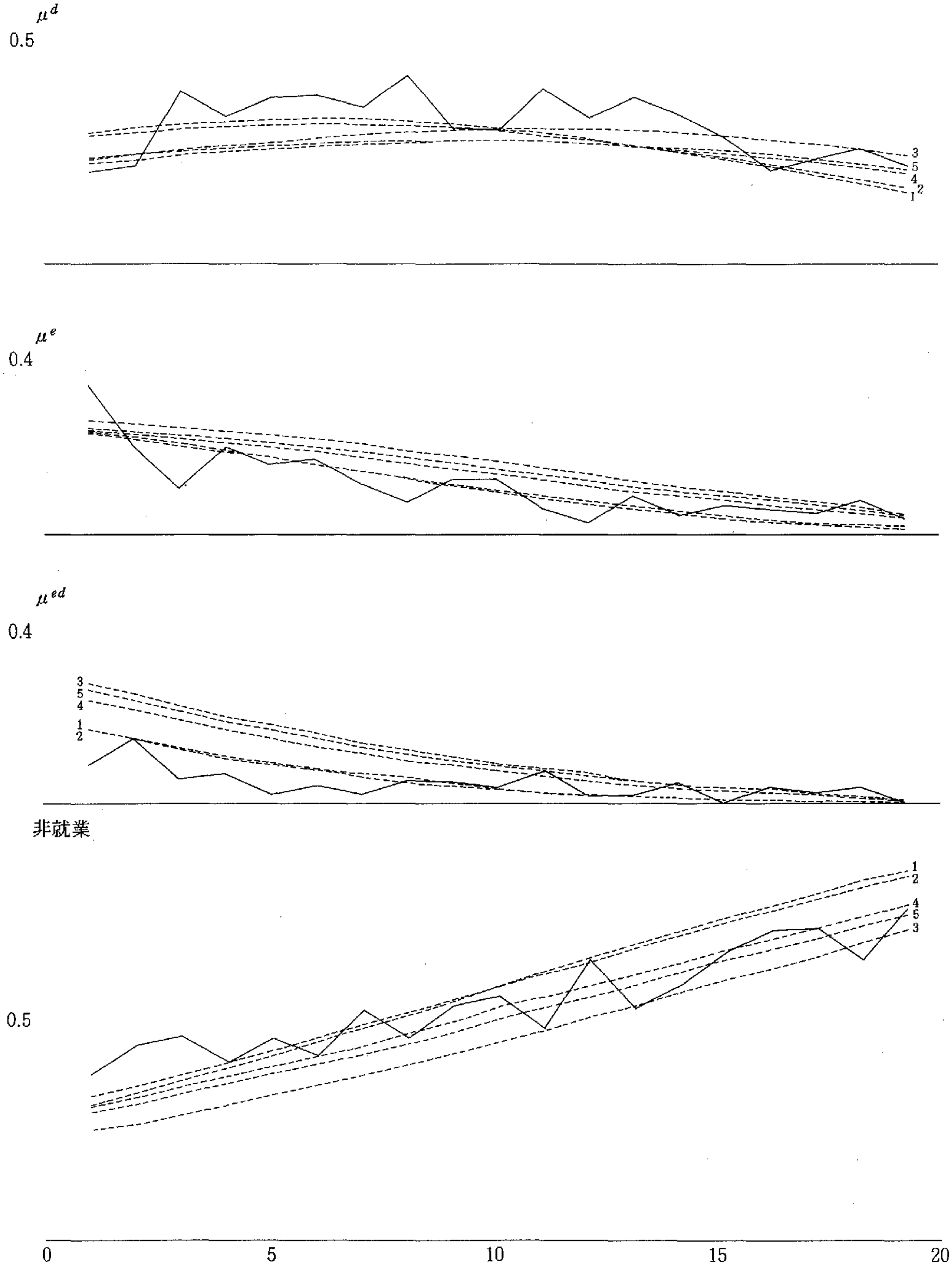
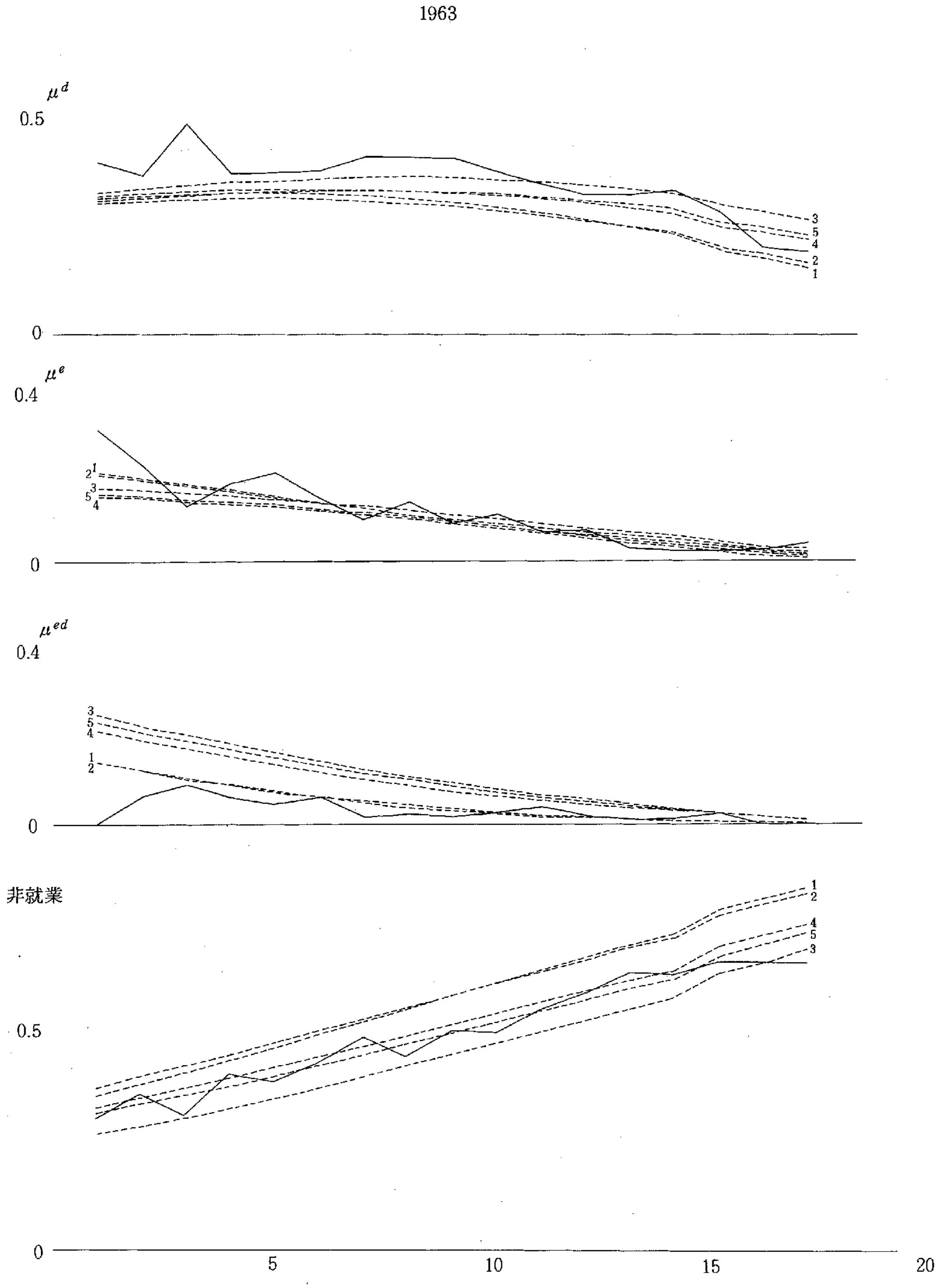
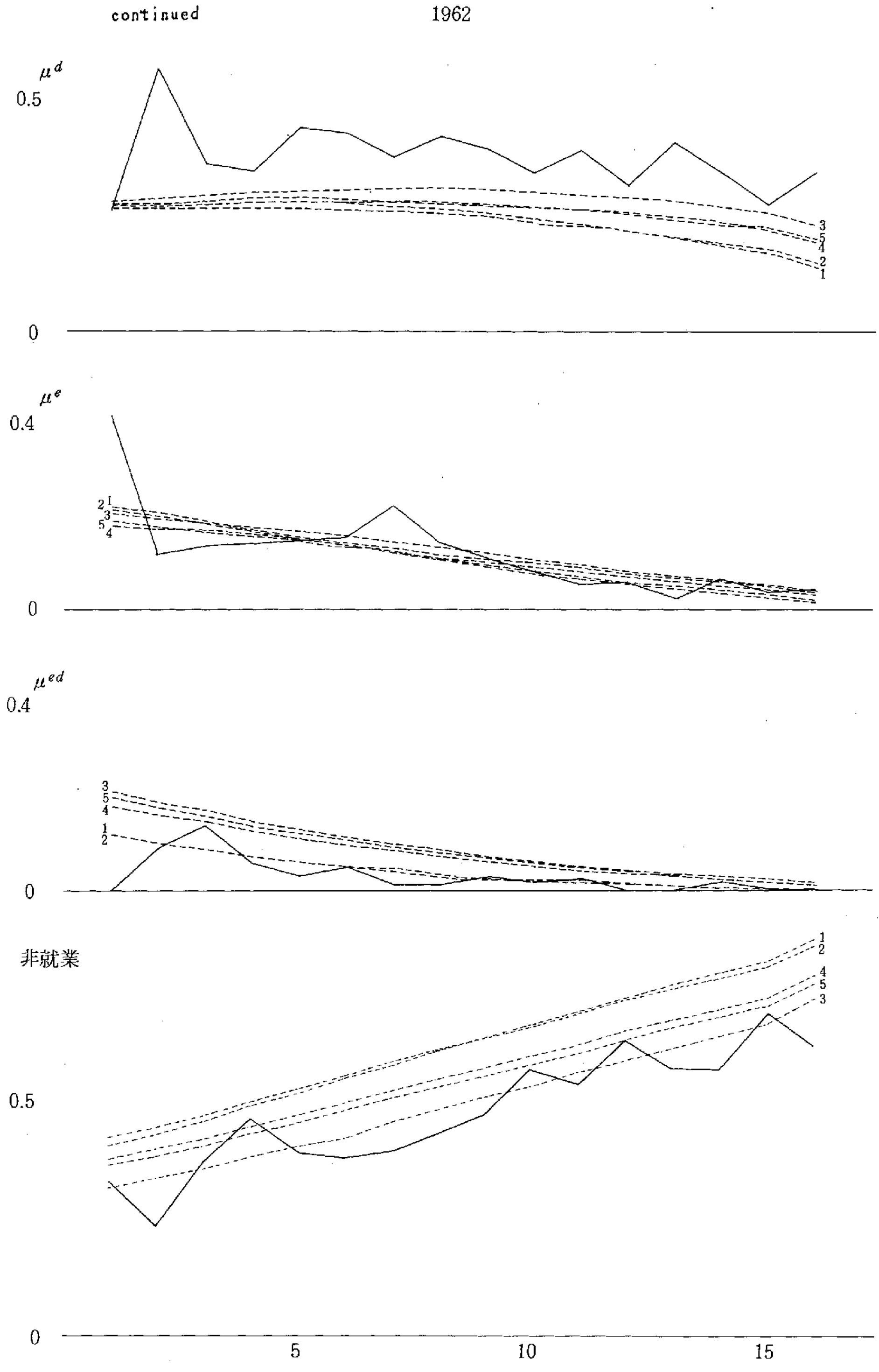


Fig. Ⅵ-14

1964　（#79—08—11）




1963
μ^d
0.5
0
3
5
4
2
1
μ^e
0.4
0
1
2
3
5
4
μ^ed
0.4
0
3
5
4
1
2
非就業
0.5
0
1
2
4
5
3
5
10
15
20



continued
1962
μ^d
0.5
0
3
5
4
2
1
μ^e
0.4
0
2 1
3
5
4
μ^ed
0.4
0
3
5
4
1
2
非就業
0.5
0
1
2
4
5
3
5
10
15

1961

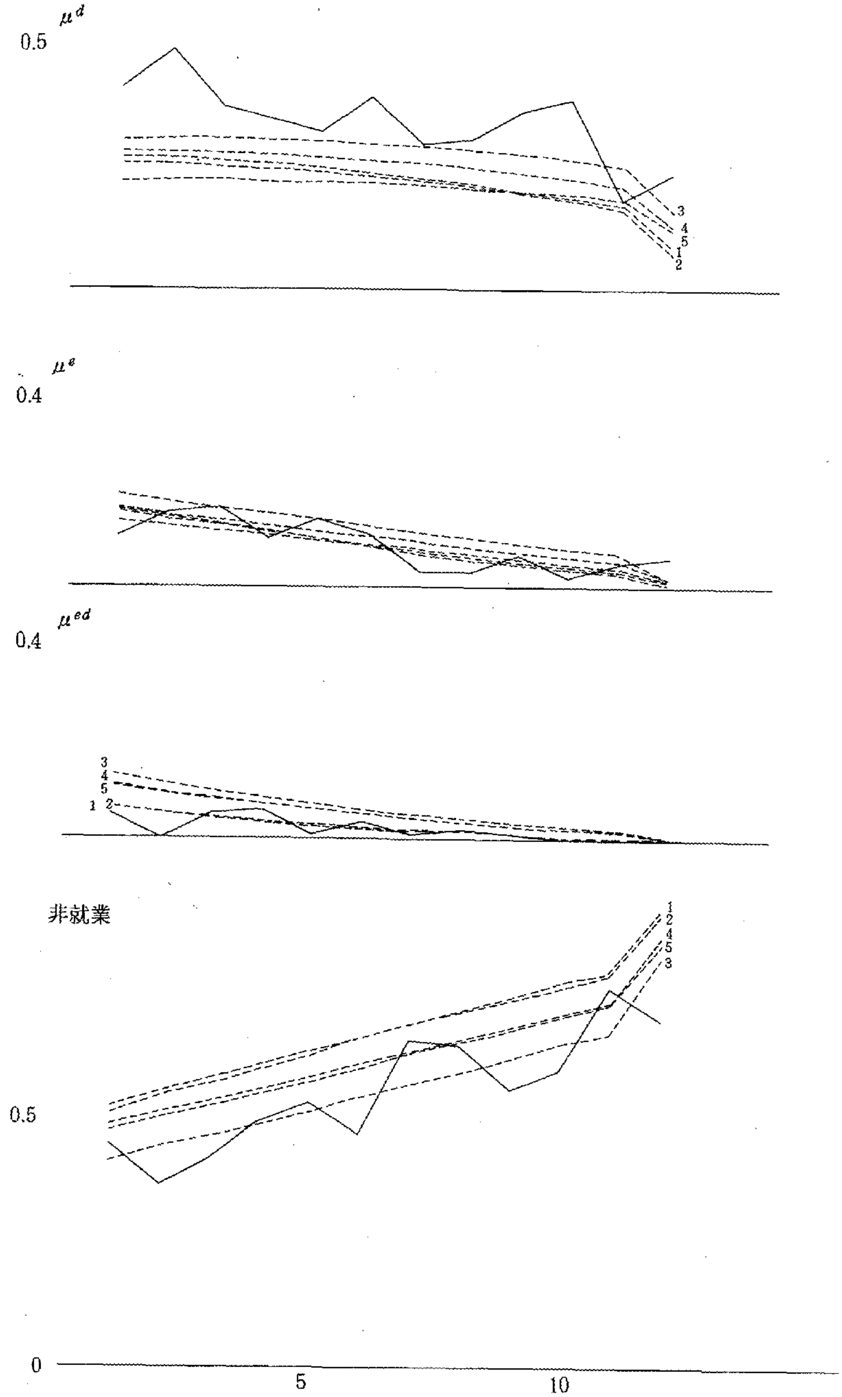


Table. Ⅵ—4 （# 4—26）

1. $r_2=150$, $r_3=0$, $r_4=11000$, $r_5=-5400$, $\sigma=0.188$

年	v	r^2_{NON}	$r^2\mu^d$	$r^2\mu^e$	$r^2\mu^{ed}$	TU_{NON}	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$	AP_{NON}	$AP\mu^d$	$AP\mu^e$	$AP\mu^{ed}$	$TU\mu^{deed}$	$TU_{全}$	$AP\mu^{deed}$	$AP_{全}$	OBF
39	44	0.830	0.359	0.673	0.477	0.109	0.268	0.171	0.271	16.6	69.7	52.9	220.3	0.711	0.820	342.9	359.4	380.0
	45	0.833	0.312	0.675	0.475	0.099	0.199	0.171	0.289	14.9	48.5	68.9	186.8	0.659	0.758	304.2	319.1	280.8
	46	0.835	0.269	0.680	0.474	0.090	0.142	0.245	0.313	13.7	31.6	117.4	161.0	0.700	0.791	310.0	323.7	246.5
38	36	0.960	0.148	0.840	0.264	0.208	0.762	0.203	0.402	35.7	705.2	37.2	252.4	1.368	1.576	994.8	1030.5	1410.9
	37	0.960	0.011	0.839	0.268	0.196	0.674	0.189	0.363	34.1	416.5	35.2	199.3	1.225	1.421	651.0	685.1	554.9
	38	0.961	0.324	0.839	0.272	0.184	0.586	0.169	0.335	32.3	283.9	32.5	157.0	1.091	1.275	473.5	505.8	300.8
	39	0.961	0.530	0.839	0.276	0.171	0.500	0.147	0.323	30.4	203.1	29.5	125.6	0.970	1.141	358.2	388.6	178.7
	40	0.961	0.601	0.839	0.281	0.158	0.417	0.124	0.326	28.4	147.5	27.8	102.6	0.866	1.024	277.9	306.2	111.1
	41	0.961	0.627	0.840	0.285	0.144	0.335	0.118	0.341	26.1	106.5	30.1	87.1	0.794	0.938	223.6	249.8	72.5
	42	0.961	0.639	0.842	0.289	0.130	0.256	0.166	0.366	23.7	74.5	45.9	75.7	0.787	0.917	196.0	219.7	53.6
	43	0.961	0.642	0.845	0.294	0.116	0.178	0.305	0.395	21.0	48.1	101.6	69.3	0.879	0.995	219.0	240.0	66.5
	44	0.960	0.637	0.853	0.298	0.102	0.108	0.741	0.428	18.1	27.7	723.1	65.7	1.278	1.380	816.5	834.6	1392.5
37	34	0.836	0.005	0.554	0.337	0.242	0.786	0.215	0.497	39.9	774.5	34.8	172.2	1.498	1.740	981.4	1021.3	1255.9
	35	0.837	0.044	0.553	0.340	0.231	0.701	0.211	0.445	38.4	460.0	34.5	139.2	1.357	1.588	633.6	672.0	434.2
	36	0.838	0.143	0.553	0.343	0.219	0.616	0.208	0.401	36.9	313.9	35.3	114.0	1.224	1.443	463.1	500.0	214.9
	37	0.838	0.156	0.553	0.346	0.207	0.531	0.208	0.367	35.2	224.8	37.1	94.9	1.106	1.313	356.8	392.0	120.9
	38	0.839	0.152	0.553	0.349	0.194	0.449	0.220	0.346	33.3	163.7	42.3	81.4	1.014	1.209	287.3	320.6	74.4
	39	0.839	0.146	0.553	0.351	0.181	0.369	0.254	0.339	31.3	118.6	56.5	71.6	0.961	1.142	246.7	278.0	52.6
	40	0.840	0.141	0.554	0.354	0.168	0.290	0.331	0.346	29.0	84.0	91.3	67.5	0.967	1.135	242.8	271.8	53.3
	41	0.840	0.134	0.555	0.357	0.154	0.217	0.505	0.363	26.6	57.3	218.3	64.8	1.084	1.237	340.4	367.0	144.7
36	30	0.736	0.304	0.549	0.340	0.215	0.889	0.234	0.620	35.4	4627.8	78.6	266.6	1.744	1.959	4973.0	5008.4	127496.8
	31	0.736	0.020	0.550	0.342	0.206	0.807	0.226	0.556	34.2	843.7	81.4	205.3	1.589	1.795	1130.4	1164.6	1091.4
	32	0.735	0.159	0.552	0.344	0.197	0.720	0.214	0.491	32.9	510.6	86.2	158.4	1.426	1.623	755.2	788.1	435.5
	33	0.734	0.341	0.553	0.346	0.187	0.633	0.200	0.430	31.5	348.9	94.5	125.0	1.263	1.449	568.3	599.8	256.4
	34	0.733	0.413	0.555	0.349	0.176	0.546	0.184	0.376	29.9	250.0	108.7	99.1	1.106	1.282	457.8	487.7	198.4
	35	0.731	0.445	0.557	0.351	0.165	0.460	0.175	0.335	28.2	182.5	131.1	81.1	0.970	1.135	394.6	422.8	205.9
	36	0.730	0.464	0.559	0.353	0.154	0.375	0.194	0.312	26.4	133.0	173.3	69.3	0.882	1.035	375.6	402.0	297.4
	37	0.728	0.477	0.561	0.356	0.142	0.291	0.284	0.309	24.3	94.7	271.0	60.6	0.885	1.027	426.3	450.7	656.2
	38	0.726	0.489	0.563	0.358	0.129	0.205	0.548	0.325	22.1	62.5	745.0	55.4	1.078	1.208	862.9	885.0	418.9

2. $\gamma_2=150$, $\gamma_3=0$, $\gamma_4=12000$, $\gamma_5=-6400$, $\sigma=0.188$

