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Abstract	In this paper I shall be using three different units of laborsupply; labor supply in man units, in hourly units, and in man-hour units.The first category, labor supply in man units, indicates the quantity oflabor supplied as measured by the number of persons who are willing to work.Labor supply in hourly units refers to the quantity of percapita labor supplymeasured in hourly units. Finally, labor supply in man-hour units indicatesthe quantity of labor supplied by a group of persons measured in man-hours.While Classical theory mainly treated the quantity of labor supplied inman units, Neo-Classical theory mainly discussed labor supply in hourly unitsor in man-hour units. These various dimensions of labor supply have not yetbeen fully unified analytically, and it is the goal of the models presented inthis book to present a framework in which to treat explicitly all threedimensions of labor supply.
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KEIO ECONOMIC OBSERVATORY
OCCASIONAL PAPER

April 1987

Observations vs. Theory of Household Labor Supply(*)

Vol. I

Keiichiro Obi

CALAMVS GLADIO FORTIOR

KEIO ECONOMIC OBSERVATORY
(SANGYO KENKYUJO)
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This paper is preliminary, and the comments and discussions are wellcome.

Contents

Contents

Contents

Contents

Contents

Introduction

[1.1] Dimensions of labor supply in Classical and Neoclassical theories.

The quantity of labor supplied can be expressed as the product of three factors of different dimensions: (1) the size of population in the economic system P, (2) the ratio of persons willing to work to the size of population X, and (3) the average hours worked by individuals H. Putting the question of labor quality aside, the product of these three factors determines the labor supply L in man-hours, that is,

$$L = P . X . H$$

In retrospect it appears that at various stages in the historical development of the analysis of labor supply, different or different combinations of the three dimensions have been emphasized by different theories.

(1.1.1) In Classical theory, for example the "Malthusian Law", changes in labor supply meant the unavoidable demographic increases and decreases induced by changes in the price of labor. The number of persons who want to work, P.X, was assumed implicitly by Classical Economics as being approximately equal to the working age population. Hence, the size of the working age population and the quantity of labor supplied were not clearly distinguished in the classical theory.

(1.1.2) It has only been since the emergence of Neoclassical theory that the third factor, an individual worker's supply of labor hours, has been explicitly treated. S.Jevons proposed an analytical framework in which individuals, the decision making units of labor supply, adjust their hours of labor supply, to what they regard as the optimal amount. In other words, he introduced into the analysis the utility maximization principle. This gave rise to the concept of optimal hours of labor supply, a concept which had not been explicitly treated in classical theories. On the other hand, the first factor, the size of the population, was regarded as given. In this sense, the quantity of labor supplied was distinguished from the size of the total population and the working age population.

Thus Neoclassical theory was more autonomous and consistent in the sense that it was deduced from the utility maximization principle,e,g., the theories of S.Jevons.

(1.1.3) In this paper I shall be using three different units of labor supply; labor supply in man units, in hourly units, and in man-hour units. The first category, labor supply in man units, indicates the quantity of labor supplied as measured by the number of persons who are willing to work. Labor supply in hourly units refers to the quantity of percapita labor supply measured in hourly units. Finally, labor supply in man-hour units indicates the quantity of labor supplied by a group of persons measured in man-hours.

While Classical theory mainly treated the quantity of labor supplied in man units, Neo-Classical theory mainly discussed labor supply in hourly units or in man-hour units. These various dimensions of labor supply have not yet been fully unified analytically, and it is the goal of the models presented in this book to present a framework in which to treat explicitly all three dimensions of labor supply.

(*) I would like to acknowledge Prof. W.Leontief for his valuable discussions and suggestions concernig the early formulation of the model. Prof.s A.Maki, A.Seike, Dr. Jim Vestal (Keio Economic Observatory, Keio University) and Mr. T.Miyauchi (graduate school of economics, Keio University) gave indispensable assistance for preparing this paper. Thanks are due to them all.

[1.2] Neo-Classical theories of an Individual's Supply of Labor

(1.2.1) The Neo-Classical theory of an individual's supply of labor may be stated as follows.

(1.2.1.1) An individual's supply of labor in hourly units

Let the utility indicator function of an individual be

$$(1.2\text{-}1)\quad U^t = U(l^t,\ q_1^t, \cdots\cdots, q_k^t),$$

where l^t stands for leisure in a given time period, $t(t=1,\cdots\cdots,T)$ and $q_k(k=1,\cdots\cdots,K)$ stands for the quantity of commodity k consumed in the same time period, t. Let T be the individual's total amount of disposable time for each given time period. This quantity T is of course constant over time. Now, assuming no savings we have the identity

$$(1.2\text{-}2)\quad W^t(T-l^t) = \sum_{k=1}^{K} p_k^t\, q_k^t$$

where $T-l^t$ equals the hours of labor supplied.

$P_k^t(k=1,\cdots\cdots K)$ stands for the price of the k-th commodity in time period t and W^t is the wage rate for the individual in time period t. Deleting hereafter the superscript t, the equilibrium condition obtained through maximizing (1.2-1) under the constraint (1.2-2) is given by

$$(1.2\text{-}3)\quad \frac{\partial U}{\partial l}/W = \frac{\partial U}{\partial q_k}/p_k\ ,\quad k=1,\cdots\cdots,K\ .$$

Solving (1.2-2) and (1.2-3) simultaneously with respect to hours of labor supplied, $T-l$, we get

$$(1.2\text{-}4)\quad H = f(W, p, \cdots\cdots, p_k),$$

where H represents hours of labor supplied, i.e.,

$$H \equiv T - l\ .$$

(1.2-4) is the labor supply function of the individual in hourly units.

(1.2.1.2) Application of the theory of an individual's labor supply to a set of individuals.

The scheme mentioned in (1.2.1.1) could be applied to a set of individuals.

Let the utility function of the i-th individual be

$$(1.2\text{-}1') \quad U^i = U(l^i,\ q_1^i, \cdots\cdots, q_k^i,\ v_i)$$

where v_i stands for a random variable characterizing the i-th individual's preference among l^i and the various q_k^i's$(k=1,\cdots\cdots,K)$.

We have

$$(1.2\text{-}2') \quad W(T-l^i) = \sum_{k=1}^{K} p_k q_k^i$$

corresponding (1.2-2) in (1.2.1.1), where
W and p_k, respectively, stand for the wage rate and prices of goods and services. The equilibrium condition corresponding to (1.2-3) in (1.2.1.1) is given by

$$(1.2\text{-}3') \quad \frac{\partial U}{\partial l_i} \Big/ W = \frac{\partial U}{\partial q_k^i} / p_k.$$

The optimal hours of labor supplied which maximize the i-th individual's utility function (1.2-1′) can be obtained by solving (1.2-2′) and (1.2-3′) with respect to $T-l^i$:

$$(1.2\text{-}4') \quad H_i = f(W,\ p_1, \cdots\cdots, p_k,\ v_i)$$

where

$$H_i \equiv T - l^i .$$

In order to clarify the core of the problem, let us specify simply that the random variable in (1.2-4′) is additive:

$$(\mathrm{A}) \quad H_i = g(W,\ p_1, \cdots\cdots, p_k) + u_i$$

where

$$u_i = u_i(v_i),$$

and

$$E(u_i) = 0.$$

Rewriting (A) gives

$$(1.2\text{-}4'') \quad H_i = g(W) + u_i$$

where the p_k's in (A) are deleted for simplicity. This is the i-th individual's labor supply function in hourly units.

From (1.2-4″) it can been seen that different individuals supply different hours of labor at the same wage rate, w, because of the presense of u_i, the magnitude of which is specific to the i-th individual.

Let us exactly determine the quantity of labor supplied by a set of individuals under a common wage rate, w. To begin, obtain mean values of both sides of (1.2-4″):

$$(1.2\text{-}5)\quad \frac{1}{n}\sum_{i=1}^{n} H_i = g(W),$$

where n stands for the number of individuals. Alternatively we have

$$(1.2\text{-}5')\quad \hat{H} = g(W)$$

where

$$\hat{H} \equiv \frac{1}{n}\sum_{i=1}^{n} H_i \quad .$$

(1.2-5′) is the supply function in hourly units of an "average individual" or a "representative individual" of the set of individuals. The quantity of labor supplied by the set of individuals, L, is obtained by summing up (1.2-5′):

$$(1.2\text{-}6)\quad L = n\cdot g(W)$$

where

$$L \equiv n\hat{H}.$$

(1.2-6) gives the quantity of labor supplied in man-hours for the set of n individuals when the wage rate is common to all.

(1.2.2) Per capita hours of labor supply and the number of persons supplied.

——Effective suppliers of the first kind——

Equation(1.2-5′) which gives the quantity of labor supplied in terms of hours, $\hat{H}$, is usually thought to be an equation for an average individual or representative individual. However, it should be noted that the quotient obtained by dividing L, the quantity of labor supplied in man hours given by equation (1.2-6), by $\hat{H}$, the quantity of labor supplied in hours described by equation (1.2-5′), does not necessarily give the number of individuals whose hours of supply are positive for a given value of the wage rate, w. In other words, the value of the quotient will be larger than the number of individuals with positive hours of supply except for special cases. This is becausse there may exist individuals whose optimal hours of supply are zero for the given wage rate, w.

(1.2.2.1) With respect to (1.2-4″) let the number of individuals with positive H_i be n_s^1, where

$$(1.2\text{-}8) \quad n_s^1 \leqq n.$$

We shall call n_s^1 "the number of effective suppliers of the first kind". "Effective supplier of the first kind" indicates an individual whose optimal hours of supply for the given wage rate is positive.

If $H_i > 0$ for all $i(i=1,\cdots\cdots,n)$

we have

$$(1.2\text{-}9) \quad n_s^1 = n,$$

that is, the number of effective suppliers of the first kind, n_s^1, equals the total number of individuals, n. Only for this special case, in (1.2-5′), can $\hat{H}$ be regarded as the hours of supply per one effective supplier which we denote by $\hat{H}_s$. Hence in this case we have

$$(1.2\text{-}10) \quad \hat{H}_s = \hat{H}$$

(1.2.2.2) When $H_i > 0$ for some of the n individuals only and $H_i = 0$ for the remaining individuals, we have

$$(1.2\text{-}11) \quad n_s^1 < n .$$

That is, the number of effective suppliers is less than the total number of individuals considered. In this case, since $\hat{H}$ in (1.2-5′) represents the average hours supplied by each of n individuals, the average hours supplied by each of n_s^1 effective suppliers, $\hat{H}_s$, is larger than $\hat{H}$:

$$(1.2\text{-}12) \quad \hat{H} < \hat{H}_s .$$

(1.2.2.3) Making use of the notion of effective suppliers of the first kind, "the supplier ratio of the first kind", m_1, can be defined as

$$(1.2\text{-}13) \quad m_1 \equiv n_s^1/n$$

In (1.2.2.1), we have

$$(1.2\text{-}14) \quad m_1 = 1$$

and in (1.2.2.2), we have

$$(1.2\text{-}15) \quad m_1 < 1 \ .$$

The number of effective suppliers of the first kind equals the product of the total number of individuals, n, and the corresponding supply ratio of the first kind, m_1, namely

$$(1.2\text{-}16) \quad n_s^1 = n \cdot m_1 \ .$$

Also, the quantity of labor in units of man-hours supplied by the n_s^1 effective supplier is given by

$$(1.2\text{-}17) \quad \hat{H} n_s^1 = \hat{H} n\text{-}m_1 \ ,$$

where $\hat{H}$ is given by (1.2-5′) in (1.2.1.2).

(1.2.3) Effective supplier of the second kind

The notion of effective supplier of the first kind would be a useful one for the purpose of describing the supply of labor in man units or the number of suppliers only if individuals could easily adjust actual working hours to optimal levels for a given wage rate. In fact, in the case of self-employed workers, the adjusment mentioned above would be fairly easy, and hence the notion of the effective supplier of the first kind would be useful. However, in the case of employees in a firm, this notion of the first kind is not particularly useful in describing the supply of labor in man units.

(1.2.3.1) In a modern labor market, where individuals work mainly as employees, the adjustment of actual working hours to their optimal levels is difficult because the former is normaly assigned by the employers. Although there does exist some leeway for adjustment of working hours, actual working hours in a given time period (e.g. a day, a week) are practically determined and restricted by the firm. Hence, it can be said that in a modern labor market, working conditions are not given by the wage rate only but rather by a combination of the wage rate and assigned working hours. Denoting assigned working hours and the wage rate by h and w respectively, the i-th individual's optimal hours of work, H_i, for W in (1.2-4″) would rarely be equal exactly to h. As long as this difference is not very large, the i-th individual will accept the given working opportunity with wage rate, W, and assigned working hours, h.

The size of the discrepancy between h and H_i, $h-H_i$, depends upon the value of u_i in (1.2-5′) which affects the characteristics of the individual's preference between leisure and commodities. The Neo-Classical theory of an individual's labor supply does not explicitly deal with the question of the size of this difference, $h-H_i$, which is the criterion by which an individual accepts or rejects a given work opportunity. Although we shall clarify this point later, it is useful to make a preliminary and introductory discussion here.

Suppose among the n individuals whose H_i's are positive or zero, $H_i \geq 0$, n_s^2 individuals accept the work opportunity characterized by the wage rate, W, and assigne working hours, h. (henceforth we shall denote this work opportunity by (w, h)). We shall call those who accept the work opportunity (w, h) "effective suppliers of the second kind". The number of effective suppliers, of course is n_s^2. This stands for the quantity of labor supplied in man units for the work opportunity (w, h). In man-hours, this quantity of labor supplied is given by $n_s^2 \cdot h$. It is clear that

$$(1.2\text{-}18) \quad n_s^2 \leq n_s^1.$$

That is, the number of effective suppliers of the second kind for the opportunity(w, h) is equal to or less than the number of effective suppliers of the first kind for the common wage, rate W.

(1.2.3.2) Employing the notion of the number of effective suppliers of the second kind we can define "the supplier ratio of the second kind",

$$(1.2\text{-}19) \quad m_2 \equiv n_s^2 / n$$

where n stands for the total number of individuals considered. From (1.2-14), (1.2-15),(1.2-18) and(1.2-19) we obtain

$$(1.2\text{-}20) \quad m_2 \leqq 1.$$

The quantity of labor suppliers of the second kind among n individuals in man-hours, hn_s^2, is given by

$$(1.2\text{-}21) \quad hn_s^2 = n \cdot m_2 \cdot h.$$

(1.2.3.3) In the cases where $n_s^1 < n$ and $n_s^2 < n$, to what extent n_s^1 and n_s^2 differ from n ultimately depends on the characteristics of the distribution of u_i in equation (1.2-4″) in (1.2.1.2). In other words, the magnitudes of the supplier ratios of the first and second kinds are determined by the distribution of u_i. Hence, theory of labor supply which explicitly deals with the distributionof the random variable u_i , in individuals' labor supply functions, (1.2-4″), is indispensable if one wishes to analyze the quantity of labor supplied in man units for the following cases:

(1) there exist individuals whose optimal hours of supply equal zero for a given wage rate and actual working hours can be adjusted to their optimal hours, or

(2) working conditions are given by a combination of wage rates and assigned working hours as is common in a modern labor market.

[1.3] On Implicit Design of Experiments

(1.3.1) As was discussed in (1.2.1) through (1.2.3), Neo-Classical theories of labor supply deal with the determination of the individual's optimal supply of labor in terms of hours. Hence, quantitative analyses of labor supply have often measured the individual's labor supply in hours. When these quantitative analyses are carried out, however, problems sometimes occur between data availablity and data requirements generated by theory.

The most suitable data for the Neo-classical theory of an individual's labor supply might be given by information (if any) obtained from relevant items in household income and expenditure surveys. However, in usual household income and expenditure surveys in Japan, working hours of household members are rarely available although very recently a modest amount of this kind of information has been accumulated. Normally, the information most relevant to the supply of labor is the number of household members gainfully employed together with household members' earnings. Hence, in early studies which used household income and expenditure surveys, the number of working persons and their earnings in a household were taken as a measure of labor supplied.*(1)
In some cases, the ratio of the wife's earnings to her husband's full time earnings was taken as a proxy of the wife's working hours, while in other cases, the ratio of the number of wives gainfully employed in a group of households to the total number of wives in the group was regarded as a measure of <u>labor supply in hours</u> per wife in the group.*(2)

(1.3.2) We shall examine the plausibility of employing the ratio of wives gainfully employed to the total number of wives in a group of households as a measure of <u>supply of labor in hours per wife</u>. Several conditions which will be given in (1.3.3) are necessary in order that the ratio of the number of wives gainfully employed to the total measure of hours of work per wife in the group.

(*)(1) Rosett Richard.N, "Working Wives: An Econometric Study." in <u>Study in Household Economic Behavior</u>, Yale Univ. Press 1958

(*)(2) The pioneering work which makes use of the participation ratio is J. Mincer: "Labor Force Participation of Married Women" in <u>Aspects of Labor Economics</u> Princeton Univ. Press 1962

(1.3.2.1) Suppose a person whose hours of labor supply is defined for a given time interval (horizon) which is measured by some unit period. To make the argument simple while keeping the core of the problem, let the time unit be a month and the time interval by which the quantity of labor supplied is defined be four months: in other words, the person's quantity of labor supplied is defined as the four month total because his "horizon" of supply planning is four months.

Suppose the person wants to work for the given wage rate, w, in two months freely chosen by him(her) during the four months. Here, let us assume hours ofw ork (supply of labor in hours) in each month are the same in all months in which he(she) would like to work. (e.g.160 hours per month). Therefore, the hours of labor supplied by the person should be propotional to the number of months he wants to work. Suppose further that the time distribution of the labor supply of the person in a given time interval is random: that is, his/her decision in which two months he/she will want to work(160 hours for a month) is made at random.

(1.3.2.2) Given the assumptions in (1.3.2.1), the possible ways in which the person chooses two months out of four number six in total. These are listed below,

(1) in the first and the second months, or
(2) in the first and the third months, or
(3) in the first and the fourth months, or
(4) in the second and the third months, or
(5) in the second and the fourth months, or
(6) in the third and the fourth months.

Suppose a group of K persons who have the same characteristics as the person mentioned above. By assumption, these K persons' horizons are the same, namely four months, and the number of months in which they want to work are the same, two months. Hence the time allocation of the supply of labor in the interval for the K persons is totally random.

Should these assumptions be fullfilled, we would find that the number of persons distributed to each case (1) through (6), is approximately$\frac{1}{6}$K each. The number of persons who want to work at least in the first month is obtained by summing the number of persons distributed to cases, 1, 2, and 3, which is given by

$$(1.3\text{-}1)\quad \frac{1}{6}K \times 3 .$$

Similarly, the number of persons who do not want to work in the first month can be obtained by summing the number of persons who are distributed to cases 4, 5 and 6: which amounts to

$$(1.3\text{-}2)\quad \frac{1}{6}K \times 3 .$$

The supplier ratio or the ratio of the number of persons who want to work in the first month to the number of all the persons considered, K, is given by $(\frac{1}{6}K)\times 3 \div K = \frac{1}{2}$.

It can be easily seen that the value of the supplier ratio, in this case, $\frac{1}{2}$, equals the ratio of the number of months in which any person's labor is supplied, namely two months, to the total number of months in a given interval, four months. The latter ratio may be regarded as a measure of labor supply in hourly units for any individual.

Suppose each person in the group considered wants to work only one of four months. In this case, as is seen from Fig(I-1) labor supplied in hourly units is half that of the previous case. Applying the same argument, the supply ratio will also be half of what it is above. Hence, the supply ratio with respect to the group of persons could be regarded as a plausible measure of labor supply in hourly units for each person in the group in the sense that the former is proportional to the latter.

(1.3.2.3) Let us restate the above argument in analytical form.

Let a person's horizon be n unit periods, and let the number of periods in which he wants to work be *r*.
The ratio,

$$(1.3\text{-}3)\quad \frac{r}{n},$$

can be regarded as a measure of his supply of labor in hourly units (under the assumption that in each of r periods, hours of work are a given constant).
As to the number of patterns of the time distribution of r periods during a

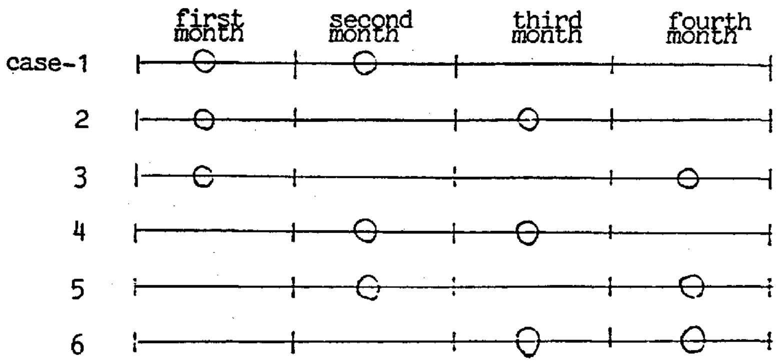
horizon consisting of n periods, we have

(1.3-4) ${}_{n}C_{r}$

which corresponds to the six cases in (1.3.2.2) when n=4 and r=2.
The number of cases that labor is supplied in an arbitrarily given period, e.g., in the first period, is given by

(1.3-5) $\frac{r}{n}\cdot{}_{n}C_{r}.$

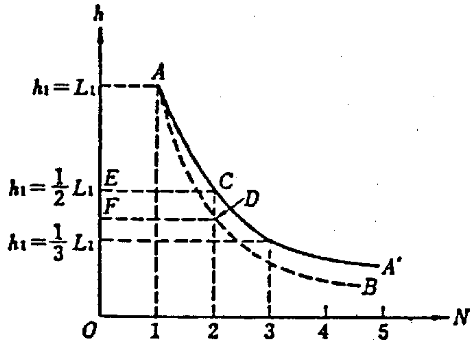
Fig (I-1)



Eaech month stands for a unit interval

for calculating participation rate.

Fig. I -2



Among K persons, the number who choose to supply labor in one of the nCr cases is give by

(1.3-6) K／nCr.

Hence, the number of persons who work in a given unit time period is shown by

$$(1.3\text{-}7)\quad (K/nCr)\cdot\frac{r}{n}\cdot nCr = K(\frac{r}{n})$$

From (1.3-7) we have the ratio of the number of persons who work in the given time period, $K(\frac{r}{n})$, to the total number of persons, K,

$$(1.3\text{-}8)\quad K(\frac{r}{n}) \div K = \frac{r}{n}$$

This is the supplier ratio with respect to the group of K persons for a given unit time period. Comparing (1.3-8) with (1.3-3) we can see the supplier ratio for a given period is proportional to the supply of labor in hourly units of a person arbitrarily chosen from the group of K persons.

(1.3.3) From the argument developed in (1.3.2), it was found that the following conditions have to be fulfilled for the supplier ratio in an arbitrarily chosen unit period to be a plausible measure of an arbitrarily chosen person's supply of labor in hourly units.

(a) The time horizon of a person considered is longer than the unit time period in which the supplier ratio is observed. That is, when the unit time period in which the supplier ratio is measured is a month, the individual's time horizon is equal to or longer than two months.

(b) For each unit time period in which the supplier ratio is measured, the labor hours supplied by a person must be the same, i.e., hours of work in each month must be the same.

(c) Labor hours supplied in a unit time period are the same for all the persons of a group for which the supplier ratio is measured, e.g. labor hours supplied in a month are 160 hours a month per person.

(d) The length of the time horizons for all the persons considered is the same, e.g. each person's time horizon equals four months.

(b), (c) and (d) together imply that each person's labor supply in hourly units is the same for each unit time period as well as for each person's horizon.

(e) The time distribution or allocation of each person's supply of labor is random.

As long as conditions (a)~(e) are fulfilled, the supplier ratio of a group of persons could be regarded as an indicator of each individual person's supply of labor in hourly units for a given wage rate. If this holds true, the Neo-Classical theory of labor supply in hourly units can be applied to data in which the measure of labor supply is given by the supplier ratio only. But are the above mentioned conditions really plausible?

Condition (a), together with (b), implies (1) that each respective person in the group under consideration has a planning horizon which is longer than the unit time period (2) that he(she) adjusts actual hours of supply to the optimal level for a given wage rate during the horizon and (3) that the length of the unit time period for which the supplier ratio of the group is measured is nothing more than a part of horizon. However, we do not have any a priori information about the length of the horizon. Therefore, there is no way to judge the plausibility of requirement (a) without examining the results of analyses employing these conditions as hypotheses.

Condition (b) might be realistic if both (1) working hours in a unit time period are assigned by the employer and (2) adjustment of the quantity of labor in hourly units to the optimal quantity is carried out not in a single unit time period but during a horizon consisting of plural unit periods. There is of course no a priori information to judge if this requirement can be fulfilled. Conditions (d) and (e) are also hypotheses which require testing. One other consideration to take into account is that it may be necessary for all persons in the group to be homogenous.

(1.3.4) Given that there is no evidence that conditions (a) through (e) hold, Neo Classical theory might not be applicable to supplier ratios. For instance, in some analyses a person's life time was regarded as his(her) horizon or planning period and a year, say, was taken as the unit time period. In this type of analysis, the plausibility of condition (e) seems to be quite dubious: Does a supplier allocate his(her) optimal hours of work during the planning period, his life time, at random? This is clearly not the case; for example, a woman's life-time allocation of working hours is thought to be fairly systematic because it is well known that the age profile of the labor force participation ratio of women seems to peak at middle age.

(1.3.5) As was previously explained, it is difficult to apply a theory of hourly labor supply of individuals to data in which labor supply is measured in man units or earnings. In particular, the experimental design is difficult. That is, in order to make theoretical concepts correspond to observed variables, dubious assumptions must be introduced.

Those difficulties, at first glance, seem to stem partly from the inadequacy of accumulated data. However, they originate from the fact that though we have an analytical theory of labor supply in hourly units, we do not have an adequate theory describing quantity of labor supplied in man units. We do not have a theory which considers the mechanism of labor supplied in acceptance or rejection of working opportunities based on consideration of the wage rate and assigned working hours. It is this deficiency of existing theory which must be corrected.

Such a theory of labor supply in man units must, of course, be stated explicitly in relation to the traditional theory in hourly units. By basing a theory of labor supply in man units on the traditional theory of preference among income and leisure, we could deduce quantitatively a Neo-Classical supply function in hourly units, from data in which the measure of labor supply is reported only in man units. Such a theory of labor supply in units of man is needed not only because of a lack of available data, but also for analytical completeness of economic theory.

In addition, a man-unit theory of labor supply is needed to clarify the effects of the shortening of assigned working hours or the adoption of flex time on the supply of labor. Even when data on individuals' hours of work is available,the traditional theory of optimal labor supply in hourly units alone might not be sufficient to obtain a labor supply function in terms of hours of work because , individuals' actual working hours may be seriously affected by the hours of work assigned by firms. Hence, observed working hours might not necessarily be equal to the individual's optimal hours of supply.

ADDENDUM TO CHAPTER 1

[1] Division of Labor and the Dimensions of Labor Input

In chapter 1 we claimed a unified theory of labor supply describing two aspects of supplier's behavior simultaneously: the one being concerned with supplier's optimal hours of work for a given wage rate and the other being concerned with supplier's acceptance or rejection of working opportunities depending upon the wage rate and assigned working hours. In the former, the dimension of the quantity of labor supplied is "hours"; dimension in the latter is "man".

Dividing the quantity of labor, man-hours, into man and hours, however, is necessary and important not only for treating the supply of labor but also for attaining a fundamental consistency in economic theory: that is, the necessity for the above menioned division of dimension is closely related to the existence of the "merit of division of labor". This point can be clarified by observing the classical example of manufacturing pins given by Adam Smith: Suppose a set of instruments necessary for a person to make pins is given. Let the hours of work needed for making X_1 unit of pins be h_1 when one laborer is engaged in the work. If there were no merit to the division of labor, when two (three) laborers with one half (one third) of the set of instruments are engaged in making pins, working hours per one laborer necessary for making X_1 pins would be $\frac{1}{2}h_1$, ($\frac{1}{3}h_1$ etc.). Hence, when we depict the relation between the number of laborers, n, and per capita labor hours, h, needed for the production of X_1 pins, the relation is shown by rectangular hyperbola as is shown by curve AA′ in Fig(I-2).

If the above case holds, the labor input, L, in terms of man hours required to produce X_1 pins woulld be aconstant ; i.e.,

A-1) $L=\frac{1}{n}h_1 \cdot n=h_1.$

The equation of curve AA' is given by

A-2) $L=h_1=n \cdot h.$

Contrary to the above example, the existence of the merit of division of labor requires that the greater the number of laborers employed, the less the total working hours required to produce X pins. Thus, the relation between n and h is no longer a rectangular hyperbola. The curve would be like the contour

AB passing through point A: that is, except for point A, the contour will lie below curve AA′ .

If the true shape of the contour were depicted by a rectangular hyperbola, marginal productivities of labor inputs both in units of man and hour would be equal to each other: i.e.,

$$\text{A-3)} \quad \frac{\partial X}{\partial n} = \frac{\partial X}{\partial h} ,$$

where X stands for the quantity of output. As far as the merit of division of labor exists, however, marginal productivities measured in hours and in men do differ from each other. Hence, the existence of the merit of division of labor apparently contradicts the relation shown by (A-2). Here, it is not adequate to describe labor input, L, in units of man hours as

$$L=n\ h.$$

Rather L should be denoted as L=(n,k). This most fundamental empirical law of the division of labor divides man-hours into both units of man and hours.

The existence of the merit of division of labor induces collaboration among various production processes. Introduction of this collaboration would be one of the main reasons why working hours are assigned by firm. There exists a close relationship between the fact that working opportunities are given by the coupling of wage rate and assigned hours of work and the existence of the merit of the division of labor or the difference between marginal productivities in terms of hours and in terms of men.

[2] Unemployment and the Dimensions of Labor Input

In traditional Neo-Classical theory, the unit of quantity of labor was thought to be one man-hour where no distinction was made between 1 man × 1 hour or 2 man × $\frac{1}{2}$ hour .

An exceptional case is where "unemployment" is discussed. In this case Neo classical theory implicitly divides man-hours into men and hours e.g., Pigou states in his Employment and Unemployment, that in order to argue the quantity of unemployment, per capita hours of work must be given.

§ II The Empirical Laws of the Labor Force (or Job Participation) and Their Implications

Several empirical laws concerning the labor force or job participation have been found, and the theory of labor supply has to be constructed so as to be consistent with these laws.

[2.1] Interdependency of Household Members' Participation Behavior

(2.1.1) Some Classical Findings

P.H.Douglas' original findings are shown in Table 1.
Figures in the table are the correlation coefficients between labor force participation rates of various groups of persons classified by age and sex and the male adults' wage rates. The correlation coefficients were obtained by cross sectional studies on 41 cities in the U.S.

(2.1.1.1)From Table 1, it can be seen that the participation rates of groups which consist of males under 19 years old and 65 years old and over are negatively correlated with male adults' wage rates; that is, the participation rates of those groups are higher in the cities where the adults' wage rates are lower, and vice versa. As well, the participation rate of groups of females under 19 years old and 25 years old or over are negatively correlated with adults' wages.

(2.1.1.2) From those observations it seems that

(1) the individual's labor supply behavior is not determined independently but is mutually dependent on the household to which he/she belongs,

(2) the participation behavior of adults of 20 years through 64 years is insensitive to the wages of those adults,

(3) the higher the wage rates of the male adults in a group of households, the lower the participation rates of the younger and older females of the group.

(2.1.1.3) Douglas's main findings have been confirmed by C.D. Long's comprehensive statistical regression analysis. Based on these empirical facts, it can be concluded that the theory of labor supply describing the supply behavior of household members is not in terms of each member's utility function but is in terms of the collective utility function of the household members. (cf. (1) in 2.1.1.2)

Hence, a theory of household labor supply which assumes that earnings of the principal earner are exogenous to the supply behavior of other household members, but further assumes that other members' labor supply is dependent on the principal earner's income seems to be consistent with the observed facts. As well, facts support the contention that the principal earner's participation behavior is insensitive to his/her own wage rates.
(cf. (2) in 2.1.1.2)

Table Ⅱ－1

Coefficients of correlation between average money earnings per "Equivalent male" in 41 cities in manufacturing in 1919 and proporations of age and sex geoups gainfully employed

age geoup	male	female
14	−.60	−.46
15	−.56	−.36
16	−.35	−.13
17	−.24	+.04
18～19	−.22	+.07
20～24	−.18	−.20
25～44	−.08	−.47
45～64	−.25	−.48
65 and over	−.43	−.55

guoted from P. H. Douglas : <u>The Theory of Wages</u>

(2.1.2) Findings from the Family Income and Expenditure Survey

(2.1.2.1) Suppose the household members' supply of labor in man units in a given group of housholds is negatively correlated with the household's principal earner's income, as was suggested by (3) in 2.1.1.2. The participation ratio, defined as the ratio of the number of persons gainfully employed to the number of household members, of the group of households whose principal earners' incomes are higher will conseqently be lower than the participation ratio of the group of households whose principal earners' incomes are lower. This was confirmed for Japan in 1954 by H.Arisawa(*) who used the Family Income and Expenditure Survey (FIES). This same tendency was found in an analysis of labor force participation rates of wives in the last half of 1950's.(* *) Finally, Massachusetts data of the 19th century also confirms that this tendencyis a fairly universal one.(cf. Tab.2) Hence, as far as these observations are concerned, the theoretical framework suggested in (2.1.1.3) is quite consistent with empirical facts.

(*) H.Arisawa: Chinginkozo to Keizaikozo (The wage structure and the Economic structure) In Ed. I.Nakayama "Chingin Kihonchosa" (Basic Survey on the wages), Toyokeizaishinposha, 1956

(* *) Long Clarence D
The labor force under changing income and employment Princeton Univ. Press,1958

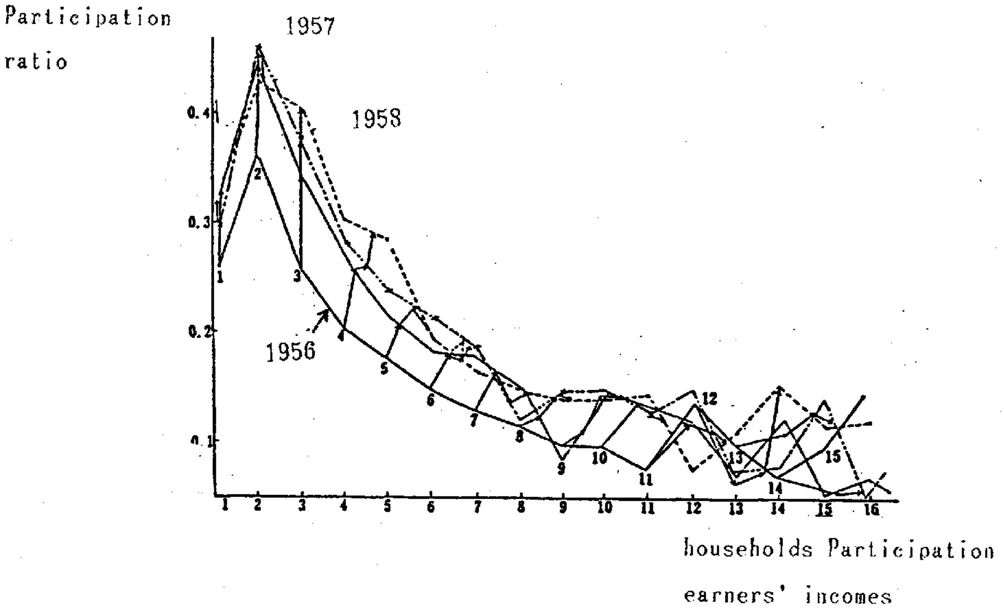
Table

Participation ratio of 393 families of Maaaschusetts in 1875.

	Father's yearly wages.	Participation ratio of wife and children
Skilled workshop handicraftsmen	752.36	0.0526
Metal workers	739.30	0.074
Building trades	721.32	0.074
Teamsters	630.02	0.091
Mill operatives	572.10	0.200
Shoe and Lesther workers	540.00	0.2110
Metal workers' labores	458.09	0.2045
Workshop laborers	433.06	0.1864
Outdoor laborers	424.12	0.2051
Mill laborers	386.04	0.2222

Calculated from statistics in ; Sidney and Beatrice Webb "Industrial Democracy"

Fig. II −1



The participation ratios, for the groups households at the same percentile positions on the distributions of households ; hesds' incomes, for each yesr are connected by the segments denoted by arrows.

(2.1.2.2) Time serial movement of cross sectional relationships between the participation ratio and principal earner's income

It has been noted that cross sectional relations between participation ratios and principal earners' incomes shift from year to year as is shown in Fig (II-1). These curves were obtained from Japanese F.I.E.S. data., with households surveyed grouped by principal earner's income. For each group of households, the ratio of the number of household members gainfully employed to the total number of household members was calculated. The participation ratios thus obtained for various principal earner's incomes for each year are shown on the curves for 1955 through 1958 in Fig (II-1). Among the households surveyed in the FIES, the households whose members consist of three persons of 15 years old were selected.

FIES is a longitudinal survey of households which are followed for six months. Some households are surveyed from January to June while others are surveyed from February through July and so on. Out of all the households surveyed, households which were not surveyed in the month of December were discarded.This is because more wages and salaries are paid in December than in the other months and the former considerably affects the amount of yearly earnings. This treatment of data will to some extent suppress the effect of transitory variation in earnings.

In addition to the fact that the cross sectional participation ratio curve tends to be downward sloping, an interesting fact is observed from time serial shifts of the cross section curves. The groups of households in various years are not the same because each household is surveyed only for six months. Hence we do not have longitudinal observations on each household for the four years, 1955 through 1958. However, if the yearly growth rates of principal earners' income of the households in the i-th group,(*) do not exceed that of the neighbouring group, i+1th, we could approximately pursue the same percentile positions on the size distributions of the principal earners' income for each of the years, 1955 through 1958.

(*) By "the households in i-th group" we mean both the surveyed households and the households which are not surveyed but whose principal earners' incomes are to be classified as i-th group.

The segments dotted by arrows line in Fig(II-1) were drawn by connecting the points of which the coordinates on the abcissa are the principal earner's income levels corresponding to the same percentile position for four years, 1955-1958. The corresponding participation ratios on the arrows were obtained by interpolation. As is mentioned above, any one arrow is interpreted as showing approximately the time serial movement of the participation ratio of any one group of households in which the same households are included. From this, we can observe that the participation ratio curve is downward sloping, as was expected and that the curve shifts from year to year reflecting changes in opportunities of non principal earners and in principal earners' income as is explained below.

(1) in groups whose principal earners' incomes are relatively low, the participation ratios were sharply augmented,

(2) in the higher principal earners' income groups, the participation ratio, on average, grew relatively slowly during four years.

(3) among the upper income groups mentioned in (2), there are a few groups in which participation ratios even declined (1957 and/or 1958)

As will be precisely explained below, these observations suggest (a) that the participation ratio is affected by two main causes, that is, the principal earner's income and the economic opportunities of the members of the household other than the principal earner, and (b) that the augmentation of the former (principal earner's income) tends to decrease the other members participation ratio and (c) that favorable movement of the latter tends to increase the ratio.

In 1956, the Japanese economy was recovering from the depression of 1954 and 1955, and after the boom year 1957, a recession occurred in 1958. During these phases of the business cycle, the principal earner's income in the lower income group did not grow much as is shown by dotted lines in Fig(Ⅱ-1). Hence it may be said that in the lower income groups, the effect of increases in principal earners' incomes was suppressed by the effect of growing economic opprtunities opened to non principal earners. These growing opportunities increased participation ratios, so that in these lower income groups the ratio grew rapidly. On the other hand, in the upper income groups, principal earners' incomes grew quite rapidly and the effect of growing principal earners' incomes loweringthe participation ratio overwhelmed the non principal earner's economic opportunity effect. Hence, in the upper principal earner's income groups the ratios at most grew a little or even declined.

(2.1.2.3) The results of the observations in (2.1.2.2) are summarized by the propositions below.

(2.1.2.3.0) In a household there exists a principal earner whose income can be considered to be exogenous to the labor supply behavior of other members of the household.

(2.1.2.3.1) Given the earning opportunities, wage rates and so on, of non principal earners, the participation ratio of the group of households in which households' principal earners' incomes are appoximately the same will decline as principal earners' income grow.

(2.1.2.3.2) Given the principal earners' income, the participation ratio of the group of households increases when non principal earners' earning opportunities become more favorable.

Let us call proposition (2.1.2.3.1) the first law of the household's participation ratio and the proposition (2.1.2.3.2) the second the low of household's participation ratio.

[2.2] Observations on Wives' Participation Ratio

(2.2.1) In order to set out a theoretical model of household labor supply measured both in men and in hours, it is appropriate to begin with a simple case. Hence we would like to consider the type of household whose members consist of a couple and an unspecified number of children under fifteen years of age . We shall call this type of household "type A" hereafter.

In a household of this type, the wife can be identified as a non principal potential earner and the husband as the principal earner.

(2.2.2) Among the households surveyed by FIES for four years, 1961 through 1964, only households of type A were selected. Out of the households thus selected, households which were not surveyed in December were discarded. The rest of type A households were stratified by principal earners' income. The participation curves of type A households thus selected are shown in Fig(Ⅱ-2).

§III A Model of Labor Supply of Type A Haouseholds (1)

……… Empolyee Opportunity Model ………

[3.1] Households' Preference Functions Between Leisure and Income

As can be seen from the discussion in §II-2.1.2, an individual's supply of labor is not independent from but is rather connected with those of other members of the household. It would then be appropriate to construct a collective preference function with respect to household income and the individual's leisure. Consider a household with P persons. Let their utility indicator functions be

$$U_1(\lambda_1, X), U_2(\lambda_2, X), \cdots\cdots, U_p(\lambda_p, X),$$

where λ $(i=1,2,\cdots\cdots,P)$ stands for leisure and X stands for the household's income in contant price. The collective utility indicator function would then be written as

$$3.1\text{-}1)\quad \omega=\omega_1[U_1(\lambda_1, X),\cdots\cdots,U_p(\lambda_p, X)]$$
$$=\omega_2[\lambda_1,\cdots\cdots,\lambda_p,X]$$

Letting T be the individual's total number of hours in the defined period (day, month, etc.) and denoting the quantity of labor supplied by the i-th individual by h_i, there exists the identity,

$$3.1\text{-}2)\quad \lambda_i\equiv T-h_i \quad , i=1,2,\cdots\cdots,P.$$

Substituting 3.1-2) for 3.1-1) we obtain

$$3.1\text{-}3)\quad \omega \equiv\omega_2[(T-h_1), (T-h_2),\cdots\cdots,(T-h_p),X].$$

Again from Douglas' and Long's finding (cf. §II-2.1.1.2) it is hypothesized that in each household there exsists a person on whose income the other members' labor supply behavior depends. We call this member of the household "the principal earner". The members of the household other than the principal earner,except for children, we call "non principal potenial earners". (in short, potential earners or non principal earners.)

For the Type A household (cf. §II-[2.2]) with S children we have

$$P=S+2.$$

The hours of work for the children are institutionally restricicticted to zero, namely

$$3.1\text{-}4)\quad h_t = 0 \quad (t = P, P-1,\cdots,P-S+1).$$

Letting the first and the second members be husband and wife respectively we obtain from 3.1-3) and 3.1-4)

3.1-5) $\omega = \omega_3[(T-h_1),(T-h_2),X]$.

It is appropriate to assume that the wife's supply of labor is dependent upon the husband's income. This means that husband and wife are identified as principal earner and potential earner respectively. Denoting the husband's earnings by I, we have

3.1-6) $W_1 h_1 \equiv I$,

where W_1, the husband's wage rate, is an exogenous variable. The hours of work of gainfully employed household members seem to vary depending upon their wage rates. However, in reality, as is discussed in §I-<1.2.3>, institutional factors prevent them from choosing their optimal working hours which may differ from those assigned by employers. As a first approximation, it is not unreasonable to assume that h_1 equals $\bar{h}_1$, which stands for fixed hours assigned by the employer. Now, from 3.1-5) and 3.1-6),we obtain

3.1-7) $\omega = \omega_3[(T-\bar{h}_1),(T-h_2),X]$,

$X \equiv I + wh_2$,

where I is an exogenous variable, and h_2 equals $\bar{h}_2$ or zero depending upon whether the wife is gainfully employed or not.