年	v	r^2_{NON}	$r^2\mu^d$	$r^2\mu^e$	$r^2\mu^{ed}$	TU_{NON}	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$	AP_{NON}	$AP\mu^d$	$AP\mu^e$	$AP\mu^{ed}$	$TU\mu^{deed}$	$TU_{全}$	$AP\mu^{deed}$	$AP_{全}$	OBF
39	44	0.833	0.337	0.676	0.475	0.097	0.217	0.164	0.260	15.0	51.2	48.1	189.9	0.640	0.737	289.2	304.2	254.2
	45	0.836	0.293	0.679	0.474	0.087	0.159	0.180	0.267	13.3	35.6	65.4	163.9	0.606	0.693	264.8	278.1	196.1
	46	0.838	0.252	0.685	0.472	0.079	0.118	0.270	0.283	12.0	22.8	115.4	142.2	0.672	0.750	280.4	292.3	184.6
38	34	0.959	0.373	0.839	0.270	0.217	0.818	0.212	0.507	36.7	38464.9	38.4	332.4	1.537	1.754	38835.7	38872.4	40120769.0
	35	0.960	0.229	0.839	0.273	0.206	0.739	0.203	0.455	35.4	620.1	37.2	266.6	1.397	1.603	923.9	959.2	1136.1
	36	0.960	0.022	0.839	0.276	0.195	0.659	0.190	0.408	33.9	389.4	35.5	213.3	1.257	1.452	638.2	672.1	502.7
	37	0.960	0.138	0.839	0.279	0.184	0.579	0.174	0.367	32.3	273.4	33.3	171.5	1.120	1.304	478.3	510.5	284.4
	38	0.960	0.432	0.839	0.282	0.172	0.501	0.155	0.336	30.5	200.2	30.8	137.5	0.992	1.164	368.4	398.9	174.1
	39	0.960	0.561	0.839	0.286	0.159	0.425	0.134	0.318	28.7	148.8	27.4	111.0	0.877	1.036	287.3	316.0	110.6
	40	0.960	0.610	0.840	0.289	0.146	0.351	0.118	0.313	26.7	110.4	25.5	91.2	0.782	0.928	227.1	253.7	72.2
	41	0.960	0.629	0.841	0.293	0.133	0.279	0.127	0.320	24.5	80.3	29.1	78.1	0.727	0.860	187.4	211.9	49.2
	42	0.960	0.638	0.843	0.296	0.119	0.209	0.190	0.338	22.1	55.8	46.2	68.8	0.737	0.857	170.8	192.9	38.8
	43	0.959	0.640	0.847	0.300	0.106	0.141	0.336	0.363	19.6	34.8	104.2	62.9	0.840	0.945	201.8	221.4	54.3
	44	0.958	0.633	0.854	0.303	0.092	0.089	0.759	0.392	16.9	20.7	728.2	60.1	1.240	1.332	809.0	825.8	1256.4
37	33	0.837	0.017	0.554	0.343	0.240	0.765	0.216	0.557	39.5	686.3	33.8	191.9	1.537	1.777	912.1	951.6	1027.4
	34	0.837	0.005	0.554	0.345	0.229	0.687	0.212	0.505	38.1	431.3	33.4	156.4	1.404	1.633	621.1	659.2	402.5
	35	0.838	0.104	0.554	0.347	0.218	0.610	0.208	0.456	36.7	302.9	33.2	128.9	1.274	1.492	465.0	501.7	209.2
	36	0.838	0.152	0.553	0.349	0.207	0.533	0.207	0.413	35.1	222.0	33.1	107.2	1.153	1.360	362.3	397.5	121.0
	37	0.839	0.154	0.553	0.351	0.195	0.459	0.212	0.377	33.4	165.4	34.8	91.2	1.047	1.242	291.4	324.8	74.7
	38	0.839	0.149	0.554	0.353	0.182	0.386	0.229	0.352	31.6	123.3	40.5	79.6	0.966	1.149	243.4	275.0	49.6
	39	0.839	0.144	0.554	0.355	0.170	0.315	0.270	0.338	29.6	90.4	55.4	70.1	0.923	1.093	215.9	245.5	38.2
	40	0.839	0.139	0.555	0.357	0.157	0.247	0.353	0.337	27.4	65.0	92.3	64.6	0.937	1.094	221.9	249.3	43.7
	41	0.839	0.132	0.557	0.359	0.143	0.182	0.530	0.347	25.1	43.8	221.2	62.4	1.059	1.202	327.4	352.5	131.8
36	30	0.730	0.092	0.542	0.352	0.202	0.785	0.232	0.606	33.5	730.9	66.1	234.2	1.623	1.825	1031.2	1064.8	860.6
	31	0.729	0.054	0.543	0.353	0.193	0.705	0.221	0.547	32.3	465.1	67.7	184.6	1.473	1.667	717.4	749.7	365.1
	32	0.728	0.263	0.545	0.354	0.184	0.626	0.208	0.487	31.0	326.2	70.1	146.0	1.320	1.504	542.3	573.3	204.0
	33	0.727	0.369	0.547	0.356	0.174	0.546	0.192	0.430	29.5	238.5	76.0	117.8	1.169	1.342	432.3	461.8	136.4
	34	0.725	0.418	0.548	0.357	0.163	0.468	0.178	0.379	27.9	177.4	87.6	95.5	1.025	1.189	360.4	388.3	111.6
	35	0.724	0.444	0.550	0.359	0.153	0.392	0.174	0.338	26.2	132.1	106.5	80.1	0.904	1.057	318.7	344.9	118.4
	36	0.722	0.461	0.553	0.361	0.141	0.317	0.203	0.312	24.3	96.9	141.9	68.4	0.832	0.974	307.2	331.5	172.3
	37	0.720	0.474	0.555	0.363	0.130	0.243	0.303	0.304	22.3	68.4	223.5	60.5	0.850	0.979	352.4	374.7	383.8
	38	0.718	0.486	0.558	0.365	0.118	0.169	0.567	0.312	20.1	43.5	619.3	56.3	1.048	1.166	719.2	739.3	2389.8

3. $\gamma_2=150$, $\gamma_3=10$, $\gamma_4=11000$, $\gamma_5=-5400$, $\sigma=0.188$

年	v	$r^2{}_{NON}$	$r^2\mu^d$	$r^2\mu^e$	$r^2\mu^{ed}$	TU_{NON}	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$	AP_{NON}	$AP\mu^d$	$AP\mu^e$	$AP\mu^{ed}$	$TU\mu^{deed}$	$TU_{全}$	$AP\mu^{deed}$	$AP_{全}$	OBF
39	44	0.826	0.330	0.690	0.488	0.120	0.241	0.185	0.287	18.0	69.0	120.1	977.2	0.713	0.832	1166.4	1184.4	10982.3
	45	0.830	0.299	0.693	0.486	0.111	0.181	0.178	0.294	16.9	49.5	163.4	783.1	0.653	0.764	996.0	1012.9	7087.9
	46	0.834	0.266	0.699	0.485	0.105	0.143	0.242	0.312	17.0	35.1	273.0	630.8	0.696	0.801	938.9	955.9	4919.7
38	35	0.956	0.072	0.850	0.200	0.219	0.827	0.229	0.526	37.2	1416.4	44.3	1362.4	1.582	1.801	2823.1	2860.3	27454.1
	36	0.958	0.077	0.849	0.206	0.207	0.735	0.218	0.474	35.6	580.3	44.2	1027.6	1.427	1.634	1652.1	1687.7	12011.0
	37	0.959	0.404	0.849	0.213	0.195	0.642	0.203	0.428	33.8	376.2	44.3	782.1	1.273	1.468	1202.6	1236.5	6863.9
	38	0.961	0.556	0.848	0.220	0.183	0.552	0.184	0.392	31.9	265.4	45.0	597.4	1.127	1.310	907.8	939.7	4017.3
	39	0.962	0.609	0.848	0.227	0.170	0.465	0.159	0.368	29.7	193.4	47.8	459.9	0.993	1.163	701.0	730.8	2394.4
	40	0.962	0.631	0.848	0.233	0.157	0.382	0.132	0.360	27.4	142.2	53.6	363.5	0.874	1.031	559.2	586.6	1543.3
	41	0.963	0.641	0.849	0.240	0.144	0.303	0.117	0.365	24.8	103.7	67.4	278.1	0.784	0.928	449.2	474.0	916.3
	42	0.963	0.647	0.850	0.247	0.130	0.228	0.154	0.381	22.0	73.4	104.5	221.3	0.763	0.893	399.3	421.3	629.8
	43	0.963	0.648	0.853	0.254	0.117	0.162	0.292	0.404	18.9	48.5	215.6	180.9	0.858	0.975	445.1	464.0	640.4
	44	0.962	0.641	0.857	0.261	0.104	0.127	0.736	0.433	15.5	35.9	1408.5	149.9	1.296	1.401	1594.3	1609.8	11013.2
37	33	0.831	0.001	0.556	0.291	0.251	0.851	0.232	0.602	41.1	1610.9	48.0	620.3	1.684	1.936	2279.2	2320.3	10627.7
	34	0.833	0.068	0.555	0.297	0.240	0.762	0.228	0.545	39.7	651.0	49.7	467.6	1.534	1.775	1168.3	1208.0	2458.1
	35	0.835	0.152	0.554	0.302	0.229	0.672	0.222	0.490	38.1	420.4	52.4	354.5	1.384	1.613	827.4	865.5	1292.1
	36	0.837	0.161	0.554	0.308	0.217	0.584	0.216	0.440	36.4	295.9	57.4	272.0	1.241	1.457	625.3	661.7	735.2
	37	0.838	0.157	0.553	0.313	0.205	0.499	0.213	0.400	34.5	215.4	66.0	212.0	1.112	1.316	493.4	527.9	438.9
	38	0.839	0.153	0.553	0.319	0.192	0.417	0.220	0.371	32.3	158.6	80.5	168.8	1.008	1.199	407.9	440.3	281.4
	39	0.840	0.149	0.553	0.324	0.179	0.339	0.248	0.357	30.0	116.1	108.7	135.8	0.943	1.122	360.6	390.6	213.4
	40	0.841	0.144	0.553	0.329	0.165	0.266	0.321	0.357	27.5	84.7	173.9	113.0	0.943	1.108	371.5	399.0	248.3
	41	0.842	0.138	0.554	0.334	0.152	0.202	0.495	0.369	24.7	60.5	396.7	98.2	1.065	1.217	555.5	580.1	792.1
36	30	0.744	0.034	0.568	0.289	0.215	0.875	0.248	0.696	35.6	1662.6	221.0	548.7	1.819	2.034	2432.3	2467.9	5795.0
	31	0.744	0.161	0.569	0.293	0.206	0.785	0.241	0.634	34.4	796.5	241.4	406.9	1.660	1.866	1444.8	1479.2	1942.3
	32	0.744	0.355	0.570	0.298	0.196	0.694	0.230	0.569	33.0	522.1	270.2	303.3	1.493	1.689	1095.6	1128.6	1471.4
	33	0.744	0.424	0.571	0.302	0.186	0.603	0.216	0.504	31.5	371.2	311.3	226.7	1.323	1.508	909.3	940.7	1484.1
	34	0.743	0.453	0.573	0.306	0.175	0.514	0.200	0.443	29.8	273.5	373.8	169.5	1.156	1.331	816.8	846.5	1870.1
	35	0.743	0.468	0.574	0.310	0.163	0.427	0.190	0.390	27.9	204.6	475.7	129.5	1.007	1.170	809.7	837.6	2870.0
	36	0.741	0.478	0.574	0.315	0.151	0.343	0.206	0.352	25.8	153.2	661.8	100.2	0.901	1.052	915.1	940.9	5476.0
	37	0.740	0.486	0.574	0.319	0.139	0.263	0.292	0.332	23.5	113.1	1166.5	87.9	0.887	1.026	1367.5	1391.0	16843.1
	38	0.738	0.494	0.570	0.323	0.127	0.188	0.553	0.333	21.0	79.9	3545.7	74.6	1.075	1.201	3700.2	3721.2	152432.2

4. $r_2=150$, $r_3=10$, $r_4=12000$, $r_6=-6400$, $\sigma=0.188$

年	v	$r^2{}_{NON}$	$r^2\mu^d$	$r^2\mu^e$	$r^2\mu^{ed}$	TU_{NON}	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$	AP_{NON}	$AP\mu^d$	$AP\mu^e$	$AP\mu^{ed}$	$TU\mu^{deed}$	$TU_{全}$	$AP\mu^{deed}$	$AP_{全}$	OBF
39	44	0.830	0.314	0.693	0.487	0.107	0.197	0.173	0.284	16.2	50.9	100.6	694.1	0.654	0.761	845.6	861.8	4861.6
	45	0.834	0.284	0.696	0.485	0.099	0.149	0.182	0.280	15.1	35.7	138.7	571.4	0.612	0.711	745.8	760.9	3440.0
	46	0.836	0.251	0.702	0.483	0.093	0.127	0.266	0.288	15.0	26.4	235.8	472.2	0.681	0.773	734.4	749.4	2505.1
38	34	0.958	0.129	0.849	0.212	0.217	0.807	0.222	0.581	37.0	1118.3	39.8	1159.9	1.609	1.827	2318.0	2355.0	15418.0
	35	0.959	0.007	0.849	0.217	0.206	0.722	0.212	0.528	35.5	539.0	39.4	903.1	1.463	1.669	1481.6	1517.1	7652.5
	36	0.960	0.273	0.849	0.222	0.195	0.638	0.200	0.477	33.9	358.6	38.7	705.5	1.315	1.510	1102.8	1136.7	4584.8
	37	0.961	0.492	0.848	0.228	0.183	0.555	0.184	0.432	32.1	257.4	37.8	553.0	1.170	1.353	848.1	880.2	2819.9
	38	0.962	0.579	0.848	0.234	0.171	0.475	0.163	0.394	30.2	190.6	36.4	434.4	1.031	1.202	661.3	691.5	1762.4
	39	0.962	0.615	0.849	0.239	0.158	0.398	0.140	0.366	28.1	142.6	38.2	342.8	0.903	1.062	523.6	551.8	1116.9
	40	0.962	0.632	0.849	0.243	0.145	0.324	0.119	0.350	25.9	106.2	43.9	272.5	0.793	0.938	422.6	448.5	719.4
	41	0.962	0.640	0.850	0.250	0.132	0.254	0.119	0.348	23.4	77.5	55.5	217.0	0.721	0.853	350.0	373.4	478.5
	42	0.962	0.645	0.852	0.256	0.119	0.188	0.177	0.357	20.7	54.1	90.0	176.4	0.722	0.841	320.5	341.2	349.6
	43	0.961	0.645	0.854	0.262	0.106	0.132	0.324	0.375	17.8	35.0	191.2	146.9	0.830	0.936	373.1	390.9	382.1
	44	0.960	0.638	0.859	0.268	0.093	0.113	0.755	0.399	14.6	27.0	1238.7	124.4	1.267	1.360	1390.1	1404.7	6687.6
37	32	0.833	0.006	0.557	0.300	0.249	0.832	0.228	0.657	40.9	1284.5	40.7	589.2	1.716	1.966	1914.4	1955.3	6397.4
	33	0.835	0.026	0.557	0.305	0.239	0.751	0.224	0.604	39.5	605.8	41.6	456.5	1.578	1.817	1103.9	1143.4	1935.2
	34	0.836	0.128	0.556	0.309	0.228	0.668	0.219	0.551	38.1	401.1	43.0	353.9	1.438	1.666	798.1	836.1	1040.7
	35	0.837	0.158	0.556	0.314	0.217	0.587	0.214	0.499	36.5	287.3	45.1	277.7	1.300	1.517	610.1	646.6	617.9
	36	0.838	0.158	0.555	0.318	0.205	0.509	0.211	0.451	34.8	212.6	48.9	219.6	1.170	1.375	481.2	515.9	377.0
	37	0.839	0.154	0.555	0.322	0.193	0.433	0.212	0.410	32.9	159.3	56.4	176.3	1.055	1.247	392.0	424.9	239.0
	38	0.840	0.150	0.555	0.327	0.180	0.360	0.225	0.378	30.8	119.2	69.9	144.4	0.963	1.143	333.5	364.2	162.1
	39	0.840	0.146	0.555	0.331	0.167	0.292	0.262	0.357	28.5	88.2	95.9	119.2	0.911	1.078	303.2	331.8	130.1
	40	0.841	0.142	0.556	0.335	0.154	0.228	0.343	0.349	26.1	64.6	154.9	99.7	0.920	1.074	319.2	345.2	159.5
	41	0.841	0.136	0.557	0.339	0.141	0.175	0.520	0.353	23.4	46.6	358.1	88.6	1.049	1.189	493.3	516.6	518.3
36	30	0.741	0.077	0.563	0.306	0.203	0.773	0.242	0.680	33.9	698.4	146.6	436.5	1.695	1.898	1281.4	1315.3	1264.1
	31	0.741	0.288	0.564	0.309	0.194	0.689	0.232	0.622	32.6	465.7	159.5	333.4	1.542	1.736	958.6	991.1	783.4
	32	0.740	0.385	0.565	0.312	0.184	0.605	0.219	0.561	31.1	334.6	177.9	255.4	1.385	1.569	767.9	799.1	620.1
	33	0.739	0.428	0.567	0.316	0.173	0.523	0.204	0.500	29.6	248.7	203.9	195.9	1.228	1.401	648.5	678.0	611.7
	34	0.738	0.450	0.568	0.319	0.162	0.443	0.190	0.443	27.8	187.6	246.7	159.2	1.076	1.238	593.5	621.3	766.2
	35	0.736	0.464	0.569	0.322	0.151	0.366	0.185	0.392	25.9	141.8	313.2	126.4	0.943	1.094	581.3	607.2	1161.4
	36	0.734	0.473	0.570	0.326	0.140	0.292	0.212	0.353	23.9	106.0	437.9	100.7	0.857	0.996	644.7	668.5	2218.4
	37	0.732	0.482	0.570	0.329	0.128	0.222	0.309	0.329	21.6	77.1	733.6	84.1	0.860	0.988	894.8	916.4	6105.2
	38	0.730	0.492	0.568	0.333	0.116	0.158	0.574	0.322	19.2	52.2	2170.3	72.6	1.054	1.170	2295.1	2314.2	51074.0

5. $r_2=150$, $r_3=20$, $r_4=12000$, $r_5=-6400$, $\sigma=0.188$

年	v	$r^2{}_{NON}$	$r^2\mu^d$	$r^2\mu^e$	$r^2\mu^{ed}$	TU_{NON}	$TU\mu^d$	$TU\mu^e$	$TU\mu^{ed}$	AP_{NON}	$AP\mu^d$	$AP\mu^e$	$AP\mu^{ed}$	$TU\mu^{deed}$	$TU_{全}$	$AP\mu^{deed}$	$AP_{全}$	OBF
39	44	0.827	0.302	0.707	0.494	0.119	0.190	0.188	0.310	18.2	56.3	242.4	3677.3	0.687	0.807	3975.9	3994.1	197708.3
	45	0.831	0.279	0.710	0.493	0.113	0.154	0.190	0.298	18.5	42.0	348.5	2818.3	0.642	0.755	3208.7	3227.2	114794.7
	46	0.835	0.251	0.716	0.491	0.108	0.148	0.264	0.299	19.3	36.0	607.6	2191.0	0.711	0.819	2834.6	2854.0	270888.9
38	34	0.954	0.010	0.855	0.156	0.218	0.796	0.239	0.639	37.0	886.7	57.3	1745.1	1.674	1.892	2689.1	2726.2	17843.3
	35	0.956	0.300	0.854	0.163	0.207	0.706	0.230	0.587	35.4	516.1	59.7	4667.0	1.523	1.730	5242.8	5278.2	338556.0
	36	0.959	0.513	0.854	0.171	0.195	0.618	0.218	0.535	33.7	357.2	62.8	3371.7	1.371	1.566	3791.7	3825.4	172931.6
	37	0.960	0.591	0.854	0.178	0.184	0.533	0.202	0.487	31.7	261.3	67.5	2496.0	1.222	1.406	2824.9	2856.6	93296.1
	38	0.962	0.622	0.853	0.185	0.172	0.452	0.182	0.445	29.6	195.8	74.0	1851.6	1.079	1.250	2121.3	2150.9	50685.0
	39	0.963	0.637	0.853	0.193	0.159	0.376	0.158	0.411	27.3	147.8	83.9	1387.1	0.945	1.104	1618.8	1646.1	28271.0
	40	0.963	0.644	0.853	0.200	0.147	0.303	0.133	0.389	24.7	111.1	104.3	1050.3	0.826	0.973	1265.7	1290.4	15991.3
	41	0.964	0.648	0.854	0.208	0.134	0.237	0.126	0.379	21.9	82.1	144.4	800.4	0.742	0.876	1026.8	1048.7	9345.7
	42	0.963	0.651	0.854	0.215	0.122	0.179	0.172	0.381	18.8	58.4	230.0	613.9	0.732	0.854	902.3	921.1	5838.1
	43	0.963	0.650	0.855	0.223	0.111	0.138	0.314	0.393	15.8	43.5	463.1	475.5	0.845	0.956	982.0	997.8	5035.9
	44	0.962	0.641	0.855	0.231	0.100	0.143	0.751	0.413	14.1	37.9	3154.3	373.0	1.306	1.406	3565.2	3579.3	88802.5
37	32	0.828	0.021	0.557	0.253	0.249	0.823	0.244	0.696	40.8	1025.0	64.7	2182.2	1.762	2.011	3271.9	3312.7	61981.1
	33	0.831	0.130	0.556	0.260	0.239	0.736	0.240	0.644	39.4	583.5	68.5	1565.9	1.619	1.858	2217.9	2257.3	30653.4
	34	0.834	0.162	0.555	0.267	0.228	0.651	0.234	0.590	37.8	400.7	73.9	1153.0	1.475	1.702	1627.6	1665.4	16210.1
	35	0.836	0.163	0.554	0.274	0.216	0.568	0.228	0.537	36.0	291.8	82.2	850.9	1.332	1.548	1225.0	1261.0	8671.6
	36	0.838	0.159	0.554	0.281	0.204	0.488	0.222	0.486	34.1	218.2	94.2	632.5	1.196	1.401	944.9	979.0	4745.7
	37	0.839	0.155	0.553	0.287	0.192	0.412	0.220	0.442	32.0	164.7	113.0	480.8	1.074	1.266	758.6	790.5	2697.9
	38	0.841	0.152	0.552	0.294	0.180	0.341	0.229	0.406	29.6	124.3	144.8	366.1	0.975	1.155	635.0	664.6	1626.2
	39	0.842	0.149	0.552	0.301	0.167	0.276	0.260	0.380	27.1	93.8	203.9	283.9	0.916	1.083	581.6	608.6	1144.4
	40	0.842	0.145	0.552	0.307	0.154	0.219	0.336	0.367	24.3	70.2	331.8	223.5	0.922	1.076	625.5	649.8	1273.2
	41	0.842	0.139	0.552	0.314	0.142	0.179	0.511	0.367	21.7	53.8	770.4	180.0	1.057	1.199	1004.2	1025.9	4460.4
36	30	0.745	0.270	0.572	0.260	0.203	0.760	0.259	0.738	34.0	775.4	486.8	957.5	1.757	1.960	2219.6	2253.7	6537.2
	31	0.746	0.391	0.573	0.264	0.194	0.672	0.250	0.683	32.7	541.1	552.5	703.5	1.604	1.798	1797.1	1829.7	5783.1
	32	0.746	0.436	0.574	0.269	0.184	0.585	0.238	0.624	31.1	399.4	645.4	516.6	1.446	1.630	1561.3	1592.4	6394.1
	33	0.746	0.457	0.574	0.274	0.173	0.501	0.223	0.562	29.4	302.9	769.2	388.7	1.287	1.459	1460.8	1490.2	8291.7
	34	0.746	0.469	0.575	0.279	0.162	0.420	0.209	0.502	27.5	232.6	999.6	291.3	1.131	1.293	1523.4	1550.9	13529.5
	35	0.745	0.476	0.573	0.284	0.150	0.344	0.203	0.446	25.4	179.3	1371.4	218.8	0.993	1.143	1769.5	1795.0	25314.5
	36	0.744	0.482	0.573	0.289	0.139	0.272	0.226	0.399	23.1	137.6	2099.0	166.2	0.897	1.036	2042.8	2426.0	59286.2
	37	0.742	0.488	0.570	0.294	0.127	0.208	0.318	0.364	20.6	103.9	3937.5	129.6	0.890	1.017	4171.1	4191.7	208664.5
	38	0.740	0.495	0.561	0.299	0.116	0.158	0.579	0.346	17.9	78.8	12552.2	102.2	1.083	1.199	12733.1	12751.0	2116374.1

Tab. VI—5 (# 4—27)

		1		2		3		4		5	
	r_2	150		150		150		150		150	
	r_3	0		0		10		10		20	
	r_4	11000		12000		11000		12000		12000	
	r_5	−5400		−6400		−5400		−6400		−6400	
	σ	0.188		0.188		0.188		0.188		0.188	
			v		v		v		v		v
39	TU_{μ}^{deed}	⑤ 0.659	45	① 0.606	45	④ 0.653	45	② 0.612	45	③ 0.642	45
39	TU全	④ 0.758	45	① 0.693	45	⑤ 0.764	45	② 0.711	45	③ 0.755	45
	AP_{μ}^{deed}	② 304.2	45	① 264.8	45	④ 938.9	46	③ 734.4	46	⑤ 2834.6	46
	AP全	② 319.1	45	① 278.1	45	④ 955.9	46	③ 749.4	46	⑤ 2854.0	46
	OBF	② 246.5	46	① 184.6	46	④ 4919.7	46	③ 2505.1	46	⑤ 70888.9	46
38	TU_{μ}^{deed}	⑤ 0.787	42	② 0.727	41	④ 0.763	42	① 0.721	41	③ 0.732	42
	TU全	⑤ 0.917	42	③ 0.857	42	④ 0.893	42	① 0.841	42	② 0.854	42
	AP_{μ}^{deed}	② 196.0	42	① 170.8	42	④ 399.3	42	③ 320.5	42	⑤ 902.3	42
	AP全	② 219.7	42	① 192.9	42	④ 421.3	42	③ 341.2	42	⑤ 921.1	42
	OBF	② 53.6	42	① 38.8	42	④ 629.8	42	③ 349.6	42	⑤ 5035.9	43
37	TU_{μ}^{deed}	⑤ 0.961	39	③ 0.923	39	④ 0.943	39 40	① 0.911	39	② 0.916	**39**
	TU全	⑤ 1.135	40	③ 1.093	39	④ 1.108	40	① 1.074	40	② 1.076	40
	AP_{μ}^{deed}	② 242.8	40	① 215.9	39	④ 360.6	39	③ 303.2	39	⑤ 581.6	39
	AP全	② 271.8	40	① 245.5	39	④ 390.6	39	③ 331.8	39	⑤ 608.6	39
	OBF	② 52.6	39	① 38.2	39	④ 213.4	39	③ 130.1	39	⑤ 1144.4	39
36	TU_{μ}^{deed}	③ 0.882	36	① 0.832	36	④ 0.887	37	② 0.857	36	⑤ 0.890	37
	TU全	⑤ 1.027	37	① 0.974	36	④ 1.026	37	② 0.988	**37**	③ 1.017	37
	AP_{μ}^{deed}	② 375.6	36	① 307.2	36	④ 809.7	35	③ 581.3	35	⑤ 1460.8	31
	AP全	② 402.0	36	① 331.5	36	④ 837.6	35	③ 607.2	35	⑤ 1490.2	**33**
	OBF	② 198.4	34	① 111.6	34	④ 1471.4	**32**	③ 611.7	33	⑤ 5783.1	33

Among the sets of parameters, those with relatively small values for Φ are listed in Tab. Ⅵ-5. It can be seen that the ranges of v found to be consistent with the theory (i.e. satisfying the theoretical restrictions) are fairly stable among the various sets of preference parameters, ① through ⑤. Secondly, minimum and maximum values for the ranges for each year slightly increase from 1961 through 1964. This seems to be consistent with the experience in Japanese economic growth during those years. We can see that plausible values for v appear to be 45 and 46 for 1964, 41, 42 and 43 for 1963, 39 and 40 for 1962, and, 32, 33, 34, 36 and 37 for 1961. The underlined figures are those which appear most frequently among groups of the parameters (γ_i, σ_u) 1 through 5 for each year.

Fig. Ⅵ-13(b) indicates that the sums of μ^e, μ^{ed}, and μ^d are underestimated. Hence, it appeared necessary to augument the intercept of the marginal utility curve of income γ_2 and to reduce that of leisure γ_4. Before doing so, a preliminary test was conducted, making use of data for 1964, to examine if restrictions 1 through 7 were violated by slight shifts in parameters $\overline{\gamma_4}$, γ_5 and γ_3. The results were;

(a) Shifting γ_2 from 150 to 195 (intervals are 5) does not violate the restrictions

(b) Shifting $\overline{\gamma_4}$ from -6400 to -6700 (intervals are 100) does not violate the restrictions

(c) Shifting γ_3 from 0 to 10 (intervals are 2) does not violate the restrictions.

By taking advantage of results (a) (b) and (c), we set the trial level of parameters as shown in Tab. (A), where intervals between testing levels are narrowed compared to previous ones.

Tab. (A)

γ_2	150 155 160 165 170 175
γ_3	0 2 4 6 8 10
$\overline{\gamma_4}$	12000 11900 11800 11700 11600 11500
γ_5	-6400 -6500 -6600 -6700
σ_u	0.188 0.193 0.198
$\bar{h}$	1/3

By making combinations of numerical values of the parameters listed in the table, we obtained parameter sets. For each of these sets, we computed theoretical values for μ^e , μ^{ed} and μ^d. Among those results, sets of parameters with favorable Φ , TU and AAPE were selected as shown in the following table (B).

Tab. (B)
Results for the year 1964 (*)

	TU_1	TU_2	$AAPE^1$	$AAPE^2$	Φ	γ_2	γ_3	$\overline{\gamma_4}$	γ_5	σ_u
1	.573	.646	297.5	310.3	359.7	150	6	12000	-6700	.188
2	.577	.643	207.8	218.9	125.3	150	2	12000	-6700	.188
3	.813	.912	116.8	141.0	15.0	165	0	12000	-6700	.188
4	.737	.792	119.9	131.0	15.4	160	0	11900	-6400	.198
5	.797	.860	118.8	132.7	13.9	165	0	12000	-6400	.198

	v_{64}	v_{63}	v_{62}	v_{61}
1	45	42	39	36
2	45	42	39	36
3	45	43	40	37
4	45	43	40	37
5	45	43	40	36

(*) Suffixes 1 and 2, respectively indicate the values when non participation probabilities are excluded and included.

The estimated values for μ^e, μ^{ed} and μ^d are depicted in Fig. VI-14. It can be seen that the fitting for μ^e has, to some extent, improved but considerable systematic discrepancies between theoretical and observed values for μ^d remained. Hence, in order to reduce these discrepancies, the Newton method was used to estimate a better set of values of preference parameters making use of the values shown in Tab. (B) as initial values for the computation. However, the results of appling the Newton method did not seem to be successful, because at the point where the objective function attained its local maximum, the initial values of parameters did not change sufficiently, so that estimated μ^e , μ^{ed} and μ^e did not closely approach to the observed values.

Hence, we might suspect that the discrepancy between the estimated and the observed values did not stem from the estimation method employed but from some inadequacy in the model itself. However, it seems that we should not discard the basic characteristics of the model under consideration, because we have succeeded, at least to some extent, in following the basic characteristics of the observed data; that is, the upward convexity of the μ^d curve and downward sloping μ^{ed} and μ^e curves. Neverthless, it seemed that we would not be able to proceed further without altering some part of the present model because the ranges of the parameters satisfying the theoretical restirictions are fairly narrow and we cannot expect any further sets of parameters will contribute to reducing discrepancies between estimated and observed values of μ^e, μ^{ed} and μ^d.

In fact the model seems to have one point, at least, that needs to be modified; that is, γ_4, the intercept of the marginal utility line of leisure, has been assumed to be written as $\overline{\gamma}_4 \cdot u$, with the (density) distribution function of u being log-nomal. This is equivalent to assuming that the minimum value of γ_4 is zero, an assumption which connot be expected to result in favorable approximation. Hence, we shall rewrite the model taking into account this point.

[6.3.2] Introduction of γ_4^0 and the estimation of the parameters

We rewrite the model replacing $\overline{\gamma}_4 \cdot u$ by

$$1)\quad \gamma_4^0 + \overline{\gamma}_4 \cdot u, \qquad \gamma_4^0 > 0$$

where γ_4^0 stands for the minimum value of γ_4 distributed among households. Hence, γ_4's in the previous model are replaced by 1). Making use of this rewritten model, we shall reestimate the parameters of the preference function.

Now, parameters other than γ_4^0 have been estimated in the previous section. We use those estimates as initial values for obtaining second approximation estimates of the parameters together with the newly introduced γ_4^0.

First we must determine the plausible range for γ_4^0 satisfying restrictions 1 through 7. We tentatively set this range from 0 to 1920. Computation results indicated plausible values for γ_4^0 were from 0 to 800.
Next, we narrowed down the range of tentative values for γ_4^0.
The values 0, 10, 40, 120, 320 and 800 were adopted and, together with the values for γ_4^0, the numerical values for γ_5 were simultaneously varied from -6000 through -6800, the intervals being 100. The values for the other parameters tentatively assigned are shown in the table (C).

Making use of combinations of the values for γ_4^0 and γ_5 mentioned above, estimates or theoretical values for μ^e, μ^{ed} and μ^d and values for the objective function Φ were computed. Among those results, cases satisfying the restrictions are shown in Tab. Ⅵ-6.(*) However, it should be noted that plausible sets of parameters might have been excluded because of the large size of intervals for tentatively assigned values of the parameters. In order to check this point, we alternatively took 0, 2, 4, 6, 8 and 10 for γ_4^0 and 0.178, 0.180, 0.182, 0.184, 0.186 and 0.188 for σ_u.

(*) other parameters are given on the next page.

Tab. (C)

case	v_{64}	v_{63}	v_{62}	v_{61}	γ_2	γ_3	$\overline{\gamma_4}$	σ_u	$\bar{h}$
1	45	42	39	36	150	6	12000	0.188	1/3
2	45	42	39	36	150	2	12000	0.188	1/3
3	45	43	40	37	165	0	12000	0.188	1/3
4	45	43	40	37	160	0	11900	0.198	1/3
5	45	43	40	36	165	0	12000	0.198	1/3

These values are reproduced from the table (B) on the previous page.
Assigned values for γ_5 and γ_4^0 are as follows.

γ_5 -6000, -6100, -6200, -6300, -6400, -6500, -6600, -6700, -6800

γ_4^0 0, 10, 40, 120, 320, 800 (*)

---

foot note

(*) The values of $\overline{\gamma_4}$ in Tab.(C) were used for the initial values of $\overline{\gamma_4}$.

Tab. VI—9

(1) r_2 150、 r_3 6、 r_4 12000、 σ .188

n	r_4^0	r_5
1	0	− 6300
2		− 6400
3		− 6500
4		− 6600
5		− 6700
6	10	− 6300
7		− 6400
8		− 6500
9		− 6600
10		− 6700
11	40	− 6300
12		− 6400
13		− 6500
14		−66600
15		− 6700
16		− 6800
17	120	− 6300
18		− 6400
19		− 6500
20		− 6600
21		− 6700
22		− 6800
23	320	− 6300
24		− 6400
25	320	− 6500
26		− 6600
27		− 6700
28		− 6800
29	800	− 6300
30		− 6400
31		− 6500
32		− 6600
33		− 6700
34		− 6800

(2) 150、 2、 12000、 .188

n	r_4^0	r_5
1	0	− 6300
2		− 6400
3		− 6500
4		− 6600
5		− 6700
6	10	− 6300
7		− 6400
8		− 6500
9		− 6600
10		− 6700
11	40	− 6300
12		− 6400
13		− 6500
14		− 6600
15		− 6700
16	120	− 6300
17		− 6400
18		− 6500
19		− 6600
20		− 6700
21		− 6800
22	320	− 6300
23		− 6400
24	320	− 6500
25		− 6600
26		− 6700
27		− 6800
28	800	− 6300
29		− 6400
30		− 6500
31		− 6600
32		− 6700
33		− 6800

(3) 165、 0、 12000、 .188

n	r_4^0	r_5
1	0	− 6300
2		− 6400
3		− 6500
4		− 6600
5		− 6700
6	10	− 6300
7		− 6400
8		− 6500
9		− 6600
10		− 6700
11	40	− 6300
12		− 6400
13		− 6500
14		− 6600
15		− 6700
16	120	− 6300
17		− 6400
18		− 6500
19		− 6600
20		− 6700
21		− 6800
22	320	− 6300
23		− 6400
24	320	− 6500
25		− 6600
26		− 6700
27		− 6800
28	800	− 6300
29		− 6400
30		− 6500
31		− 6600
32		− 6700
33		− 6800

(4) 160、0、11900、
.198

n	r_4^0	r_5
1	0	− 6300
2		− 6400
3	10	− 6300
4		− 6400
5	40	− 6300
6		− 6400
7	120	− 6300
8		− 6400
9		− 6500
10	320	− 6300
11		− 6400
12	320	− 6500
13		− 6600
14		− 6700
15	800	− 6300
16		− 6400
17		− 6500
18		− 6600
19		− 6700
20		− 6800

(5) 165、0、12000、
.198

n	r_4^0	r_5
1	0	− 6300
2		− 6400
3	10	− 6300
4		− 6400
5		− 6500
6	40	− 6300
7		− 6400
8		− 6500
9	120	− 6300
10		− 6400
11		− 6500
12		− 6600
13	320	− 6300
14		− 6400
15	320	− 6500
16		− 6600
17		− 6700
18		− 6800
19	800	− 6300
20		− 6400
21		− 6500
22		− 6600
23		− 6700
24		− 6800

Using combinations of the given values for γ_4^0 and σ_u, we examined if the theoretical restrictions 1 through 5 were violated. It was found that no case violated the restrictions. To further substantiate this conclusion, we extended the range for γ_4^0 from 0 through 40. The intervals of tentative values were 8. The trial levels for σ_u were the same as the previous ones. Combinations of the values for γ_4^0 and σ_u were checked against restrictions 1 through 5. Again, the results showed that there were no cases violating the restrictions.

Taking into account the results of these preliminary test, it was thought that there was little chance that any combination of parameters would violate the restrictions for the ranges checked even though combinations actually tested were limited in number. Hence, we proceeded to computation for estimating parameters making use of the steepest ascent method. Initial values of the preference parameters were tentatively chosen as,

$\sigma_u=0.188$, $\gamma_2=150$, $\gamma_3=2$, $\overline{\gamma_4}=12000$, $\gamma_5=-6700$ and $\gamma_4^0=40$
with other parameters given as $\bar{h}=0.33$, $w_{39}=47.4$, $v_{39}=45$, $w_{38}=44.10$
$v_{38}=42$, $w_{37}=41.70$, $v_{37}=39$ $w_{36}=38.4$ and $v_{36}=36$.

The objective function χ^2 was computed by employing the entire data set from 1961 through 1964 and the corresponding estimated values for the parameters.

However, before we proceed to the estimation results, one point should be made. According to the experience of previous estimation and the preliminary estimation which used the above mentioned set of initial values, it was found (1) that when we allow all the parameters to vary, some parameters, sometimes, clearly do not attain their optimal (minimizing χ^2) value for ranges fulfilling the restrictions and (2) that when we allow σ_u and $\gamma_4{}^0$ to vary, other parameters being fixed at initial values, the speed of convergence for $\gamma_4{}^0$ is extremely slow. These experiences show that some parameters barely attain convergence when their initial values and/or initial values for the other parameters are not appropriate.

Consequently, to begin with, we shall vary numerical values of a few parameters to which initial values are attached. In the first place, we shall allow v only to vary because we have some information for the values to be estimated. That is, the observational period under consideration is a period of fairly steady growth as shown by the growth of w as well as the growth rate of GNP. Hence, the parameter v to be estimated is expected to grow. At the very least, descending values or radical random movement in v can be ruled out. This constitutes information for estimating v. That is, if we have estimates (or convergence values) for v that exhibit counter-intuitive movement, it is probable that the initial values for other parameters are inadequate. Consequently, we should allow parameters other than v to vary in order to minimize χ^2, and after that, we have to vary v by employing the newly obtained values of other parameters as given. After this, we should examine if the estimated values for v are consistent with the other information.

As a "postulate" for the estimation, we consider (1) the parameters γ_1, γ_2, γ_3, $\overline{\gamma}_4$, γ_5, γ_4^0 and σ_u to be constant for all the observational years and for all the principal earner's income classes and (2) w, v and $\hbar$ are considered to vary from year to year but are considered to be constant cross-sectionally. We minimize

$$\Phi_t \qquad (t=1961,\ldots\ldots,64)$$

instead of $\sum_t \Phi_t$. After obtaining Φ_t's thus minimized we calculate $\Sigma\ \hat{\Phi}_t$, where $\hat{\Phi}_t$ stands for the minimized value of Φ_t for each year.

Experiment I

As was mentioned previously, we start from the estimation of v.

	1964	63	62	61	
γ	-1	-1	-1	-1	
γ'	150	150	150	150	
γ_3	2	2	2	2	
$\bar{\gamma}_4$	12000	12000	12000	12000	
γ_5	-6700	-6700	-6700	-6700	
γ_4^0	40	40	40	40	
σ_u	0.188	0.188	0.188	0.188	Φ
$\bar{h}$	0.333	0.333	0.333	0.333	
w_t	47.4	44.1	41.7	38.4	543.298
v_t	45 *(45.51)	42 (42.63)	39 (40.35)	36 (37.48)	↓ (464.330)

Parameters except for v are held constant at their initial values. v_t's are varied so as to minimize Φ_t 's. The result of the estimation is shown in the table. The estimates for v_t's seem to satisfy estimation postulate.

(*) values in the parentheses are the convergence values.

Experiment 2.

	1964	63	62	61	Φ
v	45→45.48	42→42.63	39→40.35	36→37.48	543.298
σ_u	0.188→0.13675	0.188→0.17675	0.188→0.17675	0.188→0.17675	↓
γ_4^0	40→25.3	40→25.3	40→25.3	40→25.3	431.099

Initial values for γ_2, γ_3, $\bar{\gamma}_4$ and γ_5 are held constant at the same levels as in experiment 1, and σ_u ane γ_4^0 together with v, are varied. Estimates for the parameters are shown in the table below. It should be noted that the estimates v_t's are similar to those obtained in experiment 1. Also, the direction of changes in the theoretical (estimated) values for μ_t^e, μ_t^d and μ_t^{ed} stemming from changes in the values for σ_u, γ_4^0 and v_t's is same as that observed in experiment 1.

The switching algebraic sign of γ_4^0 and the low speed of convergence which was experienced in preliminary estimation before experiment 1, did not occur in this experiment.
In the preliminary experiment γ_4^0, together with σ_u, was varied, but in this experiment v_t's were allowed to vary together with γ_4^0 and σ_u. Hence, allowing v_t's to vary caused γ_4^0 to have a stable sign and also eliminated the problem of convergence speed.