$T-\bar{h}_1$ being an institutionally given constant, the collective utility indicator function of the household is written as

3.1-8) $\omega = \omega[(T-h),X]$,

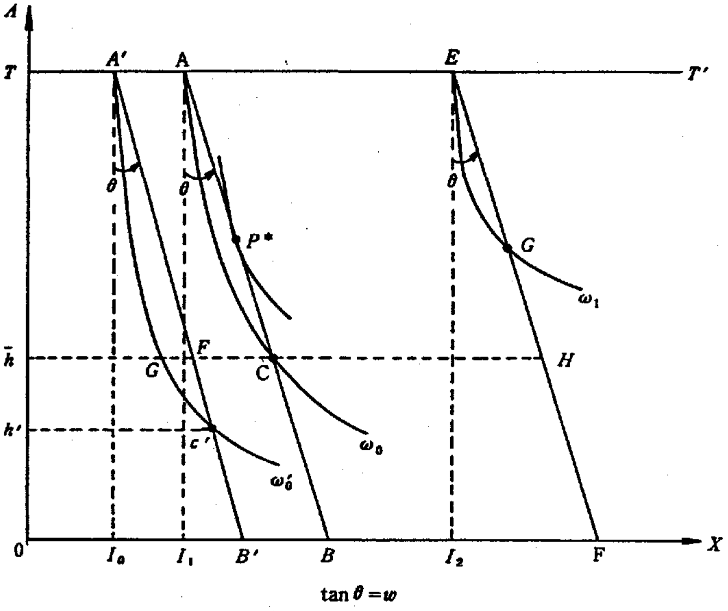
where h stands for the wife's hours of work. (We drop the subscript 2 attached to h for simplicity.) For Type A households, therefore, the utility indicator is fully described by a function of the wife's leisure hours $T-h$, and the household's income X.

[3.2] A Model for Labor Supply Probability in Terms of Income-Leisure Preference Functions

<3.2-1> Optimal Hours of Work

Let us consider a household of Type A. We have here one principal earner (husband) and one potential earner (wife).

Fig. Ⅲ－1



Let the income-leisure preference function of the household be, as shown by (1-8),

3.2-1) $\omega = \omega(X, \Lambda)$

where,

3.2-2) $\Lambda \equiv T - h$,

h being the potential earner's (wife's) supply of labor.

Household income is defined by

3.2-3) $X \equiv I + wh$

where W, the potential earner's wage rate and I, the principal earner's income, are given. Substituting 3.2-2) and 3.2-3) into 3.2-1), we can obtain the value of h which maximizes ω by solving the equation,

3.2-4) $\frac{d\omega}{dh} = 0.$

This system is shown in Fig. Ⅲ-1. Leisure, Λ, and the household's total income, X, are scaled on the vertical axis and the abcissa respectively. Total disposable hours for the potential earner, T, depend on the time interval for which X, Λ, and I are defined; e.g. if these variables are defined for a 24 hour period, T is 24.

Let I be the given value of the principal earner's income, and let $\tan\theta$ be the potential earner's wage rate as given by the employer. The potential earner's optimal working hours and the household's total income are shown by the coordinate of P^*, which is the tangency point of the income-leisure contour, ω, and the income line, AB. Obviously, h^* on the vertical axis in Fig.Ⅲ-1 is the solution of equation 3.2-4), and it could be written as:

3.2-5) $h^* = h^*(I, W, \alpha_1, \cdots, \alpha_p)$,

$\alpha_i (i=1,\cdots,p)$ being parameters of the preference function 3.2-1). The value of h^* varies as I, W, and parameters of the preference function, $\alpha_1 \cdots\cdots \alpha_n$ change. Equation 3.2-5), corresponding to the locus of P^* in Fig.Ⅲ-1, is the supply schedule of the potential earner.

Now, if the supplier were able to determine working hours in accordance with his supply schedule, he would work exactly h^* hours under the wage rate assigned by the employer. However, in reality, workers have to accept the institutionlly assigned normal working hours $\bar{h}$, in order to be employed. These normal working hours ($\bar{h}$) need not be equal to the optimal hours (h^*).

<3.2-2> The Principal Earner's Critical Income

In this section, we will discuss the range of the principal earner's income over which the potential earner accepts work under the condition that both the wage rate and the working hours, w and $\bar{h}$, are assigned by the employer.

Let the principal earner's income be I_2, which is higher than I_1, in Fig.Ⅲ-1. Suppose that the wage rate and the assigned working hours are tan θ (=w) and T$\bar{h}$ (=$\bar{h}$) respectively. If the potential earner were to accept this work, the household's position with regard to income and leisure would be shown by point H. At this point the household is obviously worse off than point E, where the potential earner does not work at all and the household's total income is equal to the principal earner's income, I_2. Hence, so long as the principal earner's income is higher than I_1, the potential earner does not accept employement at tan θ and T$\bar{h}$ respectively. In the same manner, it can be shown that the household is better off if it accepts work under these conditions when the principal earner's income is less than I_1. When the principal earner's income is exactly I_1, the household is indifferent to the choice between acceptance and rejection of this job. Let us call the principal earner's income I_1 the critical level of principal earner's income with regard to this specified employment opportunity, or in short, the principal earner's critical income (PECI).

As can be seen from Fig.Ⅲ-1, the principal earner's critical income varies with changes in the assigned working hours $\bar{h}$, the wage rate w, and the shape of the contour of the preference map. For instance, if the assigned working hours were less than what is shown by point C, the potential earner would work because the household would be better off. Therefore, I_1 could no longer be the principal earner's critical income of the household. Consequently, denoting the principal earner's critical income by I^*, we have

3.2-6) $\quad I^* = I^*(w, \bar{h}, \alpha_1, \alpha_2, \cdots, \alpha_p).$

Analytically, function(3.2-6) can be derived as follows. Let the preference function be (3.2-1) in [3-2-1]. That is

3.2-1) $\quad \omega = \omega(X, \Lambda).$

where, as shown in [3-2-1],

3.2-2) $\quad \Lambda \equiv T - h,$

and,

3.2-3) $X \equiv I + wh$.

In the first place we will obtain an equation of the contour passing through the point A in Fig.Ⅲ-1. At point A we have

3.2-7) $\Lambda = T$,

and,

3.2-8) $X = I$,

by applying $h=0$ to 3.2-2) and 3.2-3). Inserting 3.2-7) and 3.2-8) into 3.2-1)

we get

3.2-9) $\omega_0 = \omega(I)$.

where T, a constant, is deleted for brevity. This is the indicator of the contour passing through point A.

To obtain the equation for the contour passing through point C in Fig.Ⅲ-1, we set h equal to $\bar{h}$, the assighned hours of work, in 3.2-2) and 3.2-3), and insert these relations,

3.2-7′) $\Lambda = T - \bar{h}$

3.2-8′) $X = I + w\bar{h}$

into 3.2-1); That is, we have

3.2-10) $\omega_0' = \omega(\bar{h}, w, I)$,

where T is deleted again.

By the postulate that I in (3.2-9) and (3.2-10) is the principal earner's critical income, I^*, we have

3.2-11) $\omega_0 = \omega_0'$.

Appling this condition to (3.2-9) and (3.2-10), we get

3.2-12) $\omega(I^*) = \omega(\bar{h}, w, I^*)$,

where I's in (3.2-9) and (3.2-10) are replaced by I^*.

Solving (3.2-12) with respect to I^*, we obtain equation (3.2-6).

Given w and $\bar{h}$, the value of the principal earner's critical income, I^*, of the specific household is determined by a set of values of the preference parameters, $\alpha_i (i=j,\cdots,p)$, specific to the household. Therefore, I^*, the principal earner's critical income, can be viewed as a parameter characterizing a specified household with respect to its preference between income and leisure where the wage rate, w, and the assigned hours of work, $\bar{h}$, are specified.

<3 - 2 - 3> Size Distribution of the Principal Earner's Critical Income

(3-2-3-1) Let us consider a group of m households of type A in which the principal earner's income and assigned working hours open to each household's potential earner are the same. If we were able to single out, among m households, m′ ($\leq$ m) households whose preference among income and leisure are exactly same, it would be obvious from equation 3-2-6), that their principal earner's critical income, $I^*_j (j=1, \cdots\cdots, m')$ must be equal (a constant I_I). Since it is difficult identify households whose preference functions are exactly the same, it is nesessary to introduce the probability density distribution of the critical income(PECI), I^*_k $(k=1, \cdots\cdots, m)$,

$$3.2\text{-}13) \qquad g(I^*_k \mid \bar{h}, w; \ \pi)$$

where π is a set of parameters of the distribution function g. The elements of π are, respectively, functions of the preference parameters. The functional form of the probability distribution g depends on the differences in the shape of indifference curves among households.

(3.2.3.2) The magnitude of I^* of an arbitarily chosen household depends on the form of the indifference curves of the household, the wage rate w and assigned hours of work h. Hence, given the distribution function of preference parameters among households, the form of the PECI distribution g is also determined. Let us examine how the latter is deduced from the former.

In the first place, we shall assume that only one parameter, out of p parameters which characterize the shape of income—leisure preference curve of each household, differs among houthholds. The reason why we adopt this assumption is that (1) it simplifies the relation between the PECI distribution and the distribution of preference parameters without impairing the core of the problem, and (2) such a simple model was found to be consistent with the observations.

Hence, let the preference function of the ith household be

$$3.2-14) \qquad \omega = \omega (X, \Lambda, \alpha^i_1, \cdots\cdots\cdots, \alpha^i_p), \qquad (i=1, \cdots\cdots, m)$$

where α^i_1 differs among m households, and the magnitudes of p−1 preference parameters $\alpha_2^i,\cdots\cdots,\alpha_p^i$ are respectively assumed to be common to all households; that is,

3.2−15) $\alpha_s^i=\alpha_s.\quad (s=2,\cdots,p)$
$(i=1,\cdots,m)$

3.2−14) can be rewritten as

3.2−16) $\omega=\omega(x,\Lambda,\alpha_1^i,\alpha_2,\cdots\cdots,\alpha_p)$,

where α_1^i ,differs among m households.

Let the differences in α_1^i ,be noted by the density distribution

3.2−17) $\phi(\ \alpha_1,\theta\ )$

where θ stands for the set of parameters of the distribution function.

The PECI for the ith houthhold, I^*, in equation (3.2-6) in [3.2.2] can be rewritten, by taking into account the parameters in 3.2−16),

3.2−18) $I_i^*\ =I^*(\ w,\bar{h},\alpha_1^i,\alpha_2,\cdots\cdots,\alpha_p\ )$,

Solving 3.2−18) for α_1, we have

3.2−19) $\alpha_1^i=G(\ I^{*i},w,\bar{h}\ ,\alpha_2,\cdots\cdots,\alpha_p\ )$,

where G stands for the inverse function of I^* in 3.2−18)

Inserting 3.2−19) into 3.2−17), we have

3.2−20) $\phi[G(I^*,\ w\ ,\ \bar{h},\ \alpha_2\ ,\cdots\cdots,\ \alpha_p\ )\ ,\ \theta\ ]$.

Let the probability element of the density distribution ϕ be

3.2−21) $\phi(\ \alpha_1\ ,\ \theta\ )\cdot d\alpha_1$

From 3.2−19), $d\alpha_1$ in 3.2−21) can be written as

3.2−22) $d\alpha_1=(\partial G/\partial I_1^*)\cdot dI^*$

where the super script i is deleted.

Replacing α_1 in 3.2−21) by G in 3.2−20), and $d\alpha_1$ in 3.2−21) by $d\alpha_1$ in 3.2−22), we have

3.2−23) $\phi[G(I^*, w, \bar{h}, \alpha_2, \cdots\cdots, \alpha_p),\ \theta]\,|\partial G/\partial I^*|\cdot dI^*$

This is the probability element of the I^*distribution. Hence, the density distribution of PECI, I^*, or the PECI distribution for short, can be written as

3.2−24) $\Phi[\ G(I^*, w, \bar{h}, \alpha_2, \cdots\cdots, \alpha_p), \theta\ ]\,|\partial G/\partial I^*|$

This is nothing but equation 3.2−13) in 〔3.2.3.1〕. Comparing 3.2−24) and 3.2−13), it can be seen that the set of parameters π in 3.2−13) includes the parameters, α_2 ,……, α_n and θ ,that is, 3.2−13) can be rewritten as

3.2−25) $\phi[\ I^*, w, \bar{h}, \pi(\alpha_2, \cdots\cdots, \alpha_p, \theta)\ ]$

The equation of the probability of labor supply can be obtained by integrating the probability density distribution of principal earner's critical income, (3.2−13). However, in this process of deriving a probability-of-supply equation, it is necessary to induce new notions of maximum hours of labor supply, and of its distribution.

<3.2.4> Maximum Hours of Labor Supply and its Distribution

The notion of maximum hours of labor supply (MHLS) is closely related to principal earner's critical income (PECI). Hence, in order to define MHLS we can again use Fig(Ⅲ−1) by which the definition of PECI was given.

(3.2.4.1) Suppose a household of type A with principal earner's income, I_0. Let the wage rate offered to the nonprincipal potential earner by the firm be W. (see Fig(Ⅲ−1)). If the firm wants to induce the potential earner to work at the wage rate, W, the hours of work assigned by the firm have to be less than Th' in Fig(Ⅲ−1). We shall call the hours of work, Th' , the maximam hours of labor supply(MHLS).

If the hours of work assigned by the firm are Th′ , a potential earner is indifferent about working. Should the potential earner be employed, her position would be shown by point C′ which is the intersection of the income line A′ B′ and the indifference curve ω_0' passing through point A′ which shows the potential earner's position when she does not work. When the assigned hours of work is less than Th′ , the potential earner is better off accepting the offered earning oppotunity because the potential earner's position will be somewhere between A′ and C′ on income line A′ B′ . Such a position is on an indifference curve with a higher utility indicator than that of ω_0' which passes through point A'. Finally, when the assigned hours of work exceed Th′ , she will be worse off accepting the earning opportunity because her position is on an indifference curve where the utility indicator is lower than ω_0' . Thus, the assigned hours of work, Th′ ,is her maximam hours of labor supply.

(3.2.4.2) Given the wage rate, W, and the principal earner's income, I, the maximum hours of labor suppoly (MHLS) depend on the shape of the household's indifference map, especially the shape of the contour passing through point A′ . Hence the value of MHLS of a household could be viewed as a parameter which expresses the characteristics of the preference between income and leisure specific to the household, under the given wage rate and the principal earner's income.

(3.2.4.3) MHLS Locus

The value of MHLS varies with the wage rate offered to nonprincipal potential earners for a given principal earner's income. That is, in Fig(Ⅲ-1), it can be seen that an increase in principal earner's income from I_0 to I_1

changes the value of MHLS from Th' , for the given wage rate W. In Fig(Ⅲ-1), the locus of points, C' , C, G, whose ordinates give the values of MHLS's corresponding to the various principal earner's income levels, is obtained. This is named the MHLS locus.

Making use of the MHLS locus, the relation between the PECI (principal earner's critical income) and MHLS can be shown. Let the assigned hours of work be $T\bar{h}$ in Fig(Ⅲ-1). The dotted line passing through point $\bar{h}$ parallel to the abscissa is drawn. Let the ineresection of the line and the MHLS locus be point C. We can easily determine the potential level of principal earner's income I, corresponding to the MHLS shown by the ordinate, $T\bar{h}$, of point C. The potential level of a principal earner's income is, by definition the PECI under the assinged hours of work, $T\bar{h}$, and the wage rate, W. In other words, a PECI under specific assigned hours of work and wage rate is a principal earner's potential and critical income level by which MHLS is made equal to the assigned hours of work. This level of principal earner's income is potential in the sense that the actual level of the principal earner's income does not matter in its derivation.

(3.2.4.4) MHLS curve

If the locus of the MHLS is an upward sloping in income-leisure plane as is shown in Fig.Ⅲ-2, the relation between the MHLS and the principal earner's income is downward sloping in the MHLS and principal earner's income plane as is depicted in Fig.Ⅲ-2. We shall call this the MHLS curve. The equation for the MHLS curve of any household of type A can be derived asfollows.

Let the utility function of a household be

3.2-26) $\omega = \omega(X,\Lambda)$.

The utility indicator ω_0 of the contour passing through point A in Fig.Ⅲ-1 is given by

3.2-27) $\omega_0 = \omega(I,T)$,

noting that $X = I$, $\Lambda = T$ at point A. Hence the equation of the contour passing through point A is given by

3.2-28) $\omega(I,T) = \omega(X,\Lambda)$

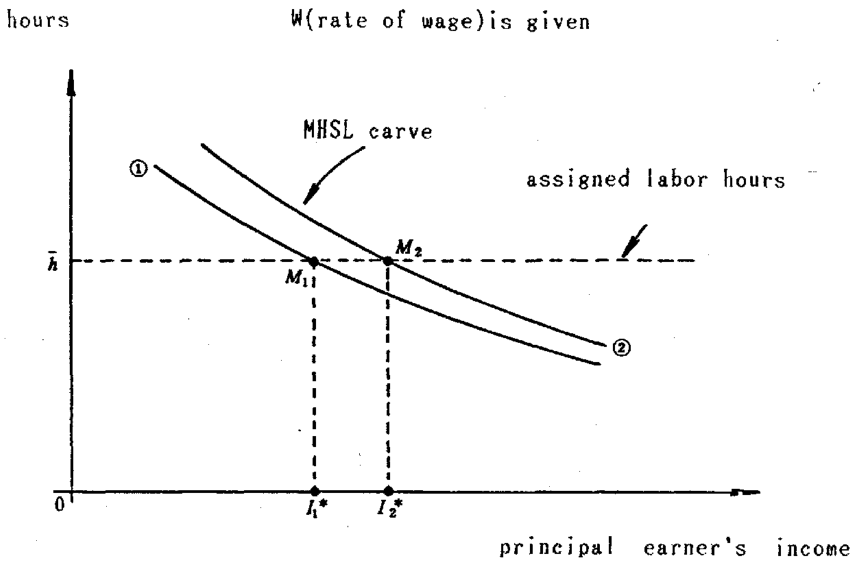
where the value of I is given. Making use of the relation

3.2-29) $\Lambda = T-h$,

3.2-28) reduces to

3.2-30) $\omega(I,T) = \omega(X,T-h)$.

Fig. Ⅲ－2



maximum hours for supply of labor.

By solving simultaneously equations 3.2-30) and

3.2-31) $I + wh = X$

We have

3.2-32) $h = h(I,W,T)$, and

3.2-33) $X = X(I,W,T)$.

3.2-32) is the equation for the MHLS curve. Equations 3.2-32) and 3.2-33) simultaneously give the coordinates of the MHLS locus in Fig.Ⅲ-2.

In Fig.Ⅲ-2, the value of the abscissa, I_1^* , of the intersection point, M_1 of the MHLS curve and the straight line parallel to the abscissa axis is the principal earner's critical income when the wage rate, w, and the hours of work, $\bar{h}$, are assigned. The value of I_1^* is analytically obtained by making h equal $\bar{h}$ and w equal $\bar{w}$ in equation, 3.2-32) and solving with respect to I, that is

3.2-34) $I_1^* = f(w,\bar{h},T)$.

(3.2.4.5) As is easily seen from Fig.Ⅲ-1, the MHLS for the given values of W and I varies among households due to the difference in the shape of the indifference curves among the households. Hence the MHLS curves also vary among different households. This implies that the level of the principal earner's income for a household which is determined by the intersection of the given horizontal line and the MHLS curve varies from household to household.

Therefore we explicitly introduce a set of preference parameters, αi, which are specific to each of the households into eqation 3.2-34). That is

3.2-35) $\quad I_i^* = f(w, \hbar, T \ \alpha i).$

I_i^* is the principal earners critical income for the i-th household when the assigned wage rate, w, and hours of work, $\hbar$, of nonprincipal earner equals w and $\hbar$, respectively.

The distribution of I_i^* in 3.2-35) has been given by 3.2-13) in 3-2-3, or equation 3.2-25) in 3.2.3.2

<3.2.5> Rsestrictions on the Shape of the MHLS Curve

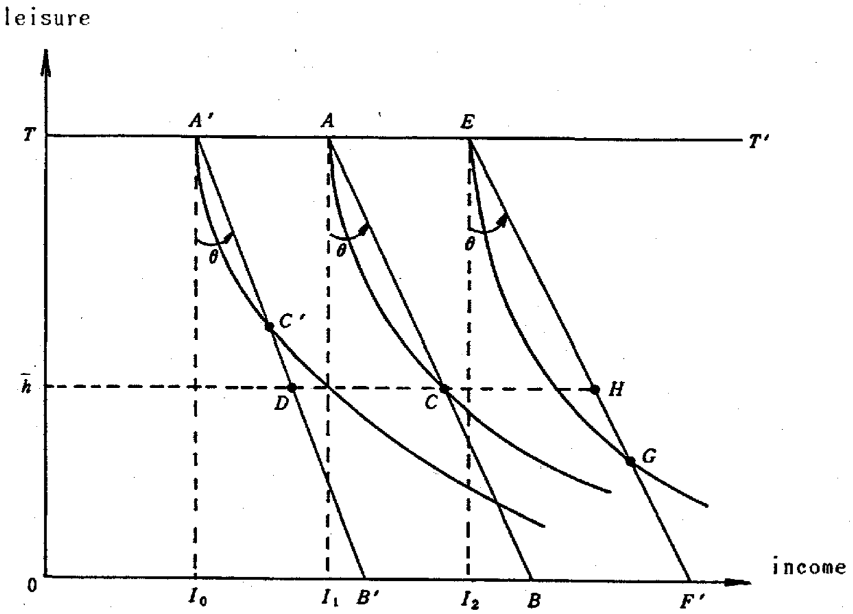
(3.2.5.1) Given the wage rate and the assigned hours of work, the non-principal potential earner is indifferent to work if the principal earner's actual income just equals the PECI.

(3.2.5.2) When the MHLS curve is downward sloping, as shown in Fig. Ⅲ-2, the non-principal potential earner does not (does) work if the principal earner's actual income, I, is larger (smaller) than the PECI, I^*, for the given wage rate and the assigned hours of work for the nonprincipal potential earner. That the MHLS locus in the income-leisure plane is ascending means that the MHLS curve in the MHLS-principal earner's income plane is downward sloping.

The case of an ascending MHLS locus is shown in Fig. Ⅲ-1. In FIg. Ⅲ-1, the household's PECI is shown by point I_1 on the abscissa. Let the principal earner's actual income level be I_2 which is higher than the PECI. Given the nonprincipal potential earner's wage rate, $w = (\tan\theta)$, and given the assigned hours of work, the household will be at point H on the income line E F when the nonprincipal earner accepts work. When she does not work, the household will be at point E. The utility indicator of the indifference curve passing through point E is apparently higher than that passing through point H. Hence point E is chosen, i.e., the nonprincipal potential worker does not accept the opportunity to work.

(3.2.5.3) The opposite case, a downward sloping MHLS locus in the income-leisure plane, is shown in Fig. Ⅲ-3. The locus obtained by connecting the points C' C G etc. is the MHLS locus. Hence the MHLS curve in the MHLS-principal earner's income plane is ascending. Given the nonprincipal potential earner's wage rate, w ($= \tan\theta$), and the assigned hours of work h(Th), the PECI of this household is shown at point I_1, on the abscissa. If the principal earner's actual income is larger than PECI as is shown by point I_2 on the abscissa, the household's position with respect to income and leisure is shown at point H on the income line E F′ . Contrary to the case of an ascending MHLS locus as shown in Fig. Ⅲ-1, the intersection, G, of the contour passing through point E and the income line lies below point H, in Fig. Ⅲ-3. Hence,the household will be better off if the nonprincipal potential worker accepts the opportunity to work because the indifference curve passing through point H (not shown) lies above the curve which passes through point E. In this case, the

Fig. Ⅲ－3



W, θ : given h = assigned working hours.

nonprincipal potential earner will work if the principal earner's actual income execeeds the PECI. When the actual income is less than PECI (see point I_0 on the abscissa) the nonprincipal potential earner does not work because if the nonprincipal earner were to work,the household's position would be shown at point D, and the contour which passes through the point will be below the one which passes through point A'.

Hence, if the MHLS locus is downward sloping and the MHLS curve is ascending, the nonprincipal potential worker works (does not work) when the principal earner's actual income is larger (less) than PECI.

(3.2.5.4) When the nonprincipal earner's wage rate and hours of work assigned by firm are given, the condition of the principal earner's actual income which will determine the nonprincipal earner's choice between work and non-work is summarized below.

Let I^* and I be the PECI and the principal earner's actual income of a household respectively, and suppose the wage rate, w, and assigned hours of work, $\bar{h}$, are given.

a) if the MHLS curve is downward sloping, when $I < I^*$, the nonprincipal potential earner works, and when $I > I^*$, the nonprincipal potenteal earner does not work.

b) if MHLS curve is ascending, when $I < I^*$, the nonprincipal potential earner does not work, and when $I > I^*$, the nonprincipal potential earner works.

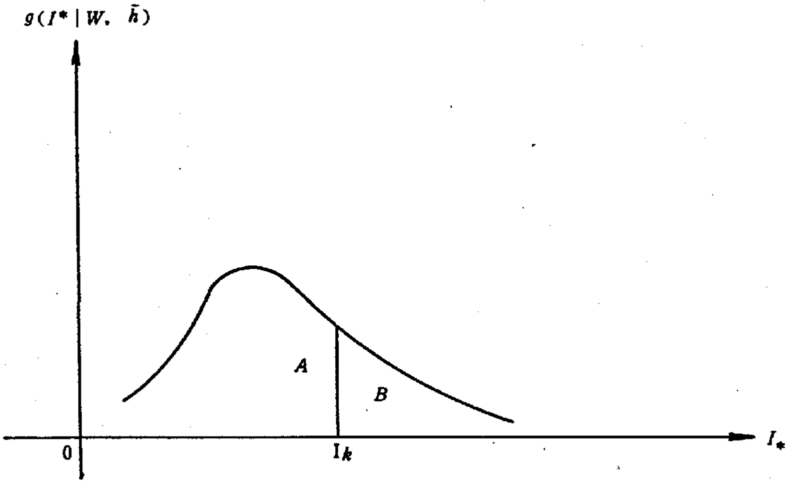
<3.2.6> Probability-of-Supply Equation and Distribution of Principal Earner's Critical Income

(3.2.6.1) Probability-of-Supply Equation

For the given wage rate, w, and assigned working hours, $\bar{h}$, the distribution of I^* is uniquely determined. Suppose we have K groups of households where within each group the principal earner's income, I_k, the wage rate, W_k, and assigned working hours, $\bar{h}_k$, are the same. Let the number of households in each group (k=1,2,……,K) be N_k.

The characteristics of the distribution function of I^*〔equation 3.2.13 in 3.2.3.1〕,g, are assumed to be common to the K groups. For the K-th group the density distribution of I^* and the level of the principal earner's income, I_k, is shown in Fig.Ⅲ-4. Area B stands for the probability of $I^* > I_k$ holding for any one household.

Fig. Ⅲ－4



(3.2.6.1−1) The case of a downword sloping MHLS curve

In the household where $I_k < I^*$ the potential earner is gainfully employed. (cf.3.2.5.4) For this group, the number of households in which one potential earner is gainfully employed is equal to N multiplied by the value of the probability designated by the area B. The probability designated by area B is the probability of supply, μk, of the households in the k−th group and is given by the following equation.

$$3.2\text{-}36) \qquad \mu k = 1 - \int_a^{I_k} g(I^*, \bar{W}_k, \bar{h}_k, \pi(\alpha_2, \cdots, \alpha_p, \theta))dI^*$$

where a is the lower limit of integration. Thus, the μ_k of the k-th group of households depends on the principal earner's income I_k, the wage rate W_k and assigned working hours $\bar{h}_k$ which is shown by

$$3.2\text{-}37) \qquad \mu_k = G(I_k, W_k, h_k, \alpha_2, \cdots\cdots, \alpha_p, \theta)$$

This equation can be exactly interpreted as the supply function in terms of the probability of supply. Multiplying μ_k by N yields the supply function in terms of persons.

As the definite integral of the second term of the right hand side in equation 3.2-36) is an increasing function of the principal earner's income, I_k, μ_k in 3.2-37) is a decreasing function of I_k.

Hence, we have,

$$3.2\text{-}37'\,) \qquad \frac{\partial \mu_k}{\partial I} = \frac{\partial G}{\partial I_k} < 0$$

This relation means that the larger the principal earners' income of the group of households, the smaller the non principal earners' probability of supply. The above proposition is clearly consistent with the empirical law mentioned in (2.1.2.3.1)

In the following discussion, we call μ_k the participation ratio (taking into account the approximation mentioned below). The notion of Supply probability μ_k may be, exactly speaking, a little different from the participation ratio. This will be clarified later in (3.2.8) and in the addendum to section Ⅲ, but we mention here that the latter can be regarded as an approximation of the former.

(3.2.6.1-2) The Case of an Ascending MHLS Curve

When the MHLS curve is ascending, i.e., the participation ratio is given by the area A in Fig. Ⅲ-4, as is explained in 3.2.5.4(b). In this case the participation ratio is given by

$$\mu_k = \int_a^{Ik} g(I^* | w_k, \bar{h}_k, \alpha_2, \cdots\cdots, \alpha_\rho, \theta) dI^*$$

As the definite integral of right hand side is an increasing function of the principal earner's income, I_k, we have

(3.2-39) $\partial \mu / \partial I > 0$

Hence, the greater the principal earners' income, the greater the non principal potential earners' participation ratio for the group of households. This clearly contradicts the empirical law in (2.1.2.3.1). Hence, it can be said that a utility function with an ascending MHLS curve (a downward sloping MHLS locus in income-leisure plane) is not consistent with previous observations.

(3.2.6.1-3) From the above argument the following proposition is obtained. As far as the observed participation ratios for the years 1961 to 1964 are concerned, the MHLS curves which are deduced from the households' income-leisure utility functions must be downward sloping. Then, equations (3.2-36), (3.2-37), or (3.2-37') in (3.2.6.1-1) will provide relevant participation ratios.

<3.2.7> The shift of the Participation Equation and the PECI Equation.

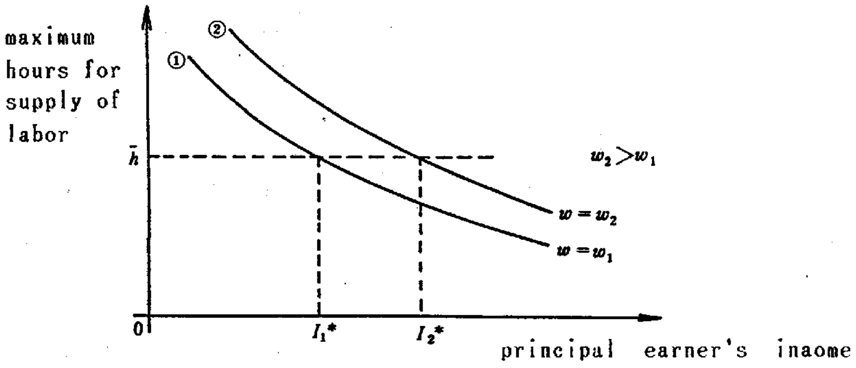
(3.2.7.1) The shift of the size distrubution of PECI induced by non principal potential earners' wage rate.

(3.2.7.1—1) Given the principal earner's actual income, the magnitude of MHLS increases in accordance with the size of the non principal potential earner's wage rate. This is shown in Fig. Ⅲ-1. Let the principal earner's actual income be I, as is shown in the figure. When the non principal earner's wage rate is increased, the slope of AB, $\tan\theta$, grows, and hence the intersection, C, of the income line, AB, and the contour, ω_0, passing through the point A, moves downward along the contour ω_0. That is, the MHLS is augmented. This applies to any level of the principal earner's actual income, so that increases in the non principal earner's wage rate shifts the MHLS income-leisure locus downward.

(3.2.7.1-2) Let the MHLS curve be downward sloping. When the non principal earner's wage rate is w, the MHLS curve of a household will be depicted as is shown in Fig. Ⅲ-5. For a wage rate W_2 which is larger than W_1, the MHLS curve moves upward. Given the assigned hours of work, $\bar{h}$, the level of the household's PECI is increased from I_1^* to I_2^* along with the growth of the non principal earner's wage rate from W_1 to W_2. The above argument applies to all the households with various levels of PECI (principal earner's critical income), given the non principal earner's assigned hours of work. Hence we get the following proposition. An increase in the wagerate offered to non principal earners shifts the distribution of the PECI to the right.

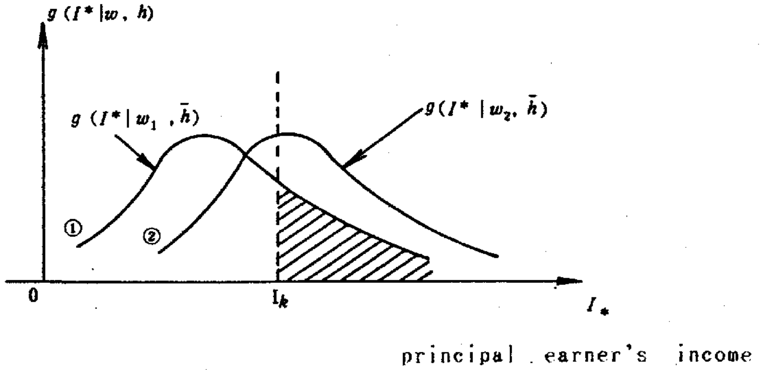
This is shown in Fig. Ⅲ-6. The curve ① shows the size distribution of PECI when $W = W_1$, and curve ② shows that of PECI when $W = W_2 > W_1$, the assigned hours of work, $\bar{h}$, being given. Suppose a group of households, k, whose principal earners' levels of actual income are the same. Let us denote the actual income level by I_k. When the non principal earner's wage rate is W, the right hand side of the area (hatched) enclosed by the vertical line passing through point I_k, and the density distribution curve ① gives the non principal earners participation ratio. Whenthe wage rate is increased to W_2, the density distribution curve of PECI is shown by curve ②. The participation ratio given by the area enclosed by the vertical line and the density distribution curve ②

Fig. Ⅲ－5



By MHSL, we mean maximum hours of work which the employers can assign.

Fig. Ⅲ－6



is clearly larger than what is given by the curve ①. Hence we have the following porposition.

When the non principal earners' wage rate offered by the firm increases, then the non principal earners' participation ratio of a group of households for which the level of principal earners' actual income are identical also increases. That is, with respect to the participation equation (3.2-37) in (3.2.6.1-1), we have

$$\partial \mu_k / \partial W = \partial G / \partial W > 0$$

This proposition is consistent with the empirical law given in (2.1.2.3).

(3.2.7.1-3) Contrary to the case mentioned above, suppose an ascending MHLS curve. Increasing the non principal earner's wage rate, W, decreases any household's PECI for the given assigned hours of work, as is shown in Fig. Ⅲ-7.

Hence, the PECI distribution shifts toward the left in accordance with the increase in the wage rate. In the case of an ascending MHLS curve, the left hand side of the area enclosed by the vertical line passing through point I_k (see Fig. Ⅲ-6) and the distribution curve gives the participation ratio (see 3.2.5.4(b)). Therefore, the shift of the PECI distribution to the right caused by an increase in the wage rate makes the participation ratio of a group of similar households increase.

By the above argument, preference functions which yeild an ascending MHLS curve clearly contradict the empirical law given in (2.1.2.3.2). (See also 3.2.6.1-3)

(3.2.7.1-4) To conclude, to be consistent with the empirical facts, the indifference curves with respect to income and leisure are required to have the following properties.

a) downward sloping and convex to the origin

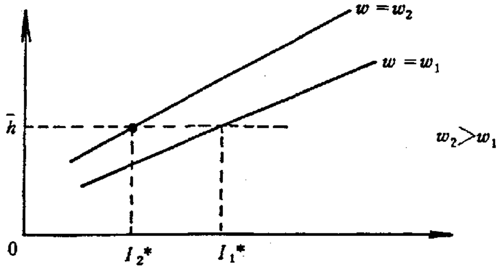
b) yielding downward sloping MHLS curves.

Hereafter, we shall consider only the case of downward sloping MHLS curves.

(3.2.7.2) Shifts of PECI Distribution generated by Changes in assigned Hours of Work

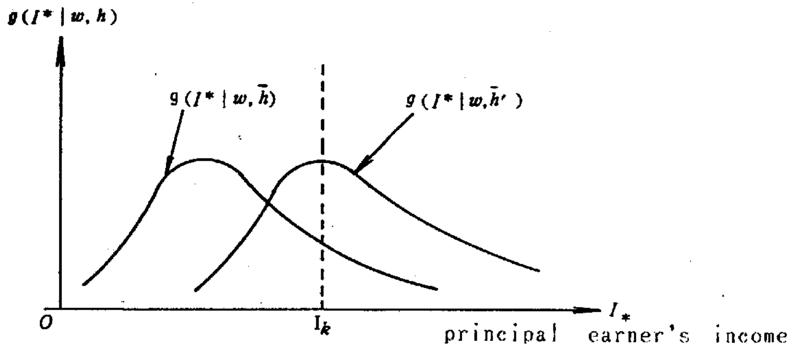
Assuming a downword sloping MHLS-curve, a decrease (increase) in the assigned hours of work increases (decreases) the level of PECI of any household. This is easily seen from Fig. Ⅲ-5 in (3.2.7.1-2)or directly observed from Fig. Ⅲ-1 in (3.2.4.1). Making use of the latter figure, we shall verify the above point.

Fig. Ⅲ－7



employed model for labor supply households

Fig. Ⅲ − 8



w : given $\bar{h} > \bar{h}'$

Suppose a household with an actual level of principal earner's income I_1 equal to the PECI of this household, assigned hours of work and wage rate being given by $\bar{h}$ and $\tan\theta$ respectively as shown in FigⅢ-1. The non principal earner of this household is, of course, indifferent to whether he/she works or not.Now, suppose, that assigned hours of work are shortened. When the non principal potential earner works, the household's position with respect to the income leisure plane is shown at a point (not shown in the Figure)somewhere above point C along the income line AB. This point lies on an indifference curve (not shown) which has a utility indicator higher than that of contour ω_0 passing through point A. Hence, shorter assigned hours of work for the non principal earner induce the non principal earner who does not work under the previously longer assigned hours of work, $T\bar{h}$, to work. This also means that the increase in PECI caused by the shortened assigned hours of work does make the principal earner's actual income, lower than PECI. Hence, given the non principal earner's wage rate, a decrease (increase)in the assigned hours of work shifts the distribution of PECI to the right (left).As a result, the participation ratio of a group of households whose principal earners' incomes are the same increase (decrease) when the assigned hours of work, $\bar{h}$, decrease (increase). This is easily seen from Fig.Ⅲ-8. Hence we have proposition that with respect to the participation ratio equation (3.2.37) in (3.2.6.1-1), the relation

$$3.2\text{-}40)\quad \frac{\partial \mu k}{\partial \bar{h}} = \frac{\partial C}{\partial \bar{h}} < 0$$

holds.

<3.2.8> The Relationship Between the Supply-Probability Equations and the Observed Participation Curves

Equation 3.2-36) or 3.2.-37) in (3.2.6.1-1) describe the probability of a non principal earner's supply of labor for a given earning opportunity.
We shall discuss the relationship between this equation and the observed participation curve shown in [2.1] and [2.2].

(3.2.8.1) The Theoretical Counterpart of the Observed Participation Curve

Let the wage rate proposed by the firm and the assigned hours of work be w_1, and $\bar{h}$ respectively. Inserting these values into the supply-probability equation 3.2-37), we have the relationship

3.2-41) $\mu_k = G(I_k, w_1, \bar{h}, \alpha_2, \cdots\cdots, \alpha_p, \theta)$.

This is an equation describing the relationship between the probability of supply μ_k and the principal earner's income, I_k, when w_1 and $\bar{h}$ are given. This equation is depicted by the curve C_1 in Fig.Ⅲ-9

Given w_2 and w_3 respectively,
we have

3.2-42) $\mu_k = G(I_k, w_2, \bar{h}, \alpha_1, \cdots\cdots, \alpha_p, \theta)$

and

3.2-43) $\mu_k = G(I_k, w_3, \bar{h}, \alpha_1, \cdots\cdots, \alpha_p, \theta)$,

where

$$w_3 < w_2 < w_1$$

and $\bar{h}$ is common to all households considered. These equations are shown as the curves C_1, C_2 and C_3 in Fig.Ⅲ-9.

Suppose in a group of households', principal earners' incomes are at the same level, I. Let the number of households in the group k be Nk. Hence, the number of non principal earners in the group equals Nk because the households are of type A. Further suppose three kinds of working opportunities are given to the Nk non principal earners by firms. Let the wage rates of the

opportunities be w_i
$(i=1,2,3)$ where

$$w_3 < w_2 < w_1,$$

assigned hours of work being the same, $\bar{h}$.

Let the number of the opportunities with wage rate w_i be N_i^d $(i=1.2.3)$ (Fig. Ⅲ-10). In this situation, firms want to hire N_i^d persons among N_k non principal earners by paying a wage rate w_i, assigned hours of work being the same for all the firms.

Who receives the highest wage rate, w_1, among N_k persons will depend mainly upon the choices of firms because potential supplyers (N_k persons) are not homogeneous from the firms' points of view. It will also depend partly on the amount of information on working opportunities posessed by the potential suppliers as well as on the distribution of information among N_k persons.

Firms select among N_k non principal potential earners, N_1^d persons for whom employment opportunities with wage rate w_1 are given. However, only some of these N_1^d persons will accept opportunities offered by firms. Letting the ratio of the number of persons who accept work opportunities to N_1^d persons be $\mu(I_k, w_1)$, the maganitude of which is shown by the ordinate of the point m_1 in Fig. Ⅲ-9, the number of persons gainfully employed is $N_1^d \cdot \mu(I_k/w_1)$.

The same arguement applies to the persons who can be employed at wage rates w_2 and w_3 if they so wish. Thus the number of persons gainfully employed at wage rates w_1, w_2, w_3, is given by

$$\mu_1 N_1^d + \mu_2 N_2^d + \mu_3 N_3^d \quad .$$

The participation ratio, i.e., the ratio of the number of persons gainfully employed to the number of non principal earners of the group of households whose principal earners' income are I_k is given by

$$(\mu_1 N_1^d + \mu_2 N_2^d + \mu_3 N_3^d)/N_k \; .$$

In general, let the number of potential work opportunities provided by firms be N_j^d, $(j=1,\cdots\cdots,J)$. The corresponding wage rates are w_j's and assigned hours of work are the same, $\bar{h}$. Among the J kinds of potential opportunities given by firms, there might be some opportunities for which the probability of supply, μ, is zero because of low wage rates. In this sense, the work opportunities may be said to be potential. Each of the N_k non principal earners of the group of type A households whose principal earners' incomes are the same, I_k, has her (his) own potential work opportunities. Some have opportunities at both high and low wage rates. Some have opportunities at low wage rates either because they are unskilled labor from the firms' point of view or because they possess only poor information on employment opportunities. Hence, each potential non principal earner in the group of households, k, has her own most favorable employment opportunity (defined in terms of wage rate) given by the firm (or firms).

Owing to her (his) quality, assessed by the order by which they are selected by firms, some non principal earners with most favorable working opportunities have high wage rates while others have to accept those with lower wage rates.(＊) In this sense, it is reasonable to presume that all the N_k non principal earners have their own available employment opportunities. Hence we have

$$(3.2\text{-}44) \quad \sum_{j=1}^{J} N_j^d = N_k \ .$$

(＊) See §3.3 below for the detail of the mechanism.

As is discussed above, the observed, participation ratio is given by

$$(3.2\text{-}45)\quad \mu^{o}(I_k) = \sum_{j=1}^{J} \mu(I_k/w_j)\cdot N_j^d/\ N_k$$

Substituting (3.2-44) into (3.2-45), we have

$$(3.2\text{-}46)\quad \mu^{o}(I_k) = \sum_{j=1}^{j} \mu(I_k/w_j)\cdot N_j^d/ \sum^{j} N_j^d\ .$$

Hence, the observed participation ratio, $\mu^{o}(I_k)$, of the group of households whose principal earners' incomes, I_k, are the same, is a weighted average of the supply probabilies of various wage rates, $\mu(I_k/w_j)$. The distribution of available working opportunities is nothing but the distrubution of wage rates offered by firms. This is shown in Fig. Ⅲ-10 for the case of three different of wage rates.

Let the size distrubution of potential work opportunities be continuous and denoted by

$$(3.2\text{-}47)\quad \lambda(w).$$

The observed participation ratio of the group of households whose principal earners' incomes are I_k can be written as

$$(3.2\text{-}48)\quad \mu^{o}(I_k) = \int_{w_j=0}^{\infty} \mu(I_k \mid w_j)\lambda(w)\cdot dw\quad ,$$

or, making use of 3.2-36) in (3.2.6.1-1), we have

$$(3.2\text{-}36')\ \mu^{o}(I_k)=1-\int_{w=0}^{\infty}\int_{I^*=a}^{I^*=I_k} g(I^*\mid w,h,\alpha_2,\cdots,\alpha_p,\theta)\cdot\lambda(w)dI^*\cdot dw$$

More generally, the size distribution of potential work opportunities is written as follows taking into account asssigned hours of work, h.

$$(3.2\text{-}47')\quad \lambda(w,h)$$

Hence the more general expression will be

$$(3.2\text{-}36'')\quad \mu^{o}(I_k)=1-\int_{h=0}^{\infty}\int_{w=0}^{\infty}\int_{I^*=a}^{I_k} g(I^*,w,h,\alpha_2,\cdots,\alpha_p,\theta)\cdot\lambda(w,h)dI^* dw\cdot dh$$

The last equation is the theoretical counterpart of the observed participation curve given in [2.1] and [2.2].

Hereafter we shall call this the participation equation.

When wage rates of work opportunities open to N_k non-principal earners increases, the mean value of $\lambda(w,h)$ with respect to w also increases, h being given. Hence, the mean value of supply probabilities which are weighted by λ increases. This is consistent with the second empirical law concerning labor market participation. It can also be seen that the shorter the assigned working hours, the less the mean value of $\lambda(w,h)$ with respect to h, and the participation ratio, $\mu^{o}(I_k)$, increases for the given wage rates, w. This means that a part-time system enables firms to induce more non-principal earners to work without increasing their wage rates.

(3.2.8.2) Approximation of The Participation Equation

If variation in assigned working hours h among job opportunities is negligible for all households, that is, the variance of h in the distribution function of opportunities, $\lambda(w,h)$, is negligible, we can approximately place h equal to $\bar{h}$, a constant common to all opportunities. Futher, if the variance of w_k is so small that w_k's approximately equal $\bar{w}$, a constant, we obtain an approximation from (3.2-36″)

$$3.2\text{-}49)\quad \mu_k^{o} \simeq 1-\int_a^{Ik} g(I^{*} \mid \bar{w},\ \bar{h},\ \overline{\alpha}_1,\cdots,\overline{\alpha}_n)\cdot dI_{*}\ ,$$

or $$3.2\text{-}50)\quad \mu_k^{o} \simeq F(I_k,\ \bar{w},\ \bar{h},\ \overline{\alpha}_1,\cdots,\overline{\alpha}_n)\ ,$$

where, $$\frac{\partial F}{\partial I} < 0,\quad \frac{\partial F}{\partial \bar{w}} < 0,\quad \frac{\partial F}{\partial \bar{h}} < 0\ .$$

Equation (3.2-50) is an approximation of the participation equation 3.2-36) in section 3-2-3

The observed data mentioned in 1.2.3 consist of households of Type A. This means that the non principal earners' wage are restricted within a fairly narrow range. Hence, the plausibility of approximation by 3.2-49) or 3.2-50) is greater than would be otherwise.

3.3 An Equilibrium Model of Continually Heterogeneous Labor Market. *

In this section a model of the labor market, where wage differentials among the firms of various scales exist, is presented. The term "firm of various scale" is used to indicate that the heights of marginal productivity curves for labor are different for different firms.

If the labor market is competitive, a unique wage rate prevails so long as the labor force is homogeneous from the firm' s point of view. If the labor force is heterogeneous, but can be split into three groups A, B and C, where firm "a" exclusively recruits workers from group A and the members of group A exclusively apply to a, and so on, we have three independent labor markets and the notion of non-competing groups can be applied to determine wages within each market. However, if the firms a,b and c respectively recruit among all the members of the groups A,B and C simultaneously, then the notion of non-competing groups is not applicable to the labor market. Since the actual labor market we observe has such a nature, we need to construct a model which can describe the performance of a competitive and heterogeneous labor market.

By heterogenity, we mean the existence of various grades (or labor ques) among applying workers from the firm' s point of view. The grades or ordering of applicants might be directly or indirectly correlated with their work experience, educational background, age, and / or sex. However, even if those characteristics or qualities are controlled, there may yet exist some ordering or differences in grades of applying workers. In fact, statistical data shows that there are wage differances among workers of firms of different sizes when controling for these characteristics of the workers.

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* The main part of this model originally appeared in "A Model of Household Labor Supply--Estimation of the parameters of income-leisure preference function-"in proceedings of 6th conferrence in econometric research (Japanese), 1967] The model and the results of numerical experimenes were presented in 「A model of Labor Market—Theory of generation and changes of wage differentials by firm size—」 Mitagakkaizassh (Mita Journal of Economics) vol.71 No.4 August, 1978 (Japanese)

This observed fact suggests that firms recongnize different grades among workers of the same age, sex, work experience, and / or educational backgrounds. Any reason for the paying of higher wages by large firms, whose labor productivities are higher than smaller ones, cannot be found as long as the grades of workers are the same across firms. In fact, large scale firms with higher productivities offer comparatively favorable work conditions (higher wages and shorter hours of work) and as a result attract many applicants of various grades. The firms recruit what they perceive as the most favorable ones among those who applied. Smaller firms with lower productivity can offer only less favorable terms and recruit among the residual applicants who fail to be employed by the large scale firms. This is the common experience of high school and college graduates in Japan.

In the following section we present a model of the labor market making use of the notion of grades* of labor in order to realistically approximate the labor market in Japan. The model is suitably simplified. Although the labor supply actually consists of members of self-employed households (e.g. farmers' households) and employee households whose principal earners are employees, only the latter type of household is taken into account. As well, the investment behavior of firms is not explicitly treated. These simplifications will not impair the basic characteristics of the model which remain sufficiently autonamous. The performance of the model is tested by numerical examples.

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* The notion of this kind, that is, labor que, is used in L.C.Thurow : "Generating Inequalities" (Basic Books, 1975)

Models of wage and employment determination with respect to a firm (or a group of firms) have been developed elswhere.* In this kind of model, individual labor supply and labor demand functions for a firm are assumed; that is, the notion of a kind of local labor market is introduced in the models. However, the relation between the individual labor supply function for the specific firm considered and the supply function of the market as a whole is not explicitly discussed. Such an individual supply function is, to some extent, an ad hoc relation just as is the individual demand function for a firm' s product in an oligopolistic market. The wage level of the firm considered and the average wage level of other firms appear as explanatory variables of the individual labor supply function (The ratio of the both variables is adopted in some cases).

Elasticities of labor supply with respect to those variables or coefficients of those variables for each firm change, reflecting changes in the conditions of the labor market as a whole including changes in the degree of of competition, the labor suppliers' conjecture with respect to the recruitment policy of firms other than the firms to which the suppliers are applying. However, the mechanism of such interdependent changes of elasticities or coefficients of individual supply function has not been clarified. In this sense, models using individual supply functions lack autonomy.

Individual labor supply functions for each firm are not used in the model presented below. Instead, two basic relations are introduced. Instead of an individual supply function for a firm, which describes the relation between the number of applicants for the specific firm and the wage rate the firm offers, we use the labor supply function for the whole market describing the quantity of labor supplied, the wage rate being given. That is, the supply function used in the following model does not specify the distribution of the quantity of labor supplied among firms. The distribution itself is determined by the model including firm demand functions for labor.

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C.A.Pissarides : Labor Market Adjustment: Microeconomic Foundations of Shortrun Neo classical and Keynesian Dynamics, London Cambridge Univ. Press 1976

[1] Basic Equations of the Model

1. Distribution Function of Grades of Labor

Let the indicator of the grade of the worker be G_i, where

$$i=1, 2, \cdots\cdots, m,$$

and m is the total number of people of working age.
The range of G_i is supposed to be

$$\varepsilon \leqq G_i \leqq 1,$$

where ε is some positive small number.

The cumulative distribution (cumulative from the top of G, where G=1) function of G is designated by $\nu(G)$ and the density distribution by $\nu'(G)$.

2. Labor Supply Probability Function

Suppose among n persons, n′ persons accept the employment opportunity at wage rate w, and assigned hours of work h, offered by firms. The ratio n′/n is the supply ratio with respect to the employment opportunity.

$$\underset{n \to \infty}{\mathrm{P\ lim}}\ n'/n \equiv \mu$$

is defined as the supply probability, which is a function of w and h.

3 Distribution of Minimum Supply Price of Labor

The minimum supply price of labor is defined as a critical wage rate below which suppliers reject the employment opportunity, assigned hours of work h being given. The minimum supply price of labor (MSPL) is denoted by $\underline{w}$.

Any supplier′s level of MSPL depends on following three factors:

a) the shape of his/her income-leisure preference curve

b) the level of his guaranteed income Xg which he/she can obtain without working
(e.g. principal earner′s income is a guaranteed income for non principal earners),

c) hours of work assigned by firms, h.

Hence, we have

$$1-1')\quad \underline{w}^i=\underline{w}(x_g^i,\ h^i,\ \alpha^i)\ ;\ i=1, \cdots, n$$

where α^i stands for the set of preference parameters of the ith supplier. x_g^i and h^i can be regarded as exogenous variables for the ith supplier. The value of α^i is specific to ith supplier; that is, the value of α^i differs among each of the n suppliers. Hence, we have the density distribution function $\psi(\alpha)$.

Now, suppose a group of persons have the same level of guaranteed income $\bar{x}_g$; that is,

$$1-2)\ x_g^i = x_g^{i+1} = \bar{x}_g$$

From 1−1), 1−2) and $\psi(\alpha)$, we have

$$1-3)\ g_{f\psi}\ (\underline{w} \mid \bar{x}_g, h)$$

which is the density distribution function of MSPL, h for brevity being assumed a common value for all persons considered. Subscript f and ψ denote the fact that the analytical form of the function g depends on f and $\psi.^{(*)}$ Integration of g,

$$1-4)\ \mu = \int_{w=0}^{w} g(w \mid \bar{x}_g, h)\ dw = \mu(w \mid \bar{x}_g, h),$$

gives the supply-probability function μ of the group of persons with $\bar{x}_g$ and h.

Multiplying by n, the number of persons in the group, we have the number of suppliers L^S, namely,

$$1-5)\ L^S = n\ \mu(w \mid \bar{x}_g, h),$$

When Xg and h are destributed as a joint density distribution

$$1-6)\ \Psi\ (x_g, h),$$

we have

$$1-7)\ \mu(w) = \int_{w=0}^{w} \int_{x_g=c}^{d} \int_{h=a}^{b} g(\underline{w}, x_g, h),\ \Psi(x_g, h) \cdot dh \cdot dx_g \cdot d\underline{w}$$

where a, b, c, d and e are the values standing for regions of integration for the relevant variables, h, x_g and $w.^{(**)}$

[2] The Outline of the Model

Let the production function of the i th sector (or firm) be

2-1) $Q_i = F(L_i, \bar{G}_i, A_i)$, $(i=1, \cdots, n)$

where A and $\bar{G}$, respectively, stand for the set of firm parameters and the index of the grade of workers employed in the ith sector. Further, $\bar{G}_i$ can be written as

2-1') $\bar{G}_i = \bar{G}_i(G_i^{min}, G_i^{max})$

where G_i^{max} and G_i^{min} are indicators of the highest grade of workers (most preferable workers among applicants from the firm's point of view) and the lowest grade of workers. It is supposed that

$$\partial F/\partial \bar{G}_i > 0, \ \partial F/\partial L_i > 0$$

Let the supply probability equation 1-7) be

2-2) $\mu = \mu(w, \overline{\lambda})$

where $\overline{\lambda}$ is a set of parameters of individuals, and for the sake of brevity assigned hours of work, h, is excluded and the guaranteed income level xg is supposed to be included in the set $\overline{\lambda}$.

The (cumulative) distribution function of G is denoted by

2-3) $\nu_G = \nu(G)$

where

2-4) $\epsilon \leqq G \leqq 1$

Let us suppose that the analytical form of the function ν is common to all the sectors under consideration. Hence, by letting the number of potential suppliers be N, the number of suppliers with grade G and over, N_G, is given by

2-5) $N_G = N \cdot \nu_G = N \cdot \nu(G)$

The number of suppliers with grade G and over going to the ith sections, L_S^i, is written as

2-6) $L_S^i = N \cdot \nu(G) \cdot \mu(\omega_i, \overline{\lambda})$

where w_i stands for the wage rate offered by the ith sector.(*)

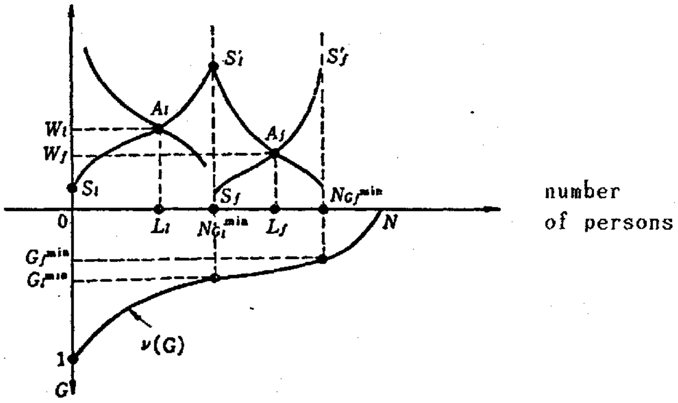
(1) <u>Behavior of the Leader</u>(*)

Imagine a sector (or firm) which offers the most favorable wage in comparison to other sectors in order to attract a number of potential suppliers. This sector can recruit workers of higher grades comparing to other sectors which offer less favorable working conditions. We shall call this sector a leader sector (firm) or a leader for short. Residual sectors are followers. Among those residual sectors, we can distinguish leaders and followers in accordance with the wage differentials each sector is willing to pay. That is, if we have three sectors with wage rates w_1, w_2 and w_3 where $w_1 > w_2 > w_3$, sector 2 plays the role of the follower of sector 1, while sector 2 plays the role of leader of sector 3. Follower sector 2, against leader sector 1, recruits workers with relatively higher grades amongst residual applicants which the leader has left for followers to employ because those applicants are not fully suitable for employment from the leader's point of view. Sector 2 as a leader against sector 3 will again leave undesirable labor suppliers. This pattern can be viewed as continuing indefinitely, with sector 3 acting as a leader to sector 4, and so on.

Let us imagine a labor market which consists of two sectors to simply present the basic characteristics of the model, where one of the sectors is able to attain a given level of production $Q_i (i=1,2)$ by varying G_i and L_i in the production function (2-1).

The distribution function (2-5), $N \cdot \nu(G)$, is depicted in the fourth quadrant in Fig 1. The curve GN is the cumulative distribution curve from the top labor grade $G(=1)$. Suppose firm ℓ(we denote leader by ℓ) wishes to recruit workers with grades higher than G_ℓ^{min}. In this case, the labor supply curve for firm ℓ can be depicted by curve S_ℓ S_ℓ' in the 1st quadrant. This curve stands for equation (2-6) where G^{min} is inserted for G. Now, $G^{max}(=1)$ and G^{min} being given for the firm ℓ, the demand curve for labor is derived from the production function 2・1) and (2-1') by applying the condition of cost minimization. This is depicted by curve D_ℓ in the first quadrant. The intersection of the

Fig. 1



supply curve $s_\ell\ s_\ell'$ and D_ℓ gives the wage rate w_ℓ and the demand for labor L_ℓ by firm ℓ necessary to attain the given level of production Q_ℓ.

If firm ℓ were content to recruit workers with lesser grades, e.g. $[G^{min}] < G_\ell^{min}$, the curve $s_\ell\ s_\ell'$ would be less steep and stretched to the right. Hence, the required grade of workers would be less and the number of workers employd would increase. At any rate, given the production function (2-1), the grade distribution function (2-3), and the supply probability function (2-2), the number of workers and the required grade to attain production level Q_ℓ are detemined by the procedure of cost minimization.

From 2-5) and 2-2) the number of potential applicants with grade G_ℓ^{min} and over, $L^{min}_{G_\ell}$, is given by

$$2\text{-}6'\ )\ L^{min}_{G_\ell} = N_\ell \cdot G_\ell{}^{min} \cdot \mu = N \cdot \nu(G_\ell^{min}) \cdot \mu(w_\ell\ ,\ \overline{\lambda}),$$

which is a function of w_ℓ. Eq. 2-6') is depicted by the curve $s_\ell\ s_\ell'$ in Fig. 1. We have, for the leader, $G^{max}=1$ in 2-1'). Hence, 2-1') is written as $\bar{G}_\ell = \bar{G}_\ell(G_\ell^{min},1)$. Substituting this function and 2-6') into 2-1) gives the leader's production function.

$$2\text{-}1''\ )\ Q_\ell = F[N \cdot \nu(G_\ell^{min}) \cdot \mu(w_\ell\ ,\ \overline{\lambda}),\ \overline{G}_\ell(G_\ell^{min},\ 1),\ A_\ell]$$

where the subscript i in 2-1) has been replaced by ℓ to denote that eq. 2-1" refers to the leader.

Definition of cost is given by

$$2\text{-}7)\ C_\ell = C_o^\ell + w_\ell \cdot L_{G_\ell}^{min} = C_o^\ell + w_\ell N \cdot \nu(G_\ell^{min}) \cdot \mu(w_\ell, \overline{\lambda})$$

where C_o^ℓ stands for capital cost which is regareded as given.

We can obtain W_ℓ and $G_\ell{}^{min}$ by minimizing C_ℓ in 2-7) under the constraint 2-1″), Q_ℓ being given:

Letting

$$2\text{-}8)\ \Psi_\ell = C_\ell + k\ \ Q_\ell - F[N \cdot \nu(G_\ell^{min}) \cdot \mu(w_\ell, \overline{\lambda}),\ \overline{G}_\ell\,(G_\ell^{min}; 1),\ A_\ell]$$

where k is Lagrangian multiplyer, and C_ℓ is given by 2-7), we have

$$2\text{-}9)\ \frac{\alpha \Psi_\ell}{\partial G_\ell^{min}} = \frac{\alpha \Psi_\ell}{\partial w_\ell} = 0.$$

Solving 2-1″) and 2-9) simultaneously for G_ℓ^{min} and W_ℓ , we obtain.

$$2\text{-}10)\ G_\ell^* = G_\ell^*\ (\overline{\nu}_o, \overline{\lambda}, A_\ell, Q_\ell\ )$$

and

$$2\text{-}11)\ w_\ell^* = w_\ell^*(\overline{\nu}_o, \overline{\lambda}, A_\ell, Q_\ell)$$

where $\overline{\nu}_o$ is a set of parameters in the grade distribution function 2-3). Equations 2-10) and 2-11) give optimal values for $G_\ell{}^{min}$ and w_ℓ both minimizing cost G_ℓ for the given production level Q_ℓ . The solution for employment LG_ℓ^{min} can be calculated by substituting 2-10) and 2-11) into (2-4) for G and w respectively. We shall call the number of workers thus obtained, and G_ℓ^* and w_ℓ^* given by 2-10) and 2-11) the "leader solution."

(2) Follower's Behavior

The highest grade of workers available to the follower is G_{ℓ}^{min} which is the lowest grade for the leader. Let the lowest grade of people in the group of potential applicants for the follower be G_f^{min}. The number of people with grades between G_f^{min} and G_{ℓ}^{min}, which we denote by $N_{G_f}^{min}$, is given by

2-12) $$N_{Gf}^{min} = N \cdot \nu(G_f^{min}) - N \cdot \nu(G_{\ell}^{min}),$$

which is shown by the length of $N_{Gf}^{min} \sim N_{G\ell}^{min}$ in Fig.1. Hence, the number of suppliers to the follower LG_f^{min} is written as

2-13) $$L_{Gf}^{\overline{min}} = N_{Gf}^{min} \cdot \mu = N\ [\nu(G_f^{min}) - \nu(G_{\ell}^{min})] \cdot \mu(w_f, \overline{\lambda})$$

Substituting 2-13) into 2-1), we have the production function of the follower;

2-14) $$Q_f = F\{N \cdot [\nu(G_f^{min}) - \nu(G_{\ell}^{*})] \cdot \mu(w_f, \overline{\lambda}), \overline{G}_f\ (G_f^{min}; G_{\ell}^{*}), A_f\}$$

where A_f is the set of parameters of the follower's production function, and G_{ℓ}^{*} is given by 2-10).

The definition of follower's cost C_f is given by

2-15) $$C_f = C_0{}^f + w_f \cdot L_f = C_0^f + w_f \cdot N\ [\nu(G_f^{min}) - \nu(G_{\ell}^{*})] \cdot \mu(w_f, \overline{\lambda}),$$

where C_0^f is capital (fixed) cost and 2-13) is substituted for L_f.

Let us minimize C_f in 2-15) under the constraint of 2-14) where the level of Q_f is given.

2-16) $$\Psi_f = C_f + j\ [Q_f - F\{\cdot\}\ ],$$

where j is the Lagrangian multiplier.

The minimization condition is as follows.

$$2\text{-}17)\quad \frac{\partial \Psi_f}{\partial G_f^{min}} = \frac{\partial \Psi_f}{\partial wf} = 0$$

Solving 2-17) for G_f^{min} and w_f,
we have

$$2\text{-}18)\quad G_f^* = G_f^* \,(\overline{\nu}_0,\ \overline{\lambda}_1,\ A_f,\ Q_f,\ \ G_\ell^*)$$

$$2\text{-}19)\quad w_f^* = w_f^*(\overline{\nu}_0,\ \overline{\lambda},\ A_f,\ Q_f,\ G_\ell^*,)$$

where G_ℓ^* is already given by the leader's solution 2-10). L_f can be obtained from 2-13) by inserting 2-18) and 2-19). We shall call this employment level and 2-18) and 2-19) the "follower's solution."

(3) Succession Equilibrium

When we have three or more firms (sectors), we can successively apply the above leader-follower relationship. The market thus generated is characterized by succession equilibrium. Let us suppose two firms are in a state of succession equilibrium. Now, suppose relative or absoluate changes in the production level of the leader cause a "leader's solution" with a wage rate w lower than the follower's.Then, of course, the initial state of the market cannot be sustained. A new leader-follower relation has to be established. The former follower succeeds to the position of leader and the former leader now becomes a follower. However, alternative cases could be considered. If the initial leader expects that he will not be able to hold the position of leader without augmenting the marginal productivity of his workers and if he finds losing his leader position is not profitable, he might invest in capital to augment his workers' productivity.

[3] Simple Model

1. Basic Equations

(1) Production Function

We shall specify the analytical form of production functions 2-1) and 2-11") in 2.3.2 as

3-1) $Q_i = b_i \, L_i^{\alpha i} (\overline{G}_i)^{\gamma i}, \ \overline{\alpha}_i > 0, \ \gamma_i > 0. \quad (i=1.2.3.\cdots)$

3-1′) $\overline{G}_i = (G_{i+1} \cdot G_i)^{\frac{1}{2}}, \ G_{i+1} < G_i,$

where G_i and G_i+1 respectively stand for the highest and the lowest values of G among the workers the ith firm employs.

(2) The Distribution Function of Grade Indicator

Simplifying the distribution function $\nu(G)$ without impairing the basic characteristics of the model, we use

3-2) $\nu(G) = \nu_0 + \nu_1 G,$

where ν_0 and ν_1 are parameters. $\nu(G)$ is the ratio of the number of potential applicants with grade G and over to the total number of potential applicants (the number of the people of working age). The magnitudes of G's the potential suppliers with the highest and lowest grades amongst all potential suppliers are respectively defined to equal unity and ε, ε being some small positive number. Hence, we have

3-3) $\nu(G) = 1 \quad \text{if} \quad G = \varepsilon$

and

$\nu(G) = 1/N \quad \text{if} \quad G = 1,$

where N stands for the number of the total potential suppliers.

By applying 3-3) to 3-2) we have

3-4) $\nu_1 = -(1-1/N)/(1-\varepsilon)$

3-5) $\nu_0 = 1+\varepsilon(1-1/N)/(1-\varepsilon)$

from 2.3.3-2).

Hence, the distribution function 3-2) is(*) written as

3-2′) $$\nu(G) = 1 + \frac{\varepsilon(1-\frac{1}{N})}{1-\varepsilon} - \frac{1-\frac{1}{N}}{1-\varepsilon} G$$

By adopting the magnitude of ε as

$$\varepsilon = 1/N,$$

3-2′) is written as

3-2″) $$\nu(G) = 1 + \frac{1}{N} - G$$

If N is sufficiently a large number we have

3-2″ ′) $$\nu(G) \cong 1 - G\ .$$

The number of persons with G higher than Gj, N (G≧Gj), is given by

3-6) $$N\ (G \geqq Gj) = N \cdot \nu(Gj)$$

Hence, applying 3-2'), we obtain

3-7) $$N\ (G \geqq Gj) = N\ [1+(1-\frac{1}{N})\frac{\varepsilon}{1-\varepsilon} - (1-\frac{1}{N})\frac{\varepsilon}{1-\varepsilon}Gj]$$

Making use of the relation $\varepsilon = 1/N$, 3-7) is written as

3-7′) $$N\ (G \geqq Gj) = N+1-NGj = N\ (1-Gj)+1$$

From 3-2″ ′) we have, as a good approximation for 3-7′),

3-7″) $$N\ (G \geqq Gj) \simeq N\ (1-Gj)$$

(3) Equation of Supply-probability

We specify the supply-probability equation 2-2) as a linear function of w

3-8) $\mu = \lambda_0 + \lambda_1 w$

where as shown later

3-9) $\lambda_0 < 0$, $\lambda_1 > 0$ and $0 \leqq \mu \leqq 1$

are postulated. In order to make our model simple without impairing its basic characteristics, we use a linear function as a supply probability function. This simplification means that we implicitly employ a rectangular distribution for the minimum supply price of labor, $\underline{w}$.

In equation 3-8), we have $w = -\frac{\lambda_0}{\lambda_1}$ when $\mu = 0$, hence,

$$3\text{-}10) \quad \begin{aligned} \mu &= 0 \qquad \text{if } w \leqq -\frac{\lambda_0}{\lambda_1} \\ \mu &= 1 \qquad \text{if } w \geqq -\frac{1-\lambda_0}{\lambda_1} \end{aligned}$$

and for the range of w,

$$-\frac{\lambda_0}{\lambda_1} < w < \frac{1-\lambda_0}{\lambda}$$

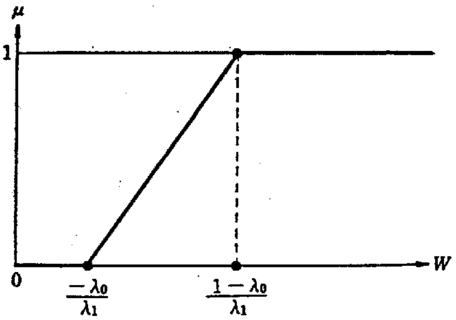
3-8) holds. The supply probability curve with (3.8) and (3.10) is depicted in Fig. 2.

The numerical value of $-\lambda_0/\lambda_1$, stands for the minimal value of the range of distributed values of $\underline{w}$. This minimal values of $\underline{w}$ must be positive, and

3-11) $-\lambda_0/\lambda_1 > 0$

must hold. On account of the nature of the distribution function, λ_1 must be positive. Hence, from 3-11) we have

Fig. 2



3-12) $\lambda_0 < 0$

2. Behavior of the Leader in the Simple Model

(1) Basic Equations

Let the leader's production function be 3-1), and we have

$$(\mathrm{L}\text{-}1) \quad Q_\ell = b_\ell \cdot L_\ell^{d\ell} \cdot (\overline{G_\ell})^{\gamma\ell}$$

where the suffix i in 3-1) is replaced by ℓ to show that the equation is that of leader. From 3-1') we have

$$(\mathrm{L}\text{-}1') \quad \overline{G}_\ell = (G_\ell^{\max} \quad G_\ell^{\min})^{\frac{1}{2}}$$

where $G_\ell^{\max} = 1$. Applying (L-1') to (L-1) we have

$$(\mathrm{L}\text{-}1'') \quad Q_\ell = b_\ell (G_\ell^{\max})^{\frac{1}{2}\gamma_\ell} \cdot L_\ell^{\alpha_i} \cdot G_\ell^{\frac{1}{2}\gamma_\ell} = b_\ell \cdot L_\ell^{\alpha_\ell} \cdot G_\ell^{\frac{1}{2}\gamma_\ell}$$

where the superscript of G_ℓ^i, min, is deleted for the sake of brevity.

The leader's cost is defined by

$$(\mathrm{L}\text{-}2) \quad C_\ell = w_\ell \cdot L_\ell + C_o^\ell$$

The number of suppliers to the leader is given by

$$(\mathrm{L\text{-}3}) \qquad L_{\ell}^{S} = \overline{N}\,(1-G_{\ell})\cdot(\lambda_0+\gamma_1 w_{\ell})$$

which corresponds to 2-6). This equation states that effective suppliers to the leader must be the ones with at least grade $G\ell$.

Given $Q\ell$, and by applying (L-3) to (L-1), we obtain

$$(\mathrm{L\text{-}4}) \qquad Q_{\ell} = b_{\ell}\,\{\overline{N}(1-G_{\ell})\,(\lambda_0+\lambda_1 w_{\ell})\}^{\alpha_{\ell}}\,G_{\ell}^{\frac{1}{2}\gamma_{\ell}}$$

where ${G_{\ell}^{\max}}^{\frac{1}{2}\gamma_{\ell}}$ is deleted because its numerical value equals unity.

Under the constraint of (L-4), we minimize C_{ℓ} in (L-2) with respect to w_{ℓ} and G_{ℓ} . The cost minimizing condition is given by

$$(\mathrm{L\text{-}5}) \quad \lambda_0(\alpha_{\ell}+\ \gamma\tfrac{1}{2})G_{\ell}+\lambda_1(\alpha_{\ell}+\ \gamma_{\ell}\ )G_{\ell}\,w_{\ell}-\lambda_1\gamma_{\ell}\,w_{\ell}-\tfrac{1}{2}\gamma_{\ell}\,\lambda_0 = 0$$

We can obtain solutions for G_{ℓ} and w_{ℓ} from the simultaneous equations (L—4) and (L-5).

(2) Graphical Presentation of equations (L-4) and (L-5)

Solution of the simultaneous equations (L-4) and (L-5) gives the wage rate w_{ℓ} and the grade indicator G_{ℓ} which the leader prefers. It can be seen that this solution is unique by the following discussion of the graphs of equations (L-4) and (L-5).

a) Graph of Production Function (L-4)

Equations (L-4) is depicted by the curve JNRPM in Fig. 3. Rewriting equation (L-4), we have

$$(\mathrm{L\text{-}6}) \qquad w_{\ell} = \frac{H}{G_{\ell}^{\gamma_2/2\alpha_{\ell}}(1-G_{\ell})} - \frac{\lambda_0}{\lambda_1}$$

where

$$H \equiv \frac{1}{\lambda_1}\left[\frac{Q_{\ell}}{b_{\ell}}\right]^{\frac{1}{\alpha_{\ell}}}\frac{1}{N} > 0$$

The value of G_ℓ minimizing w_ℓ can be obtained by solving the equation $dw_\ell / dG_\ell = 0$, giving,

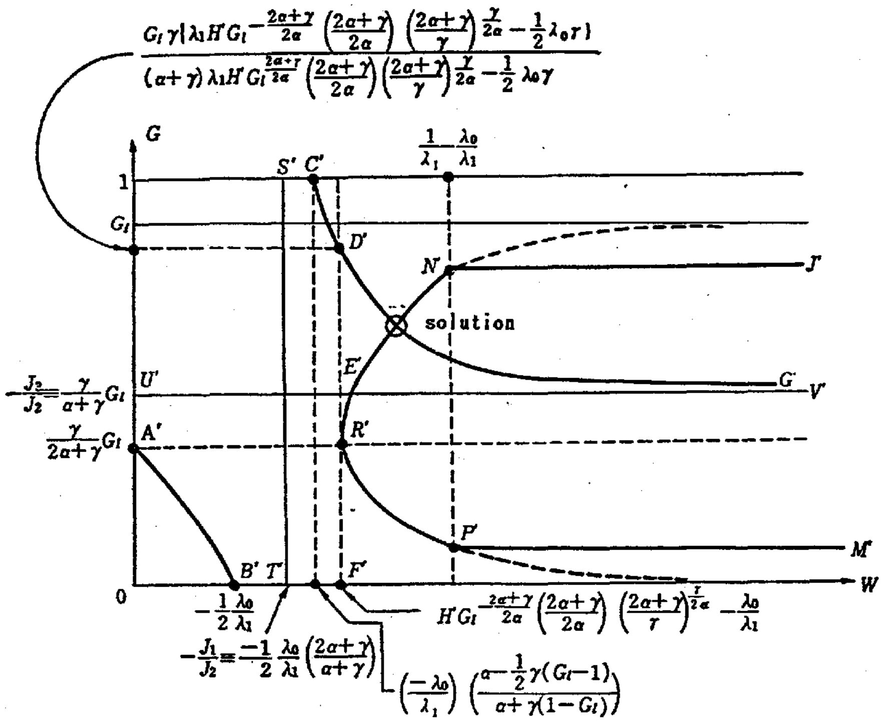
$$(\text{L-7}) \qquad G_\ell = \frac{\gamma_\ell}{2\alpha_\ell + \gamma_\ell}$$

This value of G_ℓ is depicted by the vertical value of point R in Fig.3.
The value of w_ℓ corresponding to G_ℓ in (L-7) is obtained by inserting (L-7) into (L-6); giving,

$$(\text{L-8}) \qquad w_\ell = \frac{H}{\left[\frac{\gamma_\ell}{2\alpha_\ell + \gamma_\ell}\right]^{\frac{\gamma_\ell}{2\alpha_\ell}} \left[\frac{2\alpha_\ell}{2\alpha_\ell + \gamma_\ell}\right]} - \frac{\lambda_0}{\lambda_1}$$

The value of w_ℓ in (L-8) is shown by the abscissa value of point R in Fig. 3.
From (L-6) we have

Fig. 3



$$(\text{L-6}')\qquad \frac{dW_\ell}{dG_\ell} = \frac{-1}{\left[G_\ell^{\gamma_\ell/2\alpha_\ell}(1-G_\ell)\right]^2}\cdot\left[\frac{\gamma_\ell}{2\alpha_\ell}\cdot G_\ell^{\frac{\gamma_\ell}{2\alpha_\ell}-1}(1-G_\ell)\right.$$

$$\left.+G_\ell^{\frac{\gamma_\ell}{2\alpha_\ell}}(-1)\right]$$

Hence, the sign of dw_ℓ/dG_ℓ depends on the sign of the right hand side of (L'-6); that is,

$$\frac{dW_\ell}{dG_\ell} < 0 \quad \text{if} \quad G_\ell < \frac{\gamma_\ell}{2\alpha_\ell+\gamma_\ell}$$

and,

$$\frac{dW_\ell}{dG_\ell} > 0 \quad \text{if} \quad G_\ell > \frac{\gamma_\ell}{2\alpha_\ell+\gamma_\ell}$$

Now, when $G_\ell = 1$ or 0, we have $dw_\ell/dG_\ell = \infty$. Hence, asymptotic lines of the curve shown by (L-4) are the horizontal axis and the horizontal line starting from point 1 on the vertical axis. However, effective parts of the curve RN and RP are further restricted by a theoretical requirement. That is, for the range of w_ℓ,

$$-\frac{\lambda_0}{\lambda_1} < w_\ell < \frac{1}{\lambda_1} - \frac{\lambda_0}{\lambda_1}, \text{ where, } \lambda_0 < 0,$$

there holds

$$0 \leqq \mu \leqq 1, \text{ where } \mu = \lambda_0 + \lambda_1 w_\ell.$$

So long as w_ℓ is less than $\frac{-\lambda_0}{\lambda_1}$, μ equals zero.

Also $\mu=1$ if w_ℓ exceeds the upper limit $\frac{1}{\lambda_1} - \frac{\lambda_0}{\lambda_1}$.

If w_ℓ is larger than the upper limit, $\frac{1-\lambda_0}{\lambda_1}$, μ must be unity, hence, $\lambda_0+\lambda_1 w_\ell$ in (L-4) equals 1. Taking into account this, we can solve (L-4) for G_ℓ. We have two solutions for G_ℓ, and the larger and smaller one are depicted in Fig. 3 by the ordinate of the horizontal lines NJ and PM respectively. Hence, the effective part of the curve derived from (L−4) is shown by JNRPM in Fig. 3(*).

(※) In equation (L-8), $H > 0$, and therefore, the minimum value of w_ℓ (which is shown by the abcissa of point F shown in Fig. 3) is larger than $-\frac{\lambda_0}{\lambda_1}$, which is the minimum value for w in the supply probability function, the minimum value making μ positive.

b) The Graph for the Equation for Equilibrium (L-5)

Equation (L-5) stands for the hyperbola shown by AB and CG in Fig. 3. We rewrite (L-5) as

$$\text{L-5}'\ )\ K_1 G_\ell + K_2 G_\ell w_\ell + K_3 w_\ell + K_0 = 0,$$

where

$$(\text{L-5}'\ \text{-1})\ K_1 \equiv \lambda_0 \ (\alpha_\ell + \frac{1}{2}\gamma_\ell)$$

$$(\text{L-5}'\ \text{-2})\ K_2 \equiv \lambda_1 \ (\alpha_\ell + \gamma_\ell)$$

$$(\text{L-5}'\ \text{-3})\ K_3 \equiv \gamma_\ell \lambda_1$$

$$(\text{L-5}'\ \text{-4})\ K_0 \equiv -\frac{1}{2}\gamma_\ell\ \lambda_0$$

It can be shown that, owing to 3-9),

$K_1 < 0$, $K_2 > 0$, $K_3 < 0$, and $K_0 > 0$

from (L-5' -1,), (L-5' -2), (1-5' -3), and (1-5' -4), respectively. We can rewrite (L-5') in the standardized form,

$$\text{L-5}''\ ) - (G_\ell + \frac{K_3}{K_2})\ (w_\ell + \frac{K_1}{K_2}) = \frac{K_1}{K_2} - \frac{K_1 K_3}{K_2{}^2}\ .$$

The signs of K_0, K_1, K_2 and K_3 are shown in the following table.

K_0	K_1	K_2	K_3	K_1/K_2	K_3/K_2	K_0/K_2	$K_1K_3/K_2{}^2$	$\frac{K_0}{K_2} - \frac{K_1K_3}{K_2{}^2} \equiv M$
+	−	+	−	−	−	+	+	−

We have $M \equiv \frac{K_0}{K_2} - \frac{K_1K_3}{K_2{}^2} < 0$ for $\lambda_0 < 0$, and both $\frac{K_3}{K_2}$ and $\frac{K_1}{K_2}$ are negative as can be seen from the table.

Taking into account

$$\frac{K_3}{K_2} = \frac{-\gamma_\ell}{\alpha_\ell \gamma_\ell}, \text{ where } \quad \alpha_\ell > 0 \text{ and } \gamma_\ell > 0,$$

it can be shown

$$\left| \frac{K_3}{K_2} \right| < 1$$

The curves AB and CG in Fig. 3 was drawn taking into account

$$\left| \frac{K_3}{K_2} \right| = \left| \frac{-\gamma_\ell}{\alpha_\ell + \gamma_\ell} \right| < 1$$

Asymptotic lines of both AB and CDG are ST and UV.

The vertical line ST apparently lies to the left of RF. Hence, curve AB has no intersection point with the curve JNRPM, representing the production function (L-4).

Horizontal line UV lies above line PM. Hence, the simultaneous equations (L-4) and (L-5) have a unique solution, that is, curve JNRPM intersects curve CDG at point X.It can be seen from Fig. 3. that the ordinate of point X is between those of points E and D. The ordinate values of E and D are shown in Fig. 3.

3. The Behavior of followers

(1) Basic Equations

As shown in section 2, the leader has already recruited $N(G_\ell) =_\ell(1-G\ )N$ persons with grade G_ℓ and over $[N(G_\ell)-L_\ell^S]$ persons are those with minimum supply prices of labor larger than w_ℓ (see Eq. L-3). Hence, the follower who can only pay wages less than w_ℓ cannot recruit those persons. The potential number of persons (among those persons some are not gainfully employed because their supply prices of labor are higher than the w_ℓ the follower offers) whose grades are higher than the follower's minimum requisit level, G, is given by

$$\text{(F-1)}\quad N(G)-N(G_\ell) = (1-G)N-(1-G_\ell)N = (G_\ell-G)N,$$

where the suffix f for Gf is deleted for the brevity.

The probability of supply, μ, is given by

$$\text{(F-2)}\quad \mu = \lambda_0 + \lambda_1 w_f ,$$

The function (F-2) is <u>common</u> to both the leader and the follower except for notation w_ℓ and w_ℓ.

A) Follower's Production Function

The follower's production function is obtained by substituting L_i, G^{max} and G^{min} in (2-1) and (2-1') for L_ℓ, G^ℓ and G^f respectively,

That is,

$$\text{(F-3)}\quad Q_f = bL_f^{\alpha f}\ (\overline{G_f})^{\gamma f}$$

where,

$$\overline{G} = (G_\ell \cdot G_f)^{\frac{1}{2}}$$

Hereafter, suffix f standing for the follower is deleted. (F-3) is rewritten as

$$\text{(F-3')}\quad Q = bG_\ell^{\frac{\gamma}{2}} \cdot L^\alpha\quad G^{\frac{\gamma}{2}} ,$$

where the value of G_ℓ is already determined by the leader.

B) Cost Minimization Condition for the Follower

The effective supply of labor, L_S, to the follower is given by

$$(\text{F-4}\ )\quad L^S = (G_\ell - G)\ N\ (\lambda_0 + \lambda_1 w)$$

where w stands for the follower's wage rate. The follower's definition of cost, C, is written as

$$(\text{F-5}\ )\quad C = wL + C_0 = w_\ell(G_\ell - G)\ N\ (\lambda_0 + \lambda_1 w) + C_0,$$

where C_0 stands for the follower's fixed cost which is given.
Inserting (F-4) into (F-3'), we have

$$(\text{F-3}''\ )\quad Q = bG_\ell^{\frac{1}{2}\gamma}(G_\ell - G)^{\alpha} \cdot N^{\alpha}(\lambda_0 + \lambda_1 w)^{\alpha}\ G^{\frac{1}{2}\gamma}\ ,$$

where Q is given.

Cost C in (F-5) is minimized under the constraint of (F-3"):
The minimizing conditions are

$$\frac{\partial F}{\partial G} = 0, \quad \text{and} \quad \frac{\partial F}{\partial w} = 0$$

where

$$F = w(G_\ell - G)\ N\ (\lambda_0 + \lambda_1 w) + C_0 + m\{\ Q - bG_\ell^{\frac{1}{2}\gamma}(G_\ell - G)^{\alpha} N^{\alpha}(\lambda_0 + \lambda_1 w)^{\alpha} G^{\frac{1}{2}\gamma}$$

and m is the Lagrangian multiplier. Calculating $\frac{\partial F}{\partial G} = 0$,
We have

$$(\text{F-6})\quad -wN\ (\lambda_0 + \lambda_1 w) = m\ \frac{\frac{1}{2}\ \gamma\,(G_\ell - G) - \alpha G}{G(G_\ell - G)}\ Q$$

From $\qquad \frac{\partial F}{\partial w} = 0$, we have

$$(F\text{-}7) \quad (G_\ell - G)\, N(\lambda_0 + 2\lambda_1 w) = m \frac{\alpha \lambda_1 Q}{\lambda_0 + \lambda_1 w}$$

Eliminatiog m from (F-6) and (F-7), we have

$$(F\text{-}8) \quad \frac{-w}{\lambda_0 + 2\lambda_1 w} = \frac{\frac{1}{2}\gamma(G_\ell - G) - \alpha G}{G \alpha \lambda_1}$$

This is the minimizing condition for the follower.