The fitting of cross sectional estimates for μ^e, μ^d and μ^{ed} obtained by making use of the parameters estimated to the observed values in 1964 is fairly good. However, the estimates were not as good for the observed data for 1961 to 1963. In particular, μ^d and μ^e are underestimated.

Experiment 3.

A	v	w	$\bar{\gamma}_4$	Φ
1964	45.51	47.4		462.33
63	42.63	44.1	12000 ↓	↓
62	40.35	41.7	11887.5	447.71
61	37.48	38.4		

$\sigma_u=0.188$、 $\gamma_2=150$、 $\gamma_3=2$、 $\gamma_5=-6700$、 $\gamma_4^0=40$、 $\bar{h}=0.333$

B	v	w	γ_5	Φ
64	45.48	47.4		431.32
63	42.63	44.1	-6700 ↓	↓
62	40.35	41.7	-6962.5	356.12
61	37.48	38.4		

$\sigma_u=0.17675$、 $\gamma_2=150$、 $\gamma_3=2$、 $\bar{\gamma}_4=12000$、 $\gamma_4^0=25.3$、 $\bar{h}=0.333$

C	v	w	γ_5	Φ
64	45.51	47.4		462.33
63	42.63	44.1	-6700 ↓	↓
62	40.35	41.7	-6762.5	445.64
61	37.48	38.4		

$\sigma_u=0.188$、 $\gamma_2=150$、 $\gamma_3=2$、 $\bar{\gamma}_4=12000$、 $\gamma_4^0=40$、 $\bar{h}=0.333$

D	v	w	$\bar{\gamma}_4$	Φ
64	45.48	47.4		431.32
63	42.63	44.1	12000 ↓	↓
62	40.35	41.7	11775	430.33
61	37.48	38.4		

$\sigma_u=0.17675$、 $\gamma_2=150$、 $\gamma_3=2$、 $\gamma_5=-6700$、 $\gamma_4^0=25.3$、 $\bar{h}=0.333$

This experiment examined the effect of varying $\bar{\gamma}_4$ and γ_5 respectively. The results are shown for cases A through D. In cases A and C, we used estimates for v_t obtained in experiment 1, while in cases B and D those estimates obtained in experiment 2 were used. Other parameters except for $\bar{\gamma}_4$ and γ_5 are held constant during the four years, 1961 through 1964,

the values of which are shown in the table in experiment 1, and are common to all cases A through D (with the exception of σ_u).
More specifically, in cases A and D, $\overline{\gamma}_4$ is allowed to vary, while in cases B and C γ_5 is, while in cases A and C the value of σ_u is the one used in experiment 1, while in cases B and D the value of σ_u estimated in experiment 2 is employed. The purpose of using alternative values for v and σ_u is to check if the estimates for $\overline{\gamma}_4$ or γ_5, respectively, are affected by slight differences in the values of parameters which are held constant for the estimation of $\overline{\gamma}_4$ or γ_5. As shown in the table, the estimates of $\overline{\gamma}_4$ and γ_5 were fairly stable for cases A and D and cases B and C respectively.

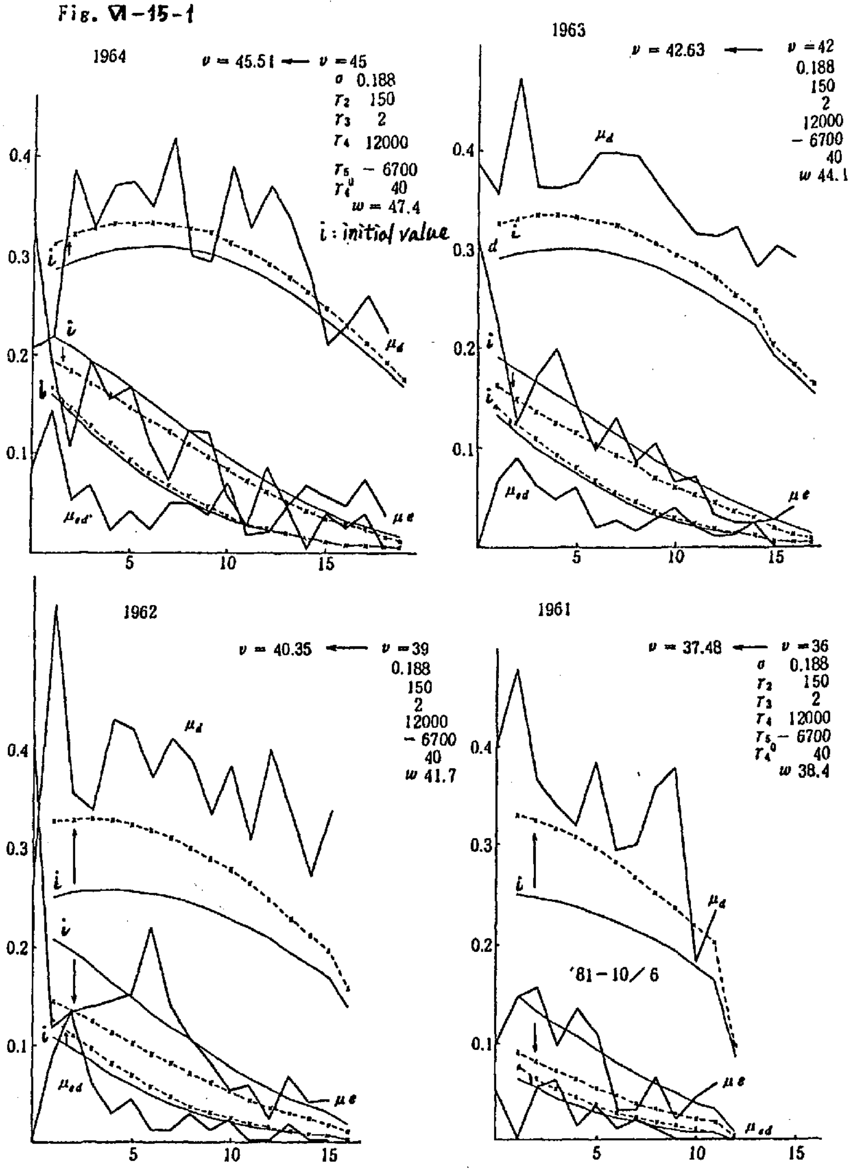
Experiment 4.

	64	63	62	61	
v	45.51	42.63	40.35	37.48	
w	47.4	44.10	41.70	38.4	
σ_u	0.188	0.188	0.188	0.188	Φ
γ_2	150	150	150	150	458.42
γ_3	2	2	2	2	↓
$\overline{\gamma}_4$	12000→12112.5	12000→11875	12000→11887.5	12000→11875	430.87
γ_4^0	40	40	40	40	
γ_5	-6700	-6700	-6700	-6700	
$\bar{h}$	0.333	0.333	0.333	0.333	

The purpose of this experiment was to check if the estimates for $\overline{\gamma}_4$ are stable for the four years, 1961 through 1964. Hence we allow the estimates for $\overline{\gamma}_4$ to vary from year to year in contrast to experiments 1 through 3. In those experiments estimates for $\overline{\gamma}_4$, as well as other preference parameters, γ_2, γ_3, γ_5, γ_4^0 and σ_u, were obtained by using the postulate that preference parameters should be stable over time.

Experiment 1

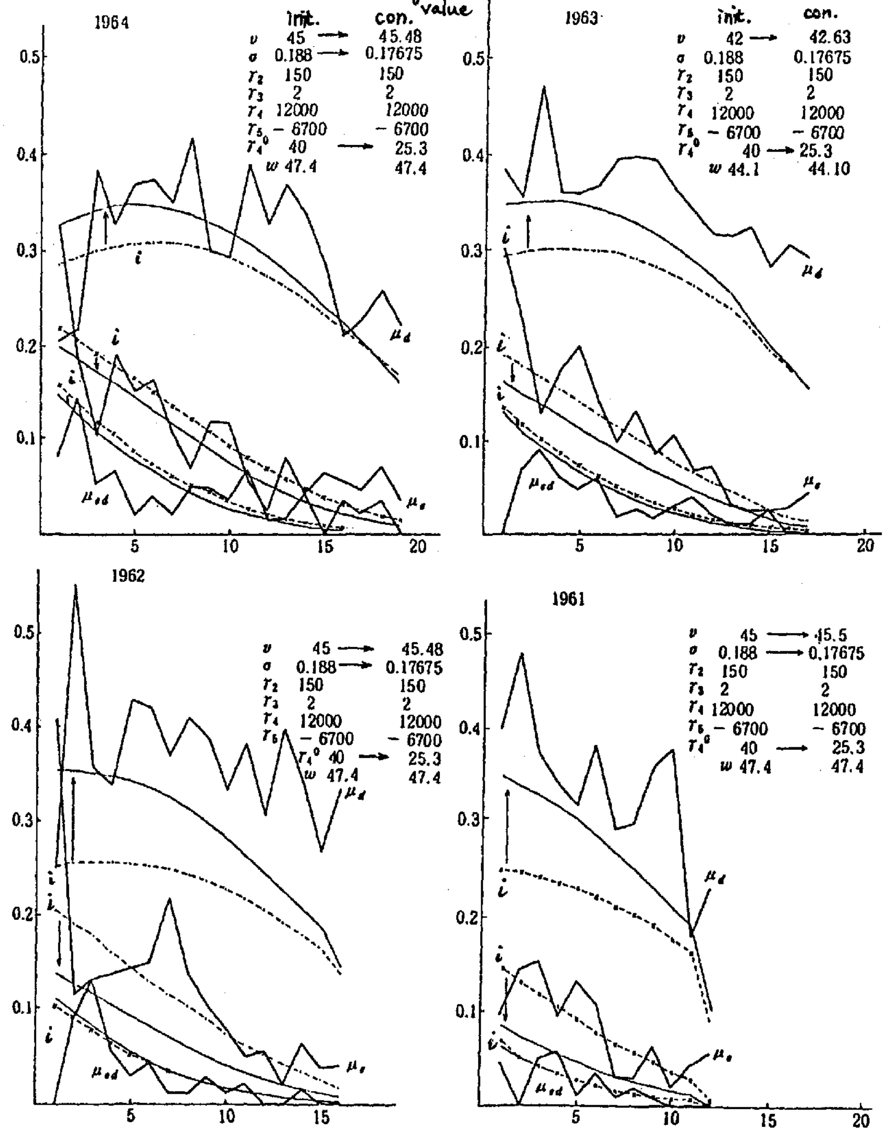
Fig. Ⅵ-15-1



Experiment 2

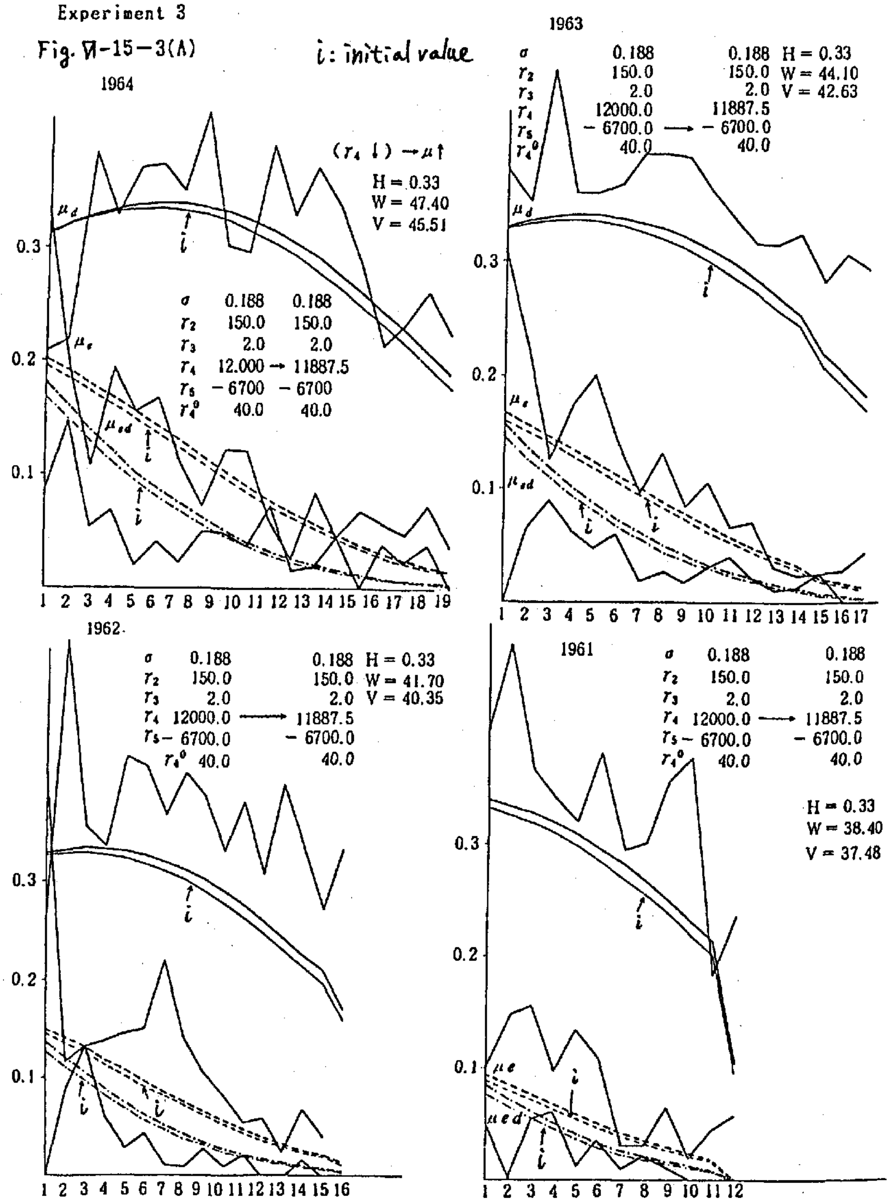
Fig. VI-15-2

init.: initial value
con.: convergence value



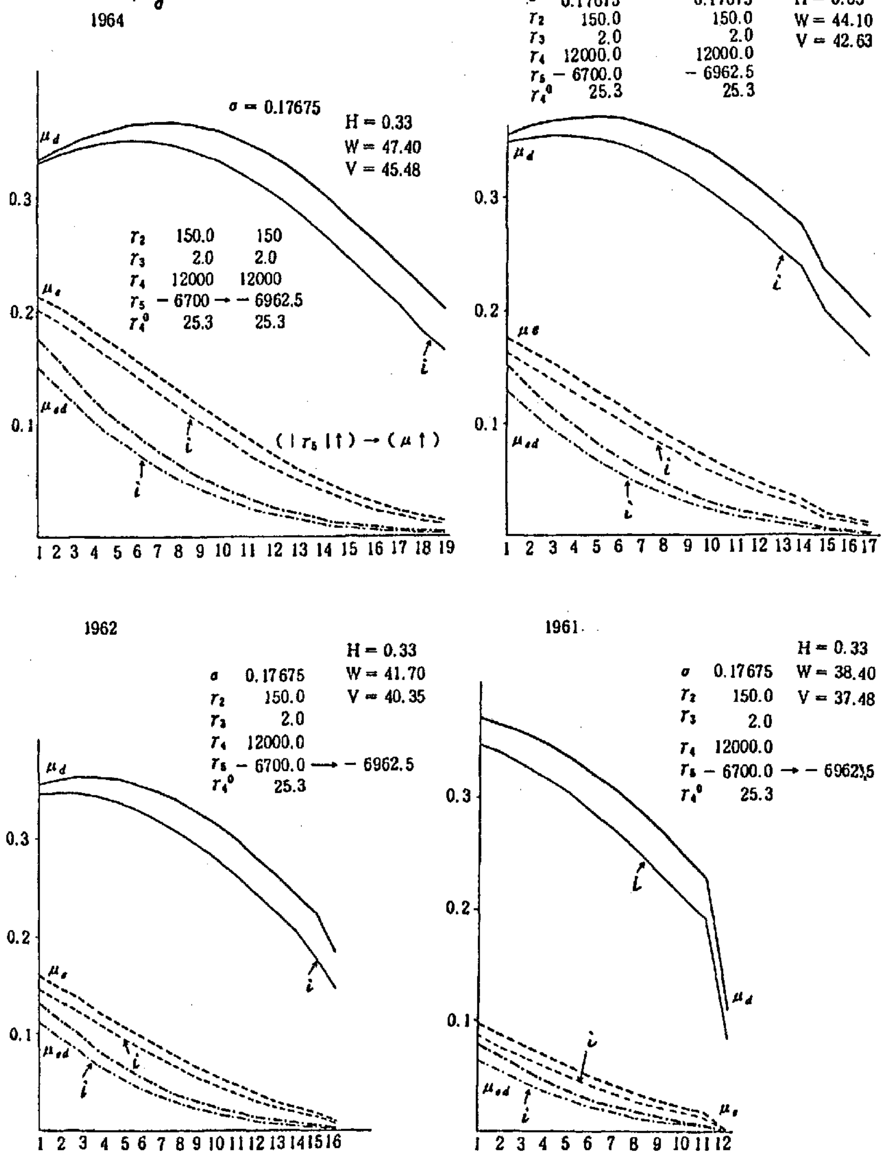
Experiment 3

Fig. VI-15-3(A) i: initial value



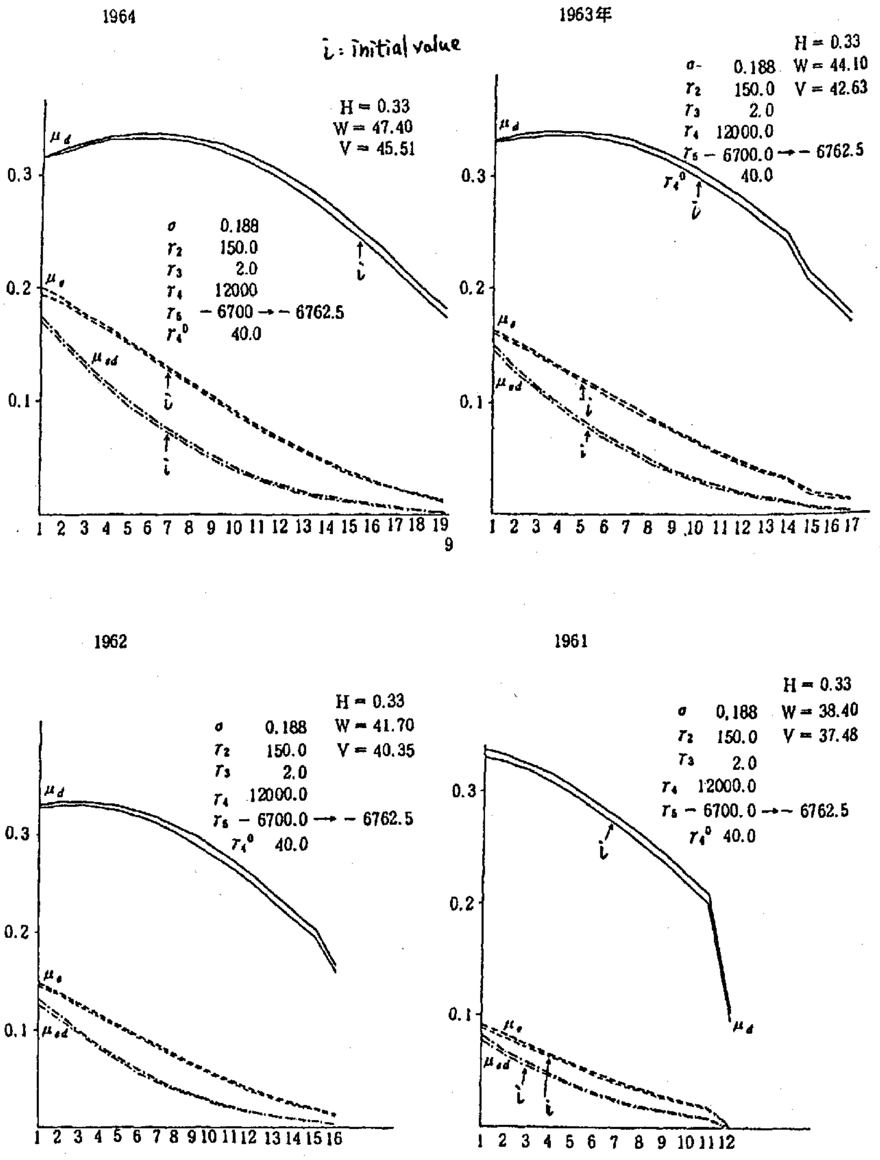
Experiment 3

Fig. Ⅵ-15-3(B)