(2) Graphical Presentation of Equations (F-3″) and (F-8)

a) Graphical Presentation of the Follower's Production Function. Solving (F-3″) for w, we have

$$(F\text{-}10) \quad w = \frac{H'}{(G_\ell - G)\, G^{\frac{\gamma}{2\alpha}}} - \frac{\lambda_0}{\lambda_1}$$

where,

$$(F\text{-}11) \quad H' \equiv \left[\frac{Q}{(bG_\ell^{\frac{1}{2}\gamma})} \right]^{\frac{1}{2}} \cdot \frac{1}{N\lambda_1}$$

Differentiating w in (F-10) by G, we obtain

$$(F\text{-}12) \quad G = \frac{\frac{1}{2}\gamma}{\alpha + \frac{1}{2}\gamma} G_\ell ,$$

which is shown by the ordinate of point R' in (Fig. 4).
Inserting (F-12) into (F-10), we have

$$(F\text{-}13) \quad w = H' G_\ell^{-\frac{2\alpha+\gamma}{2\alpha}} \cdot \frac{2\alpha+\gamma}{2\alpha} \cdot \left[\frac{2\alpha+\gamma}{\gamma}\right]^{\frac{\gamma}{2\alpha}} - \frac{\lambda_0}{\lambda_1} ,$$

which is shown by the abcissa of point R' in (Fig. 4).

Taking into account (F-12) and (F-13), the curve corresponding to (F-3″), the follower's production function, is written as J′ N′ R′ M′ in (Fig-4). The reason why N' J' and P' M' are paralleℓ to the horizontal axis is the same as in the leader's case. The values of w at both points N' and P' are the same, $\frac{1-\lambda_0}{\lambda_1}$. This value of w makes the probability of supply unity, which can be seen from (3-10)

b) Graphical Presentation of the Follower's Cost Minimization Condition (F-8)

Through rearrangement of (F-8), we have

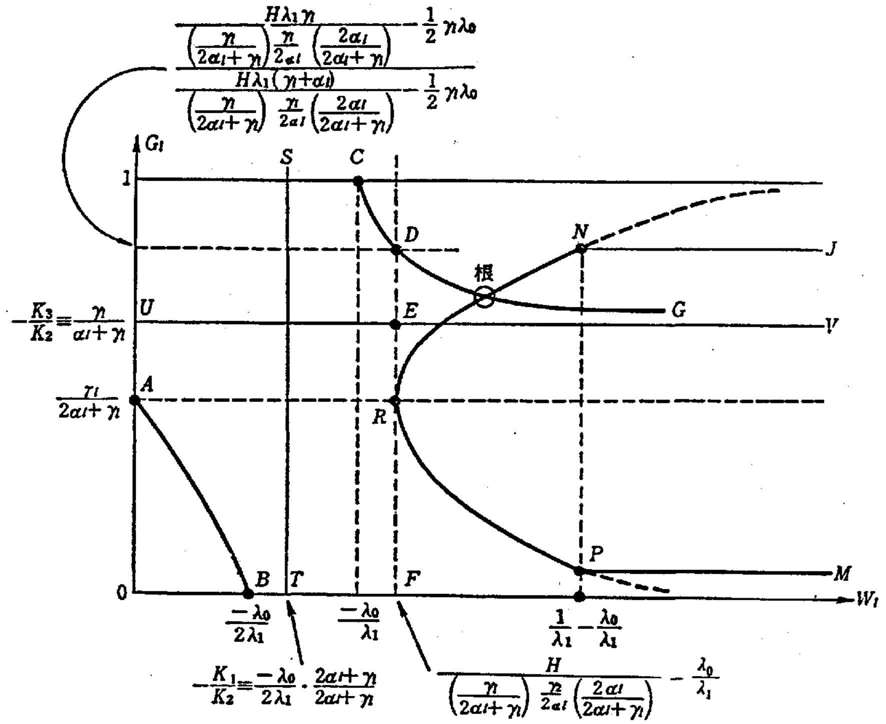
(F-8′) $J_1G + J_2GW + J_3W + J_0 = 0$,

where

F-8′ -1) $J_0 \equiv -\frac{1}{2}\lambda_0\gamma G_\ell$

F-8′ -2) $J_1 \equiv \lambda_0(\alpha+\frac{1}{2})$

Fig. 4



F-8' -3) $J_2 \equiv \lambda_1 (\alpha + \gamma)$

F-8' -4) $J_3 \equiv - \lambda_1 \gamma G_\ell$.

Also,

$$J_0 > 0, \quad J_1 < 0, \quad J_2 > 0 \text{ and } J_3 < 0,$$

which obtains from $\lambda_0 < 0$, $\lambda_1 > 0$, $\alpha > 0$, $\gamma > 0$ and $G > 0_\ell$

We can rewrite (F-8') in standardized form, that is,

$$\text{(F-9)} - \quad (G+\frac{J_3}{J_2})(w+\frac{J_1}{J_2}) = \frac{J_0}{J_2} - \frac{J_1 J_3}{J_2^2}$$

The sign of the right hand side of equation (F-9) depends on the sign of λ_0. Because λ_0 is negative,

$$M' \equiv \frac{J_0}{J_2} - \frac{J_1 J_3}{J_2^2} = \frac{\frac{1}{2}\lambda_0 \gamma G_\ell \alpha}{\lambda_1 (\alpha + \gamma)^2} < 0 \quad .$$

We have

$$\left| \frac{J_3}{J_2} \right| = \left| \frac{-\gamma G_\ell}{\alpha + \gamma} \right| < 1$$

since $G_\ell < 1$.

The constraints on the sign of parameters obtained above are summarized in the following table.

J_1	J_2	J_3	$\frac{J_1}{J_2}$	$\frac{J_3}{J_2}$	J_0	M'
−	+	−	−	−	+	−

The plausible region of solutions for the follower's minimizing condition (F-8) or (F-8′) is quite analogous to the leader's case. Equation (F-8′) can be shown by a pair of curves A'B' and C'D'G' in Fig. 4. Perpendicular line S'T' lies to the left of R'F'. Hence, the curve representing the production function J'N'R'P'M' does not intersect with A'B'. The horizontal line U'V' lies above P'M'. Hence, the curve J'N'R'P'M' intersects with the curve C'D'G' only at one point, y. That is, the solution is unique.

4. The Case of 3 or More Production Units

We have discussed the behavior of the firm offering the highest wage and the firm offering the second best wage. For a third firm paying a lesser wage rate, we can treat this firm as a follower of the firm offering the second best wage, and the analysis does not change. Hence, for the case of more firms, the roles of leader and follower can be applied successively.

[4] Application of the Simple Model

The simple model can be used to describe levels of wages and employment by sectors (firms) as well as time serial and cross sectional changes in wages and employment.

1. Determination of Numerical Values for Parameters Through Simulation

(1) Indirect observation of G_i

In the simple model, variables Q_i^t, L_i^t and W_i^t, where i and t stands for the production unit and time respectively, are directly observable from the data. However, we cannot observe the magnitude of G_i directly and we must therefore indirectly measure it making use of the model itself. We shall discuss the procedure to measure G_i bellow.

Suppose we have data on Q_i^t, L_i^t and W_i^t . With respect to the parameters of the model we have $\nu_0 = \nu_1 = 1$. Further suppose we have already estimated the parameters, λ_0 and λ_1, in the supply probability function. Labor supply curves for the leader and the follower respectively pass through points A_ℓ and A_f in Fig.1. The values of the coordinates of those points A_ℓ and A_f are known from observed data on the wages and employment of the leader and the follower. (Production units are ordered by the observed wage rates. Hence, successively, leader-follower relationship can be identified by this ordering.)
Therefor, we can obtain $N_{G\ell}^{min}$ and N_{Gf}^{min} by solving the simultaneous equations,

$$N_{G\ell}^{min} (\lambda_0 + \lambda_1 w_\ell) = L_\ell$$

$$(N_{G\ell}^{min} - N_{G\ell}^{min}) (\lambda_0 + \lambda_1 w_f) = L_f$$

where actual wages and employment W_ℓ, W_f, L_ℓ and L_f are directly obtained from the observed data and λ_0 and λ_1, are supposed to be already estimated, as mentioned above.

Applying $N_{G\ell}^{min}$ and N_{Gf}^{min} thus obtained to the left hand side of the grade distribution function 3-7") in [3], we can calculate the numerical values for G_ℓ^{min} and G_f^{min}. These are the "indirectly observed " values for $G_\ell^{min}$and G_f^{min} .

(2) Computation of leader-follower solutions for the determination of Numerical Values of the Parameters

Firstly, making use of directly and indirectly observed values of Q_ℓ , L_ℓ and G_ℓ^{min}, we can estimate initial values of the parameters of the leader's production function (L-1) by appropriate statistical estimation techniques. By an analogous procedure, we can estimate the parameters of the follower's production function (F-3′). Secondly, the initial values for the parameters of the production functions have to be refined, if necessary, by a kind of simulation technique, as follows. Suppose we have three production units (sectors or firms) and the data required of them including G_i 's are available. Comparing the observed wage rates among the three production units, in a given time unit we can identify the order of the units or the leader-follower relationships. Let this order be 1,2 and 3 successively.

Let the initial numerical (estimated) values of parameters be b^1_i, α^1_i, γ^1_i, where superscripts 1 stand for tentative (initial) values. Making use of these parameters, theoretical values for w_i and L_i can be computed, Q_i being given. If the theoretical values are fully compatible to the observed one, the estimated (initial) parameters are acceptable. However, when the order of the calculated theoretical values of w_i $(i=1,2,3)$ is different from that of the observed ones the initial values of the parameters are inconsistent with the observations.

Hence, the initial values of the parameters are slightly shifted to attain the observed orders. Even if the order of calculated w_i's itself coincides with the order of observed values, when considerable discrepancies in magnitude between observed and calculated ones occur, further slightly shifted values of the parameters have to be tried. In this manner we can refine the numerical values of the parameters.

2. NUMERICAL EXPERIMENTS

(1) The purpose

In this section, we design numerical experiments making use of the simple model of succession-equilibrium.

The main purpose of these experiments is to clarify the stability conditions of succession equilibria, where a set of numerical values of parameters in the model is given. The conditions are closely related to ranges of levels and of relative movement in output levels, Q_i's of sectors or production units, which are exogenous variables in the model. The ranges of Q_i which fulfill the stability conditions can be analytically obtained for the special case where $\lambda_0=0$ in the simple model, as is shown in [5]. However, when $\lambda_0\neq0$, analytical characteristics of the model are so complicated that we need numerical experiments to examine the ranges of production levels which fulfill stability conditions.

It is important to find those ranges of production. According to the observations, wage differentials among production sectors (units) are fairly stable over time. In some cases, changes in the differentials or changes in the order of magnitudes of wages among sectors occur. However, such changes are always gradual. Based on these observations, it is plausible to suppose that there exist some systematic machanisms governing the magnitudes of wage differentials and their changes. Hence, when these observations are explained by the succession-equilibrium model with the properly estimated parameters, the model can be regarded as reflecting basic characteristics of the market as far as the observations obtained are concerned. We can compute the ranges which fulfill the stability conditions for production by sectors. The following numerical experiments will exemplify the procedure.

The ranges of production fulfilling the stability conditions, which are computed by the model with properly estimated parameters, also provide a test for the model. That is, suppose that actual levels of production in some or all sectors falls into regions which do not fulfill the stability conditions. If we then observe some unstable phenomena occuring in the market, the plausibility of the model is proved. In contrast to this, if we observe continuing stable wage differentials, the model has to be modified or rejected. In other words, we would have to conclude that substantial changes occured in the magnitude of the parameters or that there have been some factors and/or mechanisms outside the model, which emerged as prodution levels reached an unstable region.

(2) Rules for the computation of the leader and the follower solutions

Before we proceed to the numerical example, we have to discuss rules for the computation of consistent solutions with respect to wages. Given a set of the parameters, consistent solutions for the endogenous variables, W_i, L_i and G_i must be obtained, Q_i being exogenous $(i=1,2,3)$ and numerically given. The rules and the procedure for the computation is as follows:

① One production unit is arbitrarily chosen as a leader out of the three. Suppose the unit chosen is unit 1. The leader solution for unit 1 is computed as mentioned in (1. Behavior of Leader.)

② One sector of the residual two units, unit 2 and 3, is arbitrarily chosen as a follower against the leader, unit 1. For example, suppose unit 2 is chosen. The follower solution for unit 2 is computed as mentioned in (2. Behavior of follower). Since the leader solution has already been obtained, G_1^{min} , L_1 and w_1 are treated as given constants for the computation of the follower-solution.

③ The follower-solution obtained in procedure ② is compared to the leader-solution in ①. If the values of w_i and G_i $(i=2)$ in the follower be seen that the surmised leader-follower relation between unit 1, 2 and 3 is correct.

④ The solution obtaned in ② plays the role of the leader-solution for the residual production unit which, in this case, is procuction unit 3. The follower solution is computed for production unit 3.

⑤ If the values of w's and G's obtained for the follower-solution in ④ (for production unit 3) are less than those for the leader-solution (for unit 2), then the follower-solution obtained is accepted as consistent. That is, it can be seen that the surmised leader-follower relation between unit 1, 2 and 3 is correct.

⑥ When the values of w and G obtained, in ②, for the follower-solution, are bigger than those for the producion unit considered as leader (which was obtained in ①), the initial selection (initially unit 1 is selected as a leader) must be considered as inappropriate. Consequently, unit 1 should be discarded and other units must be tried as a leader unit.

⑦ Contrary to process ⑤, if the solution is inconsistent, then the follower of unit 1 was not unit 2 but was unit 3. In this case, unit 3 should be tested as the follower of unit 1.

Examples shown in Tab. (a) are useful in clarifying the applications of those rules in the actual computation process. Suppose we have three production units 1, 2 and 3. All possible cases with respect to the leader-follower relationships are shown in cases 1 through 6 in the table. For cases 1 and 2, unit 1 is the leader, while unit 2 is a leader forcases 3 and 4, etc.. For each case the leader and follower solutions are computed. Out of the 6 cases, the case in which the order of the unit number and the order with respect to the wage rate w_i and grade G_i coincide with each other is adopted as the consistent case. For each time unit, t, the computations are carried out, Q_t being given.

(3) Conditions for Succession-Equilibrium in Labor Market

In Tab. (a), cases 1 and 3 are different in that the firms playing the roles of leader and follower are reversed. We might have, however, consistent leader-follower solutions for w and G for both cases respectively on account of specific numerical values of parameters and production levels exogenously assigned. The market is unstable when such phenomena occur. We shall examine this point in detail.

Table a

case	leader	follower		solutions of W and G		
	1	2	3	1	2	3
1	①	②	③	W_1, G_1	W_2, G_2	W_3, G_3
2	①	③	②	W_1, G_1	W_3, G_3	W_2, G_2
3	②	①	③	W_2, G_2	W_1, G_1	W_3, G_3
4	②	③	①	W_2, G_2	W_3, G_3	W_1 G_1
5	③	①	②	W_3, G_3	W_1, G_1	W_2 G_2
6	③	②	①	W_3, G_3	W_2, G_2	W_1, G_1

①, ② and ③ respectively stand for sectors

Let us concentrate on the leader unit A and the successive follower unit B. By definition we have $w_A > w_B$ and $G_A^{min} > G_B^{min}$. We shall examine the condition that guarantees a stable structure of wage differentials. we use the term "succession equilibrium" to characterize a labor market with stable wage differentials.

The necessary condition for succession equilibrium is that

(4-1) $w_\ell > w$,

where w_ℓ and w stand for leader A's and follower b's wage rate respectively. Necessary and sufficient conditions read as follows.

(a) Letting A and B be the leader and follower respectively, when (4-1) holds, the leader-follower relationship is stable, if the following condition is satisfied.

(a-1) Let B be a leader instead of A, A being a follower, and compute the leader solution for B. Let the solution for the wage rate be w_ℓ. Compute the follower solution for A. Let the solution be w. Then suppose (4-1),

$$w_\ell > w$$

does not hold. This is the necessary and sufficient condition for stable succession-equilibrium.

(a-2) When the leader-follower relationship between A and B is inverted in the computation procedure (a-1), if (4-1) holds in this case as well, then the leader-follower relationship cannot be stable. Hence it can be seen that the market is unstable when cases 1 and 3 in Tab. (a) hold simultaneously.

Now suppose that the analytical forms of the production function and the grade-distribution function are true and the estimated parameters are correct. Further suppose that numerical values of the set of parameters and the production levels of production unit A and B are such that they generate the unstable case mentioned above. On the other hand suppose, in the real labor market, a stable wage differential between unit A and B is observed. Then it must be considered that the leader-follower relationship between A and B is sustained by fctors other than those already considered ; e.g. historical or ramdom factors. Hence, the observed leader-follower position of A and B will be inverted whenever those factors change.

(b) Letting A and B be the leader and follower respectively, when (4-1) does not hold, the inverse leader-follower relationship is stable so long as the following condition is satisfied.

(b-1) Let B be a leader instead of A and compute the leader solution for B. Let the solution for the wage rate be denoted by w_ℓ. Compute the follower solutionfor A. Let the solution be denoted by w.

Next suppose (4-1) $w_\ell > w$, does not hold. In this case, it can be said that the set of estimated parameters of the model is not correct or the model itself is at fault.

(b-2) When the leader-follower relationship between A and B is inverted in the computation procedure, if (4-1) holds, then the leader-follower relationship is stable, B and A being the leader and follower respectively. However, this case, (b-2), is substantially equivalent to case (a-1), and the indepent cases are (a-1), (a-2) and (b-1). Hence, (a-1) is the necessary and sufficient condition for the stability of successive equilibrium.

(4) Numerical Exeperiments

We shall present a few numerical experiments to examine the workability of the successive equilibrium model. Let the number of production units (or firms) be two, unit 1 and 2. Numerical values of the parameters are assinged as follows.

$\alpha_1 = \alpha_2 = 1$ $\gamma_1 = 0.4$ $\gamma_2 = 0.9$ $b_1 = b_2 = 1$

$\lambda_0 = -0.5$ $\lambda_1 = 0.01$ $N = 10.000$

Suffix 1 and 2 stand for unit 1 and 2 respectively. The elasticity of production with respect to grades for unit 2, γ_2, is larger than that for unit 1. The levels of production of unit 1 and 2 are experimentally given as shown in the first and second columns of Tab. 1 through 7. These are exogenous variables in the simple model under consideration.

(1) In Tab. 1, Q_2 is increased from 150 to 300, Q_1 being constant. In this case the computation process revealed the succession-equilibrium was stable and a stable leadaer-follower relationship holds as is shown in the table ; i.e. unit 2 and 1 are the leader and follower respectively. The wage differential w_2/w_1 increases.

(2) In the second case, Q_1 and Q_2 were increased with a common rate of growth starting from $Q_1 = Q_2 = 160$, as shown in Table 2. The leader-follower relationship does not alter. The wage differential decreases, unlike that of case (1). It can be seen that the increase in the wage differential in case (1) stems from the growth and stagnation of production of unit 2 and 1 respectively.

(3) In Table 1, Q_2, the production of unit 2 which has a larger value for γ compared to unit 1, was increased. In contrast to this, production Q_1 of unit 1 is increased, Q_2 being held constant at 150, in Tab. 3. For the values of $Q_1 =$ 160, ..., 190, unit 2 occupies the position of leader, while case (b-1) appears when Q_1 exceeds 200; that is, we do not have a consistent solution for $Q_1 \geqq 200$ and $Q_2 = 150$.

(4) Next, in order to clarify the response of production unit 2 against production unit 1 with $Q_1 = 200$, we tentatively assigned Q_2 values in the range $38 \leqq Q_2 \leqq 750$. (See Table 4). It was found that the leader position switches if $Q_2 < 47$. The altered leader follower relationship is stable for $Q_1 = 200$ and $38 < Q_2 < 47$.

(5) A test analogous to (4) is shown in Tab. 5. Here, Q_2 is held constant at 150, while Q_1 is varied between $63 \leqq Q_1 \leqq 1250$. The leader role switches when Q_1 reaches 1250.
(6) Analogous to (5), we take $Q_1 = 250$ and $38 < Q_2 < 750$. For $Q_2 > 250$, unit 2 and 1 play the leader and follower respectively. If $Q_2 \leqq 54$, the relationship alters. Between $Q_2 = 54$ and $Q_2 = 250$, we do not have stable succession equilibrium (consistent solutions).
(7) Analogous to (6), we vary Q_2 between 38 and 750, Q_1 being 300. For $Q_2 > 250$, unit 2 and 1 are the leader and follower respectively. However, for $Q_2 \leqq 63$ this relationship alters.

	Q_1/b_1 followerQ	Q_2/b_2 leaderQ	leader sector	follower sector	L_1 (leader)	L_2 (follower)	G_1 (leader)	G_2 (follower)	W_1 (leader)	W_2 (follower)
第1表	150	150	2	1	166.9	179.5	0.888	0.639	57.89	56.62
	150	160	2	1	178.6	180.0	0.885	0.634	58.24	56.65
	150	170	2	1	190.3	180.5	0.882	0.629	58.57	56.72
	150	180	2	1	201.9	180.9	0.880	0.626	58.95	56.73
	150	190	2	1	213.8	181.4	0.877	0.621	59.26	56.80
	150	200	2	1	225.8	182.0	0.874	0.616	59.56	56.83
	150	250	2	1	273.5	183.7	0.865	0.602	60.86	56.98
	150	300	2	1	346.1	186.1	0.853	0.584	62.71	57.17
第2表	160	160	2	1	178.6	192.4	0.885	0.630	58.24	56.98
	170	170	2	1	190.3	205.6	0.882	0.622	58.57	57.31
	180	180	2	1	201.9	218.6	0.880	0.615	58.95	57.64
	190	190	2	1	213.8	232.0	0.877	0.607	59.26	57.99
	200	200	2	1	225.8	245.5	0.874	0.599	59.56	58.34
	250	250	2	1	285.4	313.4	0.863	0.568	61.18	60.07
	300	300	2	1	346.1	383.3	0.853	0.542	62.71	61.84
第3表	160	150	2	1	166.9	191.9	0.888	0.635	57.89	56.91
	170	150	2	1	166.9	204.3	0.888	0.632	57.89	57.23
	180	150	2	1	166.9	216.9	0.888	0.628	57.89	57.51
	190	150	2	1	166.9	229.4	0.888	0.625	57.89	57.80
	200	150	…	…						
	250	150	…	…						
第4表	200	750	2	1	266.3	916.8	0.489	0.800	59.99	75.47
	200	375	2	1	254.6	438.7	0.517	0.840	59.04	64.90
	200	250	2	1	248.4	285.4	0.582	0.863	58.55	61.18
	200	188	2	1	244.5	210.8	0.605	0.878	58.27	59.20
	200	150	…	…						
	200	50	…	…						
	200	47	1	2	217.3	74.4	0.813	0.598	56.41	56.22
	200	44	1	2	217.3	69.9	0.813	0.600	56.41	55.96
	200	42	1	2	217.3	65.8	0.813	0.602	56.41	55.79
	200	39	1	2	217.3	62.2	0.813	0.603	56.41	55.60
	200	38	1	2	217.3	59.0	0.813	0.604	56.41	55.43
第5表	1,250	150	1	2	1,474.2	383.9	0.662	0.352	76.24	74.67
	625	150	…	…						
			(この間解なし)							
	208	150	…	…						
	170	150	2	1	215.0	166.9	0.629	0.888	57.48	57.89
	156	150	2	1	187.2	166.9	0.636	0.888	56.81	57.89
	139	150	2	1	165.7	166.9	0.643	0.888	56.29	57.89
	125	150	2	1	148.5	166.9	0.650	0.888	55.87	57.89
	⋮		2	1						
	66	150	2	1	76.7	166.9	0.682	0.888	53.86	57.89
	63	150	2	1	72.7	166.9	0.684	0.888	53.74	57.89
第6表	250	375	2	1	321.3	438.7	0.534	0.840	60.67	64.90
	250	250	2	1	313.4	285.4	0.568	0.863	60.07	61.18
	250	188	…	…						
			(この間解なし)							
	250	58	…	…						
	250	54	1	2	273.9	89.2	0.796	0.568	57.47	57.14
	250	50	1	2	273.9	83.0	0.796	0.570	57.47	56.83
	250	47	1	2	273.9	77.6	0.796	0.572	57.47	56.57
	250	44	1	2	273.9	72.8	0.796	0.573	57.47	56.29
	250	42	1	2	273.9	68.7	0.796	0.574	57.47	56.05
	250	39	1	2	273.9	64.9	0.796	0.576	57.47	55.89
	250	33	1	2	273.9	57.7	0.796	0.577	57.71	57.48
第7表	300	750								
	300	375	2	1	388.8	438.7	0.523	0.840	62.22	64.90
	300	250	2	1	379.3	285.4	0.556	0.863	61.52	61.18
	300	188	…	…						
			(この間解なし)							
	300	68	…	…						
	300	63	1	2	331.0	108.5	0.782	0.542	58.52	58.25
	300	58	1	2	331.0	99.8	0.782	0.544	58.52	57.79
	300	54	1	2	331.0	92.3	0.782	0.546	58.52	57.45
	300	50	1	2	331.0	85.9	0.782	0.548	58.52	57.13
	300	47	1	2	331.0	80.3	0.782	0.550	58.52	56.85
	⋮		1	2						
	300	38	1	2	331.0	63.7	0.782	0.555	58.52	55.93

$b_1=b_2=1$　$\gamma_2=0.9$　$N=10,000$　$\lambda_0=-0.5$

$a_1=a_2=1$　$\gamma_1=0.4$　$C_0=0$　$\lambda_1=0.01$

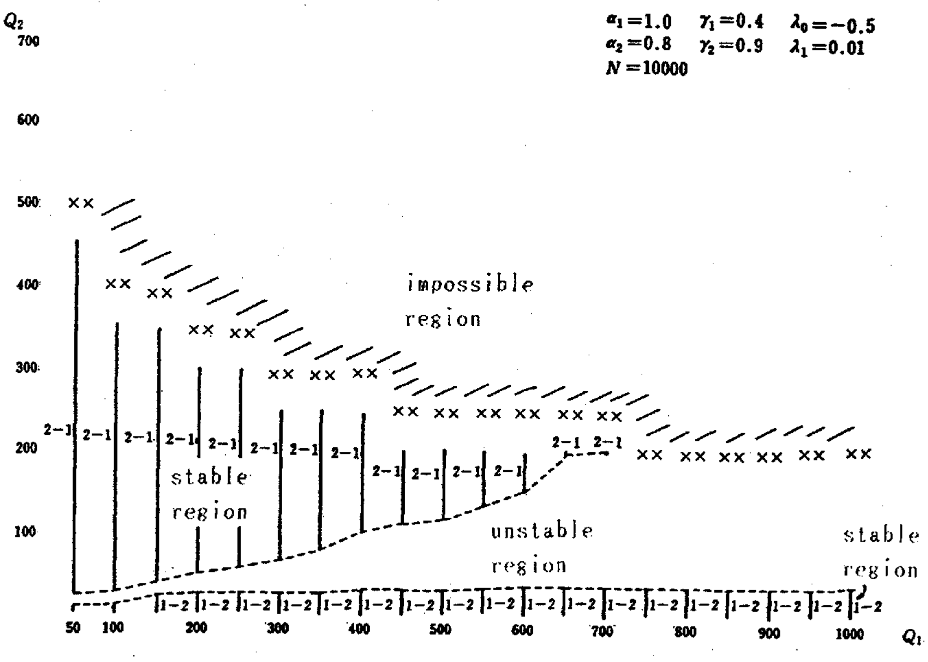
* The solution does not exist in this range of production

(5) The Ranges of Production which Guarantee Stable Succession Equilibria

The ranges for production of sectors 1 and 2, which guarantee stable succession equilibria, are depicted in Fig (5). The thick lines and dotted lines or segments respectively stand for the ranges where succession equilibria are guaranteed and not guaranteed. Thus, it can be seen that the hatched area represents (a part of) the unstable regions.

Fig. 5

Thick lines and segments stand for the regions where stable succession equilbrium holds. Attached numbers stand erd for leader follower relations, e.g.2−1 states that unit 2 and 1, respectively, play the rle leader and follower.



[5] Particular Solutions of the Simple Model

We have to solve nonlinear equations to obtain solutions of the simple model. The equations of the model were too complicated to solve analytically, hence we had to apply an iteration method. However, if the simple model is further simplified by putting λ_0 equal to zero, we can obtain analytical solutions.

1. Leader's Solution

By setting $\lambda_0=0$, equation (L-4) is reduced to

$$5\text{-}1)\ Q_\ell = b_\ell\,[N(1-G_\ell)(\lambda_1 w_\ell)]^{\alpha_\ell}\ G_\ell^{\frac{1}{2}\gamma_\ell}$$

The cost minimization condition (L-5) is simplified to

$$5\text{-}2)\ G_\ell = \gamma_\ell/(\alpha_\ell + \gamma_\ell)$$

It can be seen from (5-2) that, if $\lambda_0=0$, the grade indicator G_ℓ, which is the minimum value of G acceptable to the leader production unit, does not depend on w_ℓ and Q_ℓ but soley depends on the parameters of production function α_ℓ and γ_ℓ.

Hence, we have, from (3-7″) and (5-2),

$$5\text{-}3)\ N_{G_\ell} = N(1-\frac{\gamma_\ell}{\alpha_\ell+\gamma_\ell}) = \frac{\alpha_\ell}{\alpha_\ell+\gamma_\ell}\,N$$

That is, $N_{G\ell}$ also is independent of w_ℓ and Q_ℓ. However, this does not mean the leader's employment L_ℓ and wage rate w_ℓ are independent of Q_ℓ. This can be seen as follows.

By inserting (5-2) in the leader's production function

$$5\text{-}4)\ Q_\ell = b_\ell L_\ell^{\alpha_\ell}\ G_\ell^{\frac{1}{2}\gamma_\ell}$$

we have

$$5\text{-}5)\quad Q_\ell = b_\ell L_\ell^{\alpha_\ell}\left[\frac{\gamma_\ell}{\alpha_\ell+\gamma_\ell}\right]^{\frac{1}{2}\gamma_\ell}$$

Solving (5-5) for L_ℓ, we have the leader's employment L_ℓ, that is,

$$5\text{-}6)\quad L_\ell = \left[\frac{\alpha_\ell+\gamma_\ell}{\gamma_\ell}\right]^{\frac{1}{2}\frac{\gamma_\ell}{\alpha_\ell}} \left[\frac{Q_l}{b_\ell}\right]^{\frac{1}{\alpha_\ell}}$$

The value of w_ℓ needed to obtain the required L_ℓ is obtained by inserting (5-2) into (5-1) and by solving for w_ℓ; that is,

$$5\text{-}7)\quad w_\ell = \frac{Q_\ell^{\frac{1}{\alpha_\ell}}}{b_\ell} \quad \frac{1}{N\lambda_l} \quad \frac{\alpha_\ell+\gamma_\ell}{\alpha_\ell} \quad \frac{\alpha_\ell+\gamma_\ell}{\gamma_\ell}^{\frac{\gamma_\ell}{2\alpha_\ell}}$$

It can be seen from (5-6) and (5-7), that both L_ℓ and w_ℓ are functions of Q_ℓ.

2. Follower's Solution

The follower's production function becomes

$$5\text{-}8)\quad Q = bG_\ell^{\frac{\gamma}{2}}(G_\ell - G)^{\alpha} N^{\alpha}(\lambda_l w)^{\alpha} G^{\frac{\gamma}{2}}$$

when $\lambda_0=0$. The variables and parameters without suffixes refer to the follower.

The cost minimization equation is obtained by applying $\lambda_0 = 0$ to (F-8) and by solving for G; i, e.,

$$5\text{-}9)\quad G = \frac{\gamma_\ell}{\alpha_\ell + \gamma_\ell} \cdot \frac{\gamma}{\alpha+\gamma}$$

It can be seen that

$$G < G_{\ell}$$

by comparing (5-2) and (5-9).

Solving the follower's production function,

$$Q = bL^{\alpha} G_{\ell}^{\frac{1}{2}\gamma} \quad G^{\frac{1}{2}\gamma},$$

for L and replacing G_{ℓ} and G by (5-2) and (5-9) respectively, we have the follower's employment L, as follows.

$$\text{5-10)} \quad L = \left[\frac{Q}{b}\right]^{\frac{1}{\alpha}} \left[\frac{\alpha_{\ell}+\gamma_{\ell}}{\gamma_{\ell}}\right]^{\frac{\gamma}{\alpha}} \left[\frac{\alpha+\gamma}{\gamma}\right]^{\frac{1}{2}\frac{\gamma}{\alpha}}$$

We obtain the follower's wage w by inserting (5-9) to (5-8), i, e.,

$$\text{5-11)} \quad w = \left[\frac{Q}{b}\right]^{\frac{1}{\alpha}} \frac{1}{N\lambda_{1}} \left[\frac{\alpha_{\ell}+\gamma_{\ell}}{\alpha_{\ell}}\right]^{\frac{\gamma}{2\alpha}} \left[\frac{\alpha_{\ell}+\gamma_{\ell}}{\gamma_{\ell}}\right]^{1+\frac{\gamma}{2\alpha}}$$

$$\left[\frac{\alpha+\gamma}{\alpha}\right] \left[\frac{\alpha+\gamma}{\alpha}\right]^{\frac{\gamma}{2\alpha}}$$

3. Conditions for Succession Equilibrium

W_{ℓ} in equation (5-7) must be larger than w in (5-11) because otherwise it would be inconsistent with the proposition that the leader employs higher-grade labor by offering higher wage rate than the follower. Hence we have

$$\text{5-12)} \quad w/w_{\ell} < 1$$

Substituting w_{ℓ} and w in (5-12) by (5-7) and (5-11) respectively we have

$$5\text{-}12'\ )\quad \frac{w}{w_\ell} = \left[\frac{Q}{b}\right]^{\frac{1}{\alpha}} \left[\frac{b_\ell}{Q_\ell}\right]^{\frac{1}{\alpha_\ell}} \left[1+\frac{\gamma_\ell}{\alpha_\ell}\right]^{\frac{1}{2}\frac{\gamma}{\alpha}-1}$$

$$\left[1+\frac{\alpha_\ell}{\gamma_\ell}\right]^{1+\frac{1}{2}(\frac{\gamma}{\alpha}-\frac{\gamma_\ell}{\alpha_\ell})} \left[1+\frac{\gamma}{\alpha}\right] \left[1+\frac{\alpha}{\gamma}\right]^{\frac{\gamma}{2\alpha}} < 1$$

This inequality is the necessary condition for a succession equilibrium.

Making use of (5-12'), we can see how the parameters α_i, γ_i, b_i and production level Q_ℓ and Q must be related to each other in order that a stable leader-follower relationship is maintained. Below we concider the special case where

$$\alpha = \alpha_\ell \quad \text{and} \quad \gamma_\ell = \gamma,$$

that is, only the parameters, b_ℓ and b, and the production level, Q_ℓ and Q, of the adjacent production units differ with each other.

If we rewrite (5-12') taking into account $\alpha_\ell = \alpha$ and $\gamma_\ell = \gamma$, we have

$$\frac{w}{w_\ell} = \left[\frac{Q}{b}\right]^{\frac{1}{\alpha}} \left[\frac{b_\ell}{Q_\ell}\right]^{\frac{1}{\alpha}} \cdot 4 = \left[\frac{Q}{Q_\ell} \Big/ \frac{b}{b_\ell}\right]^{\frac{1}{\alpha}} \cdot 4 < 1$$

Hence, we have,

$$5\text{-}13)\quad \frac{Q}{b} < \left(\frac{1}{4}\right)^{\alpha} \cdot \frac{Q_\ell}{b_\ell}$$

or

$$5\text{-}14)\quad \frac{Q_\ell}{Q} > 4^{\alpha}\ \frac{b_\ell}{b}\ .$$

The level of production for the leader must be larger than $4^{\alpha} \cdot b_{\ell}/b$ times of the follower. This is the necessary condition for a stable leader-follower relationship to be sustained. In fact, this necessary condition is sufficient as well.

〔1〕 Suppose the necessary condition (5-14) holds for an arbitarily selected one of two adjacent units.

The inquality

$$5\text{-}14)\quad \frac{Q_{\ell}}{Q} > 4^{\alpha} \cdot \frac{b_{\ell}}{b}$$

is rewritten as

$$5\text{-}14'\,)\quad x > 4^{\alpha} \cdot y\,, \qquad \alpha > 0,$$

by setting

$$5\text{-}15)\quad \frac{Q_{\ell}}{Q} \equiv x\,, \quad \frac{b_{\ell}}{b} \equiv y\,.$$

We shall show that (5-14) or (5-14') no loger holds if we replace the leader unit by the other unit which we had originally adopted as a follower. Suppose that we replace the leader as mentioned above. If, after changing the leader unit, (5-14) or (5-14') were to hold, then it would have to be true that

$$5\text{-}16)\quad \frac{1}{x} > 4^{\alpha} \cdot \frac{1}{y}\,, \quad \alpha > 0\,,$$

by interchanging Q_{ℓ} and Q, and b_{ℓ} and b respectively. However, there can be no positive value of x and y satisfying (5-14) and (5-16) simultaneously. This can readily be seen as follows:

From (5-16) we have

(5-16′) $x < 4^{-\alpha}y.$

Hence, it can be seen that, if we adopt one unit, A, of the adjacent two units A and B as leader and if we find there holds a stable leader-follower relationship, then there is no stable relationship between B as a leader and A as a follower. Putting it another way, (5-14) or (5-14') is a necessary and sufficient condition for succession-eqilibrium.

〔2〕Suppose the necessary condition (5-14) does not hold for an arbitarily selected one of two adjacent units.

In this case, there are two alternative possibilities; the necessary condition (5-14) holds or does not hold the leader-follower relationship is interchanged. This depends on the value of x and y or Q_ℓ /Q and b_ℓ /b. First, note the following.

When (5-14) does not hold (5-14') must be reversed as

5-17) $x < 4^{\alpha}y , \quad \alpha > 0 .$

In order that (5-14) holds after interchanging the two units, we must have

5-18) $\frac{1}{x} > 4^{\alpha} \cdot \frac{1}{y}$

or

5-18′) $x < 4^{-\alpha} \cdot y$

(2-1) Now suppose (5-14) or (5-14') holds after interchanging.

In order that (5-14) holds after interchanging the two units, (5-17) and (5-18') must hold simultaneously. But we see (5-17) holds whenever (5-18') holds: Suppose we have two production units A and B. We adopt A, say, as a leader in trying to obtain leader-solution. Suppose we have, contrary to (5-14),

$$\frac{Q_\ell}{Q} < 4^{\alpha} \cdot \frac{b_\ell}{b}$$

where the ℓ suffix stands for unit A.

Next, we interchange the leader-follower relationship. After this arrangement we must have following inequality in order that leader follower relationship hold between A and B.

5-19) $\frac{Q_\ell}{Q} < 4^{-\alpha} \cdot \frac{b_\ell}{b}$ ……… (necessary and sufficient condition)

when ℓ stands for unit B instead of A. We can summarize this as: i) when (5-14) does not hold and (5-17) holds, x must not only be smaller than $4^{\alpha}y$ but also smaller than $4^{-\alpha}y$ in order that a succession-equilibrium holds after interchanging the leader-follower ordering. Hence (5-19) is the necessary and sufficient condition. (2-2) Suppose (5-14) or (5-14') does not hold after interchanging.

This is the case where

$$4^{-\alpha} < x < 4^{\alpha}y$$

or

$$4^{-\alpha} < \frac{x}{y} < 4^{\alpha}$$

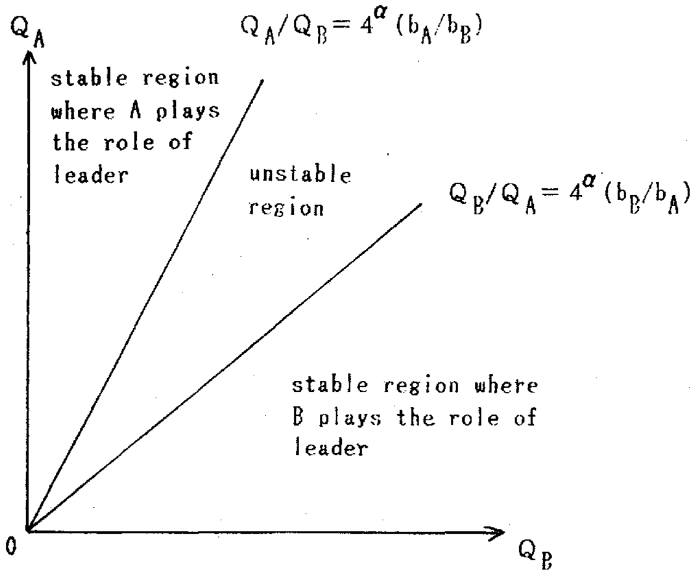
holds. Rewriting, we have

5-20) $4^{-\alpha} < \frac{Q_\ell}{b_\ell} / \frac{Q}{b} < 4^{\alpha}$,

This inequality shows the range of Q_ℓ and Q where the leader-follower relationship does not hold whatever adjacent production unit we adopt among two adjacent units as a leader. This is the case referred to as (b-1) in (3) "conditions for succession equilibrium in labor market". If such a case exists, we see that a) although the model is correct, the parameters are of incorrect value or b) the model itself is not applicable because the actual or observed leader-follower relationship is sustained by factors other than those considered here. From (5-20) it can be seen that the larger the numerical value of α the wider the range where the necessary condition for stable succession-equilibria does not exist.

Taking into account the necessary and sufficient condition, 5-19), the regions for the productions of two adjacent units (or sectors) A and B where stable succession equilibria exist and do not exist, respectively, are depicted in Fig(6).

Fig. 6



[6] correspondence between succession-equilibrium model and the observed labor participation curve

Finally let us discuss the correspondence between the succession-equilibrium model and the observed labor participation curve for type A households.

The succession equilibrium model of the labor market shown in Fig(1) can be interpreted as showing the generation of observed labor participation curve for wives (non principal potential earners) in type A households. That is, we can reinterpret ON in the figure as the number of wives in type A households with common principal earners' income I_k. Wage rates w_1 and w_2 in Fig. Ⅲ-10, respectively, correspond to w_ℓ and w_f in Fig(1). N_1^d and N_2^d in Fig Ⅲ-10, which we referred to as potential employnent opportunities, correspoud respectively, to the segments $ON_{G\ell}^{min}$ and $N_{G\ell}^{min}$ $N_{G\ell}^{min}$ in Fig. 1. The wage rate corresponding to w_3 in Fig. Ⅲ-10 is not shown in Fig(1) because only two sectors (employers) are taken as an examplein this figure. However, the counterpart of w_3 in Fig(Ⅲ-10), is analoguous to the case of w_1 and w_2.

$N_1^d\mu_1$ and $N_2^d\mu_2$ in equation (3.2-46) correspond, respectively, to the segment OL_ℓ and $N_{G\ell}^{min}L_f$ in Fig(1). The magnitudes of the ordinates of $\mu(I_k, w_1)$and $\mu(I_k, w_2)$ in Fig. Ⅲ-9, multiplied by the total number of wives having I_k respectively, correspond to the abcissa of points A_ℓ and A_f in Fig. 1 in this subsection .

Summary

1. An autonomous model of a continually heterogeneous labor market for analysing the generation and movement of wage differentials by firm size (or by sector) was developed in this section. As far as observed data are concerned, we do not find any drastic and/or random changes in wage differentials by firm size or by sector; there are no sudden and/or random inversion of the order of magnitudes of wages. These facts indicate that the notion of successive equilibrium employed in the model is realistic.

2. It was shown that, in order to measure the grade G_i of the labor force in the ith sector, which is one of the variables in the model, the numerical values of the parameters in the supply probability function should be obtained prior to the estimation of other parameters.

Hence, we see that independent measurment of the parameters in the supply probability function is extremely important. One of the ultimate purposes of the analyses in the following chapters is accordingly, to provide numerical values for the supply probability function in the succession-equilibrium model of the labor market.

3. Procedures of refining the initially given numerical values of the parameters in the model by a kind of simulation technique were discussed.

4. Making use of a simplified succesion-equilibrium model, numerical experiments were carried out. The results of the experiments showed that we can identify the regions of production levels for adjacent firms or sectors which guarantee stable succession equilibrium. Identification of those regions also provides a postulate for testing the model.

5. Finally, in the last sub-section, the relation between the observed labor participation curve and the supply probability function was placed in the perspective of the labor market as a whole.

§ Ⅳ. A Model of the Supply of Labor of Type A Households

[4.1] Specification of Income-Leisure Preference Function

The following points must be taken into account in order to specify the functional form of the preference function.First, the preference function obvioulу must fulfill the conditions required in(3.2.7.1-4). Secondly, in the present stage of analysis it is not favorable to adopt a preference function which has an analytical form that a priori specifies the algebraic sign of $\alpha h^{*}/\alpha w$ regardless of the magnitude of the nonprincipal earne'rs wage rate, w. It can not be said a priori whether the relation between the nonprincipal earner's wage rate, w, and the optimal hours of labor supply, h*, is backward bending when the principal earner's income, I, is given. That is, possible for $\alpha h^{*}/\alpha w$ to be positive in some range of values of w but negative in some other values of w. The algebraic sign of $\alpha h^{*}/\alpha w$ should be decided after the parameters of the preference function are estimated, by making use of a suitable analytical form of the preference function. Only if the algebraic sign of $\alpha h^{*}/\alpha w$ is proved to be positive(negative) for a considerable wide range of variation in w, can a function which gives a priori a positive (negative) $\alpha h^{*}/\alpha w$ regardless of the magnitude of w be employed as a plausible preference function.

Among the functions which are consistent with the two requirements mentioned above, we adopt an Allen-Bowley type function (i.e. quadratic form) for the sake of simplicity.

[4.2] PECI Equation Reduced from Allen-Bowley type Preference Function

For an arbitarily chosen household of Type A, i, the preference function is specified by

$$(4.2\text{-}1)\quad \omega=\frac{1}{2}\gamma_1^i X^2+\gamma_2^i X+\gamma_3^i X\Lambda+\gamma_4^i\Lambda+\frac{1}{2}\gamma_5^i\Lambda^2$$

where γs (s^i=1, 2, ……, 5) stand for preference parameters for the i-th household.

We can obtain the PECI equation for the Allen-Bowley type preference function by applying the procedure mentioned in section <3.2.2> to equation (4.2-1). That is, substituting(3.2-7) and (3.2-8) into (4.2-1), we have

$$(4.2\text{-}2)\quad \omega_0=\gamma_1^i\frac{1}{2}I^2+\gamma_2^i I+\gamma_3{}^i I\cdot T+\gamma_4^i T+\frac{1}{2}\gamma_5^i T^2,$$

corresponding to equation (3.2-9).
Applying to (3.2-7') and (3.2-8') to (4.2-1) we get

$$(4.2\text{-}3)\quad \omega_0{}'=\frac{1}{2}\gamma_1^i(I+W\bar{h})^2+\gamma_2{}^i(I+W\bar{h})+\gamma_3^i(I+W\bar{h})(T-\bar{h})$$
$$+\frac{1}{4}\gamma_4^i(T-\bar{h})+\frac{1}{2}\gamma_5^i(T-\bar{h})^2$$

Hereafter notation $\bar{h}$ is replaced by h corresponding to (3.2-10).

By the requirement that I in (4.2-2) and (4.2-3) be PECI (Principal Earners Critical Income), (3.2-11), that is,

$$\omega_0=\omega'_0$$

We have, replacing I by I^*,

$$\frac{1}{2}\gamma_1{}^i I_*^2 + \gamma_2{}^i I_* + \gamma_3{}^i I_* T + \gamma_4 T + \frac{1}{2}\gamma_5{}^i T^2$$

$$= \gamma_1 \frac{1}{2}(I_*^i + Wh)^2 + \gamma_2^i (I_* + Wh) + \gamma_3^i (I_* + Wh)(T - h)$$

$$+ \gamma_4^i (T - h) + \frac{1}{2}\gamma_5{}^i (T - h)^2$$

using equations (4.2-2) and (4.2-3). Solving the last equation for I^*, we obtain the PECI equation for the i-th household,

$$(4.2\text{-}4)\quad I_i^* = \frac{\gamma_4{}^i - \gamma_2{}^i W - \gamma_3{}^i W(T-h) + \gamma_5{}^i (T - \frac{1}{2}h) - \frac{1}{2}\gamma_1{}^2 W^2 h}{\gamma_1{}^i W - \gamma_3{}^i}$$

which corresponds to (3.2-6).

「4.3」 The Equation of Maximum Hours of Labor Supply Reduced from the A・B Type Preference Function (*)

<4.3.1> Reduction of MHSL Equation from A・B Type Preference Function

Making use of an A・B type Preference function, (4.2-1), the equation of MHSLin <3.2-4> can be obtained. First, we shall derive the equation of the indifference curve passing through point A shown in Fig. Ⅲ-1. The utility indicator, ω_{02} of the indifference curve is given by (4.2-2). Hence the equation of the indifference curve passing through the point A is given by

$$4.3\text{-}1)\quad \frac{1}{2}\gamma_1{}^i I^2 + \gamma_2{}^i I + \gamma_3{}^i I T + \gamma_4{}^i T + \frac{1}{2}\gamma_5{}^i T^2$$

$$= \frac{1}{2}\gamma_1{}^i X^2 + \gamma_2{}^i X + \gamma_3{}^i X \Lambda + \gamma_4{}^i \Lambda + \frac{1}{2}\gamma_5{}^i \Lambda^2$$

(*) Hereafter we shall refer to the Allen-Bowley Type as the A.B. Type for brevity.

The ordinate of the point C in Fig. Ⅲ-1 can be obtained by solving (4.3-1) and $I+wh=X$ simultaneously for X and Λ taking into account the relation

$$h=T-\Lambda.$$

The solution for Λ is given by

$$(4.3\text{-}2)\quad \Lambda i=\frac{2(\gamma_1{}^iW-\gamma_3{}^i)}{\Omega}I+\frac{\gamma_1{}^iW^2T}{\Omega}-\frac{2(\gamma_4{}^i-\gamma_2{}^iW)}{\Omega}-\frac{\gamma_5{}^iT}{\Omega}$$

where,

$$(4.3\text{-}3)\quad \Omega \quad \gamma_1{}^iW^2-2\gamma_3{}^iW+\gamma_5{}^i.$$

Rewriting the left hand side of the equation (4.3-2) we have

$$(4.3\text{-}4)\quad hi=\frac{-2(\gamma_1{}^iW-\gamma_3{}^i)}{\Omega}I-\frac{\gamma_1{}^iW^2T}{\Omega}+\frac{2(\gamma_4{}^i-\gamma_2{}^iW)}{\Omega}+\frac{\gamma_5{}^iT}{\Omega}+T.$$

This is the MHSL equation for the i-th household.

<4.3.2> The Conditions for the downward sloping MHSL curve

It can be seen from (4.3-4) that the necessary and sufficient condition for a downward sloping MHSL curve for the i-th household required in (3.2.5.4) is given by

$$(4.3\text{-}5)\quad \frac{\gamma_1{}^iW-\gamma_3{}^i}{\gamma_1{}^iW^2-2\gamma_3{}^iW+\gamma_5{}^{i.}}\qquad >0$$

Because of this inequality, several restrictions are imposed on the parameters of the preference function. To obtain these restrictions we divide the numerator and the denominator of the left hand side of (4.3-5) by γ_i, and, delete, the suffix i for brevity, we have

(4.3-6) $\dfrac{W-\gamma_3'}{W^2-2\gamma_3' W+\gamma_5'} > 0,$

where,

(4.3-7) $\gamma_3' \equiv \gamma_3/\gamma i,$ and $\gamma_5' \equiv \gamma_5/\gamma_i$.

It can readily be seen from (4.3-6) that the following inequality must be satisfied.

(4.3-8) $(W-\gamma_3')(W^2-2\gamma_3' W+\gamma_5') > 0,$

The three solutions for the cubic equation,

(4.3-9) F $(W-\gamma_3')(W^2-2\gamma_3' W+\gamma_5')=0,$

for w are given by

(4.3-10-1) $W_1=\gamma_3'$,
(4.3-10-2) $W_2=\gamma_3' +\sqrt{(\gamma_3')^2-\gamma_5'}$ and
(4.3-10-3) $W_3=\gamma_3' -\sqrt{(\gamma_3')^2-\gamma_5'}$

The order of the algebraic magnitude of the solutions is given by

(4.3-11) $W_3<W_1<W_2.$

Hence two cases need to be considered, depending upon

$$\gamma_5' \gtrless 0.$$

(4.3.2.1) The case of $\gamma_5' > 0$

This case is further divided into the following two cases according to

$$\gamma_3' \neq 0 \quad \text{or} \quad \gamma_3' = 0$$

(4.3.2.1-1) The case of $\gamma_3' \quad 0$.

This case is also divided into two cases according to

$$(\gamma_3')^2 - \gamma_5' \lesseqgtr 0.$$

(1) when $(\gamma_3')^2 - \gamma_5' < 0$, equation (4.3-9) has one real root. (see curve (a) in Fig. Ⅳ-2)

In this case

$F > 0$

if $W > \gamma_3'$.

Hence we have the following proposition.

In the case of

$\gamma_5' < 0$, $(\gamma_3')^2 - \gamma_5' < 0$ and $\gamma_3' > 0$,

$F > 0$

if $W > \gamma_3'$.

In the case of

$\gamma_5' > 0$, $(\gamma_3')^2 - \gamma_5' < 0$ and $\gamma_3' < 0$,

$F > 0$

if $W > 0$.

(2) when $(\gamma_3')^2 - \gamma_5' > 0$, equation (4.3-9) has two real roots.

In this case, the following two cases are further investigated.

(2)- a the case of $\gamma_3' < 0$.

$F > 0$ holds if $W > \gamma_3' + (\gamma_3')^2 - \gamma_5'$

as $\gamma_3' + (\gamma_3')^2 - \gamma_5' < 0$, $F > 0$ when $W > 0$.

Hence, we have the following proposition.

In the case of

$\gamma_5' > 0$, $(\gamma_3')^2 - \gamma_5' > 0$ and $\gamma_3' < 0$,

$F > 0$

for all positive wage rates, $W > 0$,

(2)- b the case of $\gamma_3' > 0$.

In this case, F is positive for the two ranges for wage rates w. (see Fig. Ⅳ-4)

The ranges of w are expressed as

$$\gamma_3' > W > \gamma_3' \quad (\gamma_3')^2 - \gamma_5' \qquad \text{and}$$

$$W > \gamma_3' + (\gamma_3')^2 - \gamma_5'$$

Hence, we have the following proposition.

In the case of

$$\gamma_5' > 0, \quad (\gamma_3')^2 - \gamma_5' > 0 \quad \text{and} \quad \gamma_3' < 0,$$

$$F > 0$$

if

$$\gamma_3' > W > \gamma_3' - (\gamma_3')^2 - \gamma_5'$$

or, if $\quad W > \gamma_3' + (\gamma_3')^2 - \gamma_5'$

[b] The case of $\gamma_3' = 0$.

Inserting $\gamma_3' = 0$ to (4.3-9) we have

$$F = W(W^2 + \gamma_5') > 0$$

Taking into account $W > 0$ it can be seen that $F > 0$ if $W^2 + \gamma_5' > 0$ since we have assured $\gamma_5' > 0$.

Hence, in the case that

$$\gamma_5' > 0 \quad \text{and} \quad \gamma_3' = 0, \quad F > 0$$

if $W > 0$.

Fig. Ⅳ－1

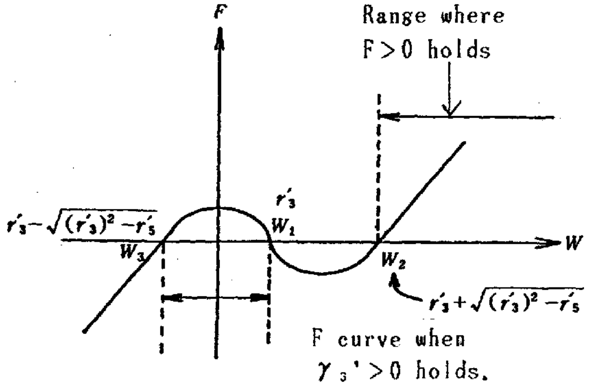


Fig. Ⅳ－4

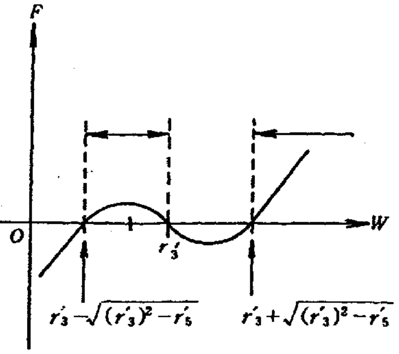


Fig. Ⅳ－2

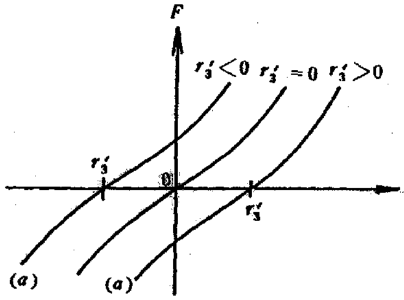


Fig. Ⅳ－5

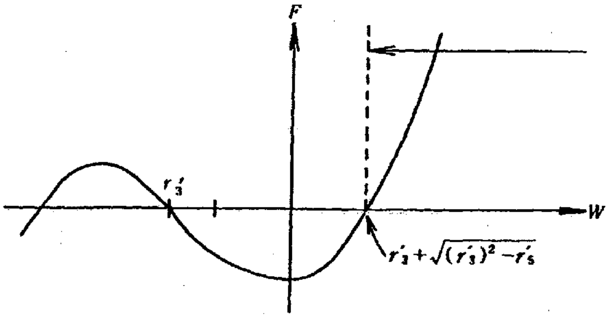


Fig. Ⅳ－3

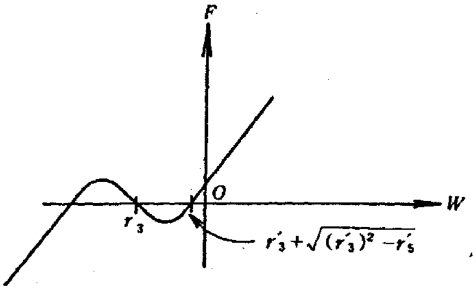
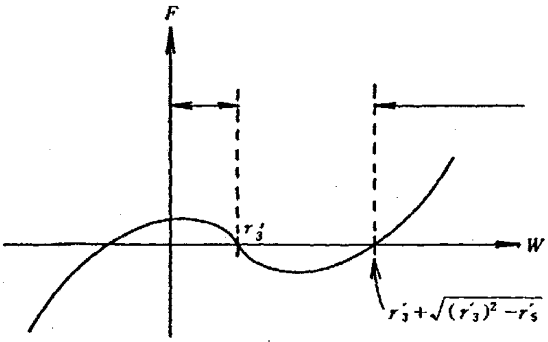


Fig. Ⅳ－6



(4.3.2.2) The case of $\gamma_6' < 0$

In this case the inequality

$$(\gamma_3')^2 - \gamma_5' > 0$$

identically holds. This means that the equation (4.3-9) has two real roots with respect to w.

(4.3.2.2.1) The case of $\gamma_3' \gtrless 0$

(a) when $\gamma_3' < 0$, we have

$$F > 0$$

if $W > \gamma_3' + (\gamma_3')^2 - \gamma_5'$

where $(\gamma_3')^2 - \gamma_5' > |\gamma_3'|$.

Hence, we have the following proposition.

In the case that

$\gamma_5' < 0$ and $\gamma_3' < 0$

$F > 0$

if $W > \gamma_3' + (\gamma_3')^2 - \gamma_5'$.

(b) When $\gamma_3' > 0$, we have

$F > 0$

if $\gamma_3' > W$

or $W > \gamma_3' + (\gamma_3')^2 - \gamma_5'$

Hence, we have the following proposition.

In the case that

$\gamma_5' < 0$ and $\gamma_3' > 0$

$F > 0$

if $\gamma_3' > W$

r $W > \gamma_3' + (\gamma_3')^2 - \gamma_5'$

] [4.3.2.2.2] the case that $\gamma_3' = 0$

Inserting $\gamma_3' = 0$ into (4.3-9) we have

$$F = W(W^2 + \gamma_5') > 0.$$

ce, taking into account $W > 0$, we have

$$W_2 > \gamma_5'$$

where $\gamma_5' < 0.$

That is, $F > 0$

if $W > -\gamma_5'$

From the above we have the following proposition.

In the case

$$\gamma_5' < 0 \quad \text{and} \quad \gamma_3' = 0,$$

$$F > 0$$

if $W > -\gamma_5'$

[4.4] Introduction of random variables into the Preference Function

<4.4.1> Size distribution of the intercept of the Marginal Utility curve

In the above sections, [4.2] and [4.3], the shape of indifference curves for the i-th household was identified by the preference parameters γ_s^i (s=1, 2, ……, 5). In order to take into account the differences in the magnitudes of the parameters between various households we shall regard γ_s^i's as random variables with respect to i. Hence it is necessary to introduce the size distributions of γ_s^i's.

We have assumed that each household, i, has a set of preference parameters, γs (s=1, ……, 5) whose numerical values are specific to it. It is also possible to assume that the numerical value of preference parameters not only vary among households but change at random over time. However, it is always desirable to specify the simplest model, as with the case of only the first assumption, which is necessary and sufficient to generate the empirical laws on the cross sectional and the time series movement of participation ratio observed in (2・1.2.3.1) and (2・1.2.3.2). Hence we regard γ_s^i's as random variables with respect to i only. Further, we shall introduce two alternative extreme cases:

The first is the case where only γ_2^i differs among households, while other parameters, γ_m^i's (m=1, 3, 4, 5) are held constant for all the households under consideration.

In the first case, marginal utilities of income and leisure are given by

$$4.4\text{-}1)\quad \frac{\partial W}{\partial X}=\gamma_2{}^i+\gamma_3\Lambda+\gamma_1 X$$

and

$$4.4\text{-}2)\quad \frac{\partial W}{\partial \Lambda}=\gamma_4+\gamma_3 X+\gamma_5\Lambda$$

respectively, where i stands for the i-th household. As to the parameters common to all the households considered, suffix i is deleted.We shall call this type of model "varying γ_2 type".

In the second case, marginal utilities of income and leisure are given by

$$4.4\text{-}3)\quad \frac{\partial W}{\partial X}=\gamma_2+\gamma_3\Lambda+\gamma_1 X$$

and

$$4.4\text{-}4)\quad \frac{\partial W}{\partial \Lambda}=\gamma_4{}^i+\gamma_3 X+\gamma_5\Lambda$$

respectively.

We shall call this type of model "varying γ_4 type".

<4.4.2> Intercept of marginal Utility Curve whose distribution is given by a log-normal distribution

As to the functional form of the size distribution of the intercept of the marginal utility curve of income, γ_2^i, or leisure, γ_4^i, the log-normal (density) distribution function is adopted. By making use of a log-normal distribution, we can exclude negative values of the intercept of the marginal utility curve, one reason why we adopt the log-normal distribution with respect to γ_2^i or γ_4^i. The other reason is as follows: At the present stage of analysis,we set out that preference parametersof household are constant over time, i.e., 1961 through 1964. However, if habit formation is operative with regard to the marginal utility of income or leisure, the present magnitude of parameter $\gamma_2{}^i$or $\gamma_4{}^i$for each household has been affected by the habit formation process in the past while the magnitude of them constant for four years, 1961 through 1964. Let us suppose that the intercept of marginal utility curve of income or leisure, γ_2^i or γ_4^i, for the i-th household has grown year after year at small growth rates which vary for different years. If this is true, after many years, the distribution of the magnitude of the intercept of the marginal utility curve of income, γ_2^i or leisure, γ_4^i, would be approximated by the log-normal distribution regardless of the initial functional form of the distribution of γ_2^i or γ_4^i. Consequently, we adopt a log-normal distribution with respect to γ_2 or γ_4^i.

Let γ_2 and γ_4 be common parameters to all the households. Parameters γ_2^i and γ_4^i can be written as

4.4-5) $\gamma_2{}^i = \gamma_2 U_2{}^i$

and 4.4-6) $\gamma_4{}^i = \gamma_4 U_4{}^i$

respectively, where $U_2{}^i$ and $U_4{}^i$ are random variables with respect to i and.

4.4-7) $E(U_2{}^i) = 1$

4.4-8) $E(U_4{}^i) = 1.$

From (4.4-5) to (4.4-8) we have

4.4-9) $E(\gamma_2{}^i) = \gamma_2$

4.4-10) $E(\gamma_4{}^i) = \gamma_4$

<4.4.3> Addendum to the relative position of the marginal Utility Curve of Income and Leisure

Making use of (4.4-1) and (4.4-2), the marginal rate of substitution between income, X, and leisure, Λ, for the i-th household is given by

4.4-11) $$\frac{\partial \Lambda}{\partial X} = \frac{\gamma_4{}^i + \gamma_3 X + \gamma_5 \Lambda}{\gamma_2{}^i + \gamma_3 \Lambda + \gamma_1 X}$$

Dividing both the numerator and denominator of right hand side of the equation by γ_1 we have

4.4-12) $$\frac{\partial \Lambda}{\partial X} = \frac{(\gamma_4{}^i)' + \gamma_3' X + \gamma_5' \Lambda}{(\gamma_2{}^i)' + \gamma_3' \Lambda + X}$$

where

4.4-12′) $\gamma_t{}^i/\gamma_1$ $(\gamma_t{}^i)'$ (t=2, 4) and $\gamma s/\gamma_1$ $\gamma s'$ (s=3, 5).
With normalization, $\gamma_1 = -1$, the marginal rate of substitution can be determined. That is, given the relative magnitude of the preference parameters, the marginal rate of substitution is uniquely defined.

It can be seen from (4.4-12) that the shape of indifference curves for the i-th household is affected both by the magnitude of the normalized intercept of the

marginal utility curve of income, γ_2^i , and by that of the marginal utility curve of leisure, γ_4^i . This means that the shape of the indifference curves may change even if the changes in $(\gamma_4^i)'$ and $(\gamma_2^i)'$ are proprotional. Consequently, a model which introduces distribution functions of $(\gamma_4^i)'$ and $(\gamma_2^i)'$

simultaneously can be definitely differentiated from those which introduce either $(\gamma_4^i)'$ or $(\gamma_2^i)'$. Hence, more general models such as

$$\frac{\partial W}{\partial X} = \gamma_2{}^i + \gamma_3\Lambda + \gamma_1 X = \gamma_2 U_2{}^i + \gamma_3\Lambda + \gamma_1 X$$

$$\frac{\partial W}{\partial \Lambda} = \gamma_4{}^i + \gamma_3 X + \gamma_5\Lambda = \gamma_4 U_4{}^i + \gamma_3 X + \gamma_5\Lambda$$

may be introduced.

However, in order to begin with a simpler case, let us set out two alternative models, each allowing for varying values either for γ_2 or for γ_4, shown by (4.4-1) and (4.4-2) or by (4.4-3) and (4.4-4) respectively.

[4.5] PECI Equations of varying-γ_2 models and varying-γ_4 models

<4.5.1> PECI Equation of varying γ_2 model

The equations of marginal utility of income and leisure with varying $\gamma 2$ are given by (4.4-1) and (4.4-2) in <4.4-1> respectively. Applying (4.4-7)to (4.4-1) we have the marginal utility equation of income for the i-th household,

$$4.5\text{-}1)\quad \frac{\partial W}{\partial X} = \gamma_2 U_2{}^i + \gamma_3 \Lambda + \gamma_1 X$$

The marginal utility of leisure is given by the equation,

$$4.5\text{-}2)\quad \frac{\partial W}{\partial \Lambda} = \gamma_4 + \gamma_3 X + \gamma_3 \Lambda,$$

which is common to all the households considered.
Replacing γ_2^i in (4.2-4) by $\gamma_2 U_2^i$ in (4.5-1) we obtain a PECI equation for a varying γ_2 model,

$$4.5\text{-}3)\quad I_i^* = \frac{\gamma_4 - \gamma_2 W U_2^i - \gamma_3 W(T-h) + \gamma_5 (T - \frac{h}{2}) - \frac{1}{2}\gamma_1 W^2 h}{\gamma_1 W - \gamma_3},$$

where the i superscript for parameters common to all households are deleted.

It can be seen from (4.5-3) that the relation between the principal earner's earner's critical income, I^*, and the random variable, U_2^i, which stands for the difference of the intercept of the marginal utility curve of income among the households, is a linear relation for the given wage rate, w, and assigned working hours, h, i.e.,

$$4.5\text{-}4)\quad I_i^* = S_0 + S_1' U_2^i \quad (U_2^i \geq 0)$$

where

$$4.5\text{-}5)\quad S_0 = \frac{\gamma_4^* - \gamma_3 (T-h) W + \gamma_5 (T - \frac{h}{2}) - \frac{1}{2}\gamma_1 W^2 h}{\gamma_1 W - \gamma_3}$$

and

$$4.5\text{-}6)\quad S_1' = \frac{-\gamma_2 W}{\gamma_1 W - \gamma_3}$$

<4.5.2> PECI equation of varying-γ_4 model

The equations of marginal utility of income and leisure in varying-γ_2 model are given by (4.4-3) and (4.4-4), respectively. The marginal utility of income is given by the equation

$$4.5\text{-}7)\quad \frac{\partial W}{\partial X} = \gamma_2 + \gamma_3\Lambda + \gamma_1 X,$$

which is common to all households considered.
Inserting (4.4-6) into (4.4-4) we obtain the marginal utility of leisure for the i-th household,

$$4.5\text{-}8)\quad \frac{\partial W}{\partial \Lambda} = \gamma_4 U_4{}^i + \gamma_3 X + \gamma_5 \Lambda.$$

Replacing γ_4^i in (4.2-4) by $\gamma_4 U_4^i$ in (4.5-8) we have the PECI equation for the model with varying-γ_4 ,

$$4.5\text{-}9)\quad I_i^* = \frac{\gamma_4 U_4{}^i - \gamma_2 W - \gamma_3 W(T-h) + \gamma_5(T-\frac{h}{2}) - \frac{1}{2}\gamma_1 W^2 h}{\gamma_1 W - \gamma_3}$$

where the i superscript of parameters which are common to all households are deleted.

It can be seen from (4.5-9) that the PECI equation for the varying-γ_4 model also is a linear relation in I_i^* and U_4^i , i.e.,

$$4.5\text{-}10)\quad I_i^* = H_0 + H_1' U_4^i \qquad (U_4^i \geq 0)$$

where

$$4.5\text{-}11)\quad H_0 = \frac{-\gamma_2 W - \gamma_3(T-h)W + \gamma_5(T-\frac{h}{2}) - \frac{1}{2}\gamma_1 W^2 h}{\gamma_1 W - \gamma_3}$$

and

$$4.5\text{-}12)\quad H_1' = \frac{\gamma_4}{\gamma_1 W - \gamma_3}$$

[4.6] Testing Empirical Plausibility of Varying-γ_4 Model versus Varying-γ_2 Model

<4.6.1> PECI equations in terms of standardized U_2 and U_4

As mentioned <4.4.2>, the differences in the intercept of the marginal utility curve of income or leisure among households considered are given by log-normal distribution. Hence, with respect to the varying-γ_2 model, we have,

(4.6-1) $\log U_2^i \sim N(m_2, \sigma_2^2)$,

where m_2 and σ_2^2 are mean and variance respectively. In the case of the varying-γ_4 model, we have,

(4.6-2) $\log U_4^i \sim N(m_4, \sigma_4^2)$,

Here, we shall introduce a random variable U_i^* which is given by standardizing $\log U_2^i$ and $\log U_4^i$. That is U_i^* is defined by

(4.6-3) $\dfrac{\log U_k^i - m_k}{\sigma_k} = U_i^*$. $(k=2, 4)$

The distribution of U_i^* is given by a normal distribution whose mean equals zero and variance equals unity regardless of k in (4.6-3),

(4.6-4) $U_i^* \sim N(0, 1)$.

From (4.6-3) we have

(4.6-5) $\log U_k^i = m_k + \sigma_k U_i^* \ (K=2, 4)$

or (4.6-6) $U_k^i = e^{mk} \cdot e^{\sigma_k U_i^*}$

Replacing k in (4.6-6) by 2 and inserting (4.6-6) into (4.5-4) we obtain the PECI equation for the varying-γ_2 model, (4.6-7) $I_i^* = S_0 + S_1' e^{m_2} e^{\sigma_2 U_i^*}$

By an analogous procedure, the PECI equation for the varying-γ_4 model is

(4.6-8) $I_i^* = H_0 + H_1' e^{m_4} e^{\sigma_4 U_i^*}$

Equations (4.6-7) and (4.6-8) are rewritten as follows.

(4.6-7′) $I_i^* = S_0 + S_1 e^{\sigma_2 U_i^*}$

(4.6-8′) $I_i^* = H_0 + H_1 e^{\sigma_4 U_i^*}$

where

(4.6-9) $S_1 = S_1' e^{m_2}$

(4.6.10) $H_1 = H_1' e^{m_4}$

Hereafter we shall call (4.6-7′) and (4.6-8′) standardized PECI equations.

Standardized PECI equations, (4.6-7′) and (4.6-8′), can be regarded as transformation functions which transform the distribution of U_* to that of I^*. Taking into account $\gamma_2 > 0$, and $\gamma_4 > 0$, it can be seen from (4.5-6) and (4.5-12) that S_ℓ in (4.6-7') and H_ℓ in (4.6-8') must have <u>opposite</u> <u>signs</u>.

<4.6.2> Preliminary discussions on the correspondence of the standardized PECI equation and the observed data.

The correspondence of the standardized PECI equation obtained in <4.6.1> with the observed data is examined in this section. In the following sections, it will be argued which of the two alternative models, varying-γ_2 model or varying-γ_4 model, is consistent with observation. If we can observe corresponding values for I_i^* and U_i^* with respect to i in(4.6-7′) or (4.6-8′) we will be able to estimate the parameters in these equations, S_0, S_1 and σ_2 or H_0, H_1 and σ_4, by some suitable estimation procedure. The following is the preliminary argument concerning this point.

If a model consistent with the observed facts is either a varying γ_2 type model or a varying γ_4 type model, the relationship between the observed I_i^* and the observed U_i^* must, at any rate, fulfill the equation.

$$(4.6\text{-}11)\quad I_i^* = A_0 + A_1 e \sigma u_i^*$$

where the algebric sign of A_1 is either positive or negative because S_ℓ in (4.6-7') and H_1 in (4.6-8') have opposite signs; thus we have two alternative cases,

$$(4.6\text{-}12)\quad A_1 > 0$$

and (4.6-13) $A_1 < 0$.

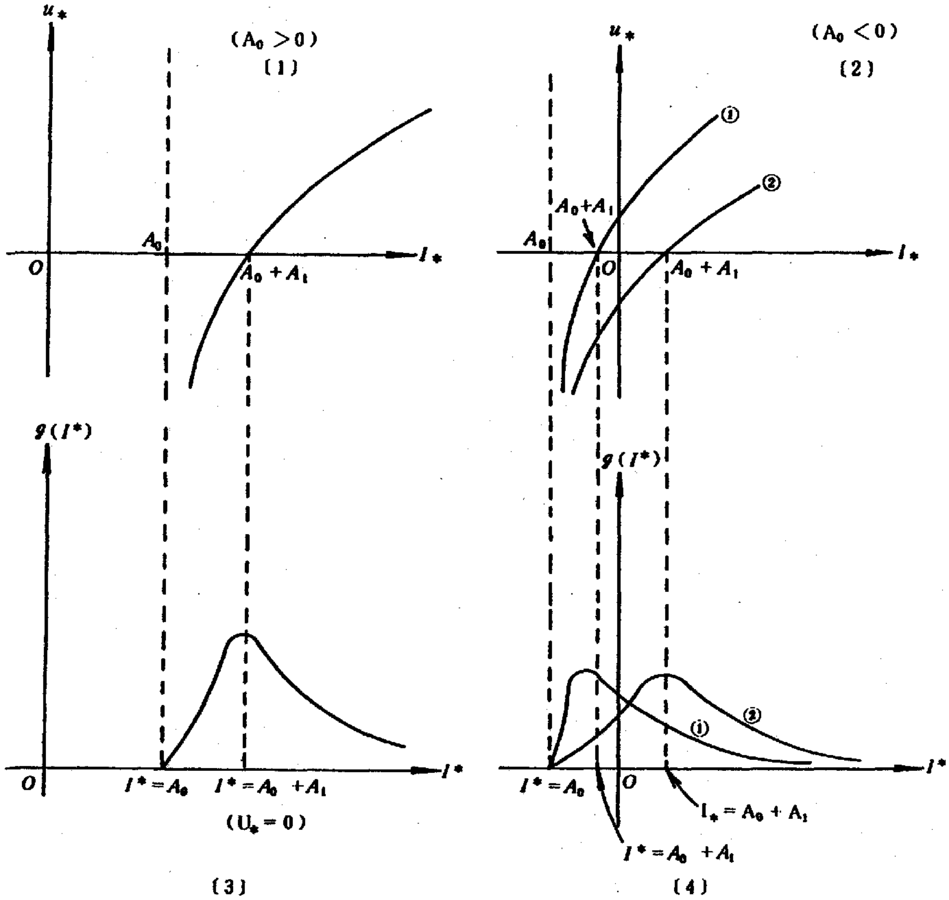
However, we should note that even if we estimated A_1 in (4.6-11) making use of the observed data and consequently determine the sign of A_1; we can not decide which model is plausible because we only know S_1 and H_1 are opposite in their signs.

(4.6.2.1) The distribution of I_* in the case where $A_1 > 0$

For the case where $A_1 > 0$, the value of PECI, I_i , for the i-th household whose U_i^* equals $-\infty$ is A_0 ; i.e., when

Fig.(Ⅳ−7)−1

$A_1 > 0$



$$U_i^* = -\infty$$

we have, from (4.6-11)

$$I_i^* = A_0$$

Hence in the group of households considered, the PECI of a household in which the value of PECI is minimum equals A₀. According to the sign of A₀ the equation (4.6-11) is depicted as shown in the upper half of [1] or [2] in Fig.4.7-1. Hence, for this case, there exists a <u>lower limit</u> of the PECI distribution curve as shown in the lower half of Fig.4.7-1, where [3] shows the case for positive A and [4] shows that for negative A₀.

(4.6.2.2) The distribution of I_* for the case where $A_1 < 0$

For the case of negative A_1, the value of PECI, I_i^*, for the i-th household whose U_i^* equal $+\infty$ is A_0 ; i.e., when

$$U_i^* = +\infty$$

We have from (4.6-11)

$$I_i^* = A_0$$

Hence, for this case, there exists the maximum value of PECI for the households considered as shown in [1] and [2] of Fig.4.7-2. Therefore, the PECI distribution curve has an upper limit point on the abscissa as shown in [3] and [4] of Fig.4.7-2.

Fig.(Ⅳ−7)−2

$A_1 < 0$

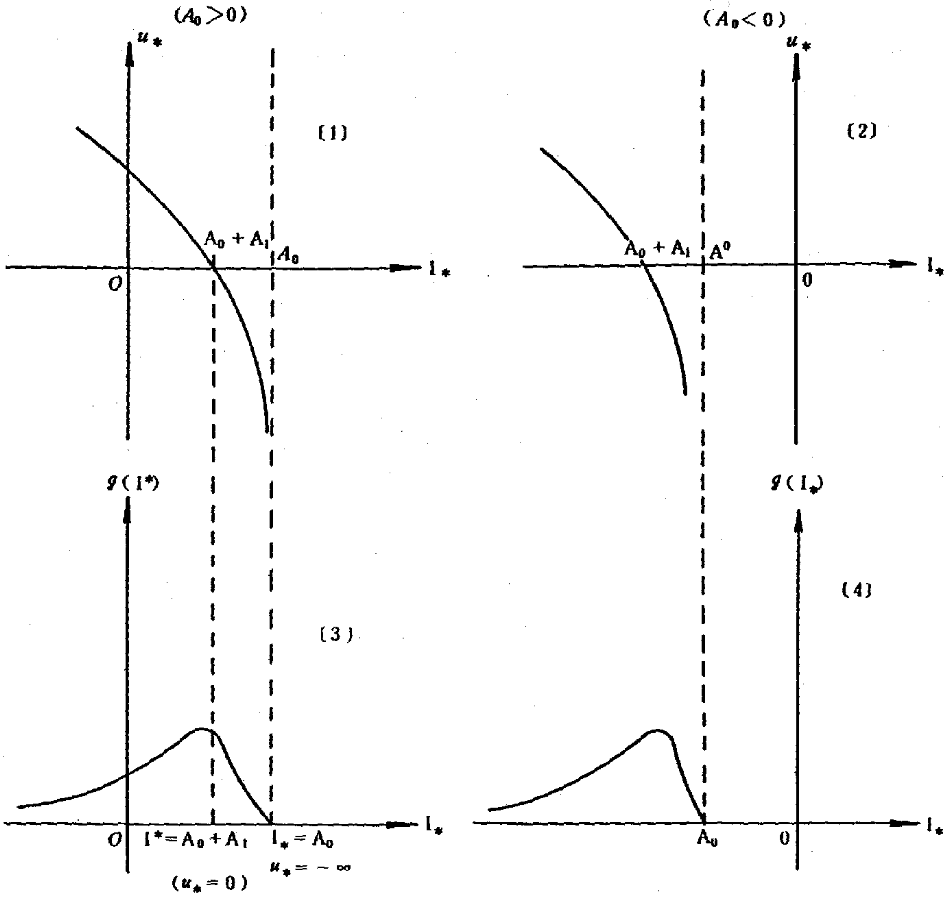
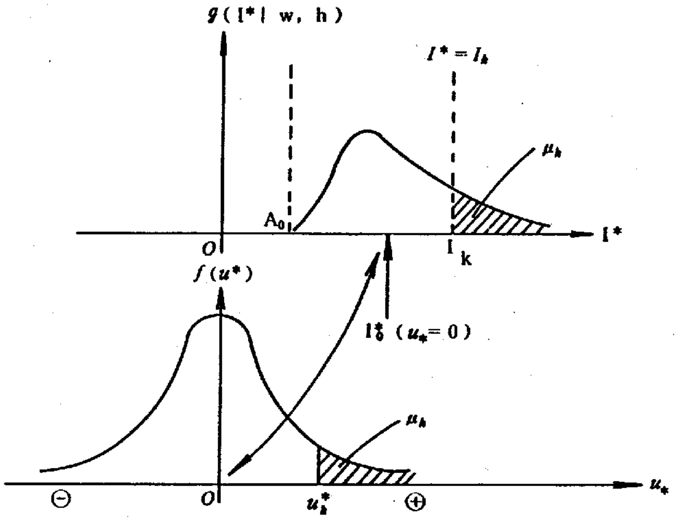


Fig.(Ⅳ−7)−3

$A_1 > 0$ and $A_0 > 0$



Now, negative values of A_0 are excluded from our analysis. Should A_0 be negative, the PECI's for all the households considered would be negative because A_i would be negative. Hence, there would be no households with a positive PECI. This means that

$$I_i > I_i^*$$

for all i's, because the actual principal earner's income, I_i, of any household, i, is positive. If this were the case, as is argued in (3.2.5.4 (a)) and (3.7.1.4), no groups of households with working non-principal earners would be observed. In other words, all the observed participation ratios for groups of households must equal zero. This contradicts observations in Japan and in the U.S. Hence, the case where $A_0 < 0$ and $A_1 < 0$ must be excluded. As will be discussed later, we can take advantage of this requirement as a condition which the estimated values of preference parameters should fulfill: A_0 can be seen as a function of w, h and γs'S (Suffix S stands for 1, 3, 4 and 5 if the varying γ_4-model applies and stands for 1, 2, 3 and 5 if the varying γ_2-model applies). Thus, it can be said that a set of parameters, γs, giving negative A_0 and A_1 is not consistent with the observation, w and h being given.

<4.6.3> Estimation of PECI equation

(4.6.3.1) The case where $A_1 > 0$

(4.6.3.1-1) Correspondence between the distribution of U^* and that of I^*

In this section we shall discuss the correspondence between the distribution of U^*, f (U_*), and that of I_*, g $(I^*/w, h)$.
Consider a group of type-A households where principal earners' income, I_k's, are the same. Let the wage rate offered and hours of work assigned by firms be common to all the non-principal earners of the households. The correspondence between f (U_*) and g $(I^*/w, h)$ differs inaccordance with the sign of A_1 in (4.6-11). Where $A_1 > 0$ and $A_0 > 0$, the correspondence betweem the two distributions is shown in Fig.4.7-3. One should be aware that the distribution of U_*, f (U_*), is known; i.e. it is given by

$$U_* \sim N\ (0,\ 1).$$

Let the observed value of the participation ratio of the group of households be μ. We can determine a point, I_k, on the axis on which I_* is measured, since we have already known that the curve has such a shape that the area of the right hand side to the vertical line at I equals μ. Also, it is quite easy to obtain a value of U_k^* such that

$$\text{Probabiltity}\ \ (U_* > U_k^*) = \mu_k,$$

because it is already known that f (U_*) is given by N (0, 1).
Hence we can determine, for any k, corresponding values of I^* and U^*, I_k^* and U^*, from the observed data.

The above argument can be restated in an analytical for as follows. The participation ratio of the k-th group of households whose principal earners' income, I_k, are common to all the households is given by

$$(4.6\text{-}14)\quad \mu_k = \int_{I^* = I_k}^{I^* = \infty} g(I^* / w,\ h) d I^*$$

We would like to know the numerical value of u_k^* of the household whose PECI equals I_k, lower limit of the integration in the above equation. As we are considering the case where A_1 >0, transformation of I^* to u^* is monotonic. Therefore the value of definite integration of the distribution function of u^* with respect to lower limit, u_k^*, through the upper limit, ∞,

$$(4.6\text{-}15)\quad \frac{1}{\sqrt{2\pi}} \int_{u^*=u_k^*}^{\infty} e^{-\frac{(u^*)^2}{2}} du^*$$

must be equal to μ_k. Hence we have

$$(4.6\text{-}16)\quad u_k = \frac{1}{\sqrt{2\pi}} \int_{u=u_k^*}^{\infty} e^{-\frac{(u^*)^2}{2}} du^*$$

Given the value of μ_k, U_k^* can be determined by this equation.
The lower limit of integration in (4.6-14), I_k, and that of (4.6-16), U_k^* must fulfill the equation,

$$(4.6\text{-}17)\quad I_k = A_0 + A_1 e^{\sigma u^{*k}} \quad , \; A_1 > 0$$

for the k-th group of households. Hence, in order to determine the values of parameters of PECI equation (4.6-17), A_0, A_1 and σ, we estimate the parameters of the regression equation, (4.6-17), making use of observed value of I_k^* and U_k the latter being given by equation(4.6-16).

Equation (4.6-17) may be shown as Fig.4.7-4. The shape of the curves in Fig 4.7-4 is quite the same as those in Fig 4.7-1 except that on the abscissa and vertical axis the observed value of U_k^* and I_k for the k-th group of households are scaled respectively. For example, I_1 stands for the value of principal earners' incomes in the first group of households and U_1^* stands for the corresponding value of the percentile position of U^* which generates the value of the observed participation ratio of the first group, μ. It should be recognized that the curves in Fig. (4.7-4) are ascending and convex in an upward direction when A_1>0.

(4.6.3.1-2) The relation between observed I_k and U_k^*

Here, we shall examine whether the requirement noted in the last part of (4.6.3.1-1) is fulfilled by observations. For the years 1961 through 1964 we can obtain the observed values of I_k and U_k^* by the method described in (4.6.3.1.-1). The relations between I_k and U_k for the four years are shown in Fig.4.7-4. It is clearly seen that the scatter is ascending but is concave in an upward direction. Hence, it must be concluded that the hypothises

$$A_1 > 0$$

does contradict the observation shown in Fig.4.7-4'.

(4.6.3.2) The case where $A_1 < 0$

In the case where $A_1 < 0$; the correspondence between the distribution of I^* and that of U^* is depicted as is shown in Fig.4.7-5. It should be stressed that the positive value of U^* is scaled from the origin to the left and negative U^* is scaled to the right on the abscissa in the figure of the distribution curve of U^*. In this case, U^* which is equal to $-\infty$ on the U^* axis corresponds to the value of I^* which is equal to A_0 on the I^* axis as shown in (4.6.2.1) and (4.6.2.2). A_0, in this case, is the maximum value of I^* on the PECI distribution.