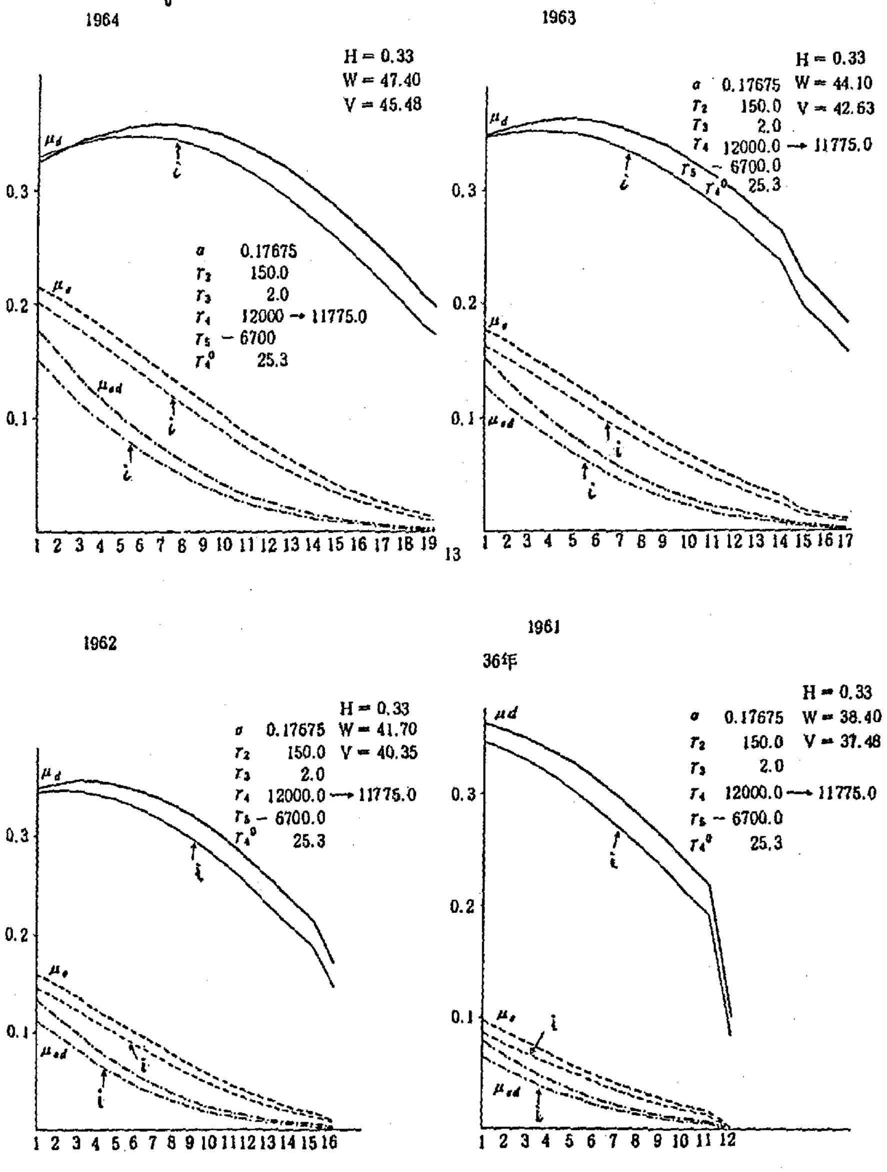
Experiment 3

Fig. Ⅵ-15-3(C)



Experiment 4

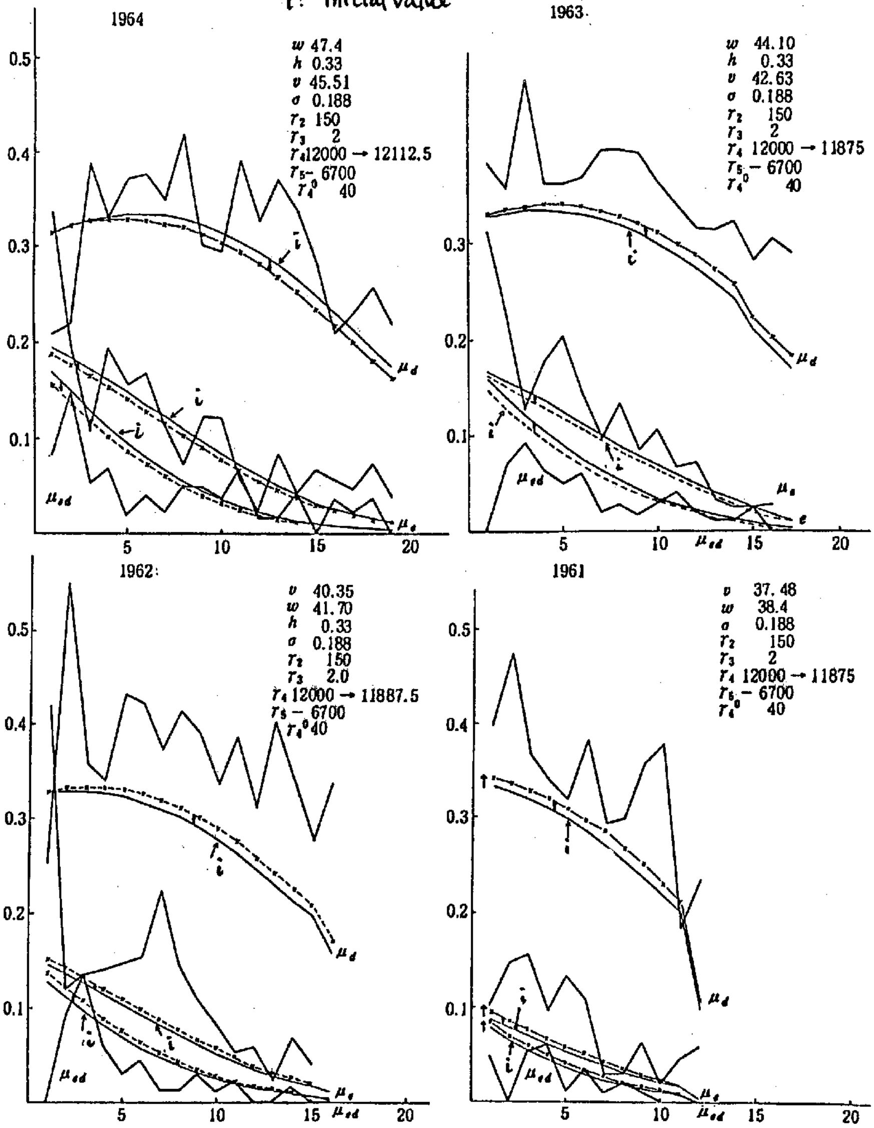
Fig. VI-15-3(D)



Experiment 4

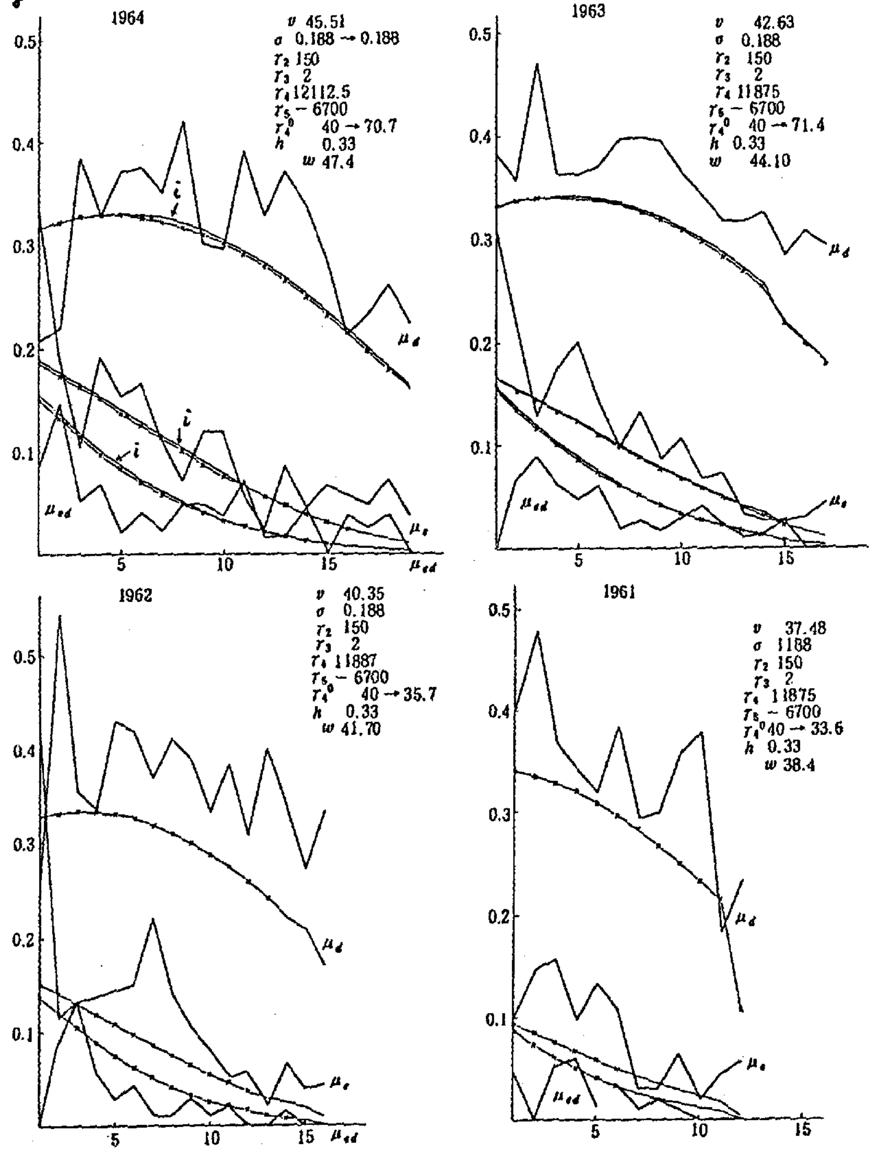
Fig VI-15-4

i: initial value

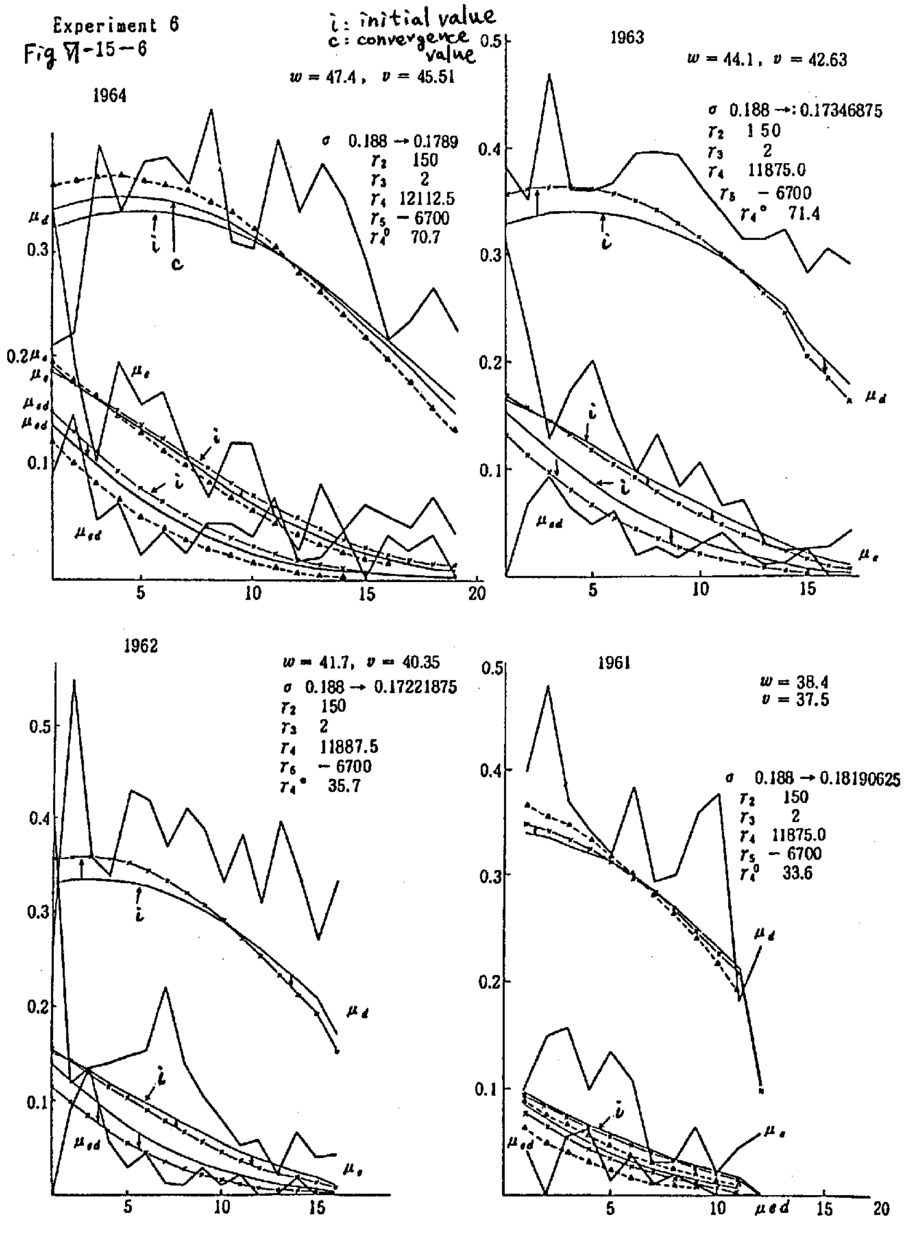


Experiment 5

Fig. VI-15-5

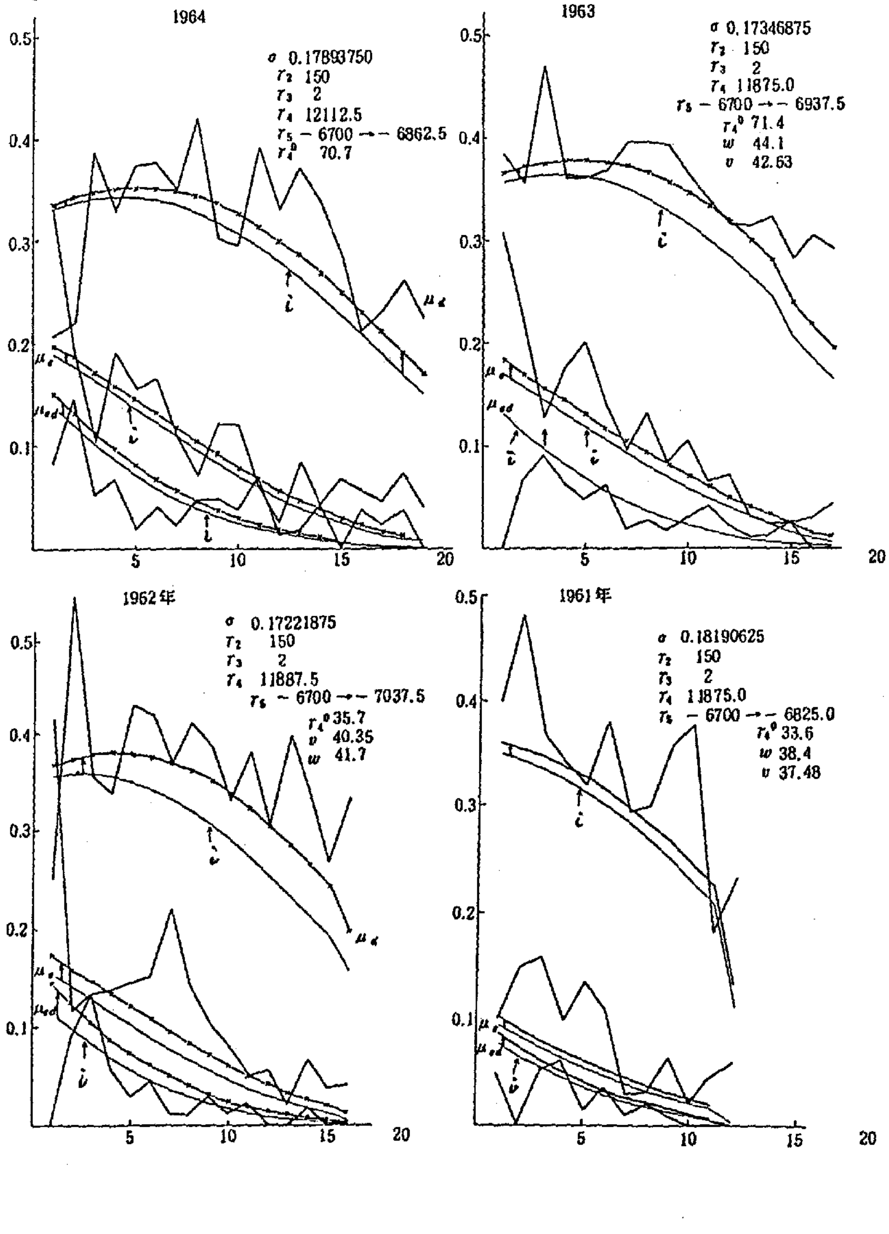


Experiment 6
Fig VI-15-6



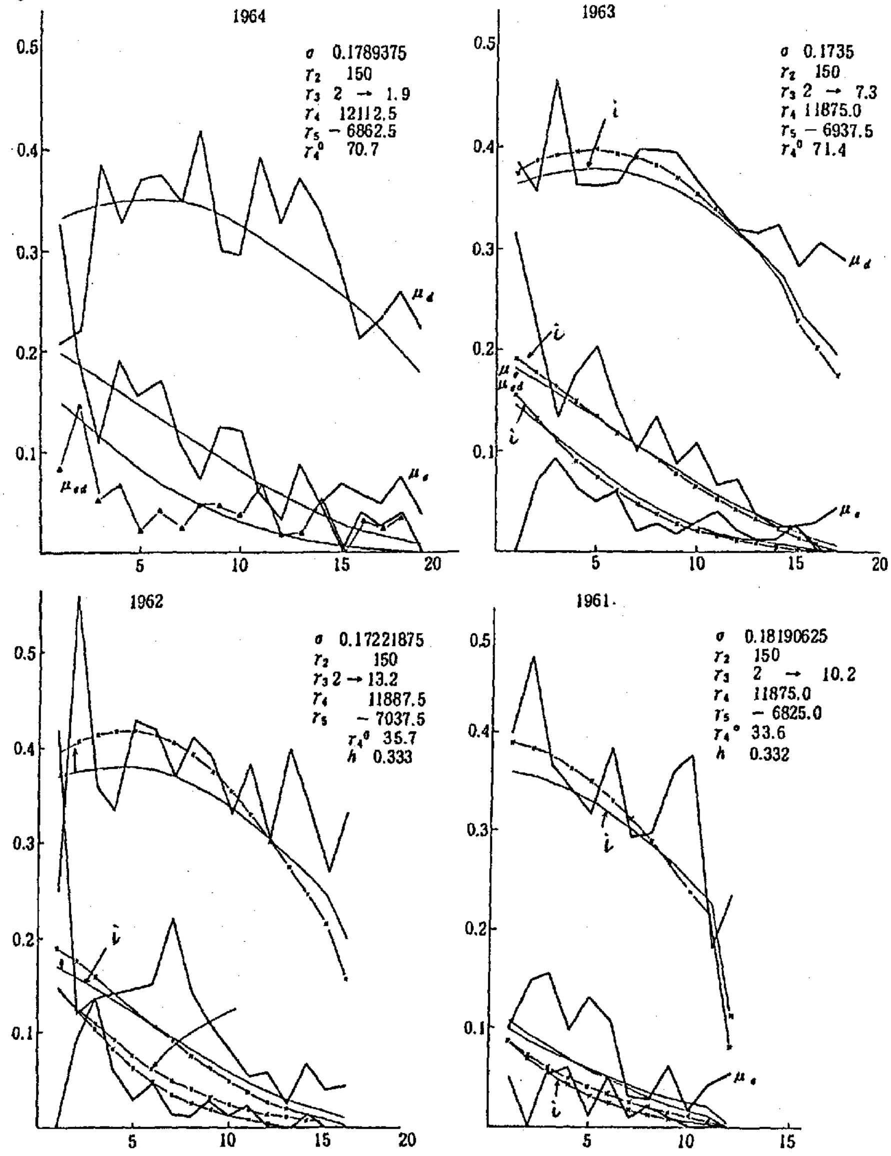
Experiment 7

Fig.Ⅵ-15-7



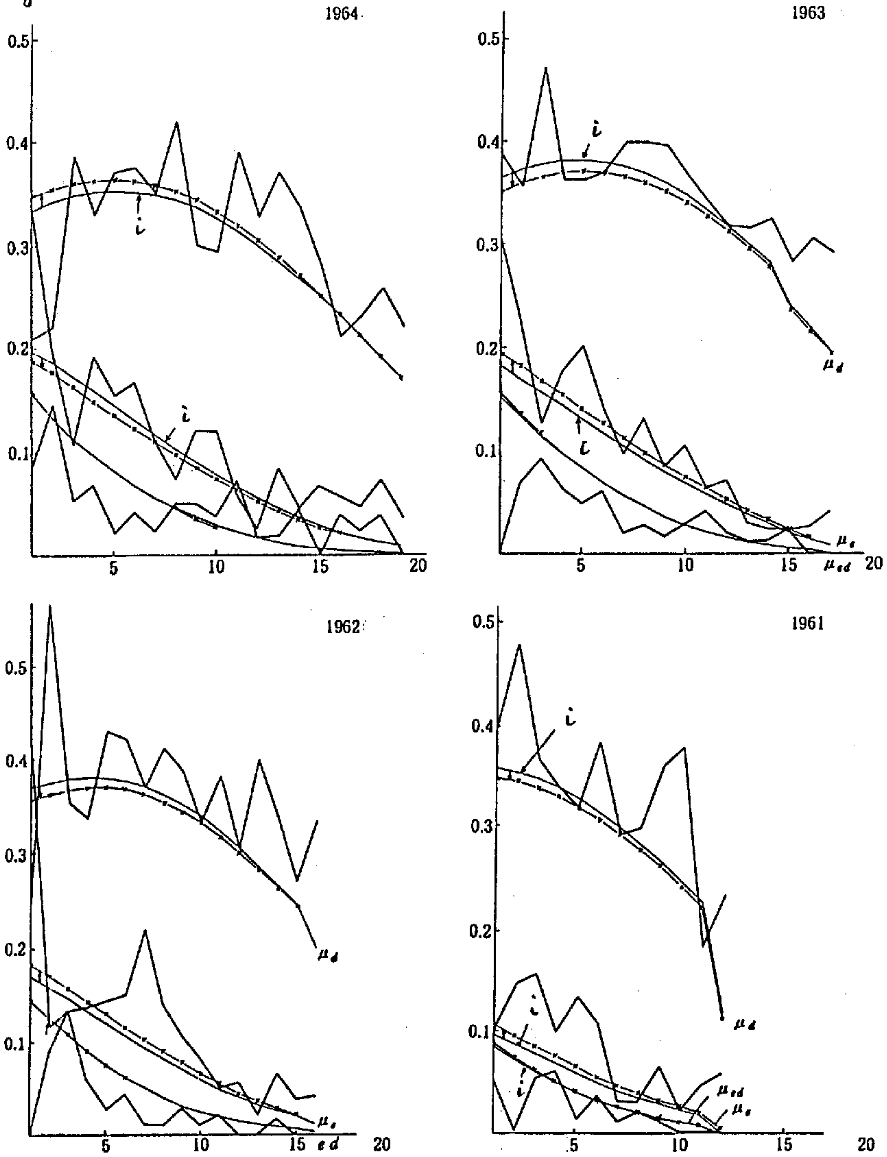
Experiment 8

Fig VI-15-8



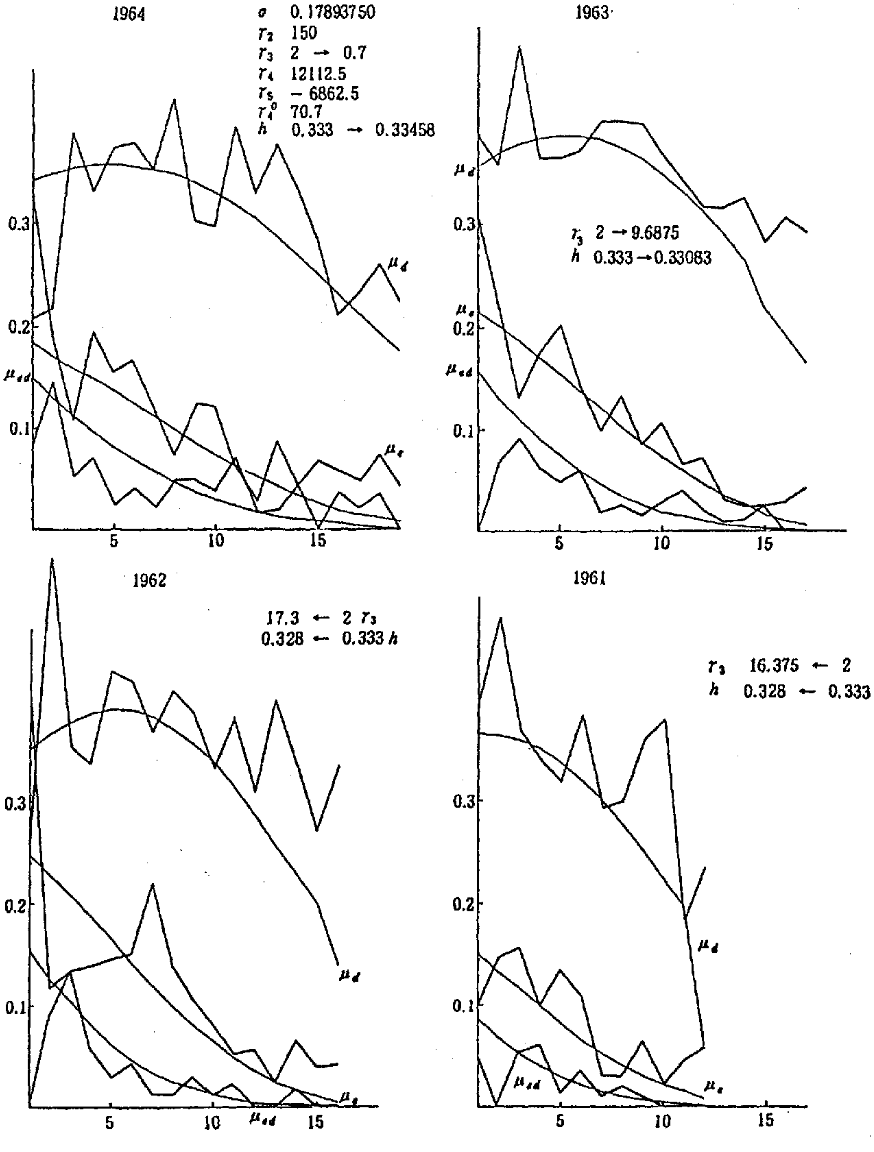
Experiment 9

Fig.Ⅵ-15-9



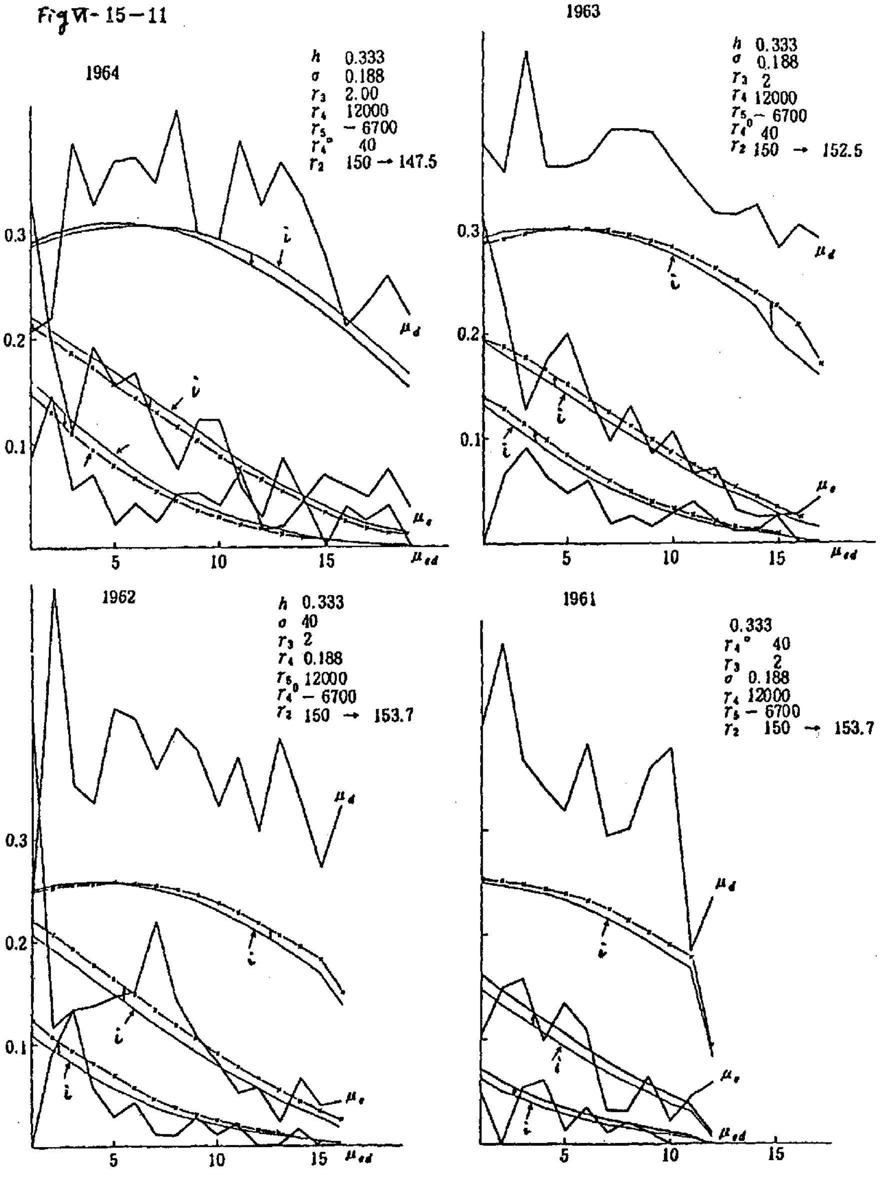
Experiment 10

Fig.Ⅵ-15-10



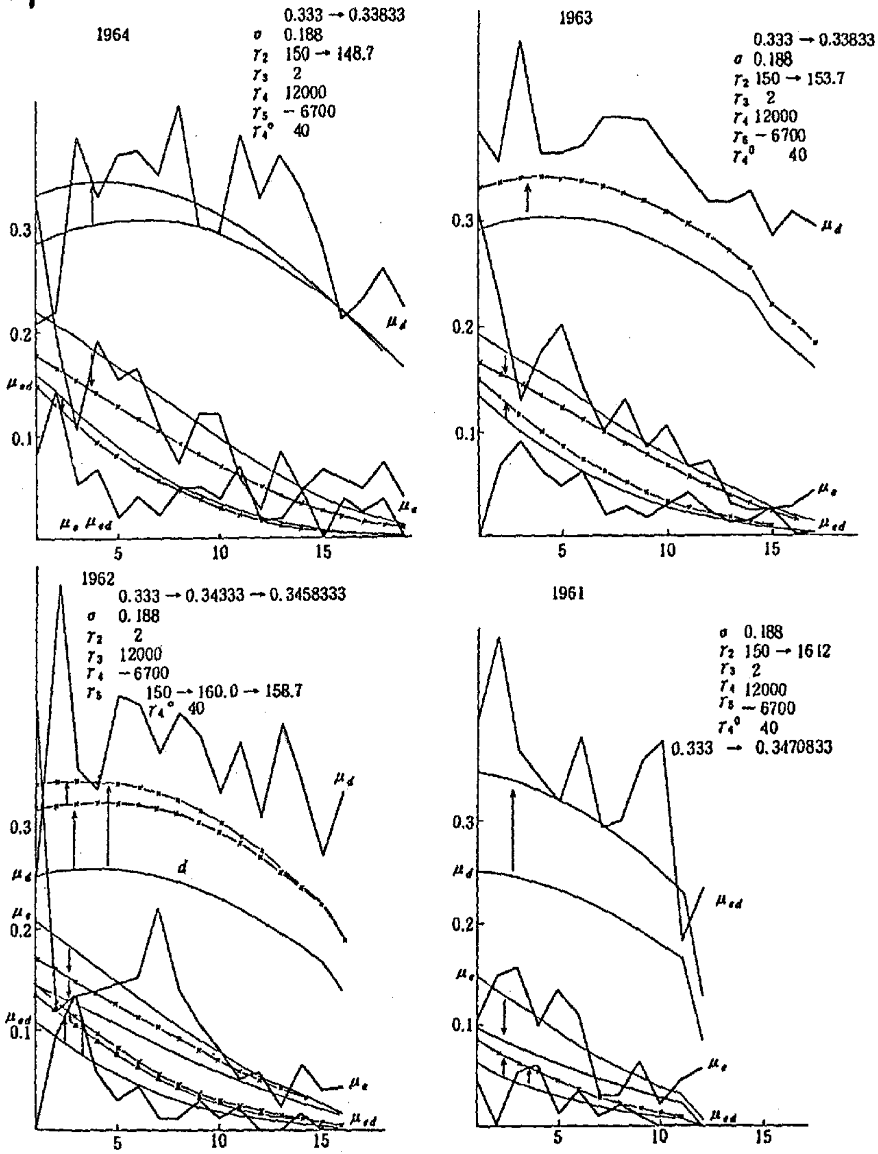
Experiment 11

Fig VI-15-11



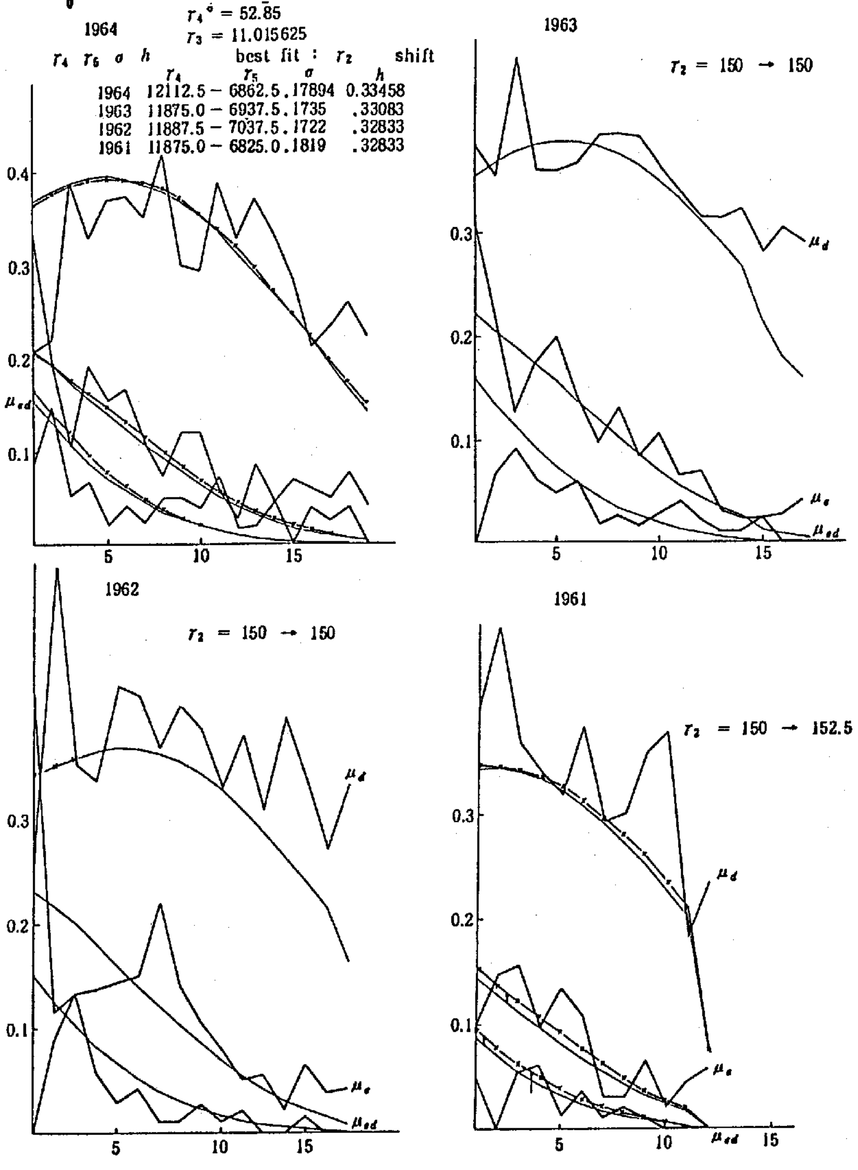
Experiment 12

Fig VI-15-12



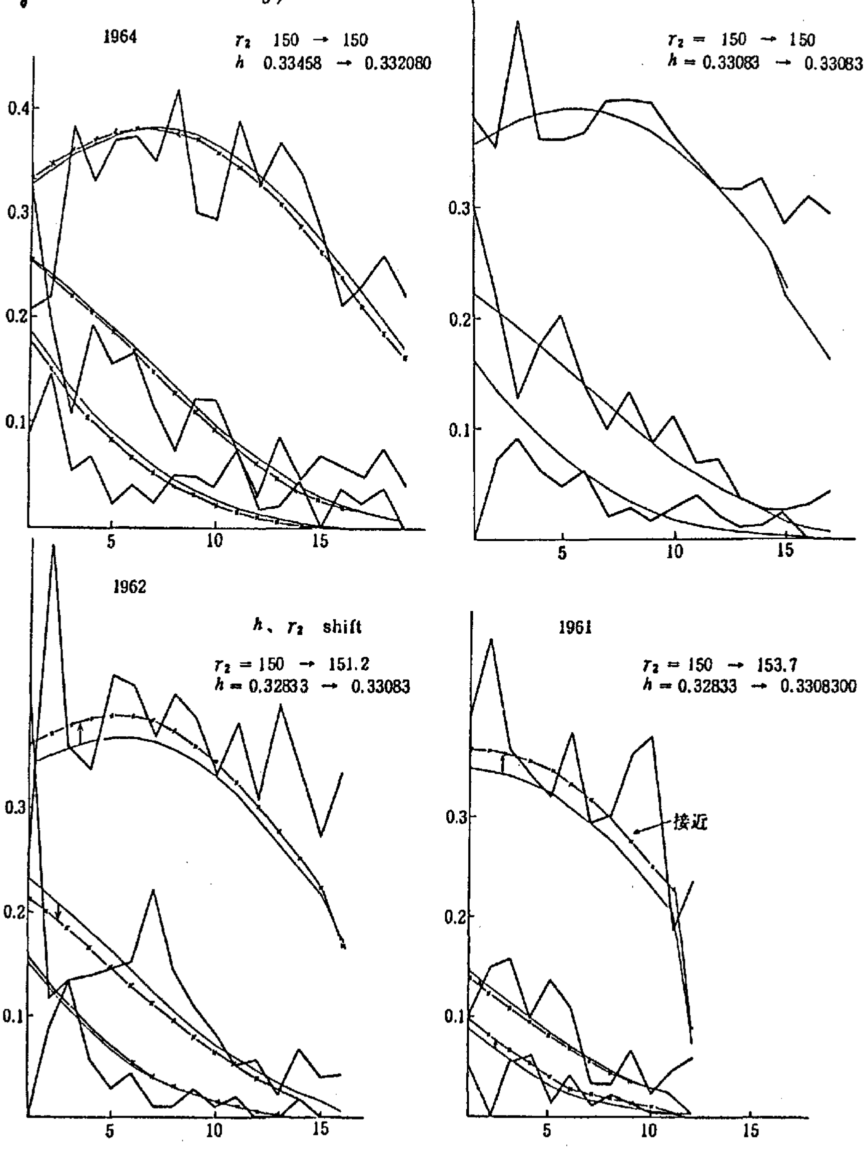
Experiment 13

Fig VI-15-13



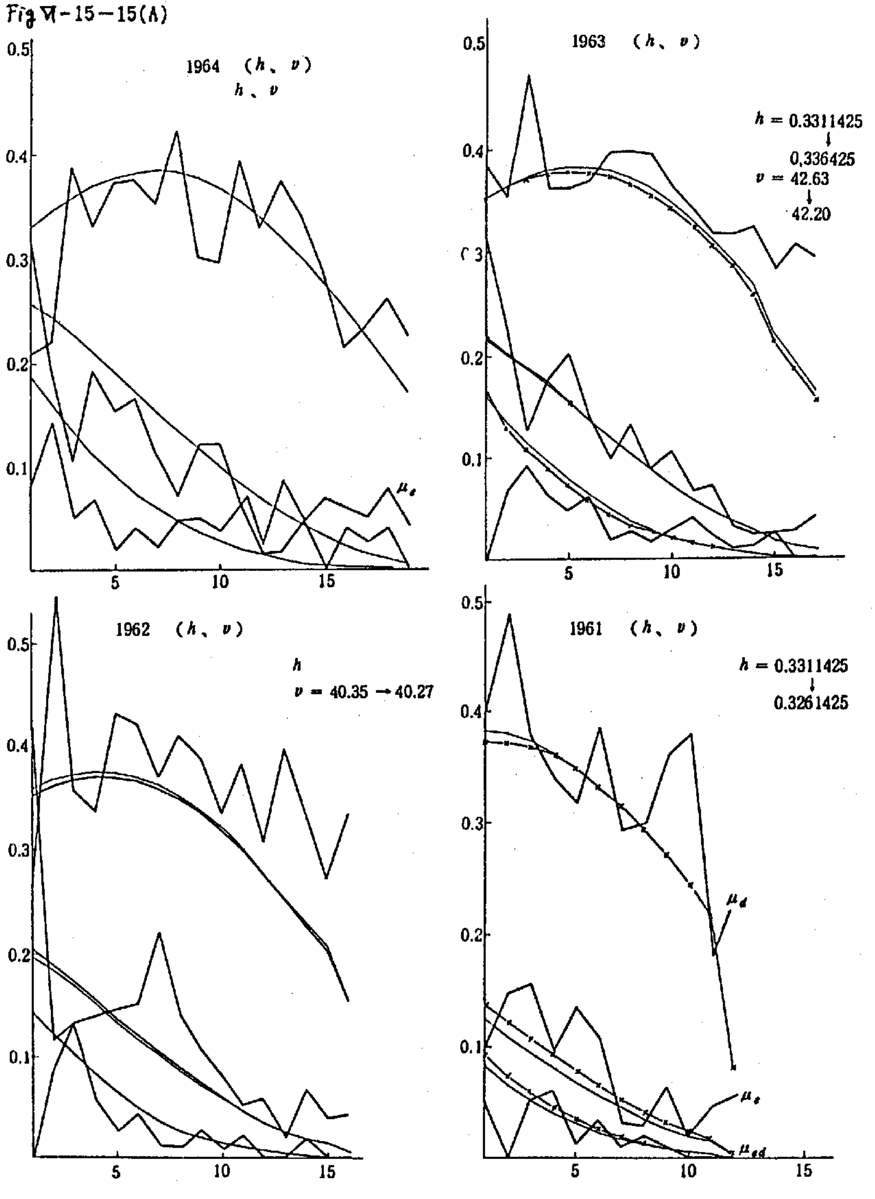
	r_4	r_5	σ	h
1964	12112.5 –	6862.5	.17894	0.33458
1963	11875.0 –	6937.5	.1735	.33083
1962	11887.5 –	7037.5	.1722	.32833
1961	11875.0 –	6825.0	.1819	.32833