Fig.(Ⅳ−7)−4

$A_1 > 0$

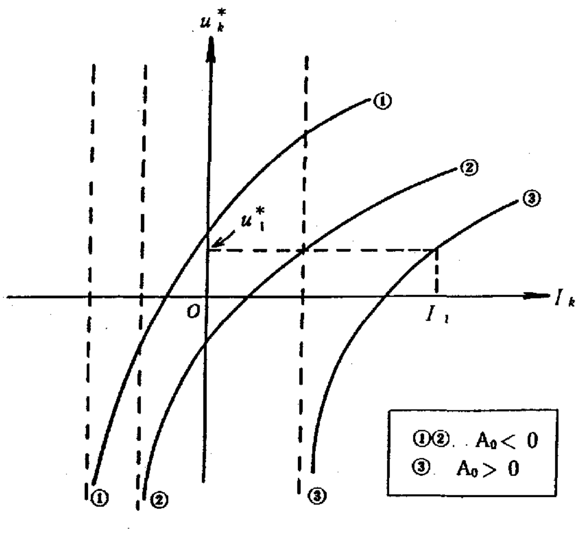
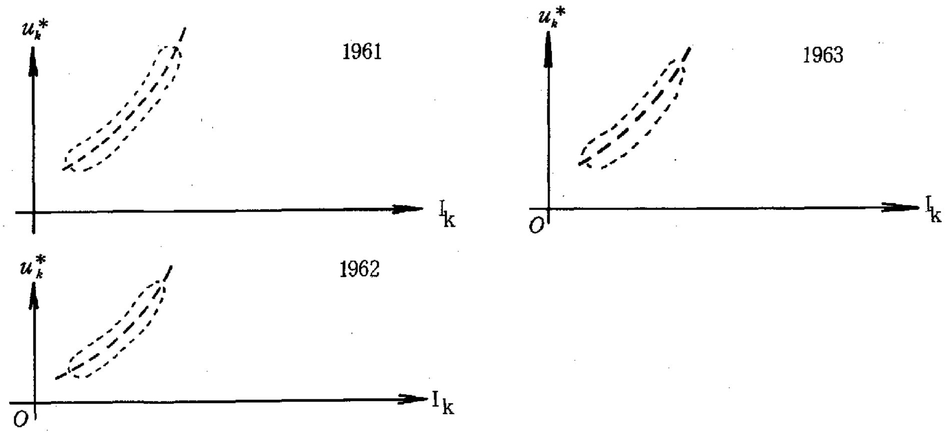


Fig.(Ⅳ−7)−4'



The participation ratio of the k-th group of households, μ_k, is given by

$$(4.6\text{-}18)\quad \mu_k = \int_{I^* = I_k}^{A_0} g(I^*/w,\ h)\,dI^*$$

Here, we would like to know the value of U_k^* which stands for the value of U^* of households whose PECI just equals the value of the lower limit of integration, I_k. The relationship between U^* and I^* is a monotonic decreasing transformation owing to the condition that A_1 is negative. Hence the value of the definite integral

$$(4.6\text{-}19)\quad \frac{1}{2\pi}\int_{u* = -\infty}^{uk^*} e^{-\frac{u*}{2}}\,du^*$$

is the value of μ_k. It should be noted that the range of integration is from $U_* = -\infty$ through $U_* = U_k^*$.

Fig.(Ⅳ-7)-5

$A_1 < 0, A_0 > 0$

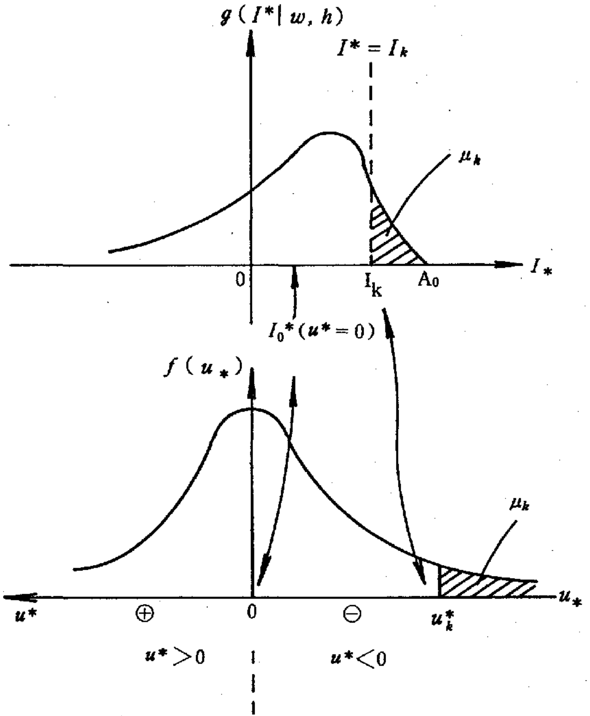
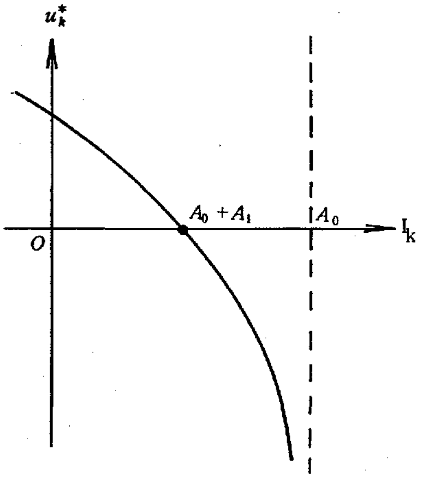


Fig.(Ⅳ-7)-6



Let us choose the value of U_k^* so that the relation,

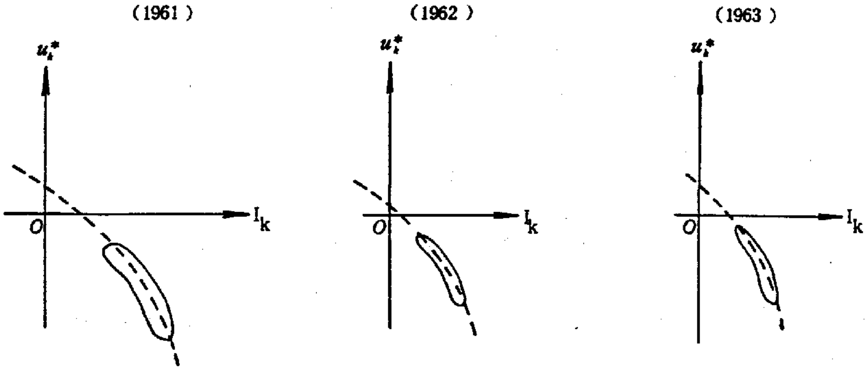
$$(4.6\text{-}20)\quad \mu_k = \frac{1}{\sqrt{2\pi}} \int_{U^*=-\infty}^{U_k^*} e^{-\frac{(U^*)^2}{2}} dU^* ,$$

holds. It is easy to choose the value of U_k^* for the k-th group of households because the distribution of U^* is known. Taking into account the above argument, the relationship between the observed U_k^* and I_k, which is expected to hold according to the theory, is depicted as is shown in Fig.4.7-6. That is, the curve in Fig.4.7-6 is theoretically expected to be downward sloping and concave to the downward direction.

In order to examine whether the above mentioned relation is actually supported by the observed data we shall obtain the values of U_k^* corresponding to I from the observed data under the hypothesis that A_1 is <u>negative</u>. Here, the following point must be taken into account to determine the algebraic sign of the observed value of U_k^*. In Fig.4.7-5, the area to the right of the vertical line passing through the point U^*_k is equal to μ_k which is observed. The magnitude of the area for the case where $A_1 > 0$, was shown in Fig.4.7-3. In the present case where $A_1 < 0$, the area hatched in Fig.4.7-5 must equal the area hatched in Fig.4.7-3. However, it should be realized that, in the present case, the values of U^*'s measured on the abscissa, in Fig.4.-5, from the origin to the left are positive,contrary to the case where $A_1 > 0$ shown in Fig.4.7-3. Hence, the absolute values of U^*_k obtained under the hypothesis that $A_1 < 0$ must be the same as those was obtained under the contrary hypothesis that $A_1 > 0$, while the algebraic sign of U^*_k must be opposite to what was obtained in the case where $A_1 > 0$.

Consequently, we obtain the scatter diagram with respect to observed I_k and U^*_k , shown in Fig.4.7-7. It is clearly observed that the relations obtained in the years 1961 through 1963 are consistent with the theoretical requirement; i.e.,the obtained curves are downward sloping and concave to the downward direction as is expected in Fig.4.7-6.

Fig.4.7-7



(4.6.3.3) From the above analysis we can conclude that in order to estimate the parameters of the PECI equation making use of the given observations,I_k's and U^*_k 's must be of the values suggested by the hypothesis that $A_1 < 0$.

<4.6.4> Comparison of Plausibilities of varying $\gamma_2$model and varying $\gamma_4$model

(4.6.4.1) In the last section, we concluded that we have to assign an algebraic sign to the observed value of U^*_k under the hypothesis that A_1 is negative. In other words, in order to regard the equation.

$$I_k = A_0 + A_1 e^{\sigma U_k^*}$$

as the PECI equation consistent with observations, we have to observe I_k and U_k^* making use of the hypothesis that $A_1 < 0$.

Next, we shall discuss what the above requirement means to the parameters of preference function and what the relation is between the above mentioned requirement and the requirement of a downward sloping MHSL curve mentioned in 3.2.6.1-3. This point is indispensable in obtaining the empirical counterpart to the PECI equation.

(4.6.4.2) The case where the condition that $A_1 < 0$ is generated by a Varying-γ_2 model making use of the requirement that MHSL curve be downward sloping.

The parameter, A_1, of the equation

$$I_k = A_0 + A_1 e^{\sigma U_k^*}$$

permits two alternative interpretations. If we employ a varying γ_2 model, A_1 corresponds to S_1' in (4.6-7′) which is a function of γ_1, γ_2, γ_3, m_2 and w, as shown in (4.6-9). That is,

$$(4.6\text{-}21)\quad A_1 = \frac{-\gamma_2 w}{\gamma_1 w - \gamma_3} e^{m_2} .$$

Dividing the numerator and denominator of the right hand side of (4.6-21) by γ_1 we have

$$(4.6\text{-}21')\quad A_1 = \frac{-\gamma_2' w}{w - \gamma_3'} e^{m_2} ,$$

where $\gamma s' \equiv \gamma s / \gamma_1 (s=2, 3).$

By employing the normalization rule

$$\gamma_1 \equiv -1,$$

we have $\gamma_2' < 0.$

Thus, <u>in order that a negative A_1 be generated from the varying γ_2 model</u>, the following inequality must hold.

(4.6-22) $W-\gamma'_3<0$ or $W<\gamma'_3$

This is one of the conditions which the preference parameters must satisfy. In addition to this constraint on the parameter γ_3, the set of preference parameters must be such that they generate a downward sloping MHSL curve. Hence, in order that the varying γ_2 model be consistent with the data, the values of γs' which fulfill the condition of a downward sloping MHSL curve must also be such that they do not contradict the requirement given by (4.6-22).

The conditions of a downward sloping MHSL curve (considered in 4.3.2) are summarized in Table Ⅳ-1. That is, for the possible range of the values of γs' , the required range of non-principal earner's wage rate which generates downward sloping MHSL curves are shown. (It should be recognized that no specific working hours, h, are assigned in the conditions for a downward sloping MHSL curve.)

Table Ⅳ-1

case	When the conditions for the downward sloping MHLS curve		The ranges of W which yield downward sloping MHLS curve	
$r'_5>0$	$(r'_3)^2-r'_5<0$	$r'_3>0$	$w>r'_3$	①
〃	〃	$r'_3<0$	$w>0$	②
〃	$(r'_3)^2-r'_5>0$	$r'_3<0$	$w>0$	③
〃	〃	$r'_3>0$	$r'_3>w>r'_3-\sqrt{(r'_3)^2-r'_5}$	④
〃	—	$r'_3=0$	$w>0$	⑤
$r'_5<0$	—	$r'_3<0$	$w>r'_3+\sqrt{(r'_3)^2-r'_5}$	⑥
〃	—	$r'_3>0$	$\left\{\begin{array}{l} r'_3>w \end{array}\right.$ および	⑦
			$w>r'_3+\sqrt{(r'_3)^2-r'_5}$	⑧
〃	—	$r'_3=0$	$w>\sqrt{-r'_5}$	⑨

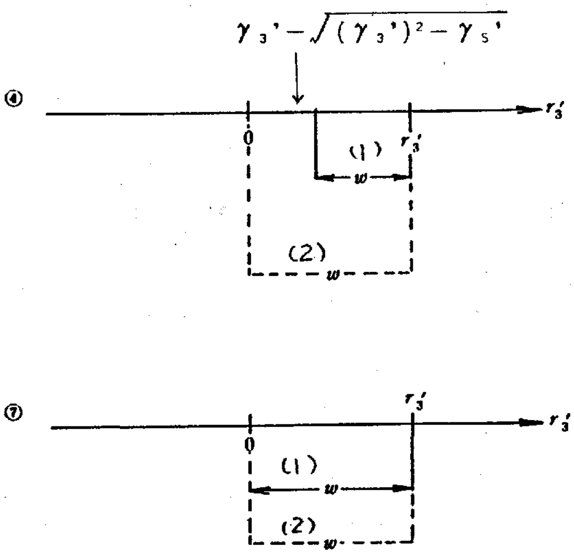
Among nine cases in the table, only two cases, case 4 and 7 require the condition that $W<\gamma_3'$. As a result, only in these cases do the conditions for a downward sloping MHSL curve not contradict the requirement given by (4.6-22). In the remaining seven cases, the condition of a downward sloping MHSL curve is not compatible with the requirement given by (4.6-22). Moreover, turning to case 4, we observe that, in order that the condition of downward sloping MHSL curve be fulfilled, the non principal earner's wage rate, w, must be within a fairly narrow positive range,

$$\gamma_3' > W > \gamma_3' - (\gamma_3')^2 - \gamma_5' , \quad \text{where} \quad \gamma_3' > 0$$

In case 7, it is required that non principal earner's wage rate w be less than γ_3, which must be positive, though this range assigned to w is not as narrow as what is required for the above case 4. (see Fig. Ⅳ-8)

Fig.(Ⅳ-8)

Case ④ and ⑦ are the cases in which $A_1 < 0$ yields from the varying γ_2 model. Other cases are the ones in which $A_1 < 0$ yields from the varying γ_4 model.



Notation(1) stands for the range of W in order that MHSL curve is downward sloping.
Notation(2) indicates the range of where $A_1 < 0$ holds.

(4.6.4.3) The case where the condition that $A_1 < 0$ is generated by varying γ_4 model making use of requirement that MHSL curve be downward sloping.

If we adopt a varying γ_4 model, the parameter A_1 in

$$I_k = A_0 + A_1 e^{\sigma u_k^*}$$

will be identified with H_1 in (4.6-8'). Hence, from (4.6-10) we have

$$(4.6\text{-}23) \quad A_1 = H_1' e^{m_4} = \frac{\gamma_4}{\gamma_1 w - \gamma_3} e^{m_4}$$

Dividing both the denominator and the numerator of the third term of (4.6-23') by γ_1, we obtain

$$(4.6\text{-}23') \quad A_1 = \frac{\gamma_4'}{w - \gamma_3'} e^{m_4} ,$$

where $\gamma s'$ $\gamma s / \gamma_1$ (s=3, 4). By normalization, $\gamma \equiv -1$,
$\gamma_4' < 0$.
Hence, in order that A_1 be negative, the following must hold.

$$(4.6\text{-}24) \quad w - \gamma_3' > 0 \quad \text{or} \quad w > \gamma_3'$$

In addition to this, the condition that the MHSL curve is downward sloping should be fulfilled. If the conditions for a downward sloping MHSL curve for the varying γ_4 model do not exclude the condition given by (4.6-24), the varying γ_4 model is consistent with the observed data. From Tab.4-1 , in seven cases out of nine, the condition of a downward sloping MHSL curve for the varying γ_4 model requires that

$$w > \gamma_3'$$

This means that, in seven cases out of nine, the downward sloping condition does not exclude (4.6-24). In these seven cases, 1, 2, 3, 5, 6, 8 and 9, assuming that a negative A_1 is generated by the varying γ_4 model dose not contradict the condition of a downward sloping MHSL curve condition for the model.

Turning to the case 1, the non principal earner's wage rate,w, must be larger than γ_3', i.e.,

$$w > \gamma_3'$$

where $\gamma_3' > 0$

In cases 2, 3 and 5, w must be positive.

$$w > 0$$

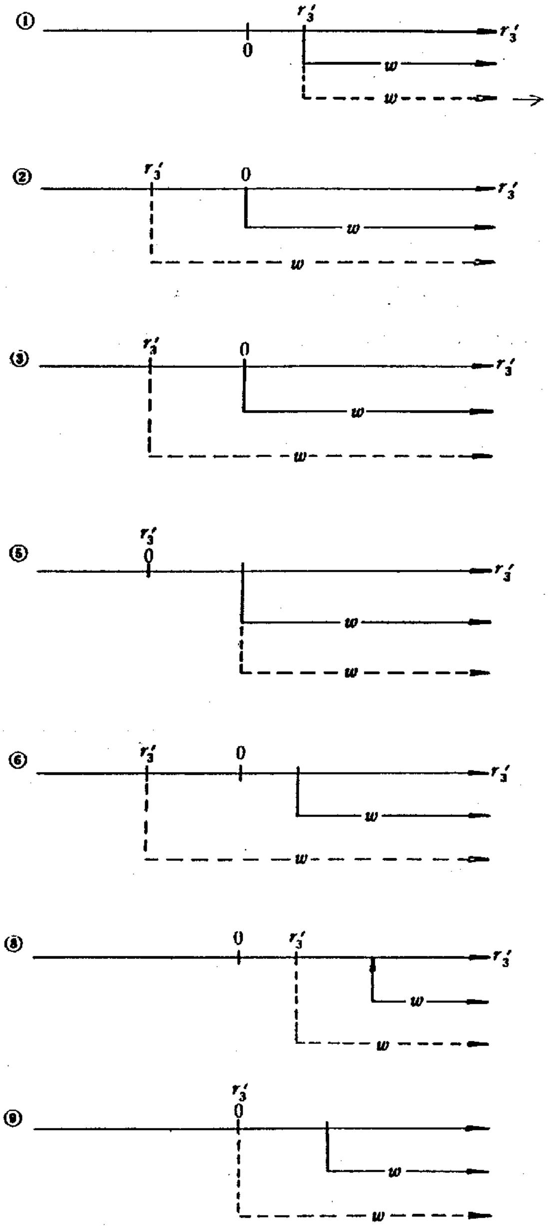
In cases 6 and 8, w must be larger than a positive value,

$$\gamma_3' + (\gamma_3')^2 - \gamma_5$$

Finally, in case 9, w is required to be larger than $-\gamma_5$. In these seven cases, it should be noticed that an upper limit for the value of w is not assingned because the MHSL curve is downward sloping. (See FigⅣ-9)

Fig.(Ⅳ−9)

Figure for testing $W > \gamma_3'$ when the conditions for downward sloping MHSL curve are fulfilled.



The range of W in order that MHSL curve is downward sloping.

→ The range of W in order that $A_1 < 0$ holds.

(4.6.4.4) Adoption of varying γ_4 model

(4.6.4.4-1) From the argument in (4.6.4.2) and (4.6.4.3) we noted that less restrictions are placed on ω adopting the varying γ_4 model rather than the varying γ_2 model.

Now, suppose we adopt the hypothesis that the varying γ_2 model holds true. Further assume, as we have seen in section 1, that the first of the empirical laws of participation has a general validity. Hence, it would be plausible to suppose, for example, that the this law has been applicable for type A households during the period nineteenth century and the mid 1950's in the U.S.. If so, we have to assume that during this long time period non principal potential earner's wage rate, w, had to be set in the ranges,

$$\gamma_3 - (\gamma_3')^2 - \gamma_5' < w < \gamma_3'$$

or $$0 < w < \gamma_3' ,$$

However, assuming such ranges as shown above for the value of w seems quite unnatural. The wage rate, w, has increased over time, implying that γ_3' had such a large value that the growth in w has not been able to exceed it, or that, by the unknown mechanism, γ_3' increased so as to be always larger than the increasing value of the wage rate, w. Of course, we have no empirical basis for assuming the second possibility as well as the first.

(4.6.4.4-2) It seems, in a sense, to be an interesting proposition that the first empirical law will no longer be valid as soon as wage rate, w, exceedes γ_3' . That is, this proposition might seem to give a criterion that discriminates between non principal earners and principal earners. When the wage rate ω of a member of a household exceeds γ_3' the member (wife in the case of a type A household) could no longer be regarded as a non principal potential earner. However, this interpretation seems unnatural. If we introduce such a definition of non principal earner that she (or he) could be indentified by the criterion

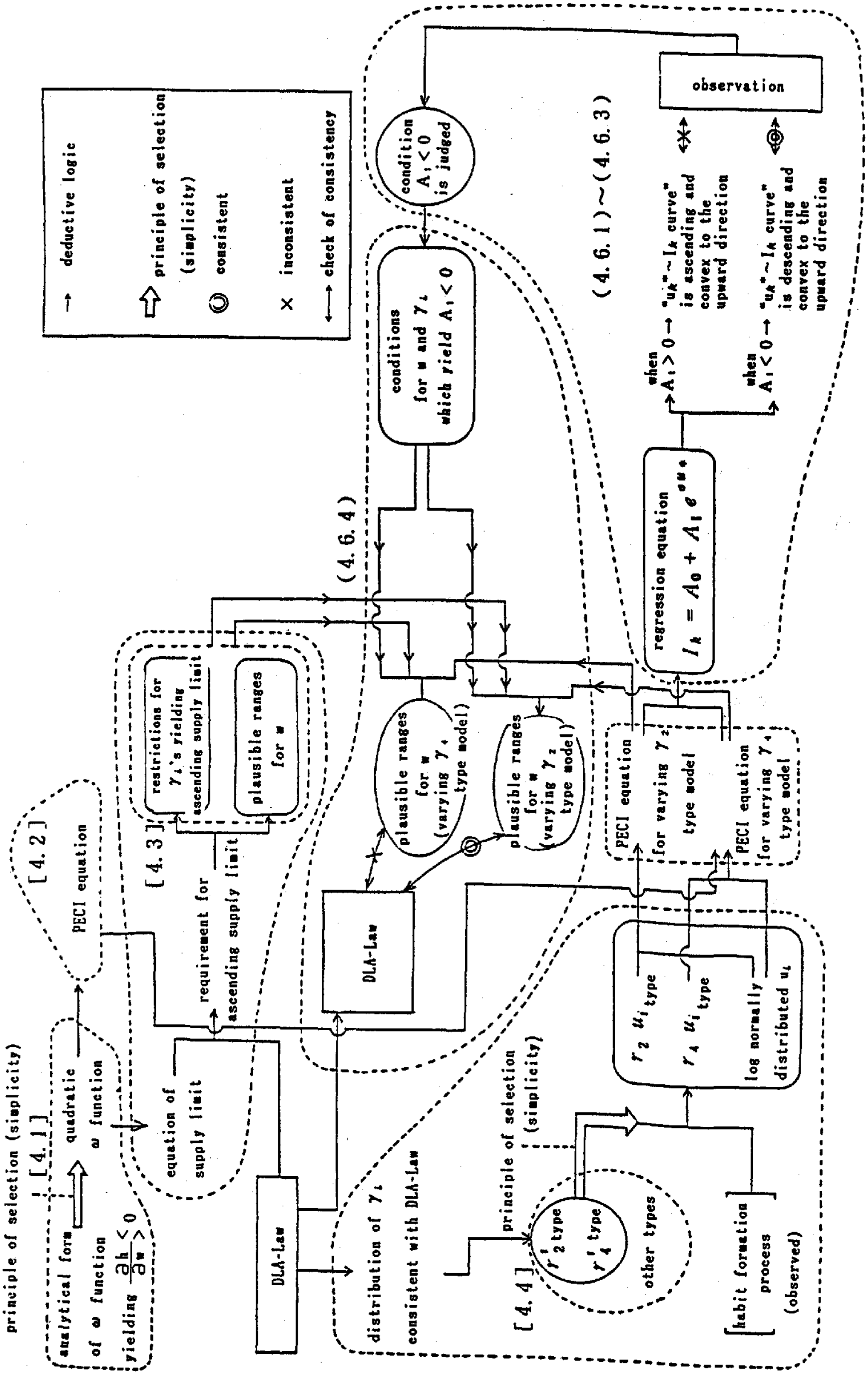
$$w < \gamma_3'$$

we could define the non-principal potential earner regardless of the wage rate (or earning) of the "principal earner". More strictly speaking, in the present stage of investigation, we regard a household member whose participation behavior is regulated by the first empirical law as a non-principal potential earner. We have such an empirical and behavioral definition only with respect to non-principal potential earners. So that, at this stage, it will be proper to avoid such a hypothesis that member of the household becomes an additional principal earner of the household as soon as his (her) w exceeds γ_3' however high the original principal earner's income level is.

(4.6.4.4-3) Given the above argument, we adopt the varying γ_4 model first for examination as one of the specific forms of an Allen-Bowley type utility function.

The process of the argument developed in [4.1] through [4.6] is depicted in Fig. Ⅳ-10.

Fig. IV-10


deductive logic
principle of selection (simplicity)
consistent
inconsistent
check of consistency
principle of selection (simplicity)
[4. 1]
analytical form of ω function yielding $\frac{\partial h}{\partial w} \lessgtr 0$
quadratic ω function
[4. 2]
PECI equation
equation of supply limit
requirement for ascending supply limit
[4. 3]
restrictions for γ_i's yielding ascending supply limit
plausible ranges for w
DLA-Law
DLA-Law
(4. 6. 4)
conditions for w and γ_i which yield $A_1 < 0$
condition $A_1 < 0$ is judged
observation
(4. 6. 1)~(4. 6. 3)
plausible ranges for w (varying γ_4 type model)
plausible ranges for w (varying γ_2 type model)
PECI equation for varying γ_2 type model
PECI equation for varying γ_4 type model
regression equation
$I_k = A_0 + A_1 e^{\sigma u_k}$
when $A_1 > 0$ → "u_k^* ~ I_k curve" is ascending and convex to the upward direction
when $A_1 < 0$ → "u_k^* ~ I_k curve" is descending and convex to the upward direction
distribution of γ_i consistent with DLA-Law
[4. 4]
principle of selection (simplicity)
r_2' type
r_4' type
other types
r_2 $u_{i\,type}$
r_4 $u_{i\,type}$
log normally distributed u_i
habit formation process (observed)

4.7 The estimation of parameters of PECI equation for the varying γ_4 model.

4.7.1 A method for solving non-linear normal equations

PECI equations could be regarded as the following type of non-linear regression equation,

$$1)\quad I = S_0 + S_2\, e^{\sigma u^*} + v,$$

where v stands for an additive shock.(*) We shall estimate the values of s_0, s_2 and σ so as to minimize $\sum v^2$.

Let

$$2)\quad V \equiv \sum v^2 = \sum (I - S_0 - S_2\, e^{\sigma u^*})^2.$$

From the conditions,

$$3)\quad \frac{\partial V}{\partial S_0} = -2\sum(I - S_0 - S_2\, e^{\sigma u^*}) = 0$$

$$4)\quad \frac{\partial V}{\partial S_2} = -2\sum(I - S_0 - S_2\, e^{\sigma u^*})\, e^{\sigma u^*} = 0$$

$$5)\quad \frac{\partial V}{\partial \sigma} = -2\sum(I - S_0 - S_2\, e^{\sigma u^*})s_2 u^* e^{\sigma u^*} = 0,$$

we have following normal equations giving estimates of the parameters, $\hat{s}_0$, $\hat{s}_2$ and $\hat{\sigma}$.

---

foot note

(*) The initial version used was

$$I = S_0 + S_2 e^{\sigma u^*} + S_1 N + v\ ,$$

where N stands for the number of children under 15 years old. However, preliminary estimation results indicated that coefficients for S_1's were not significant. This might stem from the nature of the date used. That is, in type A households, the variance of wives' ages is relatively small and hence that of the number ofchildren is small in the sample.

6) $S_0 n + S_2\ \Sigma e^{\sigma u^*} = \Sigma I$

7) $S_0 \Sigma e^{\sigma u^*} + S_2 \Sigma e^{2\sigma u^*} = \Sigma I e^{\sigma u^*}$

8) $S_0 \Sigma u* e^{\sigma u^*} + S_2 \Sigma u* e^{2\sigma u^*} = \Sigma I\ u* e^{\sigma u^*}$,

where n stands for the sample size.
Solving (6) and (7) for S_0 and S_2, we have

9) $$S_0 = \frac{\Sigma I \cdot \Sigma e^{2\sigma u^*} - \Sigma I\ e^{\sigma u^*} \cdot \Sigma e^{\sigma u^*}}{n \Sigma e^{2\sigma u*} - (\Sigma e^{\sigma u*})^2}$$

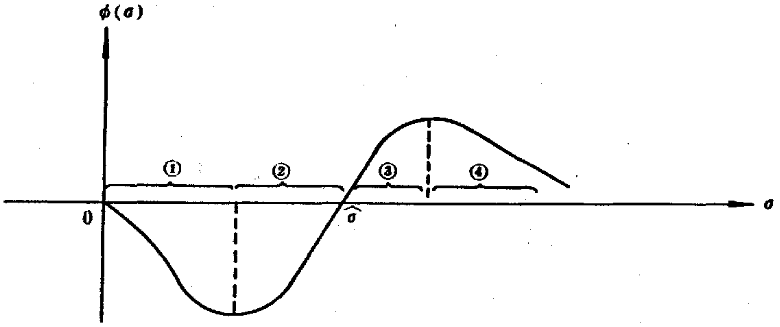
10) $$S_2 = \frac{n \Sigma I e^{\sigma u^*} - \Sigma e^{\sigma u^*} \cdot \Sigma I}{n \Sigma e^{2\sigma u*} - (\Sigma e^{\sigma u*})^2}$$

Inserting 9) and 10) into 8), we obtain the equation,

11) $$\Phi(\sigma) = (\Sigma I u^* + e^{\sigma u^*})\{n \Sigma e^{2\sigma u^*} - (\Sigma e^{\sigma u^*})^2\} + (\Sigma I)\{\Sigma e^{\sigma u^*} \cdot \Sigma u^* e^{2\sigma u^*} - \Sigma e^{2\sigma u^*} \cdot \Sigma u^* e^{\sigma u^*}\} + (\Sigma I e^{\sigma u^*})\{\Sigma e^{\sigma u^*} \cdot \Sigma u * e^{\sigma u^*} - n \Sigma u^* \cdot e^{2\sigma u^*}\} = 0,$$

where I and u^* are observables. Hence, (11) is a non-linear equation in the unknown σ. The graphical solution, σ, of the equation Φ (σ) can easily be obtained. By assigning various values for σ, the graph of Φ (σ) is depicted as shown in Fig. Ⅳ-11. As $\sigma = 0$ satisfies (11) the curve in Fig. Ⅳ-11 passes through the origin. Taking into account $\sigma > 0$, we can see that equation $\Phi(\sigma) = 0$ has a unique solution if the curve <u>is asymptotic</u> to the abscissa for $\sigma > 0$. Thus, we can obtain the first approximation for the solution, σ, by examining the graph. Employing the first approximation value, we can find the refined value of σ by the Newton method. When the first approximation lies in area 2, shown in Fig. Ⅳ-11, the refined value obtained by the Newton method will converge. Hence, it is necessary to examine a number of roots for $\Phi(\sigma) = 0$, where $\sigma > 0$.

Fig.(Ⅳ−11)



For the whole sample (including a group of households less than 50), the values of Φ for various tentative values of σ are computed for the years 1961 through 1964. It was found that the region of the solution, $\hat{\sigma}$, for each year is as follows.(*)

1961	$1.251 < \hat{\sigma} < 1.281$
1962	$0.041 < \hat{\sigma} < 0.051$
1963	$1.181 < \hat{\sigma} < 1.191$
1964	$0.691 < \hat{\sigma} < 0.701$

Employing the Newton method for the ranges of $\hat{\sigma}$ shown above, the convergence value of $\hat{\sigma}$ was obtained for each year. Inserting these values into equations (9) and (10), S_0 and S_2 were calculated respectively for each year. (see Tab. Ⅳ-2)

It is observed that the estimates $\hat{\sigma}$, vary form year to year.

<4.7.2> Estimation of non-linear regression equation when U^* is taken as a dependent variable.

(4.7.2-1) In (4.7.1), the observable I was taken as a dependent variable in the regression equation (1). However, it might be feasible to view I as an independent variable since observed data are stratified by the principal earner's income, I.

Let the PECI equation be

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* These are the estimates obtained by graphical methods.

Tab. Ⅳ－2

	1961			1962		
	σ	S_0	S_2	σ	S_0	S_2
1.	1.2745	57.398	- 103.18	0.04121	702.98	- 704.76
2.	0.93885	49.934	- 51.556	1.0245	60.490	- 86.738
3.	1.2091	50.638	- 70.020	0.49371	84.428	- 90.043
4.	1.21307	46.186	- 52.684	0.83077	63.747	- 77.303
	1963			1964		
	σ	S_0	S_2	σ	S_0	S_2
1.	1.1859	56.947	- 76.521	1.1859	75.444	- 85.037
2.	0.69536	61.731	- 62.984	0.69536	57.922	- 71.367
3.	0.70295	63.876	- 67.771	0.70295	66.943	- 79.534
4.	0.71426	60.931	- 62.690	0.71426	58.225	- 72.821

1. The case where all samples were used for estimation
2. The case where only principal earners income classes containing more than 50 households were used for estimation (selected sample)
3. All samples was used, and the moments were weighted by the sample size.
4. Selected samples were used, and the moments were weighted by sample size.

1) $I = H_0 + H_2 e^{\sigma u^*}$

which can be rewritten as

2) $u^* = \frac{1}{\sigma}[\log(H_0 - I) - \log(-H_2)]$

In equation (1) the notation H_0 and H_2 are used instead of S_0 and S_2 because the values of respective estimates for H_0 and H_2 differ from those for S_0 and S_2 on account of the difference in estimation method.
This equation can be regarded as a non-linear regression equation with the independent variable I,

3) $u^* = \frac{1}{\sigma}[\log(H_0 - I) - \log(-H_2)] + v$,

where v stands for an additive error, and

$H_0 > I$
$H_2 < 0$
$\sigma > 0$
$u'^{*} < 0$

We shall estimate σ, H_0 and H_2 so as to minimize $\sum v^2$. Denoting $\sum v^2$ by V, we have

$$4)\quad V = \sum v^2 = \sum \{u^{*} - \frac{1}{\sigma}\log(H_0 - I) + \frac{1}{\sigma}\log(-H_2)\}^2 .$$

Normal equations for the estimation of regression parameters can be obtained from the conditions,

$$5)\quad \frac{\partial V}{\partial H_0} = \frac{\alpha V}{\alpha H_2} = \frac{\alpha V}{\alpha \sigma} = 0 .$$

That is, we have

$$5\text{-}1)\quad \frac{\partial V}{\partial H_0} = 2\sum\{u^{*} - \frac{1}{\sigma}\log(H_0 - I) + \frac{1}{\sigma}\log(-H_2)\}\frac{-1}{\sigma(H_0 - I)} = 0$$

$$5\text{-}2)\quad \frac{\partial V}{\partial H_2} = 2\sum\{u^{*} - \frac{1}{\sigma}\log(H_0 - I) + \frac{1}{\sigma}\log(-H_2)\}\frac{-1}{\sigma H_2} = 0$$

$$5\text{-}3)\quad \frac{\partial V}{\partial \sigma} = 2\sum\{u^{*} - \frac{1}{\sigma}\log(H_0 - I) + \frac{1}{\sigma}\log(-H_2)\}$$
$$-\frac{\log(H_0 - I) - \log(-H_2)}{\sigma^2} = 0$$

From (5-2) we obtain

$$6)\quad \sum u^{*} - \frac{1}{\sigma}\sum\log(H_0 - I) + \frac{n}{\sigma}\log(-H_2) = 0 ,$$

Where n stands for the sample size. Equation (6) can be rewritten as

$$7)\quad \log(-H_2)=\frac{1}{n}\Sigma\log(H_0-I)-\frac{\sigma}{n}\Sigma u^*.$$

Inserting (7) into (5-1) we obtain,

$$8)\quad \Sigma\{u^*-\frac{1}{\sigma}\log(H_0-I)+\frac{1}{\sigma}[\frac{1}{n}\Sigma\log(H_0-I)-\frac{\sigma}{n}\Sigma u^*]\}\frac{1}{H_0-I}=0$$

This can be reduced to

$$\Sigma\frac{\sigma u^*}{H_0-I}-\Sigma\frac{\log(H_0-I)}{H_0-I}+\frac{1}{n}\Sigma\frac{\Sigma\log(H_0-I)}{H_0-I}-\frac{\sigma}{n}\Sigma\frac{\Sigma u^*}{H_0-I}=0$$

Solving this equation for σ we have

$$9)\quad \sigma=\frac{\Sigma\dfrac{\log(H_0-I)}{H_0-I}-\dfrac{1}{n}\Sigma\dfrac{\Sigma\log(H_0-I)}{H_0-I}}{\Sigma\dfrac{u^*}{H_0-I}-\dfrac{1}{n}\Sigma\dfrac{\Sigma u^*}{H_0-I}}$$

which is a function of H_0 .

Inserting (9) into (7), we obtain H_2 as a function of H_0, i.e.,

$$H_2=H_2(H_0).$$

Inserting this relation together with (9) into (5-3) we have an equation in the unknown H_0 only. This process of calculation is concretely shown as follows: we can rewrite (5-3) as

$$\begin{aligned}(5\text{-}3')\quad &\Sigma\sigma u^*\{\log(H_0-I)-\log(-H_2)\}-\Sigma\log(H_0-I)[\log(H_0-I)\\ &-\log(-H_2)]+\Sigma\log\{-H_2)[\log(H_0-I)-\log(-H_2)]\}=0\end{aligned}$$

From this, we have

$$\begin{aligned}&\sigma\{\Sigma u^*\cdot\log(H_0-I)-\Sigma u^*\log(-H_2)\}-\Sigma[\log(H_0-I)]^2\\ &+\Sigma\log(H_0-I)\log(-H_2)+\Sigma\log(-H_2)\log(H_0-I)-\Sigma[\log(-H_2)]^2=0\end{aligned}$$

or

$$\sigma\{\Sigma u^{*}\cdot\log(H_0-I)-\Sigma u^{*}\log(-H_2)\}-\Sigma[\log(H_0-I)]^2$$
$$+2\,\Sigma\log(H_0-I)\log(-H_2)+\Sigma[\log(-H_2)]^2=0$$

Inserting (7) into this last equation, we obtain

$$10)\quad \sigma\{\Sigma u^{*}\log(H_0-I)-\Sigma u^{*}[\frac{1}{n}\Sigma\log(H_0-I)-\frac{\sigma}{n}\Sigma u^{*}]\}$$
$$-\Sigma[\log(H_0-I)]^2+2[\frac{1}{n}\Sigma\log(H_0-I)-\frac{\sigma}{n}\Sigma u^{*}]\Sigma\log(H_0-I)$$
$$-n\{\frac{1}{n}\Sigma\log(H_0-I)-\frac{\sigma}{n}\Sigma u^{*}\}^2=0$$

Rewriting (8) we have

$$11)\quad \sigma\,\Sigma u^{*}\log(H_0-I)-\frac{\sigma}{n}\Sigma u^{*}\,\Sigma\log(H_0-I)$$
$$-\Sigma[\log(H_0-I)]^2+\frac{1}{n}[\Sigma\log(H_0-I)]^2=0$$

Inserting σ from (7) into (11), we obtain the following equation

$$12)\quad \left\{\Sigma\frac{\log(H_0-I)}{H_0-I}-\frac{1}{n}\Sigma\frac{\Sigma\log(H_0-I)}{H_0-I}\right\}\left\{\Sigma u^{*}\log(H_0-I)\right.$$
$$\left.-\frac{1}{n}\Sigma u^{*}\,\Sigma\log(H_0-I)\right\}-\left\{\Sigma\frac{u^{*}}{H_0-I}-\frac{1}{n}\Sigma\frac{u^{*}}{H_0-I}\right\}\left\{\Sigma[\log(H_0-I)]^2\right.$$
$$\left.-\frac{1}{n}[\Sigma\log(H_0-I)]^2\right\}=0$$

This is an equation in the unknown H_0 only, where observed variables, I and u^{*}, and sample size, n, can be regarded as given parameters.

Hence (12) will be written as

$$12')\quad F\,(H_0\mid I,\ u^{*})=0,$$

The root, $\hat{H}_0$, of equation (12') or (12)

is an estimate of H_0 minimizing Σv^2 in equation (3).

Inserting the $\hat{H}_0$ thus obtained into (9) we get an estimate, $\hat{\sigma}$, of σ.

Inserting $\hat{H}_0$ and $\hat{\sigma}$ into (7) we have an estimate, $\hat{H}_2$, of H_2. Hence, the essential problem in estimating regression parameters in (3) turns out to be finding a solution for equation (12).

(4.7.2-2) A Method of solving $F\ (H_0 \mid I,\ u^*)=0$

We shall obtain the first approximation for the solution, $\hat{H}_0$, of equation (12') by graphical methods; i.e. various numerical values of H_0 are assigned and the corresponding values of $F\ (H_0)$ are computed. Assigned values of H_0 have to satisfy the following inequality,

1) $I_{max} < H_0$,

where I_{max} stands for the observed maximum value of the principal earner's income among income classes with positive μ. This inequality stems from the nature of the model: H_0 in PECI equation (1) in (4.7.2.1) with negative H_2 is the maximum value of PECI among households under consideration and hence. (1) has to be satisfied. (cf.4.6.2.2 Fig.4.7-2).
Starting from the first approximation of H_0 thus obtained we can augment the degree of approximation by some suitable method, e.g., the Newton Method.

Results of the graphical solutions are shown in Tab.Ⅳ-8. For example, let us take the case shown by the first row in the table. This is the result for the year 1962. Two values in the second column, 113 and 114, stand for the two levels of H_0 between which the algebraic sign of $F(H_0)$ changes. We shall conveniently adopt the central value of these two values of H_0, 113.5, shown in the third column as an approximation of H_0. Corresponding values of σ, H_2 and H_2' are shown in the fourth, fifth and sixth column.

For the year 1961, the equation $F\ (H_0 \mid I,\ U^*)=0$ did not have any solution for the plausible range of H_0, $H_0 < I_{max}$.

Tab. Ⅳ−8

σ, H_2, H_2' where $F(H_0^*)=0$

	$F(H_0^1)\cdot F(H_0^2)<0$	H_0^*	σ	H_2	H_2'
1. *TW*62	113 ~ 114	113.5	0.3846	- 125.53	- 135.16
2. *TW*64	146 ~ 147	146.5	0.2781	- 152.40	- 158.41
3. *T* 62	128 ~ 129	128.5	0.4107	- 157.16	- 170.99
4. *W*62	60 ~ 61	60.5	1.0550	- 88.27	- 153.99
5. *W*63	67 ~ 68	67.5	0.6386	- 71.53	- 87.70
6. 62	60 ~ 61	60.5	1.1154	- 96.18	- 179.17
7. 63	78 ~ 79	78.5	0.4688	- 78.11	- 87.18

$$\sigma = \frac{\sum_i w_i \frac{\log(H_0^* - I_i)}{H_0^* - I_i} - \frac{1}{\sum_i w_i}\{\sum_i w_i \log(H_0^* - I_i)\}\{\sum_i \frac{w_i}{H_0^* - I_i}\}}{\sum_i w_i \frac{u_i*}{H_0^* - I_i} - \frac{1}{\sum_i w_i}\{\sum_i w_i \ u_i*\}\{\sum_i \frac{w_i}{H_0^* - I_i}\}}$$

$$H_2 = -\ exp\ (\frac{1}{\sum w_i} \sum w_i \log(H_0^* - I_i) - \frac{\sigma}{\sum w_i} \sum w_i\ u_i\ *)$$

$$H_2' = H_2\ / e^{-\frac{1}{2}\sigma^2}$$

foot note: (＊) In cases other than what are shown in the first column of Tab. Ⅳ-8, we do not have consistent solutions for H_0 which fulfill $H > I_{max}$.

(＊＊) The cases without T indicate where principal earners' income classes having less than 50 households are deleted for estimation. For instance, Tw64 indicates that all 1964 samples are employed and the variances and covarianes for the estimation are weighted by the number of households included in the income classes.

It is also possible to refine the estimates for H_0 shown in Tab. Ⅳ-8 by applying the Newton method. However, preliminary computations indicated that in this case, the estimates obtained by the Newton method are strongly affected by the variation in the initial values which are shown in the third column of Tab. Ⅳ-8. In accordance with this variation in the estimates for H_0, estimates for H_0 and H_2 also change. Hence, in order to obtain the parameters in PECI equations for the four years 1961 through 1964, the following procedure was adopted.