Experiment 14
Fig VI-15-14

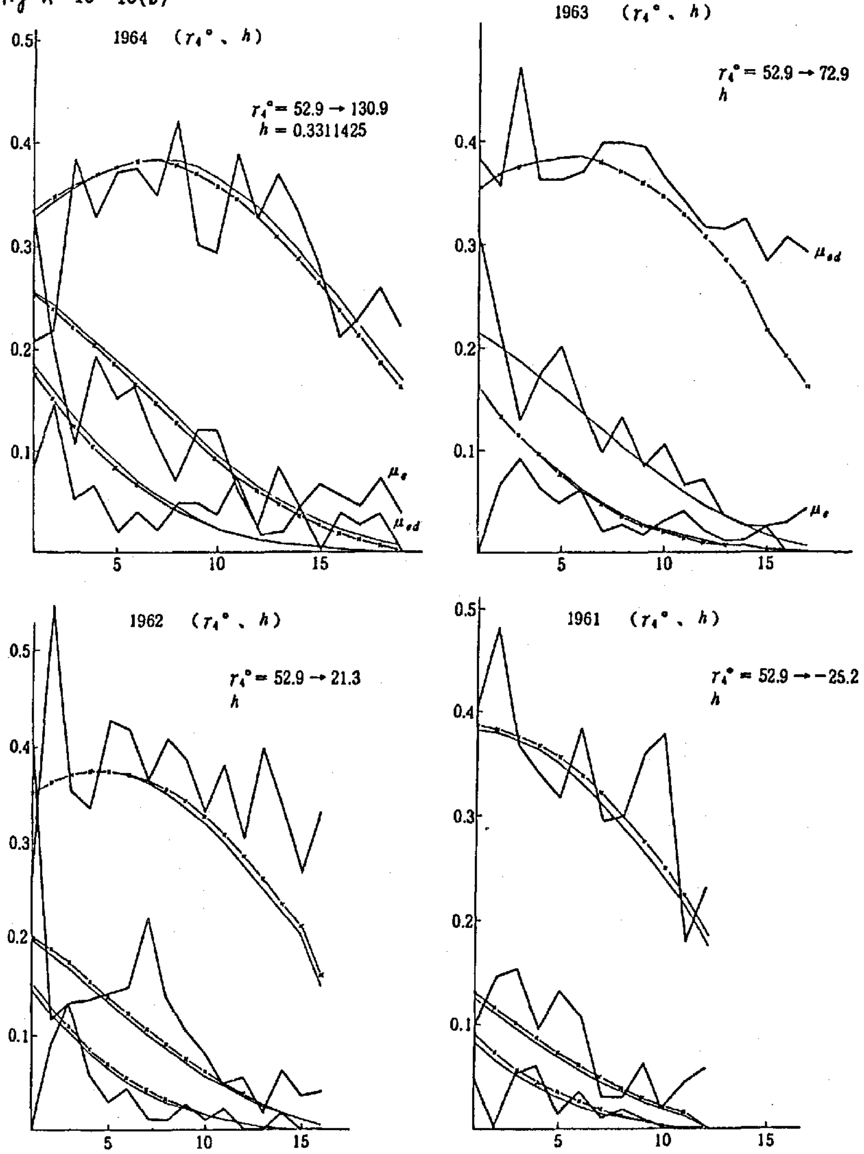


Experiment 15(A)

Fig Ⅵ-15-15(A)



Experiment 15(B)
Fig VI- 15-15(B)



Experiment 15(C)
Fig VI-15-15(C)

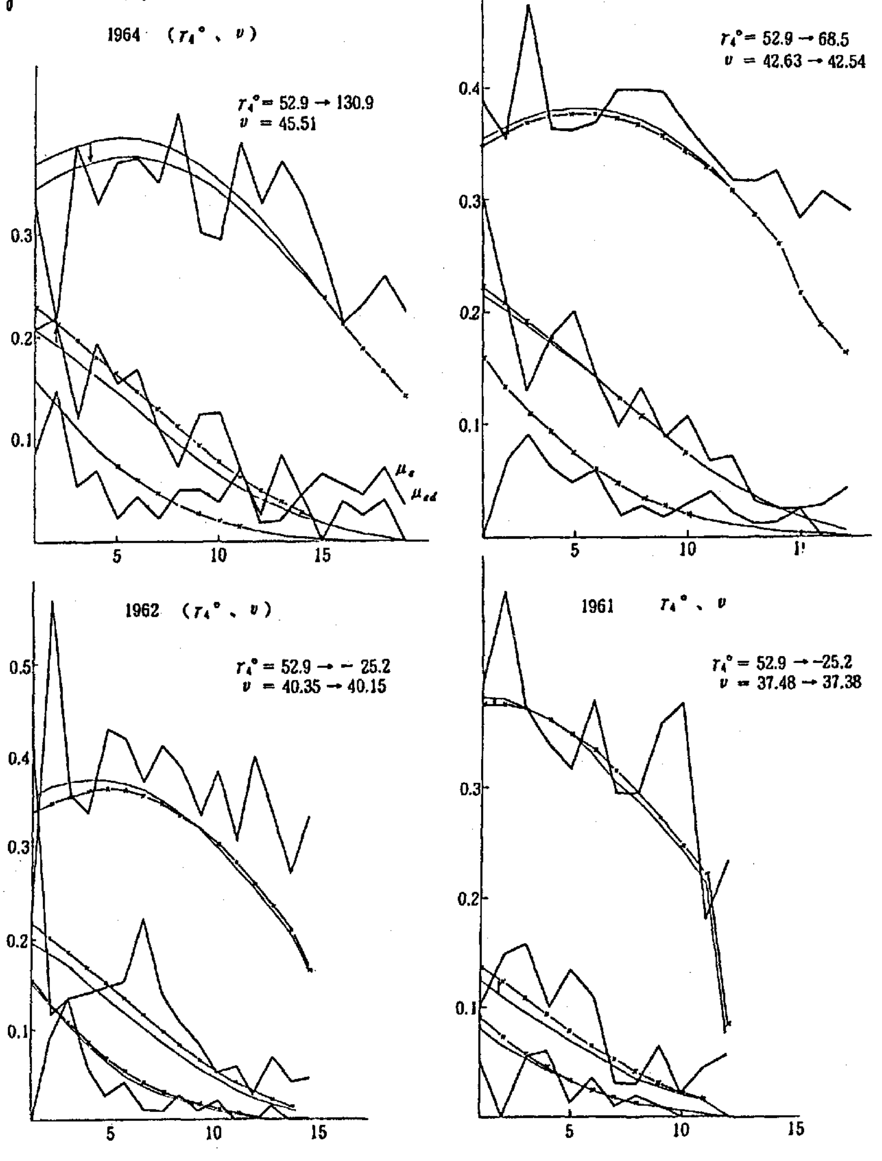
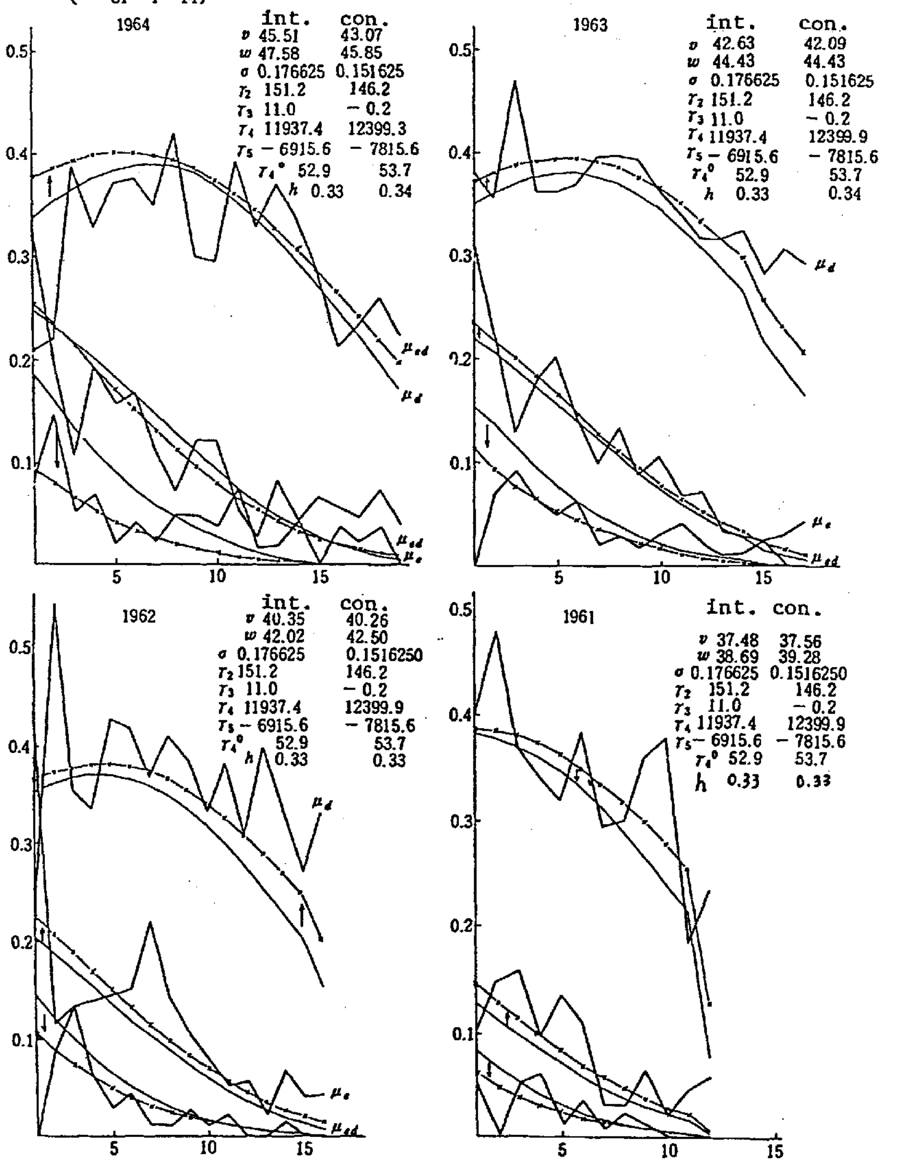


Fig. VI-15-16

int. : initial value
con. : convergence value

Experiment 16.

(81-1-14)



Numerical values for parameters which are held constant for the estimation of $\overline{\gamma}_4$ are shown in the table in experiment I. Estimates obtained for $\overline{\gamma}_4$ are indicated with an arrow. The estimates for the four years strikingly resemble each other, but on the other hand, the fitting of theoretical values to the observed values of μ^e , μ^d and μ^{ed} is not good. Hence, there remain considerable discrepancies between observed and estimated values as shown in Fig.Ⅵ-15-4 and it is therefore necessary to change the numerical values attached to the preference parameters, γ_2, γ_3, γ_5, σ_u and γ_4^0. Among those parameters we first reestimate σ_u and γ_4^0.

Experiment 5.

The initial values for w , v and $\bar{h}$

	1964	1963	1962	1961
w	47.40	44.10	41.70	38.40
v	45.51	42.63	40.35	37.48

$\bar{h}$ = 1/3

	σ_u	γ_2	γ_3	$\overline{\gamma}_4$	γ_5	γ_4^0	Φ
1964	0.188→0.188	150	2	12112.5	−6700	40→70.7	
1963	0.188→0.188	150	2	11875.5	−6700	40→71.4	431.05
1962	0.188→0.188	150	2	11887.5	−6700	40→35.7	↓
1961	0.188→0.188	150	2	11875.0	−6700	40→33.6	429.66

In this experiment σ_u and $\overline{\gamma}_4$ are allowed to vary simultaneously. For the value of $\overline{\gamma}_4$, we employ the better one obtained in experiment 4. Estimates for σ_u and γ_4^0 are shown, together with values for the parameters which are held constant, in the above table. σ_u and γ_4^0 are allowed to vary from 1961 through 1964. Results show that estimates for σ_u are almost the same as the initial values, but that estimates for γ_4^0 in 1961 and 1962 and in 1963 and 1964 respectively resemble each other. In terms of fitting, it is not clear there has been any significant improvement.

Experiment 6.

	64	63	62	61	
γ_5	-6700	-6700	-6700	-6700	
$\overline{\gamma}_4$	12112.5	11875.0	11887.5	11875.0	Φ
γ_4^0	70.7	71.4	35.7	33.6	429.66
γ_3	2.0	2.0	2.0	2.0	↓
w	47.4	44.1	41.7	38.4	389.80
v	45.51	42.63	40.35	37.48	
σ_u	0.188 ↓ 0.1789375	0.188 ↓ 0.17346875	0.188 ↓ 0.17221875	0.188 ↓ 0.18190625	

One point to be noticed among the results of experiment 5 is the apparent constancy to three decimals of the estimates of σ_u over time. However, the estimates (or convergence values) seem to be affected by the numerical values for other parameters which are held constant for estimation. In order to check this point, we tentatively replace the initial values for γ_4^0 used in experiment 5 by the ones estimated in experiment (5). We allow σ_u only to vary and let the other parameters be given as shown in the table. Estimated values for σ_u are stable over time, that is, the estimates closely resemble each other and differences between initial and convergence values are small for each year. This is a favorable result that may ensure the stability of the preference parameters over time. However, on the other hand, the fitting of theoretical values to observed values is not so easily improved, as shown in Fig. Ⅵ-15-6. In particular, there was systematic underestimation for the three years, 1961 through 1963.

Experiment 7.

	1964	63	62	61
γ_2	150	150	150	150
γ_3	2	2	2	2
$\overline{\gamma}_4$	11875.0	11875.0	11875.0	11875.0
γ_5	-6700 ↓ -6862.5	-6700 ↓ -6937.5	-6700 ↓ -7037.5	-6700 ↓ -6825.0
γ_4^0	70.7	71.4	35.7	33.6
σ_u	0.17893750	0.17346875	0.17221875	0.18190625
v	45.51	42.63	40.35	37.48
w	47.40	44.10	41.70	38.40

In the prvious experiments, 1 through 6, we did not allow γ_5 to vary. Here, γ_5 is allowed to vary in order to improve the fitting for 1961 through 1963. In this experiment, we do not use the a priori information that γ_5 should be of the same value during the years, 1961 through 1964, because we want to test the stability of estimated values for γ_5.

1961 estimates for μ^d and $\hat{\mu}^d$, approach the observed values, while $\hat{\mu}^{ed}$'s exceed those values. $\hat{\mu}^d$ approaches the observed values but not sufficiently.

1962 $\hat{\mu}^e$ and $\hat{\mu}^d$ sufficiently approach the observed values, but μ^{ed}'s do not.

1963 the same tendency as 1962.

1964 the same as 1962.

It can be seen that the estimates for γ_5 for each year are stable during the estimating period. This is a favorable result for the postalate that preference parameters are stable across years.

Experiment 8.

	64	63	62	61	
σ_u	0.1789375	0.1735	0.17221875	0.18190625	
γ_2	150	150	150	150	Φ
γ_3	2→1.9	2→7.3	2→13.2	2→10.2	313.033
$\bar{\gamma}_4$	12112.5	11875.0	11887.5	11875.0	↓
γ_5	-6862.5	-6937.5	-7037.5	-6825.0	284.946
γ_4^0	70.7	71.4	35.7	33.6	

We have not yet allowed γ_3 to vary. In the same manner as for γ_5 in experiment 7, we allow γ_3 to vary in this experiment. γ_5 is fixed at the values obtained in experiment 7, and the values for other parameters, except for γ_3, are given as in experiment 7.

	1961	62	63	64
γ_3	10.2	13.2	7.3	1.9

As shown in Fig.Ⅵ-15-8, the effect of changing the value of γ_3 is remarkable. Considerable improvement in the fit is observed, for the first time, for years 1961 through 1963.

Experiment 9.

	64	63	62	61	
σ_u	0.1789375	0.1735	0.17221875	0.18190625	
γ_2	150	150	150	150	
γ_3	2	2	2	2	Φ
$\bar{\gamma}_4$	12112.5	11875.0	11887.5	11875.0	313.0334
γ_5	-6862.5	-6937.5	-7037.5	-6825.0	↓
γ_4^0	70.7	71.4	35.7	33.6	310.8691
$\bar{h}$	0.333 ↓ 0.33458	0.333 ↓ 0.332	0.333 ↓ 0.332	0.333 ↓ 0.332	

In this experiment, parameter $\bar{h}$ only is allowed to vary. Other parameters are fixed at the values for experiment 7.

1961	62	63	64
0.332*	0.332	0.332	0.335

(* initial values are all 0.333)

The estimates of $\bar{h}$ for four years are stable over time. However, no remarkable improvement can be seen in the estimates obtained.

Experiment 10.

	64	63	62	61	
γ_3	2→0.6875	2→9.6875	2→17.3125	2→16.375	Φ
$\bar{h}$	0.333→0.33458	0.333→0.33083	0.333→0.32833	0.333→0.32833	313.0334
$\bar{\gamma}_4$	12112.5	11875.0	11887.5	11875.0	↓
γ_5	-6862.5	-6937.5	-7037.5	-6825.0	267.657
γ_4^0	70.7	71.4	35.7	33.6	
σ_u	0.17894	0.1735	0.1722	0.1819	

Taking into account the results obtained in experiment 8 and 9, we allow γ_3 and h to vary simultaneously. Changes in the estimated values for γ_3 from the initial values can bee seen especially in 1963 and 64, and the estimated values fit the observed values somewhat better.

	1964	63	62	61
γ_3	0.6875	9.6876	17.3125	16.375
$\bar{h}$	0.3346	0.3308	0.3283	0.3283

Experiment 11.

In this experiment, γ_2 only is varied, other parameters being fixed at the initial values given in experiment 1. Estimated values for γ_2 in years 1961 through 1964 are as follows:

1961	62	63	64
153.7	153.7	152.7	147.5

Over time, the estimates are fairly stable. The results show fairly good fit for the 1964 data but underestimates were obtained for the other years.

Experiment 12.

	1964	1963	1962	1961	Φ
γ_2	150→148.7	150→153.7	150→158.7	150→161.2	543.298 ↓
$\bar{h}$	0.333→0.33833	0.333→0.33833	0.333→0.3458333	0.333→0.3470833	407.7284

We allow γ_2 and h to vary simultaneously in this experiment. The values for γ_4^o, γ_3, $\bar{h}$, $\overline{\gamma}_4$, γ_5 and σ_u are fixed at those values used in experiment 11. The results are listed in the table below.

	1961	62	63	64
γ_2	161.2	158.7	153.7	148.7
$\bar{h}$	0.3471	0.3458	0.3383	0.3383

Little difference is found between the values for γ_2 estimated in this experiment and those in experiment 11. The values for γ_2's and $\bar{h}$'s are respectively fairly similar among the years.
The μ^d's for the years 1961 and 1963 are still underestimated and also are underestimated for the upper principal earners' income classes in the year 1962. The fitting of the μ_e 's are somewhat improved for 1964.

Experiment 13.

As is shown in experiment 5, discrete changes in the estimates for γ_4^o are observed as time passed; that is, those estimates for 1963 and 1964 are larger than those for 1961 and 1962. Also we have found that estimated values for other preference parameters, γ_2, $\overline{\gamma}_4$ and so on, are fairly stable over time, with the exception of γ_3. Therefore, it is reasonable to hypothesize that all the preference parameters,

γ_2, γ_3, $\overline{\gamma}_4$, γ_5, γ_4^0 and σ_u, are respectively constant over the years 1961 to 1964 ($\gamma_1 \equiv -1$). That is, significant differences in the estimates for γ_4^0 and σ_u, respectively, might stem from the inadequacy of the values for other preference parameters used in experiment 5 and 8. (The significant differences in estimates among years appeared in experiments 5 and 8). Hence, calculating mean values of the estimates for $\gamma_4{}^0$ and σ_u, respectively for the time period 1961 through 1964, we obtain $\gamma_4{}^0 = 52.85$ and $\gamma_3 = 11.0156$. Making use of those values for γ_3 and $\overline{\gamma}_4$ and those values for other preference parameters given in experiment 9, we allow γ_2 only to vary. By doing so we obtain estimates for γ_2 as follows.

1961	62	63	64
152.5	150.0	150.0	151.2

It should be noted that differences in yearly estimates of γ_2 are less than those obtained in experiment 12 where we did not use mean values for $\gamma_4{}^0$ and γ_3. Also the magnitude of the objective function Φ in this experiment is smaller than that obtained in experiment 12. Thus, it may be concluded that the results of this experiment tend to verify the above hypothesis.

Experiment 14.

	1964	1963	1962	1961	
γ_2	150.0	150.0	151.2	153.7	151.225
$\bar{h}$	0.33208	0.33083	0.33083	0.33083	0.3311425
$\overline{\gamma}_4$	12112.5	11875.0	11887.5	11875.0	11937.375
γ_5	-6862.5	-6937.5	-7037.5	-6825.0	-6915.625
σ_u	0.17894	0.1735	0.1722	0.1819	0.176625
γ_4^0	52.85	52.85	52.85	52.85	(52.85)
γ_3	11.015625	11.015625	11.015625	11.015625	(11.015625)
v	45.51	42.63	40.35	37.48	
w	47.4	44.1	41.7	38.4	

The stability of the estimates for γ_2 over time is confirmed by this experimentwhich allows γ_2 and $\bar{h}$ to vary simultaneously. The results are shown in the following table.

	1961	62	62	64
γ_2	153.7	151.2	150.0	150.0
$\bar{h}$	0.3308	0.3308	0.3308	0.3321

Although slight successive reduction in the estimates for γ_2 are observed during four yers, the estimates are fairly stable and those for $\bar{h}$ are extremely stable. The magnitude of the objective function in this experiment is also smaller compared to that value calculated with the initial values of preference parameters.

Experiment 15.

Taking into account the results of experiments 13 and 14, that estimated values for the parameters are stable over time, the values for preference parameters, $\overline{\gamma_4}$, γ_5, σ_u, γ_3 and γ_2, are respectively fixed at mean values of estimates for the years, 1961 through 1964 in this experiment.

We allow two of the three parameters $\bar{h}$, v and γ_4^0, to vary simultaneously as is shown in the following table, (a) through (c).

a) $\bar{h}$ and v are varied.