Alternative values for estimates of σ are tentatively assigned using those obtained in Tab. Ⅳ-8 to compute the parameters for H_0^t and H_2^t in PECI equation ($t=1961$, ……, 1964). H_0^t and H_2^t are reduced from parameters and are functions of w and h respectively. Consequently those values are expected to change as w_t and h_t change from year to year. In contrast to $H_0{}^t$ and $H_2{}^t$, σ in PECI equation is a structural parameter which is included in the set of preference parameters. Now, preference parameters are assumed to be constant over the years. Accordingly, although σ appears in the reduced form (PECI) equation, it too has to be assumed to be constant over the years. Hence, we use a common value of σ for the years 1961 through 1964 to compute h_0^t and h_2^t.

The alternative values for σ's were as follows

$\sigma=0.20$, 0.25, 0.2772^*, 0.30, 0.35, 0.3835^*, 0.40,
0.4104^*, 0.45, 0.4642^*, and 0.50,

where the figures with * attached are those obtained making use of revised values for H_0^*. Revised values for H_0^* were obtained by the Newton method utilising the H_0^*'s in Tab. Ⅳ-8 as initial values. (Extremely large values for σ in Tab. Ⅳ-8 were temporarily deleted from consideration.)

The results are shown in Tab. Ⅳ-14. Among these figures we have to select one set of parameters for the PECI equation for each year. One of the rules for selection would be to consult statistical measures of fitting, e.g. R^2 or Theil's U. Although magnitudes of those measures for each case do not differ much, we can adopt a selection critelian based on the theoretical consistency of the estimates.

Hence, firstly we estimated the values of the preference parameters γ_i's (structural parameters) based on each set of the values of the parameters in PECI equation (reduced form parameters) shown in Tab. Ⅳ-14. Secondly, we checked if the estimated sets of parameters fulfill the stability condition, that is, concavity of the indifference to curve to the origin because the set of the preference parameters (and accordingly set of parameters of PECI equation which are the function of the former) must be at least consistent with that condition.

The above procedure and the results are shown in the following §4.8.

Tab. Ⅳ−14(1)　　H_0, H_2, H_2', Where σ is given (1)

		(1)	(2)	(3)*	(4)	(5)	(6)*	(7)	(8)*	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. *TW*61 (11)	H_0			187.1	176.9	159.4	150.2	146.3	144.0	136.3	133.9	128.4
	H_2			−221.2	−212.8	−199.4	−193.2	−190.8	−189.5	−185.7	−184.7	−183.0
	H_2'			−229.8	−222.6	−212.0	−208.0	−206.7	−206.2	−205.5	−205.7	−207.4
	R			0.969	0.968	0.968	0.968	0.968	0.968	0.968	0.968	0.968
	$\overline{R}$			0.961	0.960	0.960	0.960	0.960	0.960	0.960	0.960	0.959
	Tu			0.122	0.122	0.123	0.123	0.123	0.123	0.124	0.124	0.124
2. *TW*62 (21)	H_0	183.3	153.9	142.4	134.5	120.8	* 113.7	110.7	108.9	102.9	101.0	96.8
	H_2	−189.7	−161.7	−151.0	−143.7	−131.6	*−125.7	−123.3	−121.9	−117.4	−116.1	−113.5
	H_2'	−193.5	−166.8	−156.9	−150.3	−139.9	*−135.3	−133.5	−132.6	−129.9	−129.4	−128.6
	R	0.982	0.982	0.982	0.982	0.982	* 0.982	0.982	0.982	0.982	0.982	0.982
	$\overline{R}$	0.979	0.979	0.979	0.979	0.979	* 0.979	0.979	0.979	0.979	0.979	0.979
	Tu	0.0948	0.0946	0.0944	0.0944	0.0942	*0.0942	0.0941	0.0941	0.0941	0.0941	0.0941
3. *TW*63 (31)	H_0	162.9	138.7	129.4	122.9	111.8	106.1	103.7	102.3	97.7	96.2	93.0
	H_2	−164.0	−141.4	−133.0	−127.3	−118.2	−114.0	−112.3	−111.4	−108.7	−108.0	−106.7
	H_2'	−167.3	−145.9	−138.2	−133.2	−125.7	−122.7	−121.7	−121.2	−120.3	−120.3	−120.9
	R	0.947	0.944	0.943	0.942	0.939	0.938	0.937	0.937	0.935	0.935	0.933
	$\overline{R}$	0.936	0.934	0.932	0.931	0.928	0.926	0.925	0.925	0.923	0.922	0.921
	Tu	0.154	0.157	0.159	0.160	0.163	0.165	0.166	0.166	0.168	0.169	0.170
4. *TW*64 (41)	H_0	189.2	158.8	* 146.9	138.6	124.2	116.7	113.5	111.6	105.3	103.3	98.7
	H_2	−193.2	−164.0	*−152.7	−145.0	−132.0	−125.5	−122.8	−121.2	−116.0	−114.5	−111.1
	H_2'	−197.1	−169.2	*−158.7	−151.7	−140.4	−135.1	−133.0	−131.8	−128.4	−127.5	−125.9
	R	0.987	0.987	* 0.987	0.987	0.987	0.986	0.986	0.986	0.986	0.986	0.986
	$\overline{R}$	0.985	0.985	* 0.985	0.985	0.985	0.984	0.984	0.984	0.984	0.984	0.984
	Tu	0.0812	0.0814	*0.0815	0.0816	0.0819	0.0821	0.0822	0.0823	0.0826	0.0827	0.0830

Tab. Ⅳ−14(2) H_0, H_2, H_2', Where σ is given (2)

		(1)	(2)	(3) (3)	(4)	(5)	(6)*	(7)	(8)*	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
5. T 61 (10)	H_0						198.1	192.3	188.9	177.4	173.8	165.6
	H_2						−268.5	−264.6	−262.4	−255.6	−253.7	−249.9
	H_2'						−289.0	−286.6	−285.4	−282.8	−282.5	−283.2
	R						0.973	0.973	0.973	0.973	0.973	0.973
	$\overline{R}$						0.966	0.966	0.966	0.966	0.966	0.966
	Tu						0.117	0.117	0.117	0.117	0.117	0.118
6. T 62 (20)	H_0		185.9	171.4	161.3	143.8	134.7	130.8	128.6	120.9	118.4	113.0
	H_2		−207.6	−194.2	−185.0	−169.7	−162.1	−159.0	−157.2	−151.5	−149.8	−146.3
	H_2'		−214.2	−201.8	−193.5	−180.4	−174.5	−172.2	−171.0	−167.6	−166.9	−165.8
	R		0.986	0.986	0.986	0.986	0.986	0.986	0.986	0.986	0.986	0.986
	$\overline{R}$		0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983
	Tu		0.0841	0.0840	0.0840	0.0839	0.0839	0.0839	0.0839	0.0839	0.0839	0.0840
7. T 63 (30)	H_0			186.3	176.4	159.1	150.2	146.3	144.1	136.5	134.1	128.6
	H_2			−209.0	−200.4	−186.4	−179.7	−177.0	−175.4	−170.7	−169.4	−166.8
	H_2'			−217.2	−209.6	−198.1	−193.4	−191.7	−190.8	−188.9	−188.7	−189.0
	R			0.965	0.965	0.964	0.964	0.964	0.964	0.963	0.963	0.962
	$\overline{R}$			0.959	0.959	0.958	0.957	0.957	0.957	0.956	0.956	0.955
	Tu			0.133	0.134	0.135	0.136	0.136	0.136	0.137	0.138	0.139
8. T 64 (40)	H_0	192.6	161.6	149.5	141.1	126.5	118.9	115.7	113.7	107.3	105.3	100.7
	H_2	−197.2	−167.4	−156.0	−148.2	−134.9	−128.3	−125.5	−123.9	−118.6	−117.0	−113.6
	H_2'	−201.2	−172.8	−162.1	−155.0	−143.4	−138.0	−135.9	−134.7	−131.2	−130.3	−128.7
	R	0.987	0.987	0.987	0.987	0.987	0.987	0.987	0.986	0.986	0.986	0.986
	$\overline{R}$	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.984	0.984	0.984
	Tu	0.0814	0.0816	0.0817	0.0818	0.0821	0.0823	0.0824	0.0825	0.0828	0.0829	0.0832

Tab. Ⅳ−14(3) H_0, H_2, H_2', Where σ is given (3)

			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)*	(9)	(10)	(11)
		σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.462	0.50
9	W61 (51)	H_0	152.3	129.0	119.9	113.5	102.5	96.7	94.2	92.8	87.8	86.3	82.7
		H_2	−154.2	−132.3	−123.9	−118.2	−108.6	−103.9	−102.0	−100.9	− 97.3	− 96.2	− 94.0
		H_2'	−157.3	−136.5	−128.7	−123.6	−115.5	−111.9	−110.5	−109.7	−107.6	−107.2	−106.5
		R	0.981	0.980	0.980	0.980	0.980	0.980	0.980	0.980	0.980	0.980	0.980
		$\overline{R}$	0.971	0.971	0.971	0.970	0.970	0.970	0.970	0.970	0.970	0.970	0.970
		Tu	0.0995	0.0996	0.0997	0.0997	0.0999	0.0999	0.1000	0.100	0.100	0.100	0.100
10	W62 (61)	H_0	161.4	135.7	125.6	118.6	106.5	100.2	97.4	95.8	90.5	88.8	85.0
		H_2	−161.8	−136.9	−127.3	−120.8	−109.7	−104.1	−101.7	−100.3	− 95.9	− 94.5	− 91.6
		H_2'	−165.0	−141.2	−132.3	−126.3	−116.6	−112.0	−110.2	−109.2	−106.1	−105.3	−103.8
		R	0.992	0.993	0.993	0.993	0.993	0.993	0.993	0.993	0.993	0.993	0.994
		$\overline{R}$	0.990	0.991	0.991	0.991	0.991	0.991	0.992	0.992	0.992	0.992	0.992
		Tu	0.0625	0.0615	0.0609	0.0605	0.0595	0.0588	0.0585	0.0583	0.0576	0.0574	0.0568
11	W63 (71)	H_0	140.0	118.6	110.3	104.4	94.3	89.1	86.8	85.5	81.0	79.6	76.4
		H_2	−135.3	−114.7	−106.8	−101.3	− 92.1	− 87.4	− 85.5	− 84.4	− 80.7	− 79.5	− 77.1
		H_2'	−138.1	−118.3	−110.9	−106.0	− 97.9	− 94.1	− 92.6	− 91.8	− 89.3	− 88.6	− 87.4
		R	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.992
		$\overline{R}$	0.990	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.991	0.991
		Tu	0.0627	0.0624	0.0622	0.0621	0.0619	0.0618	0.0618	0.0618	0.0617	0.0617	0.0617
12	W64 (81)	H_0	165.8	139.5	129.2	122.0	109.5	103.0	100.2	98.5	92.9	91.2	87.2
		H_2	−164.0	−138.6	−128.8	−122.0	−110.5	−104.6	−102.2	−100.7	− 96.0	− 94.5	− 91.3
		H_2'	−167.3	−143.0	−133.8	−127.6	−117.5	−112.6	−110.7	−109.6	−106.2	−105.3	−103.5
		R	0.987	0.988	0.988	0.988	0.988	0.988	0.988	0.988	0.988	0.988	0.988
		$\overline{R}$	0.984	0.985	0.985.	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.986
		Tu	0.0800	0.0795	0.0792	0.0789	0.0784	0.0780	0.0778	0.0777	0.0772	0.0770	0.0766

Tab. Ⅳ－14(4)　H_0, H_2, H_2', Where σ is given (4)

			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)*	(9)	(10)*	(11)
		σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
13	61 (50)	H_0	147.2	124.9	116.1	110.0	99.5	93.9	91.6	90.2	85.5	84.0	80.6
		H_2	−147.7	−126.6	−118.5	−113.1	−103.9	− 99.4	− 97.5	− 96.4	− 93.0	− 92.0	− 89.8
		H_2'	−150.6	−130.6	−123.2	−118.3	−110.5	−107.0	−105.6	−104.9	−102.9	−102.4	−101.7
		R	0.981	0.981	0.980	0.980	0.980	0.980	0.980	0.980	0.980	0.980	0.980
		$\overline{R}$	0.971	0.971	0.971	0.971	0.970	0.970	0.970	0.970	0.970	0.970	0.970
		Tu	0.0991	0.0992	0.0992	0.0993	0.0994	0.0995	0.0996	0.0996	0.0997	0.0998	0.0999
14	62 (60)	H_0	167.8	140.8	130.2	122.8	110.1	103.4	100.6	98.9	93.2	91.5	87.4
		H_2	−171.1	−145.0	−135.0	−128.1	−116.4	−110.5	−108.1	−106.6	−102.0	−100.6	− 97.5
		H_2'	−174.5	−149.6	−140.2	−134.0	−123.8	−119.0	−117.1	−116.0	−112.9	−112.0	−110.5
		R	0.993	0.993	0.993	0.993	0.993	0.994	0.994	0.994	0.994	0.994	0.994
		$\overline{R}$	0.991	0.991	0.991	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.992
		Tu	0.0598	0.0589	0.0584	0.0580	0.0572	0.0566	0.0564	0.0562	0.0556	0.0554	0.0548
15	63 (70)	H_0	138.6	117.4	109.2	103.4	93.5	88.3	86.0	84.7	80.3	* 78.9	75.8
		H_2	−133.7	−113.3	−105.4	−100.0	− 90.9	− 86.3	− 84.3	− 83.2	− 79.6	* − 78.5	− 76.0
		H_2'	−136.4	−116.9	−109.5	−104.6	− 96.6	− 92.9	− 91.4	− 90.5	− 88.0	* − 87.4	− 86.2
		R	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.992	* 0.992	0.992
		$\overline{R}$	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.991	* 0.991	0.991
		Tu	0.0623	0.0621	0.0619	0.0619	0.0617	0.0616	0.0616	0.0616	0.0616	* 0.0616	0.0616
16	64 (80)	H_0	156.7	132.2	122.6	115.9	104.4	98.3	95.7	94.2	89.0	87.4	83.7
		H_2	−153.0	−129.3	−120.2	−113.9	−103.2	− 97.8	− 95.5	− 94.1	− 89.7	− 88.4	− 85.4
		H_2'	−156.1	−133.4	−124.9	−119.2	109.7	−105.2	−103.4	−102.4	− 99.3	− 98.4	− 96.8
		R	0.988	0.988	0.988	0.988	0.988	0.988	0.988	0.988	0.989	0.989	0.989
		$\overline{R}$	0.985	0.985	0.985	0.985	0.985	0.985	0.986	0.986	0.986	0.986	0.986
		Tu	0.0784	0.0779	0.0776	0.0773	0.0768	0.0764	0.0763	0.0762	0.0757	0.0756	0.0752

4.8 Determination of parameters of the preference function making use of estimated parameters in the PECI equation

The parameters, σ, H_0 and H_2, of the PECI equation obtained in (4.7.2.2) are regarded as reduced form parameters. Among those reduced form parameters, σ is a structural parameter as well. Hence, the structural parameter σ has already been estimated. In this section we shall try to determine the values of structural parameters, (except for σ), γ_1, γ_2, γ_3, γ_4 and γ_5, making use of the relationship between structural parameters γ_i, and reduced form parameters, H_0 and H_2. After that, we shall examine the plausiblity of estimates of the structural parameters making use of the stability condition for household equilibrium.

<4.8.1> Estimation of γ_i

As is shown in <4.5.1>, the PECI equation is written as

$$1)\quad I^{*}=\frac{-\gamma_2 w-\gamma_3 w(T-h)+\gamma_5(T-\frac{1}{2}h)-\frac{1}{2}\gamma_1 w^2 h}{\gamma_1 w-\gamma_3}+\frac{\gamma_4}{\gamma_1 w-\gamma_3}u$$

Hence we have

$$2)\quad H_0=\frac{-\gamma_2 w-\gamma_3 w(T-h)+\gamma_5(T-\frac{1}{2}h)-\frac{1}{2}\gamma_1 w^2 h}{\gamma_1 w-\gamma_3}$$

and

$$3)\quad H_2'=\frac{\gamma_4}{\gamma_1 w-\gamma_3}$$

From (3) we have $\gamma_1 w-\gamma_3=\gamma_4\frac{1}{H_2},$ or

$$4)\quad w=\frac{-\gamma_3}{\gamma_1}-\frac{\gamma 4}{\gamma_1},\cdot\frac{1}{H_2}\quad ,\qquad \text{where}\quad \gamma_1\equiv -1$$

From (2) we obtain

$$5)\quad \frac{-H_0(w+\frac{\gamma_3}{\gamma_1})+\frac{\gamma_3}{\gamma_1}w(T-h)-\frac{1}{2}w^2 h}{T-\frac{1}{2}h}=\frac{\gamma_5}{\gamma_1}-\frac{\gamma_2}{\gamma_1}\cdot\frac{w}{T-\frac{1}{2}h}$$

where $\gamma_1\equiv -1$.

(4) and (5) are basic relations for estimating γ_i(i=2, 3, 4, 5). Making use of (4), the estimates of (γ_3/γ_1) and (γ_4/γ_1) are obtained because values for w and H_2' are already known for each year, 1961 through 1964: i.e. we shall

estimate the regression parameters, a and b, of

$$6)\quad w_t = a + b\left(\frac{1}{H_2{}'t}\right), \quad (t = 1961, \ldots, 1964)$$

where

$$7)\quad \hat{a} = \mathrm{est}\left(\frac{\gamma_3}{\gamma_1}\right) \qquad \text{and} \qquad \hat{b} = \mathrm{est}\left(\frac{\gamma_4}{\gamma_1}\right).$$

By normalization, $\gamma_1 = 1$, we can obtain estimates of γ_3 and γ_4 from (7).

In order to estimate $\left(\frac{\gamma_3}{\gamma_1}\right)$ and $\left(\frac{\gamma_2}{\gamma_1}\right)$ we use (5):
Inserting the estimate (γ_3/γ_1) obtained from (7) into left hand side of (5) together with the first approximation of h, $\frac{1}{3}$, the value of the left hand side in (5) for each year can be obtained because H_0 and w are known.
That is, we define

$$8)\quad y = \frac{-\hat{H}_0\left(w + \mathrm{est}\left(\frac{\gamma_3}{\gamma_1}\right)\right) + \mathrm{est}\left(\frac{\gamma_3}{\gamma_1}\right) w\left(T - \frac{1}{3}\right) - \frac{1}{2} w^2\left(\frac{1}{3}\right)}{T - \frac{1}{2} \cdot \frac{1}{3}},$$

and, from (5) and (8) we have the regression equation

$$9)\quad y = a' - b'\left(\frac{w}{T - \frac{1}{2} \cdot \frac{1}{3}}\right),$$

where,

$$10)\quad a' \equiv \frac{\gamma_5}{\gamma_1} \qquad \text{and} \qquad b' \equiv -\frac{\gamma_2}{\gamma_1}$$

From the estimates of the parameters a' and b' in regression equation 9), we can obtain γ_5 and γ_2 by normalization, setting $\gamma_1 \equiv -1$. By the above procedure all the parameters γ_2, γ_3, γ_4, and γ_5 with $\gamma_1 \equiv -1$ will be determined. To estimate parameters of the relations (6) and (9) we tried four alternative cases;

case A $\quad w = a + b\frac{1}{H_2'} \qquad \frac{A}{T - \frac{1}{2}h} = a' + b'\frac{w}{T - \frac{1}{2}h}$

B $\quad w = a + b\frac{1}{H_2'} \qquad \frac{w}{T - \frac{1}{2}h} = a'' + b''\frac{A}{T - \frac{1}{2}h}$

$$C \quad \frac{1}{H_2'} = a'' + b'' w \qquad \frac{A}{T-\frac{1}{2}h} = a'' + b'' \frac{w}{T-\frac{1}{2}h}$$

$$D \quad \frac{1}{H_2'} = a'' + b'' w \qquad \frac{w}{T-\frac{1}{2}h} = a'' + b'' \frac{A}{T-\frac{1}{2}h}$$

where $A \equiv -\hat{H}_0(w+est(\frac{\gamma_3}{\gamma_1}))+est(\frac{\gamma_3}{\gamma_1})w(T-h)-\frac{1}{2}w^2h$

The results of estimation are shown in Tab. Ⅳ-15～24. Tab. Ⅳ-25 summerizes the values of parameters obtained, where the normalization, $\gamma_1 \equiv -1$, is adopted.

Tab. Ⅳ-25

case	A	B	C	D
γ_2	140 ～ 100	160 ～ 110	68 ～ 73	270 ～ 100
γ_3	−29.6～ −29.1	−29.6～ −29.1	−16.0～ −21.6	−16.0～ −21.6
γ_4	2200 ～ 1900	2200 ～ 1900	4400 ～ 2950	4400 ～ 2950
γ_5	3400 ～ 2200	4400 ～ 2600	−2300～ 20	8000 ～ 1800

<4.8.2> Examination of the Results obtainned in 4.8.1

The values of parameters in Tab. Ⅳ-25 have to satisfy stability conditions at least in the vicinity of the observed values of X and Λ. Hence, satisfaction of the following conditions is needed:

$$1) \quad \left(\frac{\partial W}{\partial X}\right)_{X=X_i^o} > 0, \qquad \left(\frac{\partial W}{\partial \Lambda}\right)_{\Lambda=\Lambda_i^o} > 0$$

and

$$2) \quad 2\gamma_3\left(\frac{\partial W}{\partial \Lambda}\right)_{\Lambda=\Lambda_i^o}\left(\frac{\partial W}{\partial X}\right)_{X=X_i^o} - \left(\frac{\partial W}{\partial X}\right)^2_{X=X_i^o} - \left(\frac{\partial W}{\partial \Lambda}\right)^2_{\Lambda=\Lambda_i^o} > 0$$

where X_i^o and Λ_i^o are the observed value of income and leisure respectively for the i-th group of households grouped by principal earner's income.

In the above inequalities, the marginal utilities of leisure and income are given by

Tab. Ⅳ−15 Estimation of preference parameter $W_i = a + b \frac{1}{H'_{2i}} + v \quad (\begin{matrix} \gamma_3 = -a \\ \gamma_4 = -b \end{matrix})$

Case ⓐ

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0,45	0.4642	0.50
1. *T W*	a			29.57	29.50	29.38	29.31	29.28	29.27	29.21	29.20	29.18
	t_a			(3.07)	(3.14)	(3.32)	(3.44)	(3.51)	(3.55)	(3.72)	(3.79)	(3.96)
	b			−2198.5	−2122.5	−2006.7	−1958.1	−1940.7	−1931.8	−1909.7	−1905.9	−1904.2
	t_b			(1.41)	(1.45)	(1.55)	(1.62)	(1.66)	(1.68)	(1.77)	(1.81)	(1.89)
	$\overline{r}$			− 0.496	− 0.518	− 0.565	− 0.594	− 0.607	− 0.616	− 0.646	− 0.656	− 0.681
2. *T*	a						30.84	30.99	31.08	31.41	31.52	31.79
	t_a						(8.09)	(8.14)	(8.17)	(8.28)	(8.33)	(8.44)
	b						−2232.3	−2179.0	−2148.4	−2051.2	−2022.8	−1963.9
	t_b						(3.26)	(3.23)	(3.21)	(3.13)	(3.11)	(3.05)
	$\overline{r}$						− 0.873	− 0.871	− 0.869	− 0.864	− 0.862	− 0.857
3. *W*	a	44.73	42.14	40.72	39.54	36.29	35.36	34.58	34.09	32.32	31.71	30.24
	t_a	(1.50)	(1.41)	(1.36)	(1.33)	(1.28)	(1.22)	(1.20)	(1.19)	(1.15)	(1.14)	(1.12)
	b	285.1	− 102.1	− 273.9	− 403.7	− 727.3	− 806.6	− 876.7	− 919.8	−1076.3	−1130.1	−1261.6
	t_b	(0.062)	(0.026)	(0.073)	(0.113)	(0.231)	(0.261)	(0.290)	(0.309)	(0.379)	(0.406)	(0.469)
	$\overline{r}$	0.0	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0.	− 0.0	− 0.0	− 0.0
4.	a	38.51	36.40	35.26	34.32	32.32	31.05	30.44	30.07	28.71	28.25	27.14
	t_a	(1.46)	(1.38)	(1.35)	(1.32)	(1.27)	(1.24)	(1.23)	(1.22)	(1.19)	(1.19)	(1.17)
	b	− 673.2	− 855.8	− 943.7	−1013.2	−1156.2	−1246.9	−1290.5	−1317.7	−1419.0	−1454.7	−1543.9
	t_b	(0.167)	(0.248)	(0.293)	(0.332)	(0.417)	(0.475)	(0.504)	(0.523)	(0.593)	(0.619)	(0.685)
	$\overline{r}$	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0	− 0.0

Tab. Ⅳ−16 Estimation of preference parameter $\frac{A_i}{T-\frac{h}{2}} = a' + b' \frac{W_i}{T-\frac{h}{2}} + v$ $\left(\begin{array}{l} r_5 = a' \\ r_2 = b' \end{array} \right.$

Case ⓐ

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. T W	a'			3375.7	3174.4	2825.1	2643.8	2566.5	2521.2	2371.9	2326.2	2226.2
	t_a'			(1.78)	(1.80)	(1.84)	(1.88)	(1.90)	(1.91)	(1.97)	(2.00)	(2.06)
	b'			−138.4	−132.2	−121.3	−115.7	−113.3	−111.9	−107.2	−105.7	−102.4
	t_b'			(3.76)	(3.86)	(4.09)	(4.25)	(4.33)	(4.39)	(4.60)	(4.68)	(4.90)
	$\overline{r}$			−0.902	−0.907	−0.916	−0.922	−0.925	−0.927	−0.933	−0.935	−0.941
2. T	a'						1563.2	1635.1	1677.4	1818.8	1863.1	1962.9
	t_a'						(1.03)	(1.10)	(1.15)	(1.33)	(1.39)	(1.54)
	b'						−98.45	−98.33	−98.26	−98.04	−98.0	−97.8
	t_b'						(3.34)	(3.43)	(3.49)	(3.70)	(3.77)	(3.95)
	$\overline{r}$						−0.878	−0.884	−0.888	−0.899	−0.903	−0.911
3. W	a'	8778.0	7148.5	6489.6	6021.2	5091.3	4737.5	4541.1	4424.4	4026.9	3899.9	3609.4
	t_a'	(43.7)	(26.9)	(19.3)	(15.4)	(9.31)	(8.74)	(8.06)	(7.69)	(6.55)	(6.23)	(5.56)
	b'	−201.1	−176.6	−166.7	−159.6	−146.3	−140.0	−136.9	−135.1	−128.8	−126.7	−121.96
	t_b'	(51.7)	(34.3)	(25.6)	(21.1)	(13.8)	(13.3)	(12.6)	(12.1)	(10.8)	(10.5)	(9.70)
	$\overline{r}$	−0.999	−0.999	−0.998	−0.997	−0.992	−0.992	−0.991	−0.990	−0.987	−0.986	−0.984
4.	a'	7423.5	6016.6	5451.5	5051.7	4344.8	3968.1	3804.5	3707.6	3380.1	3276.3	3040.9
	t_a'	(13.4)	(9.40)	(8.09)	(7.25)	(5.92)	(5.30)	(5.05)	(4.90)	(4.43)	(4.29)	(3.99)
	b'	−192.7	−167.9	−157.9	−150.8	−138.3	−131.5	−128.5	−126.8	−120.8	−118.9	−114.4
	t_b'	(17.9)	(13.5)	(12.1)	(11.2)	(9.73)	(9.07)	(8.80)	(8.65)	(8.18)	(8.04)	(7.75)
	r	−0.995	−0.992	−0.990	−0.988	−0.984	−0.982	−0.981	−0.980	−0.978	−0.977	−0.976

Tab. Ⅳ−17 The structural parameter

(Case Ⓐ)

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. *T W*	r_1			− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.
	r_2			138.4	132.2	121.3	115.7	113.3	111.9	107.2	105.7	102.4
	r_3			− 29.57	− 29.50	− 29.38	− 29.31	− 29.28	− 29.27	− 29.21	− 29.20	− 29.18
	r_4			2198.5	2122.5	2006.7	1958.1	1940.7	1931.8	1909.7	1905.9	1904.2
	r_5			3375.7	3174.4	2825.1	2643.8	2566.5	2521.2	2371.9	2326.2	2226.2
2. *T*	r_1						− 1.	− 1.	− 1.	− 1.	− 1.	− 1.
	r_2						98.45	98.33	98.26	98.04	98.0	97.8
	r_3						− 30.84	− 30.99	− 31.08	− 31.41	− 31.52	− 31.79
	r_4						2232.3	2179.0	2148.4	2051.2	2022.8	1963.9
	r_5						1563.2	1635.1	1677.4	1818.8	1863.1	1962.9
3. *W*	r_1	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.
	r_2	201.1	176.6	166.7	159.6	146.3	140.0	136.9	135.1	128.8	126.7	121.96
	r_3	− 44.73	− 42.14	− 40.72	− 39.54	− 36.29	− 35.36	− 34.58	− 34.09	− 32.32	− 31.71	− 30.24
	r_4	− 285.1	102.1	273.9	403.7	727.3	806.6	876.7	919.8	1076.3	1130.1	1261.6
	r_5	8778.0	7148.5	6489.6	6021.2	5091.3	4737.5	4541.1	4424.4	4026.9	3899.9	3609.4
4.	r_1	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.	− 1.
	r_2	192.7	167.9	157.9	150.8	138.3	131.5	128.5	126.8	120.8	118.9	114.4
	r_3	− 38.51	− 36.40	− 35.26	− 34.32	− 32.32	− 31.05	− 30.44	− 30.07	− 28.71	− 28.25	− 27.14
	r_4	673.2	855.8	943.7	1013.2	1156.2	1246.9	1290.5	1317.7	1419.0	1454.7	1543.9
	r_5	7423.5	6016.6	5451.5	5051.7	4344.8	3968.1	3804.5	3707.6	3380.1	3276.3	3040.9

Tab. Ⅳ−18 The preference parameter $W_i = a + b\frac{1}{H'_{2i}}$ ▲(r_3, r_4)

Case Ⓑ $\left(\frac{W_i}{T-\frac{h}{2}}\right) = -\frac{\frac{a'}{b'}}{\alpha} + \frac{\frac{1}{b'}}{\beta}\ \frac{A_i}{T-\frac{h}{2}}$ $r_5 = a'$ $r_2 = -b'$

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. T W	α			27.74	27.27	26.30	25.71	25.43	25.26	24.67	24.47	24.02
	t_α			(4.35)	(4.30)	(4.22)	(4.19)	(4.19)	(4.18)	(4.19)	(4.20)	(4.25)
	β			−.633(−2)	−.667(−2)	−.736(−2)	−.778(−2)	−.798(−2)	−.810(−2)	−.853(−2)	−.867(−2)	−.902(−2)
	t_β			(3.76)	((3.86)	(4.09)	(4.25)	(4.33)	(4.39)	(4.60)	(4.68)	(4.90)
	$\bar{r}$			− 0.902	− 0.907	− 0.916	− 0.922	− 0.925	− 0.927	− 0.933	− 0.935	− 0.941
2. T	α						21.30	21.69	21.93	22.75	23.02	23.64
	t_α						(2.34)	(2.48)	(2.57)	(2.91)	(3.03)	(3.32)
	β						−.861(−2)	−.869(−2)	−.874(−2)	−.890(−2)	−.895(−2)	−.906(−2)
	t_β						(3.34)	(3.43)	(3.49)	(3.70)	(3.77)	(3.95)
	$\bar{r}$						− 0.878	− 0.884	− 0.888	− 0.899	− 0.903	− 0.911
3. W	α	43.65	40.49	38.97	37.79	34.98	34.04	33.40	33.01	31.62	31.15	30.05
	t_α	(257.3)	(118.9)	(76.0)	(55.9)	(28.61)	(25.4)	(22.7)	(21.2)	(16.9)	(15.7)	(13.4)
	β	−.497(−2)	−.565(− 2)	−.598(−2)	−.624(−2)	−.676(−2)	−.707(−2)	−.721(−2)	−.730(−2)	−.764(−2)	−.775(−2)	−.803(−2)
	t_β	(51.7)	(34.3)	(25.6)	(21.1)	(13.8)	(13.3)	(12.6)	(12.1)	(10.8)	(10.5)	(9.70)
	$\bar{r}$	− 0.999	− 0.999	− 0.998	− 0.997	− 0.992	− 0.992	− 0.991	− 0.990	− 0.987	− 0.986	− 0.984
4.	α	38.60	36.01	34.75	33.78	31.84	30.68	30.15	29.82	28.67	28.28	27.38
	t_α	(51.3)	(30.6)	(24.47)	(20.8)	(15.5)	(13.1)	(12.2)	(11.7)	(10.1)	(9.67)	(8.70)
	β	−.516(−2)	−.589(−2)	−.625(−2)	−.652(−2)	−.708(−2)	−.742(−2)	−.758(−2)	−.768(−2)	−.804(−2)	−.816(−2)	−.846(−2)
	t_β	(17.9)	(13.5)	(12.1)	(11.2)	(9.73)	(9.07)	(8.80)	(8.65)	(8.18)	(8.04)	(7.75)
	$\bar{r}$	− 0.995	− 0.992	− 0.990	− 0.988	− 0.984	− 0.982	− 0.981	− 0.980	− 0.978	− 0.977	− 0.976

Tab. Ⅳ−19 The preference parameter $W = f\left(\frac{1}{H_2'}\right)$

Case Ⓑ $W = g\left(A_i\right)$

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. T W	r_1			− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2			158.0	149.9	135.9	128.5	125.4	123.5	117.3	115.3	110.9
	r_3			− 29.57	− 29.50	− 29.38	− 29.31	− 29.28	− 29.27	− 29.21	− 29.20	− 29.18
	r_4			2198.5	2122.5	2006.7	1958.1	1940.7	1931.8	1909.7	1905.9	1904.2
	r_5			4382.1	4086.7	3572.6	3303.7	3188.2	3120.2	2893.2	2822.4	2664.6
2. T	r_1						− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2						116.1	115.1	114.4	112.4	111.7	110.3
	r_3						− 30.84	− 30.99	− 31.08	− 31.41	− 31.52	− 31.79
	r_4						2232.3	2179.0	2148.4	2051.2	2022.8	1963.9
	r_5						2473.9	2496.1	2509.5	2556.7	2572.1	2607.9
3. W	r_1	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2	201.3	176.9	167.2	160.3	147.8	141.5	138.6	136.9	131.0	129.0	124.5
	r_3	− 44.73	− 42.14	− 40.72	− 39.54	− 36.29	− 35.36	− 34.58	− 34.09	− 32.32	− 31.71	− 30.24
	r_4	− 285.1	102.1	273.9	403.7	727.3	806.6	876.7	919.8	1076.3	1130.1	1261.6
	r_5	−8785.9	7164.0	6515.9	6058.2	5170.4	4818.5	4630.6	4519.1	4140.2	4019.3	3742.8
4.	r_1	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2	193.9	169.7	160.1	153.2	141.2	134.7	131.9	130.2	124.4	122.5	118.2
	r_3	− 38.51	− 36.40	− 35.26	− 34.32	− 32.32	− 31.05	− 30.44	− 30.07	− 28.71	− 28.25	− 27.14
	r_4	673.2	855.8	943.7	1013.2	1156.2	1246.9	1290.5	1317.7	1419.0	1454.7	1543.9
	r_5	7485.5	6110.8	5562.5	5176.0	4495.0	4132.7	3975.3	3882.1	3566.0	3465.6	3236.8

Tab. IV−20 Estimation of the preference parameter

(Case Ⓒ) $\frac{1}{H_2'} = \alpha + \beta W_i + v_i$ $(\beta = \frac{1}{b} \quad b = -r_4)$

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. $T\ W$	α			.363(−2)	.405(−2)	.494(−2)	.551(−2)	.579(−2)	.596(−2)	.657(−2)	.678(−2)	.727(−2)
	t_α			(0.525)	(0.565)	(0.654)	(0.716)	(0.746)	(0.768)	(0.846)	(0.875)	(0.949)
	β			−.226(−3)	−.241(−3)	−.272(−3)	−.290(−3)	−.298(−3)	−.303(−3)	−.320(−3)	−.325(−3)	−.337(−3)
	t_β			(1.41)	(1.45)	(1.55)	(1.62)	(1.66)	(1.68)	(1.77)	(1.81)	(1.89)
	r			− 0.496	− 0.518	− 0.565	− 0.594	− 0.607	− 0.616	− 0.646	− 0.656	− 0.681
2. T	α						.108(−1)	.111(−1)	.112(−1)	−.118(−1)	.119(−1)	.123(−1)
	t_α						(2.17)	(2.15)	(2.14)	(2.10)	(2.11)	(2.09)
	β						−.377(−3)	−.385(−3)	−.390(−3)	−.405(−3)	−.410(−3)	−.419(−3)
	t_β						(3.26)	(3.23)	(3.21)	(3.13)	(3.11)	(3.05)
	r						− 0.873	− 0.871	− 0.869	− 0.864	− 0.862	− 0.857
3. W	α	−.669(−2)	−.733(−2)	−.753(−2)	−.764(−2)	−.746(−2)	−.759(−2)	−.752(−2)	−.745(−2)	−.716(−2)	−.703(−2)	−.666(−2)
	t_α	(1.44)	(1.36)	(1.31)	(1.27)	(1.12)	(1.13)	(1.10)	(1.08)	(1.01)	(0.988)	(0.925)
	β	.663(−5)	−.320(−5)	−.976(−5)	−.158(−4)	−.358(−4)	−.408(−4)	−.460(−4)	−.494(−4)	−.623(−4)	−.669(−4)	−.786(−4)
	t_β	(0.0615)	(0.0256)	(0.0732)	(0.113)	(0.231)	(0.261)	(0.290)	(0.309)	(0.379)	(0.405)	(0.469)
	r	0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00
4.	α	−.565(−2)	−.610(−2)	−.622(−2)	−.626(−2)	−.618(−2)	−.602(−2)	−.591(−2)	−.583(−2)	−.548(−2)	−.533(−2)	−.493(−2)
	t_α	(1.08)	(1.01)	(0.971)	(0.938)	(0.866)	(0.817)	(0.792)	(0.776)	(0.716)	(0.694)	(0.638)
	β	−.204(−4)	−.349(−4)	−.437(−4)	−.514(−4)	−.691(−4)	−.814(−4)	−.874(−2)	−.912(−4)	−.106(−3)	−.111(−3)	−.123(−3)
	t_β	(0.167)	(0.248)	(0.293)	(0.332)	(0.417)	(0.475)	(0.504)	(0.523)	(0.593)	(0.619)	(0.685)
	r	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00

Tab. Ⅳ−21 **Case** © $\left(\frac{A_i}{T-\frac{1}{2}h}\right)=a'+b'\left(\frac{W_i}{T-\frac{1}{2}h}\right)+v_i$ $r_5 = a'$, $r_2 = -b'$

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. TW	a'			− 2228.4	− 1839.1	− 1168.9	− 821.1	− 671.7	− 583.8	− 287.8	− 194.3	18.49
	t_a'			(0.529)	(0.483)	(0.376)	(0.299)	(0.259)	(0.234)	(0.131)	(0.0928)	(0.0987)
	b'			− 68.17	− 68.83	− 70.07	− 70.80	− 71.14	− 71.35	− 72.10	− 72.36	− 72.99
	t_b'			(0.836)	(0.933)	(1.16)	(1.33)	(1.42)	(1.47)	(1.70)	(1.78)	(2.01)
	$\bar{r}$			− 0.00	− 0.00	− 0.324	− 0.452	− 0.502	− 0.530	− 0.621	− 0.649	− 0.710
2. T	a'						246.17	348.4	408.8	613.8	679.0	828.1
	t_a'						(0.133)	(0.193)	(0.230)	(0.366)	(0.412)	(0.526)
	b'						− 79.30	− 79.61	− 79.80	− 80.46	− 80.68	− 81.21
	t_b'						(2.21)	(2.28)	(2.32)	(2.48)	(2.53)	(2.66)
	$\bar{r}$						− 0.751	− 0.763	− 0.770	− 0.794	− 0.802	− 0.819
3. W	a'	148184.8	−284684.0	− 89216.1	− 52930.4	− 20540.0	− 17206.2	− 14645.6	− 13300.3	− 9505.6	− 8502.9	− 6509.3
	t_a'	(1.46)	(1.44)	(1.42)	(1.41)	(1.28)	(1.32)	(1.30)	(1.28)	(1.22)	(1.20)	(1.14)
	b'	− 68.00	− 52.94	− 62.16	− 64.54	− 74.03	− 68.32	− 68.77	− 69.04	− 69.93	− 70.20	− 70.84
	t_b'	(0.0346)	(0.0138)	(0.0512)	(0.0886)	(0.237)	(0.271)	(0.315)	(0.344)	(0.465)	(0.512)	(0.640)
	$\bar{r}$	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00
4.	a'	− 4764.8	−25379.3	− 19177.0	− 15505.9	− 10225.7	− 7945.4	− 7054.9	− 6554.1	− 4995.4	− 4541.9	− 3578.0
	t_a'	(1.24)	(1.22)	(1.20)	(1.19)	(1.14)	(1.10)	(1.08)	(1.06)	(1.00)	(0.976)	(0.909)
	b'	− 36.58	− 51.51	− 55.99	− 58.78	− 63.03	− 65.02	− 65.84	− 66.31	− 67.83	− 68.30	− 69.33
	t_b'	(0.0490)	(0.128)	(0.181)	(0.232)	(0.362)	(0.464)	(0.519)	(0.555)	(0.701)	(0.757)	(0.909)
	$\bar{r}$	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00

Tab. Ⅳ－22　The structural parameter

Case ©

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. *TW*	r_1			− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2			68.17	68.83	70.07	70.80	71.14	71.35	72.10	72.36	72.99
	r_3			− 16.08	− 16.76	− 18.14	− 18.99	− 19.39	− 19.63	− 20.52	− 20.82	− 21.54
	r_4			4422.5	4142.1	3675.2	3445.6	3351.0	3296.7	3123.4	3072.4	2965.2
	r_5			− 2228.4	− 1839.1	− 1168.9	− 821.1	− 671.7	− 583.8	− 287.8	− 194.3	18.49
2. *T*	r_1						− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2						79.30	79.61	79.80	80.46	80.68	81.21
	r_3						− 28.58	− 28.70	− 28.78	− 29.07	− 29.17	− 29.41
	r_4						2651.9	2597.3	2566.2	2468.9	2441.1	2384.8
	r_5						246.2	348.4	408.8	613.8	679.0	828.1
3. *W*	r_1	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2	68.00	52.94	62.16	64.54	74.03	68.32	68.77	69.04	69.93	70.20	70.84
	r_3	1009.9	2287.9	771.9	484.1	208.4	186.3	163.2	150.8	114.9	105.0	84.84
	r_4	150887.0	312311.0	102456.3	63347.0	27943.7	24530.7	21717.4	20236.0	16046.5	14937.0	12730.7
	r_5	148184.8	−284684.0	− 89216.1	− 52930.4	− 20540.0.	− 17206.2	− 14645.6	− 13300.3	− 9505.6	− 8502.9	− 6509.3
4.	r_1	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0.	− 1.0	− 1.0	− 1.0	− 1.0
	r_2	36.58	51.51	55.99	58.78	63.03	65.02	65.84	66.31	67.83	68.30	69.33
	r_3	277.7	174.9	142.4	121.8	89.41	73.96	67.59	63.91	51.90	48.23	40.08
	r_4	49116.8	28658.3	22887.0	19447.7	14461.4	12292.4	11443.2	10965.3	9477.8	9045.8	8131.2
	r_5	− 47648.2	− 25379.3	− 19177.0	− 15505.9	− 10225.7	− 7945.4	− 7054.9	− 6554.1	− 4995.4	− 4541.9	− 3578.0

Tab. Ⅳ−23 The preference parameter $\left(\frac{W_i}{T-\frac{1}{2}h}\right) = -\overset{\alpha'}{\frac{a'}{b'}} + \overset{\beta'}{\frac{1}{b'}}\left(\frac{A_i}{T-\frac{1}{2}h}\right) + v_i \mid \frac{1}{H'_{2i}} = -\frac{a}{b} + \frac{1}{b}W_i + v_i$

Case Ⓓ

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. *TW*	α'			29.70	27.77	23.98	21.87	20.95	20.42	18.71	18.21	17.20
	t'_α			(1.1.13)	(1.09)	(1.01)	(0.978)	(0.969)	(0.965)	(0.966)	(0.973)	(1.00)
	β'			−.380(−2)	−.441(−2)	−.575(−2)	−.663(−2)	−.704(−2)	−.730(−2)	−.819(−2)	−.849(−2)	−.917(−2)
	t'_β			(0.836)	(0.933)	(1.16)	(1.33)	(1.42)	(1.47)	(1.70)	(1.78)	(2.01)
	$\bar{r}$			− 0.00	− 0.00	− 0.324	− 0.452	− 0.502	− 0.530	− 0.621	− 0.649	− 0.710
2. *T*	α'						17.17	17.49	17.68	18.42	18.67	19.27
	t'_α						(1.10)	(1.17)	(1.21)	(1.37)	(1.43)	(1.58)
	β'						−.894(−2)	−.906(−2)	−.914(−2)	−.937(−2)	−.944(−2)	−.961(−2)
	$t'_\beta{}'$						(2.21)	(2.28)	(2.32)	(2.48)	(2.53)	(2.66)
	$\bar{r}$						− 0.751	− 0.763	− 0.770	− 0.794	− 0.802	− 0.819
3. *W*	α'	52.75	50.96	49.53	48.07	42.40	40.77	39.00	37.83	33.18	31.46	27.12
	t'_α	(1.43)	(1.35)	(1.30)	(1.25)	(1.12)	(1.03)	(0.981)	(0.952)	(0.842)	(0.803)	(0.711)
	β'	−.880(−5)	−.181(−5)	−.211(−4)	−.606(−4)	−.369(−3)	−.517(−3)	−.686(−3)	−.810(−3)	−.140(−2)	−.165(−2)	−.240(−2)
	t'_β	(0.0346)	(0.0138)	(0.0512)	(0.0886)	(0.237)	(0.271)	(0.315)	(0.344)	(0.465)	(0.512)	(0.640)
	$\bar{r}$	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00
4.	α'	49.86	47.08	45.11	43.21	38.31	34.58	32.66	31.43	26.79	25.18	21.34
	t'_α	(1.50)	(1.36)	(1.28)	(1.21)	(1.05)	(0.948)	(0.898)	(0.868)	(0.759)	(0.723)	(0.642)
	β'	−.327(−4)	−.157(−3)	−.289(−3)	−.446(−3)	−.978(−3)	−.150(−2)	−.180(−2)	−.201(−2)	−.291(−2)	−.326(−2)	−.422(−2)
	t'_β	(0.0490)	(0.128)	(0.181)	(0.232)	(0.362)	(0.464)	(0.519	(0.555)	(0.701)	(0.757)	(0.909)
	$\bar{r}$	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00	− 0.00

Tab. Ⅳ−24

Case Ⓓ

$$\begin{cases} \dfrac{1}{H'_{2i}} = -\dfrac{a}{b} + \dfrac{1}{b} W_i + v_i \\ \dfrac{W_i}{T - \dfrac{h}{2}} = -\dfrac{a'}{b'} + \dfrac{1}{b'} \quad \dfrac{A_i}{T - \dfrac{h}{2}} + v_i \end{cases}$$

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	σ	0.20	0.25	0.2772	0.30	0.35	0.3835	0.40	0.4104	0.45	0.4642	0.50
1. TW	r_1			− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2			263.5	227.0	173.7	150.8	142.0	137.1	122.0	117.8	109.1
	r_3			− 16.08	− 16.76	− 18.14	− 18.99	− 19.39	− 19.63	− 20.52	− 20.82	− 21.54
	r_4			4422.5	4142.1	3675.2	3445.6	3351.0	3296.7	3123.4	3072.4	2965.2
	r_5			7826.9	6302.0	4165.5	3297.5	2974.8	2798.9	2283.7	2145.5	1875.6
2. T	r_1						− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2						111.8	110.3	109.5	106.7	105.9	104.1
	r_3						− 28.58	− 28.70	− 28.78	− 29.07	− 29.17	− 29.41
	r_4						2651.9	2597.3	2566.2	2468.9	2441.1	2384.8
	r_5						1919.5	1929.0	1935.8	1965.2	1976.6	2006.3
3. W	r_1	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2	113680.6	553253.7	47418.5	16512.9	2708.0	1934.3	1456.7	1234.8	716.3	605.4	417.0
	r_3	1009.9	2287.9	771.9	484.1	208.4	186.3	163.2	150.8	114.9	105.0	84.84
	r_4	150887.0	312311.0	102456.3	63347.0	27943.7	24530.7	21717.4	20236.0	16046.5	14937.0	12730.7
	r	5996958.6	28194090.	2348687.7	793830.7	114817.8	78855.6	56803.4	46713.9	23767.9	19048.5	11310.6
4.	r_1	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0
	r_2	30561.6	6377.4	3465.5	2241.8	1022.9	668.2	555.0	497.2	343.8	306.4	231.1
	r_3	277.7	174.9	142.2	121.8	89.41	73.96	67.59	63.91	51.90	48.23	40.08
	r_4	49116.8	28658.3	22887.0	19447.7	14461.4	12292.4	11443.2	10965.3	9477.8	9045.8	8131.2
	r_5	1523778.6	300275.2	156343.3	96873.7	39186.3	23107.7	18126.9	15630.5	9211.4	7713.4	5058.7

3) $\frac{\alpha W}{\alpha \Lambda} = \gamma_3 X + \gamma_4 u + \gamma_5 \Lambda$

and

4) $\frac{\alpha W}{\alpha X} = \gamma_1 X + \gamma_2 + \gamma_3 \Lambda$

respectively. Let the maximum and minimum values of u be u_{max} and u_{min}. Given a group of households with X_i^o and Λ_i^o, for a household whose marginal utility of leisure is at a maximum we have

5) $(\frac{\partial W}{\partial \Lambda})^{o\ i}_{max} = \gamma_3 X_i^o + \gamma_4 u_{max} + \gamma_5 \Lambda_i^o$,

and for household with minimun marginal utility of leisure we have

6) $(\frac{\partial W}{\partial \Lambda})^{o\ i}_{min} = \gamma_3 X_i^o + \gamma_4 u_{min} + \gamma_5 \Lambda_i^o$,

where u_{max} and u_{min} are given by

7) $E(\log u) + 3\sigma$

and

8) $E(\log u) - 3\sigma$

respectively. By the requirement,

9) $E(u) = 1$

we have

10) $E(\log u) = e^{-\frac{1}{2}\sigma^2}$

Examination of the satisfaction of conditions (1) and (2) is carried out for two kinds of households, i, e., households with

(a) $(\frac{\partial W}{\partial \Lambda})_{max}$ and $(\frac{\partial W}{\partial X})^{oi}$

and

(b) $(\frac{\partial W}{\partial \Lambda})_{min}$ and $(\frac{\partial W}{\partial \Lambda})^{oi}$

The results are given in Tab. Ⅳ-26.