	1964	63	62	61
$\bar{h}$	0.3311	0.3364	0.3311	0.3261
v	45.51	42.20	40.27	37.97

These parameters are not preference parameters which are assumed to be constant, but rather are the assigned hours of work and the earning rate for self employed work which may vary over time.

b) γ_4^0 and h are varied simultaneously. The estimates for γ_4^0 have been observed to change over time since experiment 5. In this experiment, we examine if estimates for γ_4^0 vary when using the set of parameters fixed at mean values. The results are:

	1964	63	62	61
γ_4^0	130.9	72.9	-21.3	-25.2
$\bar{h}$	0.3311	0.3311	0.3311	0.3311

The variation in the estimates for γ_4^0 reappears. However, the magnitude of the objective function is larger compared to the value obtained the value obtained in experiment 14 where γ_4^0 was held constant over time. Hence, it can be seen that allowing estimates for γ_4^0 to vary has no real merit. That is, we can obtain a better set of preference parameters by holding γ_4^0 constant and choosing better values of the other parameters.

(c) γ_4^0 and v are allowed to vary.

The estimation results are as follows.

	1961	62	63	64	Φ
γ_4^0	-25.2	-25.2	68.5	130.9	327.078
v	37.38	40.15	42.54	45.51	306.992

In this experiment variation in the estimates of γ_4^0 reappears as in experiment (b). In this case, although the maginitude of Φ is a little smaller than that in (b), it is larger than Φ in experiment 13 or 14 where γ_4^0 is held constant. Hence, it can be seen in this case also, that allowing γ_4^0 to vary over time has no merit in improving the fitting of estimated to observed valves.

Experiment 16.

	1964	1963	1962	1961
v	45.51 ↓ 43.07	42.63 ↓ 42.09	40.35 ↓ 40.26	37.48 ↓ 37.56
w	47.58 ↓ 45.85	44.43 ↓ 44.43	42.02 ↓ 42.50	38.69 ↓ 39.28
σ_u	0.176625 ↓ 0.151625	0.176625 ↓ 0.151625	0.176625 ↓ 0.151625	0.176625 ↓ 0.151625
γ_2	151.2 ↓ 146.2	151.2 ↓ 146.2	151.2 ↓ 146.2	151.2 ↓ 146.2
γ_3	11.0 ↓ -0.2	11.0 ↓ -0.2	11.0 ↓ -0.2	11.0 ↓ -0.2
$\overline{\gamma}_4$	11937.4 ↓ 12399.9	11937.4 ↓ 12399.9	11937.4 ↓ 12399.9	11937.4 ↓ 12399.9
γ_5	-6915.6 ↓ -7815.6	-6915.6 ↓ -7815.6	-6915.6 ↓ -7815.6	-6915.6 ↓ -7815.6
γ_4^0	52.9 ↓ 53.7	52.9 ↓ 53.7	52.9 ↓ 53.7	52.9 ↓ 53.7
$\bar{h}$	0.33 ↓ 0.34	0.33 ↓ 0.34	0.33 ↓ 0.33	0.33 ↓ 0.33

Φ
323.03041
↓
211.33099

Taking into account the results obtained by the previous experiments, it may be argued that there is no strong evidence contradicting the assumption of the constancy of preference parameters over time. Therefore, if the parameters are, at least locally, identifiable we will obtain more favorable estimation results by making use of the a priori information that preference parameters are constant over the years. Hence, in this experiment we use as initial values for the parameters the average values for four years with respect to preference parameters, γ_2, γ_3, $\overline{\gamma}_4$, γ_5, γ_4^0 and σ_u which are listed in the table in experiment 15. Other parameters, w, v and $\bar{h}$ are of course allowed to vary over time. Initial values for these are also listed in the table. Making use of these values as initial values for the parameters, we can estimate all the parameters by allowing all of them to vary simultaneously. All the estimates for the preference parameters are restricted to be constant over time. The results are shown in the last table.

The steepest ascent method was employed for estimation. The speed of convergence in the process of obtaining estimates was faster than that in experiment 15. It can be seen that we attained the best fitting results amongst all the estimates obtained in section Ⅵ. That is, the problem of systematic underestimation for μ^d was resolved except for the lower income classes in 1964, and fittings for μ^e and μ^{ed} were improved.

SUMMARY and CONCLUSION

1. The aims of this book have been twofold: (1) to clarify the mechanism by which the quantity of labor supplied in terms of probability and that supplied in terms of optimal hours are interrelated, and (2) to examine various mechanisms to describe this, using a quantitative and autonomous system based on income-leisure preference functions. Hence, mathematical models for describing the determination of labor supply probability were consturcted. By applying suitably designed experiments to those mathematical models, the numerical (quantitative) models were tested against data.

2. As a starting point, the most basic and simplest case was analysed. To that end, models for household with a gainfully employed husband and a wife who is either employed or is not were presented. This type of household is called type-A.

For type-A households, two kinds of models were constructed, an employment-opportunity model and an employee-self-employed model. The former is a model in which possible earning opportunities for self-employed work for wives are ignored for the sake of brevity and the latter models both earnings opportunities.

The employment-opportunity model was tested against the data for the years 1961 through 1964 and was found to not be entirely satisfactory. The latter model, the employee-self-employed model, assumes wives confront employment opportunities defined as a combination of the wage rate, w, and assigned hours of work, $\bar{h}$, and alternatively as self-employment with given earnings rates but variable hours of work. Hence, when selfemployed wives can freely choose as their actual hours of work those which maximize their utility indicator functions. The model thus describes wives' probabilities of work for employee work only, for self-employed work only, as well as for both symultaneously.(*)

(*) In the employment-opportunity model, the determination of the employment participation probability μ^e (μ^e≡number of wives gainfully employed as employees/number of wives) is discussed. In the employee-self-employed model, the participation probability for self-employed work μ^d (μ^d≡number of wives self-employed/number of wives) and the double participation probability (μ^{ed}≡number of wives gainfully employed symultaneously both as employee and self-employed) are discussed together with μ^e.

3. In the first part of this book, income-leisure preference parameters for each year, 1961 through 1964, were estimated making use of the employment-opportunity model. However, the estimates of these parameters fluctuates over times and some sets of the parameters for some years did not satisfy the necessary conditions for stability. At most, the employment-opportunity model may sometimes serves as a first approximation model.

Therefore, a more precise model, the employment-self-employed model, was introduced and the preference parameters were estimated. For estimation, initial values of the parameters were needed, and forthis the values of the parameters obtained from the employment-opportunity model were used. A summary of the results of the estimation are given below.

(a) The three kinds of participation probabilities, μ^e, μ^d and μ^{ed}, observed from cross sectional data classified by principal earners' income size for each year, change from year to year. It was shown that those changes could be explained by yearly changes in the wage rate, w, hours of work assigned by employers, $\bar{h}$, and the earning rate for self-employed work, v.

Estimates of the parameters for income-leisure preference functions were fairly stable across the four observed years 1961 through 1964.

(b) The income-leisure function used in the analysis was of the Allen-Bowley (quadratic) type,

$$\omega = \frac{1}{2}\gamma_1 \cdot X^2 + \gamma_2 \cdot X + \gamma_3 \cdot X \cdot \Lambda + \gamma_4 \cdot \Lambda + \frac{1}{2}\gamma_5 \cdot \Lambda^2$$

where X and Λ respectively stand for income and leisure, γ_i being parameters.

This Allen-Bowley type preference function was suitable for labor supply analysis for the observational periods considered.

(c) The estimated preference parameters include those for the density distribution function of preference parameters. The distribution function is assumed to describe differences in preference among the households considered.

Two alternative hypotheses, i.e., ① the value for γ_2 differs among households and ② the value for γ_4 differs among households, were examined against the data. It was found that the latter hypothesis was preferable to the former. Hence, we introduced a density distribution function for γ_4. As to the functional form of the density function, a log normal distribution was consistent with the observations.

(d) Observed values for μ^e of wives having husbands with the same income decline the higher are their husbands' income. That is, a cross sectional μ^e curve, drawn with the abscissa measuring husbands' income size, is downward sloping, though not necessarily linear. This has been widely observed in the U.S. as well as in Japan (The Douglas-Long-Arisawa effect).

On the other hand, it was found that cross sectional μ^d curves for the years 1961 through 1964 were not necessarily downward sloping, and for some years the curves were of reverse U shape; that is, both in the lower and higher husbands' income groups μ^d is less than in the middle income groups.

It was shown that our theoretical model for the simultaneous determination of μ^d, μ^e and μ^{ed} is consistent with the reversed U shaped μ^dcurve observed.

4. Qualifications

(1) In this study we only roughly controlled the type of household, limiting analysis to type-A households. The numbers of children under 15 years of age were not controlled in order to avoid decreasing the sample size of each group of households classified by husbands' income size. However, controlling for the age of childrens may contribute to better parameter estimates.

(2) We employed a linear function for the wives' earning (production) function for self-employed work. However, a nonlinear function may be more appropriate.

(3) The wage rate, w, and assigned hours of work, $\bar{h}$, have a joint density distribution ϕ(w, $\bar{h}$). In this analysis we did not explicitly introduce this joint distribution into the model but made use of the mean values of w and $\bar{h}$ respectively without refering to their variance and corariance.

(4) We plan to correct the deficiencies mentioned above by modifying and re-estimating the model using data from the Participation Structure Survey for the years 1971, 1974, 1977 and 1979.

(5) In the analysesin this book, we have focused our attention on the behavior of the labor suppliyer. However, the behavior of demand for labor was also taken into account as far as it was needed in identifying supply relations using observed data which reflects the interaction of both demand and supply. In this context a model for the labor market was presented in section 3.3.

(*) Obi Keiichiro, Observations vs. Theory of Household Labor Supply Keio Economic Observatory Occasional Paper, Vol. I (April 1987), pp.55-101

Tab. VI—10 DATA

1964

	*	I (1)	μ^{d} (2)	μ^{e} (3)	μ^{ed} (4)	N (5)
1	4	15.042	.2083	.3333	.0833	24
2	5	18.245	.2195	.1951	.1463	41
3	6	21.891	.3860	.1053	.0526	57
4	7	25.190	.3295	.1932	.0682	88
5	8	28.544	.3711	.1546	.0206	97
6	9	31.815	.3750	.1667	.0417	.96
7	10	35.035	.3507	.1119	.0224	134
8	11	38.531	.4214	.0714	.0500	140
9	12	41.768	.3006	.1227	.0491	163
10	13	45.357	.2955	.1212	.0379	132
11	14	48.662	.3922	.0588	.0719	153
12	15	51.880	.3277	.0252	.0168	119
13	16	55.253	.3714	.0857	.0190	105
14	17	58.590	.3371	.0449	.0449	89
15	18	62.005	.2838	.0676	.0000	74
16	19	65.227	.2115	.0577	.0385	52
17	20	68.541	.2326	.0465	.0233	43
18	21	72.099	.2593	.0741	.0370	27
19	22	75.269	.2222	.0370	.0000	27

(1) Principal earners' income; 10^3 Yen : 1961 price
(2) Wives' Labor Participation ratio for self-employed work
(3) Wives' Labor Participation ratio for employee work
(4) Labor Participation ratio for wives' engaging in both employee work and self-employed work
(5) Number of households
(*) income classes

1963

	*	I (1)	μ^d (2)	μ^e (3)	μ^{ed} (4)	N (5)
1	3	12.330	.3846	.3077	.0000	13
2	4	15.905	.3556	.2222	.0667	45
3	5	19.212	.4727	.1273	.0909	55
4	6	22.649	.3625	.1750	.0625	80
5	7	26.048	.3629	.2016	.0484	124
6	8	29.572	.3684	.1429	.0602	133
7	9	33.038	.3963	.0976	.0183	164
8	10	36.623	.3968	.1323	.0265	189
9	11	40.010	.3943	.0857	.0171	175
10	12	43.470	.3647	.1059	.0294	170
11	13	46.987	.3400	.0667	.0400	150
12	14	50.282	.3169	.0704	.0211	142
13	15	53.898	.3158	.0316	.0105	95
14	16	57.162	.3250	.0250	.0125	80
15	18	64.178	.2821	.0256	.0256	39
16	19	67.771	.3056	.0278	.0000	36
17	20	71.511	.2917	.0417	.0000	24

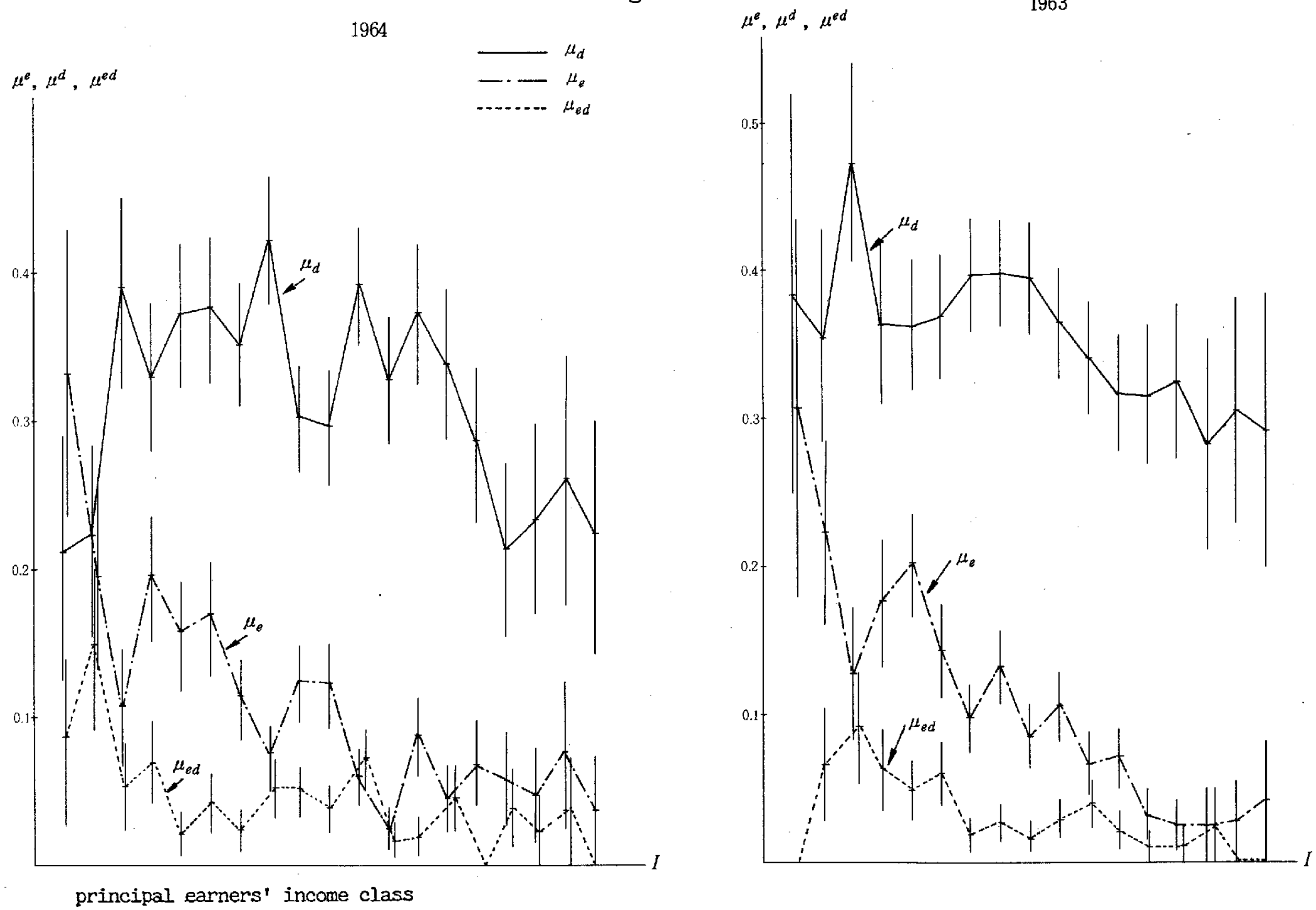
1962

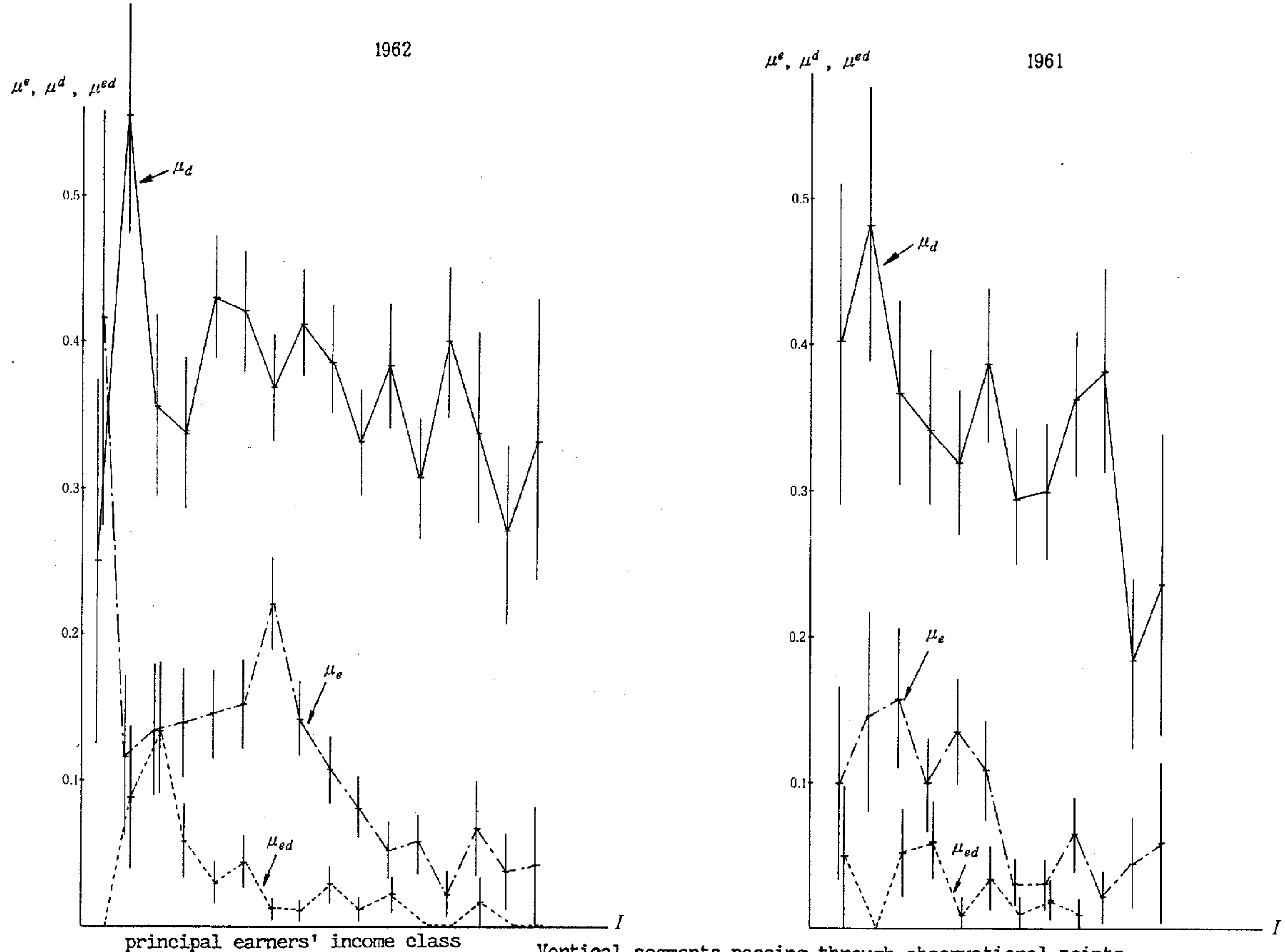
	*	I (1)	μ^d (2)	μ^e (3)	μ^{ed} (4)	N (5)
1	2	9.823	.2500	.4167	.0000	12
2	3	13.353	.5588	.1176	.0882	34
3	4	16.746	.3559	.1356	.1356	59
4	5	20.934	.3372	.1395	.0581	86
5	6	24.455	.4307	.1460	.0292	137
6	7	28.264	.4203	.1522	.0435	138
7	8	31.826	.3693	.2216	.0114	176
8	9	35.571	.4127	.1429	.0106	189
9	10	39.192	.3886	.1086	.0286	175
10	11	42.873	.3314	.0828	.0118	169
11	12	46.591	.3835	.0526	.0226	133
12	13	50.442	.3065	.0565	.0000	124
13	14	54.154	.4000	.0222	.0000	90
14	15	57.800	.3390	.0678	.0169	59
15	16	61.316	.2692	.0385	.0000	52
16	18	69.164	.3333	.0417	.0000	24

1961

	*	I (1)	μ^d (2)	μ^e (3)	μ^{ed} (4)	N (5)
1	3	14.382	.4000	.1000	.0500	20
2	4	18.452	.4815	.1481	.0000	27
3	5	22.088	.3684	.1579	.0526	57
4	6	25.946	.3415	.0976	.0610	82
5	7	29.912	.3182	.1364	.0114	88
6	8	34.152	.3855	.1084	.0361	83
7	9	37.877	.2947	.0316	.0105	95
8	10	41.881	.3000	.0300	.0200	100
9	11	45.763	.3587	.0652	.0109	92
10	12	50.091	.3800	.0200	.0000	50
11	13	53.889	.1818	.0455	.0000	44
12	19	78.371	.2353	.0588	.0000	17

Fig. —16


1964
μ^e, μ^d, μ^{ed}
μ_d
μ_e
μ_{ed}
0.4
0.3
0.2
0.1
I
principal earners' income class
1963
μ^e, μ^d, μ^{ed}
0.5
0.4
0.3
0.2
0.1
I



Vertical segments passing through observational points—show magnitudes of estimated standard deviations of sampling error.