Tab. Ⅳ −26 The results of test for the marginal utility and the stability condition. ($TW\sigma=0.2772$) №. 1

	σ	(3) 0.2772																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	test for m, U > 0					
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○	test for the stability condition					
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×						
2. TW 1962	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×				
3. TW 1963	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	×	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)$	×	×	×	×	×	×	×	×	×	×	×	×	×	×			
4. TW 1964	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{o.}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} mix, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($TW\sigma=0.30$)

No. 2

	σ	(4) 0.30																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×						
2. TW 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×				
3. TW 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×			
4. TW 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($TW\sigma = 0.35$)

No. 3

	σ	(5) 0.35																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×						
2. TW 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×				
3. TW 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×			
4. TW 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. $(TW\sigma=0.3835)$ No. 4

	σ				(6)					0.3835								
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×						
2. TW 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×				
3. TW 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×			
4. TW 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. $(TW\sigma=0.40)$ №. 5

	σ	(7) 0.40																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×						
2. TW 1962	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×				
3. TW 1963	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	×	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×			
4. TW 1964	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}mix, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition, ($TW\sigma = 0.4104$) No 6

	σ				(8)					0.4104								
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×						
2. TW 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	×				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×				
3. TW 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×			
4. TW 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. $(TW\sigma=0.45)$ No. 7

	σ	(9) 0.45																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\omega}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○						
2. TW 1962	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	×				
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×				
3. TW 1963	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○			
4. TW 1964	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	×	×
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} mix, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($TW\sigma=0.4642$) №. 8

	σ	(10) 0.4642																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○						
2. TW 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	×				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×				
3. TW 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	×	×			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○			
4. TW 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	×	×
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($TW\sigma = 0.50$) №　9

	σ	(11) 0.50																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1.　TW 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○						
2.　TW 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	×	×				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	○				
3.　TW 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	×	×			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○			
4.　TW 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	×	×	×	×
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	○

The results of test for the marginal utility and the stability condition. ($T\sigma = 0.3835$) No. 10

	σ	(6) 0.3835																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. *TW* 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	×	×	×						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○						
2. *TW* 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	○	○				
3. *TW* 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	×	×	×	×	×			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	○	○			
4. *TW* 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○	○	○

The results of test for the marginal utility and the stability condition. ($T\sigma=0.40$)

No. 11

	σ	(7) 0.40																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	×	×	×						
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max,\ \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min,\ \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○						
2. TW 1962	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	×	×	×	×				
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max,\ \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min,\ \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	○	○				
3. TW 1963	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	×	×	×	×	×			
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	○	○			
4. TW 1964	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	×	×	×	×	×	×	×
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}mix,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○	○	○

The results of test for the marginal utility and the stability condition. ($T\sigma=0.4104$) No. 12

	σ	(8) 0.4104																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	×	×	×	×						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○						
2. TW 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	○	○				
3. TW 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	×	×	×	×	×			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	○	○			
4. TW 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○	○	○

The results of test for the marginal utility and the stability condition. ($T\sigma=0.45$)

No 13

	σ	(9)							0.45									
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	×	×	×	×						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○						
2. TW 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	×	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	○	○				
3. TW 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	×	×	×	×	×			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	○	○			
4. TW 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○	○	○

The results of test for the marginal utility and the stability condition. ($T\sigma = 0.4642$) No. 14

	σ	(10) 0.4642																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	×	×	×	×						
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○						
2. TW 1962	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	×	×	×	×	×				
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	○	○				
3. TW 1963	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	×	×	×	×	×			
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	○	○			
4. TW 1964	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	×	×	×	×	×	×	×	×
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} mix, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○	○	○

The results of test for the marginal utility and the stability condition. ($T\sigma=0.50$)

No. 15

	σ	(11) 0.50																
	class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. TW 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	×	×	×	×						
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	×						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○						
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	○						
2. TW 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	×	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	○	○				
3. TW 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	×	×	×	×	×			
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	×			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○			
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	○	○			
4. TW 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} mix, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○	○	○

The results of test for the marginal utility and the stability condition. ($W\sigma=0.20,\ 0.25$) №16

	σ	(1) 0.20											(2) 0.25										
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
9. *W* 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
10. *W* 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
11. *W* 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
12. *W* 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition.($W\sigma = 0.2772,\ 0.30$) No. 17

	σ	(3) 0.2772											(4) 0.30										
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
9. W 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{max}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{min}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{max}, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{min}, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
10. W 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{max}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{min}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{max}, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{min}, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
11. W 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{max}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{min}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{max}, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{min}, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
12. W 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{max}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{min}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{max}, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} \mathit{min}, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($W\sigma = 0.35,\ 0.3835$) № 18

	σ	(5) 0.35											(6) 0.3835										
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
9. W 1961	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
10. W 1962	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
11. W 1963	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
12. W 1964	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($W\sigma$=0.40, 0.4104) No 19

	σ	(7) 0.40											(8) 0.4104										
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
9. W 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
10. W 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
11. W 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
12. W 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($W\sigma=0.45$, 0.4642) № 20

	σ	(9) 0.45											(10) 0.4642										
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
9. W 1961	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
10. W 1962	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
11. W 1963	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
12. W 1964	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}max,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi}min,\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($W\sigma = 0.50$) No. 21

	σ	(11) 0.50																					
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
9. *W* 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○															
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○															
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○															
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	○	○	○	○	○	○															
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×															
10. *W* 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial \Lambda}\right) max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial \Lambda}\right) min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×											
11. *W* 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×											
12. *W* 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	○	○	○	○	○	○	○	○	○	○											
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×											

The results of test for the marginal utility and the stability condition. (σ=0.20, 0.25) No. 22

	σ	(1) 0.20											(2) 0.25										
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
13. W 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
14. W 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
15. W 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
16. W 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($\sigma = 0.2772,\ 0.30$)　No 23

	σ	(3) 0.2772											(4) 0.30										
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
13. *W* 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
14. *W* 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
15. *W* 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
16. *W* 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($\sigma = 0.35,\ 0.3835$) №　24

	σ	(5) 0.35											(6) 0.3835										
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
13. *W* 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
14. *W* 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
15. *W* 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
16. *W* 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($\sigma = 0.40,\ 0.4104$) №　25

	σ	(7)					0.40						(8)					0.4104					
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
13. W 1961	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
14. W 1962	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
15. W 1963	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
16. W 1964	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} max, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\left(\frac{\partial\omega}{\partial\Lambda}\right)^{oi} min, \left(\frac{\partial\omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

The results of test for the marginal utility and the stability condition. ($\sigma = 0.45$, 0.4642) No. 26

	σ	(9) 0.45											(10) 0.4642										
	class	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
13. W 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	○	○	○	○	○	○					○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×					×	×	×	×	×	×	×				
14. W 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
15. W 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
16. W 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

No 27

The results of test for the marginal utility and the stability condition. ($\sigma = 0.50$)

	σ	(11) 0.50										
	class	1	2	3	4	5	6	7	8	9	10	11
13. *W* 1961	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○				
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×				
14. *W* 1962	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×
15. *W* 1963	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	×
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×
16. *W* 1964	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} max, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	○	○	○	○	○	○	○	○	○	○	○
	$\left(\frac{\partial \omega}{\partial \Lambda}\right)^{oi} min, \left(\frac{\partial \omega}{\partial X}\right)^{oi}$	×	×	×	×	×	×	×	×	×	×	×

It is observed that the conditions (1) are satisfied for most principal earner's income classes. However, condition (2) is not satisfied except for some higher income classes. Tab. Ⅳ-27 pesents a summary of types of partial satisfaction of (1) and (2),

Tab. Ⅳ-27

Type of satisfaction condition	1	2	3	4	5
$(\frac{\partial \omega}{\partial \Lambda})_{max} > 0$	○	○	○	○	○
$(\frac{\partial \omega}{\partial \Lambda})_{min} > 0$	○	○	X	X	X
$(\frac{\partial \omega}{\partial X}) > 0$	○	○	○	○	X
(2) for the case $(\frac{\partial \omega}{\partial \Lambda})_{max}$ paired by $(\frac{\partial \omega}{\partial X})^{o}$ i	X	○	○	○	○
(2) for the case $(\frac{\partial \omega}{\partial \Lambda})_{min}$ paired by $(\frac{\partial \omega}{\partial X})^{o}$ i	X	X	X	○	○

where 0 and X respectively indicate that the corresponding condition is fulfilled and not fulfilled.

In the second place, we reduced the range of u under examination, u_{max} and u_{min}: that is replacing (7) and (8) by

$$E\ (\log u)\ \pm 2\sigma$$

and

$$E\ (\log u)\ \pm \sigma$$

alternatively. The results, however, were not satisfactory.

For the other cases, (B) (C) and (D), the above mentioned test for stability conditions was carried out. These tests were conducted employing a u within the ranges given by (7) and (8). However, a satisfactory set of parameters could not be found.

Satisfaction of the theoretical requirement that the MHLS curve be downward sloping was examined as well. The result of examination was unsatisfactory. The results of the test are summarized in Tab. Ⅳ-28.
Test items are as follows.

1. MHLS curve be downwards sloping $(\frac{\alpha \Lambda}{\alpha I} > 0)$
2. marginal utility of income be positive $((\frac{\alpha \omega}{\alpha X})^{oi} > 0)$
3. $H_2' < 0$
4. $H_0 > I_{max}$
(5. $\sigma > 0$)
6. $(\frac{\alpha \omega}{\alpha \Lambda})^{ci}_{max} > 0$, $(\frac{\alpha \omega}{\alpha \Lambda})^{oi}_{min} > 0$.
7. satisfaction of relation (2) in <4.8.2>.

Tab. Ⅳ-28

set of param. type of DATA	1	2	3	4	5	6	7	8	9	10	11
1 (*TW*)			2	2	2	2	2	2	1	1	1
2 (*T*)						1	1	1	1	1	1
3 (*W*)	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	6	6	6	6	6	6	6	6

The numbers in the table stand for condition 1 through 7 by which the estimated parameters were rejected.

Hence, the method of estimation of preference parameters outlined in 4.8.1 is clearly not adequate.

While a range for the trial values of σ might be extended to obtain estimate for parameters, it seems that more fundamental problem in estimating the parameters of the PECI equation exists. This we discuss in the first part of the Chapter V, after which we shall reestimate the parameters.

[4.8.3] ADDEMDUM

The wage rate w in equation (1) in <4.8.1>, as well as in the previous sections, stands for the representative or average wage rate of employment opportunity offered to non principal potential earners' of type A households. (This point was explained in § Ⅲ (3.2.8) and 3.3.
While some wives work others of course do not although they are afforded the same wage rates. Now, wives' earnings reported in Japanese FIES data for type A households refer only to wives who are actually working. Accordingly, wives' earnings reported in FIES are not suitable for computing w in the model. Hence, we used monthly earnings for female employees as reported in the monthly wage survey (MAITSUKI KINRO TOKEI) for ωh. These values for earnings will be more suitable for computing w which stands for the average value of all the potential wage rates for employment opportunities confronting all type A household wives. By dividing earnings wh by hours of work h , w can be computed, h being a parameter to be estimated.

§ V Numerical Determination of Preference Parameters

- second estimation -

5.1 Basic procedure

5.1.1 Examination of the first Estimation

The unsuccessful results in the previous section 4.8 could be interpreted in two ways; one of which is that the model itself is inconsistent with the observations. The other is that the method of estimation adopted was inadequate. As far as the analyses in 4.1 through 4.6 are concerned, however, we have found there certainly exists considerable coincidence between our theoretical specification and the observed data. Hence, at the present stage of analysis, the estimation method should be reconsidered.

5.1.2 Need for Second Estimation

In the previous section, 4.8, parameters, H_0, H_2 and σ, of the reduced form PECI equation were estimated so as to minimize the sum of squares of residuals in the equation. If the residuals, v's, are distributed normally, the estimated parameters can be regarded as a sort of maximum likelihood estimates as well. Making use of theoretical relations between the reduced form parameters and the structural parameters, i.e., the parameters of preference function, the estimates of the latter were obtained. This procedure is shown in Fig. V-1.

Fig. V-1a shows the structural parameter space and Fig. V-1b stands for the reduced form parameter space. Parameter σ can be regarded as a structural parameter as well as a reduced form parameter, so that σ appears on the abscissa in both figures. For the convenience of graphical presentation, let us assume the sets of structural parameters (except for σ) and those of reduced form parameters are scaled on the vertical axis of both figures respectively. Let point A' in Fig. V-1b stand for a set of estimates of reduced form parameters. Corresponding to point A' we have point A in the structural parameter space which stands for a set of structural (preference) parameters obtained by the procedure mentioned in <4.8.2>. Now, in the structural parameter space, let a region satisfying the theoretical constraint mentioned in <4.8.2> be T. Suppose that a set of true values of structural parameters is shown by point α in the region T. Given point α in the structural parameter space, there exists a set of true reduced form parameters corresponding to α, which is shown by point α' in the reduced form parameter space. Because α' may be different from A', we could fail to obtain a set of true structural parameters satisfying the theoretical constraints.

Suppose the set of reduced form parameters obtained in <4.8.2> is shown by point A'. Although A' was obtained by a sort of maximum likelihood method, there is no certainty that A' should coincide with the trueset of parameters, α'. In relation to this, one point should be made. In our experience of analysis in <4.8.2>, changes in the values of structural parameters,

Fig. V−1

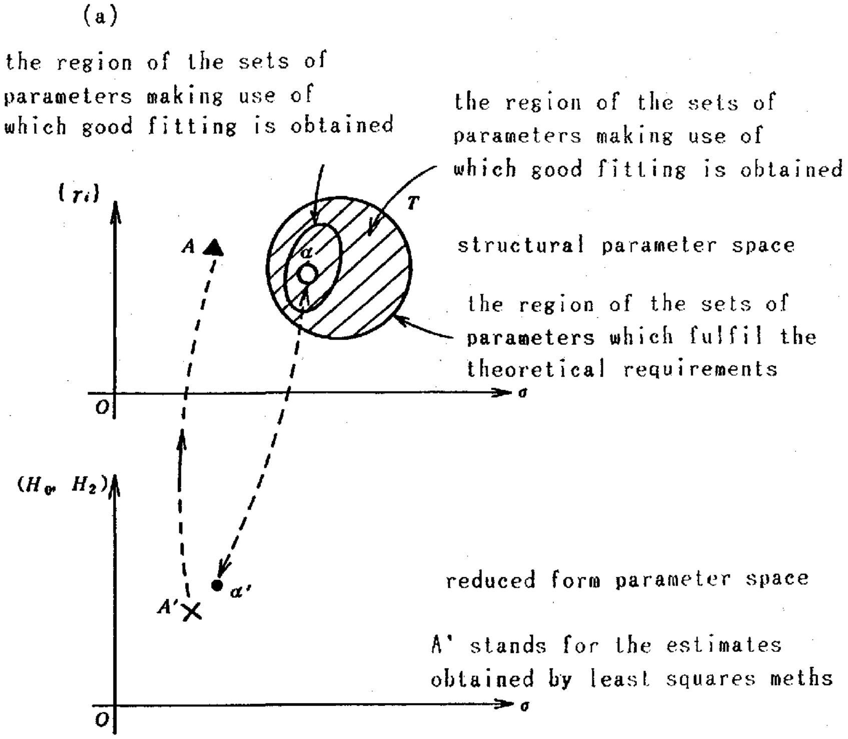
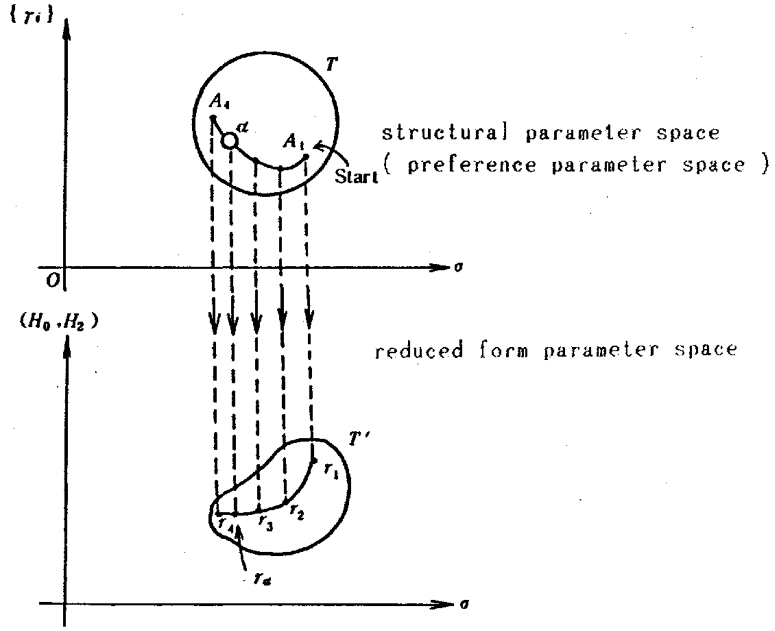

(a)
the region of the sets of parameters making use of which good fitting is obtained
the region of the sets of parameters making use of which good fitting is obtained
{γi}
A
α
T
structural parameter space
the region of the sets of parameters which fulfil the theoretical requirements
O
σ
(H0, H2)
α'
A'
reduced form parameter space
A' stands for the estimates obtained by least squares meths
O
σ

Fig. V−2


{γi}
T
A4
α
A1
Start
structural parameter space
(preference parameter space)
O
σ
(H0 ,H2)
reduced form parameter space
T'
r1
r2
r3
r4
rα
σ

γ_k (k=1, ……, 5) and σ, did not affect very much changes in the values of reduced form parameters. (This may have stemmed from the nature of observed data and partly from the analytical characteristics of the model.) Although the structural parameters are identifiable, the above mentioned fact seems to have prevented us from obtaining true values of structural parameters.

<5.1.3> Second Estimation

(5.1.3.1)

Suppose there exists a region, T, in the structural parameter space where the theoretical constraints on the parameters are satisfied. The theoretical constraints consist of stability conditions and the condition of a descending MHSL curve etc. In region T, in Fig. V-2a we shall search a set of structural parameters which gives minimum $\sum V^2$, where V stands for the residual in reduced form equation.

The following procedure is adopted. (1) Point A_1 which fulfilles the restriction T is tentatively selected in the structural parameter space shown in Fig. V-2a. Corresponding to A_1, a set of reduced form parameters, shown by point γ_1 in the reduced form parameter space shown by Fig. V-2b, can be calculated. The same procedure is applied to each point A_2, ……, A_4 etc. in the structural parameter space. Among the γ_i's (i=2,3,4) etc. which, respectively, correspond to A_i's (i=2,3,4) etc., we can choose γ_α, for example, which gives minimum values for the sum of squares of residuals. A set of the structural (preference) parameters shown by α in Fig. V-2a which corresponds to γ_α in Fig. V-2b is selected as a set of estimates for the preference parameters.

In this second estimation, we also wish to check if the values of preference parameters $\{\gamma\}$ and σ obtained are stable over the years. Accordingly, the estimates for parameters were computed for each year.

(5.1.3.2)

Theoretical requirements that structural parameters should satisfy are as follows:

a) the MHSL curve is descending

b) $\frac{\alpha\,\omega}{X} > 0$

c) $\frac{\alpha\,\omega}{\Lambda} > 0$

d) $2\gamma_3 \frac{\alpha\,\omega}{\alpha\,\Lambda} \cdot \frac{\alpha\,\omega}{\alpha\,X} - \gamma_5 \left(\frac{\alpha\,\omega}{\alpha\,X}\right)^2 - \gamma_1 \left(\frac{\alpha\,\omega}{\alpha\,\Lambda}\right)^2 > 0$

The parameters of PECI equation (reduced form) have to satisfy the following conditions:

e) $H_0 > I_{max}$,

f) $H_2' < 0$, and

g) $\sigma > 0$.

I_{max} in (e) stands for the principal earner's income of the highest observed principal earner's income class with a positive (non-zero) participation ratio in each year. In this principal earner's income class, there exists at least one household with a working wife. The principal earner's income of this household must be less than PECI of this household. H_0 stands for the maximum value of PECI for the households under consideration as shown in (4.6), and if the principal earners' income of the households in a given group exceed this value, H_0, the participation ratio, must be zero. consegvently H_0 must be larger than the principal earner's income in the highest income class with a positive (non-zero) participation ratio. Condition (f) was discussed in § Ⅳ. (g) follows from the standardization of the random variable u.

The conditions (a) through (d) should be satisfied within the following ranges of income, X, leisure, Λ, assigned hours of work, h, and random veriable, u.

1) $I_{min} \leqq X \leqq I_{max}$

2) $0.25 \leqq \Lambda \leqq 1.0$

3) $0.25 \leqq h \leqq 0.50$

4) $u_{min} \leqq u \leqq u_{max}$,

where u_{max} and u_{min} stand for E (log u) $+3\sigma$ and E (log u) -3σ respectively.

In addition to those, we shall specify contours of indifference map to be ellipsoid, that is

$$\gamma_1 < 0 \text{ , and}$$
$$\gamma_3^2 - \gamma_1 \gamma_5 < 0 \text{ .}$$
$$\gamma_5 < 0 \text{ .}$$

Hence, from condition (b), for example, we obtain a feasible region for γ_2 and γ_3 : By normalization $\gamma_1 \equiv -1$, we have

$$\frac{\omega}{X} = -X + \gamma_2 + \gamma_3 \Lambda > 0$$

or $$\gamma_3 > \frac{X}{\Lambda} - \frac{1}{\Lambda} \gamma_2 \text{ .}$$

Making use of

$$\gamma_3 = \frac{X}{\Lambda} - \frac{1}{\Lambda} \gamma_2 ,$$

we observe

$$\gamma_2 = 0 \qquad \text{if} \qquad \gamma_3 = \frac{X}{\Lambda}$$

$$\text{if} \qquad \gamma_3 = 0 \qquad \text{we have} \qquad \gamma_2 = X .$$

Here, we shall assign the regions given by the for X and Λ.
Let an arbitrary value of income be I_0, where

$$I_{min} < I_0 < I_{max}$$

We have $$I_0 / \Lambda_{max} < I_0 / \Lambda_{min} \text{, and}$$

$$I_{max} / \Lambda_{max} < I_{max} / \Lambda_{min}$$

It can be seen that the inequality

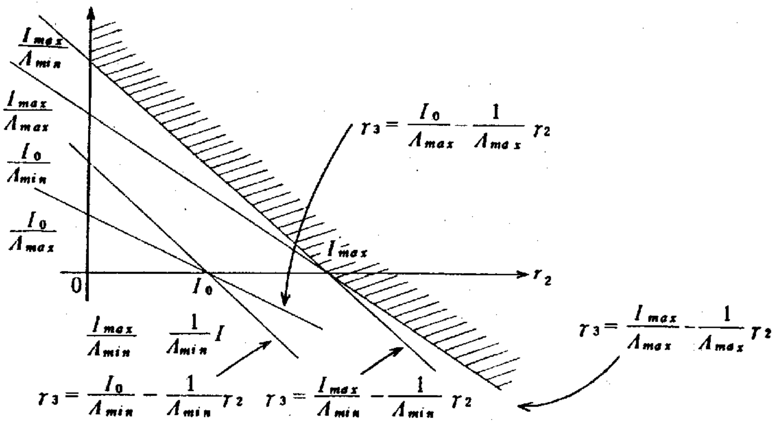
$$I_0 / \Lambda_{min} < I_{max} / \Lambda_{max}$$

holds for the observed data, so we obtain

$$0 < \frac{I_0}{\Lambda_{max}} < \frac{I_0}{\Lambda_{min}} < \frac{I_{max}}{\Lambda_{max}} < \frac{I_{max}}{\Lambda_{min}}$$

Taking into account these inequalities, we can depict a feasible region for γ_2 and γ_3, for example, as shown by the hatched area in Fig. V-3.

Fig V-3 The regions for γ_2 and γ_3 where $\partial w/\partial x > 0$ holds.



The conditions (a) through (g) in (5.1.3.2) can be rewritten in terms of γ_i $(i=1, \cdots\cdots, 5)$ and σ as follows.

(a) $\gamma_3^2-\gamma_1\gamma_5<0$ $\cdots\cdots\ \gamma_5<0$
$\gamma_1 w-\gamma_3<0$ (ellipsoid condition)

(b) when $\gamma_3 \geqq 0$,
if $\gamma_2<0$, $-I_{max}+\gamma_2+\gamma_3\Lambda_{min}>0$
if $\gamma_2>0$, $-I_{max}+\gamma_2+\gamma_3\Lambda_{min}>0.$ γ_1 -1
when $\gamma_3<0$, $-I_{max}+\gamma_2+\gamma_3\Lambda_{max}>0.$ (see Fig V−3)

(c) when $\gamma_3>0$ and $\gamma_4<0.$
$\gamma_3 I_{min}+\gamma_4 u_{max}+\gamma_5\Lambda_{max}>0.$
when $\gamma_3>0$ and $\gamma_4>0,$
$\gamma_3 I_{min}+\gamma_4 u_{min}+\gamma_5\Lambda_{max}>0.$
when $\gamma_3<0$ and $\gamma_4>0,$
$\gamma_3 I_{max}+\gamma_4 u_{min}+\gamma_5\Lambda_{max}>0.$

However, the first case in (c) contradicts the theoretical requirement

$$H_2' < 0$$

so this case should be excluded.

The condition that the countour be concave to the origin is given by

(d) $$2\gamma_3(\gamma_3 x+\gamma_4 u+\gamma_5\Lambda)(\gamma_1 x+\gamma_2+\gamma_3\Lambda)-\gamma_5(\gamma_1 x+\gamma_2+\gamma_3\Lambda)^2 -\gamma_1(\gamma_3 x+\gamma_4 u+\gamma_5\Lambda)^2>0,$$

where two extreme values of u, u_{max} and u_{min} , are applied.

With respect to Λ, the values Λ_{max} and Λ_{min} are applied. With respect to X, the values I_{max} and I_{min} are applied.

(e) $H_0 > I_{max}$,

where $$H_0 = \frac{-\gamma_2 w - \gamma_3 w(T-h) + \gamma_5(T-\frac{1}{2}h) - \frac{1}{2}\gamma_1 w^2 h}{\gamma_1 w - \gamma_3}$$

(f) $H_2' < 0$

where $$H_2' = \frac{\gamma_4}{\gamma_1 w - \gamma_3}$$

(g) $\sigma > 0$,

where σ is given by

$$\sigma = \frac{\Sigma \frac{\log(H_0 - I)}{H_0 - I} - \frac{1}{n}\Sigma \log(H_0 - I) \cdot \Sigma \frac{1}{H_0 - I}}{\Sigma \frac{u^*}{H_0 - I} - \frac{1}{n}\Sigma u^* \cdot \Sigma(\frac{1}{H_0 - I})}$$

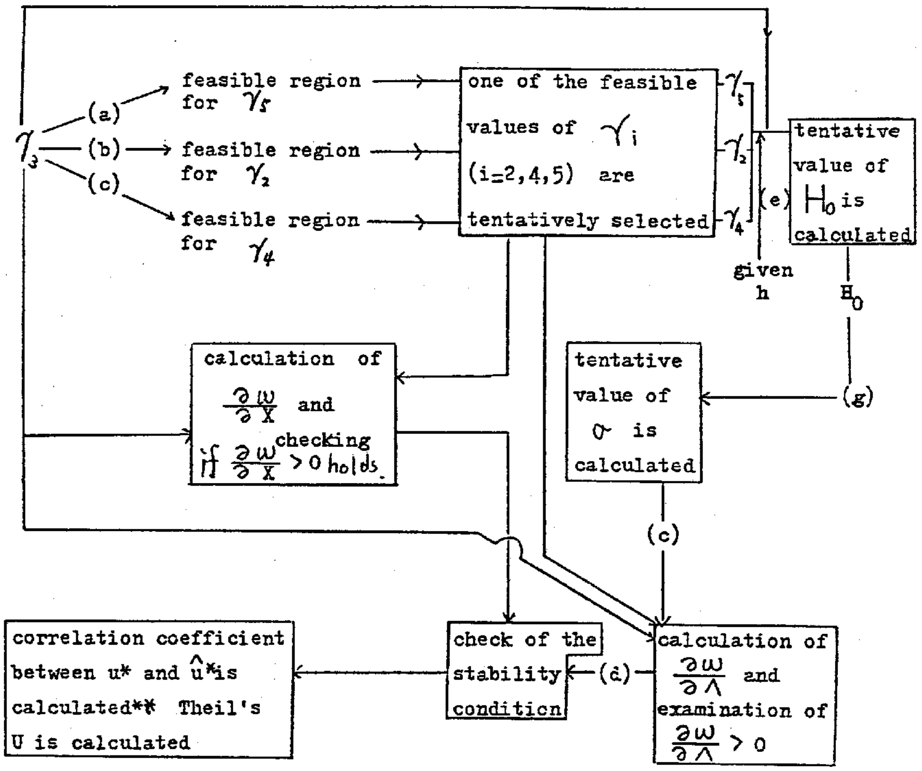
By these relations and inequalities, structural parameters are restricted. (see Tab. V-1)

Tab. V − 1

relation	variables whose values are given	parameters constraint by reiations
(a)	w	γ_3 γ_5
(b)	Imax Λmax Λmin	γ_2 γ_3 γ_5
(c)	Imax Imin Λmax Umin	γ_3 γ_4 γ_5
(d)	Umax Umin Λmax Λmin Xmax Xmin	γ_2 γ_3 γ_4 γ_5
(e)	w,h	γ_2 γ_3 γ_5
(f)	w	γ_3 γ_4
(g)	wh	γ_2 γ_3 γ_4

As can be seen from Table V-1, γ_3 is affected by all the theoretical constraints. Consequently, we first assign a trial value for γ_3. The feasible values of other parameters satisfying the restrictions are then obtained in order. (see Fig. V-4)

Fig. V−5



** cf. [4.7.3.3] equation 2)

Tentative ranges for the preference parameters were given by consulting Tab. Ⅳ-10 and Ⅳ-17 in § Ⅳ, as follows.(*)

	1	2	3	4	5
γ_2	50	100	150	200	
γ_4	6000	7000	8000	9000	
γ_5	−400	−1400	−2400	−3400	
γ_3	−10	0	10	20	30

Tab. V-2

(*) The values for σ's are computed using given set of γ_6's.

Sets of parameters satisfying the conditions for each year are shown in Tab.V-3. From Tab.V-3 distributions of the computed values for σ were obtained. These are shown in Tab.V-4. The following points should be noted.

a Obtained values of σ tend to be larger when $h=0.25$ is applied compared to the case when $h=0.5$ is applied.

b There seems to be no systematic time serial variation in σ.

c The range of calculated values of σ is smaller when we adopt $h=0.25$ for each year, than when $h=0.5$ is applied.

d Frequency of calculated values of σ is high for the range $\sigma=0.4$ through $\sigma=0.5$.

e For the year 1961, a satisfactory set of parameters computed by using the whole sample was not found. However, tentatively adopted sets of parameters are of discrete value, so that a satisfactory set might have been left between some two adjacent sets of parameters examined.

Examining the results of the experiment carried out, it was found that no observations were found which contradicted the hypothesis that parameters σ and γ_i are approximately constant over time. Consegvently, we shall adopt the sets of parameters obtained for the final year 1964 as the first approximation for a true set of preference parameters which is assumed to be common to the four years, 1961 through 1964. The sets of parameters which yield $R>0.9$ are adopted amongst those in Tab.V-5. Making use of each set obtained from the data for the year 1964, we can calculate H_0 and H_2 respectively ($t=1961$, 1962, 1963). These values are shown in lower half of Tab. V-5. Hence, we have a concrete equation for the year t,

$$I_j(t)=H_0^{(t)}+H_2^{(t)}e^{\sigma u*j(t)}$$

where j stands for the principal earner's income class.

Tab. V－3

Check for the Theoretical Requirements

$$\left[\frac{\partial \Lambda}{\partial X} < 0,\ \frac{\partial \omega}{\partial X} > 0,\quad \left(\frac{\partial \omega}{\partial \Lambda} > 0,\ \begin{matrix} H_0 > I_{max} \\ H_2' < 0 \end{matrix}\right)\right]$$

1961

No.	sample	weight	γ_3	γ_2	γ_4	γ_5	h	γ_2	γ_4	γ_5	σ	H_0	H_2	$\sum u^2$	Tu	R
1	*T*	*W*	-10	2	1	1	1	100	6000	-400	0.333	164.6	-363.8	37.31	0.406	0.637
2	*T*	*W*	0	2	1	1	1				0.753	105.3	-176.5	1.35	0.130	0.596
3	*T*		-10	2	1	1	1				0.504	164.6	-338.6	5.17	0.206	0.637
4		*W*	-10	2	1	1	1				0.181	164.6	-378.4	145.3	0.623	0.784
5		*W*	-10	2	1	1	2				0.293	115.5	-139.5	2.39	0.177	0.783
6		*W*	-10	2	1	2	1			-1400	0.132	212.7	-381.3	137.0	0.616	0.785
7		*W*	-10	2	1	2	2				0.231	136.7	-141.8	0.574	0.100	0.784
8		*W*	-10	2	1	3	1			-2400	0.104	260.8	-382.6	96.6	0.574	0.785
9		*W*	-10	2	1	3	2				0.191	158.0	-143.0	2.91	0.287	0.784
10		*W*	-10	2	1	4	1			-3400	0.0855	308.9	-383.2	46.06	0.482	0.785
11			-10	2	1	1	1			-400	0.173	164.6	-378.9	163.4	0.637	0.784
12			-10	2	1	1	2				0.279	115.5	-140.1	3.18	0.198	0.783
13			-10	2	1	2	1			-1400	0.126	212.7	-381.6	153.7	0.629	0.785
14			-10	2	1	2	2				0.221	136.7	-142.1	0.683	0.106	0.784
15			-10	2	1	3	1			-2400	0.0993	260.8	-382.7	108.9	0.588	0.785
16			-10	2	1	3	2				0.182	158.0	-143.2	2.57	0.264	0.784
17			-10	2	1	4	1			-3400	0.0819	308.9	-383.3	52.5	0.498	0.785

1962

No.	sample	weight	γ_3	γ_2	γ_4	γ_5	h	γ_2	γ_4	γ_5	σ	H_0	H_2	$\sum u^2$	Tu	R
1	*T*	*W*	-10	2	1	1	1	100	6000	-400	0.249	154.4	-326.8	100.7	0.515	0.860
2	*T*	*W*	-10	2	1	1	2				0.393	112.0	-121.8	0.895	0.101	0.860
3	*T*	*W*	-10	2	1	2	1			-1400	0.184	196.5	-331.5	91.19	0.503	0.859
4	*T*	*W*	-10	2	1	2	2				0.311	131.2	-125.4	3.00	0.208	0.860
5	*T*	*W*	-10	2	1	3	1			-2400	0.145	238.6	-333.5	58.4	0.447	0.858
6	*T*	*W*	0	2	1	1	1			-400	0.443	103.8	-195.6	16.7	0.302	0.859
7	*T*	*W*	0	2	1	1	2				0.478	99.3	-96.3	2.93	0.204	0.858
8	*T*	*W*	0	2	1	2	1			-1400	0.312	130.8	-205.6	18.94	0.315	0.860
9	*T*	*W*	0	2	1	2	2				0.376	115.1	-100.5	6.36	0.331	0.860
10	*T*	*W*	0	2	1	3	1			-2400	0.242	157.8	-209.6	12.18	0.270	0.860
11	*T*	*W*	10	2	1	1	1			-400	0.743	80.0	-120.4	0.955	0.0975	0.848
12	*T*	*W*	10	2	1	1	2				0.567	90.6	-77.9	6.16	0.325	0.856
13	*T*	*W*	10	2	1	2	1			-1400	0.473	99.9	-141.9	2.93	0.155	0.858
14	*T*	*W*	10	2	1	2	2				0.443	103.9	-82.9	9.82	0.438	0.859

* Figures in the columns for γ_3 , γ_4 and γ_5 correspond to those in the first row in Tab. V-2.

Serch for Preference Parameters
the second estimate making use of the employed-model

15	*T*		-10	2	1	1	1			-400	0.318	154.4	-320.5	44.42	0.415	0.860
16	*T*		-10	2	1	1	2				0.507	112.0	-115.7	3.34	0.227	0.860
17	*T*		-10	2	1	2	1			-1400	0.233	196.5	-328.0	40.87	0.405	0.859
18	*T*		0	2	1	1	1			-400	0.575	103.8	-182.9	3.22	0.161	0.859
19	*T*		0	2	1	1	2				0.622	99.3	- 88.9	7.61	0.386	0.858
20	*T*		0	2	1	2	1			-1400	0.400	130.8	-199.2	4.78	0.189	0.860
21		*W*	-10	2	1	1	1			-400	0.212	154.4	-329.6	137.7	0.583	0.936
22		*W*	-10	2	1	1	2				0.325	112.0	-124.8	0.825	0.0975	0.941
23		*W*	-10	2	1	2	1			-1400	0.157	196.5	-332.9	122.1	0.568	0.934
24		*W*	-10	2	1	2	2				0.261	131.2	-127.2	0.672	0.103	0.938
25		*W*	-10	2	1	3	1			-2400	0.125	238.6	-334.4	79.74	0.515	0.932
26		*W*	-10	2	1	3	2				0.219	150.4	-128.5	6.86	0.412	0.936
27		*W*	-10	2	1	4	1			-3400	0.104	280.8	-335.3	32.37	0.403	0.931
28		*W*	0	2	1	1	1			-400	0.363	103.8	-202.1	32.49	0.403	0.942
29		*W*	0	2	1	1	2				0.388	99.3	-100.1	0.379	0.0746	0.943
30		*W*	0	2	1	2	1			-1400	0.262	130.8	-208.5	32.09	0.402	0.938
31		*W*	0	2	1	2	2				0.313	115.1	-102.8	2.58	0.223	0.940
32		*W*	0	2	1	3	1			-2400	0.206	157.8	-211.3	20.92	0.352	0.936
33		*W*	0	2	1	3	2				0.262	130.8	-104.3	9.98	0.526	0.938
34		*W*	0	2	1	4	1			-3400	0.169	184.8	-212.8	7.35	0.243	0.934
35			-10	2	1	1	1			-400	0.222	154.4	-328.9	120.1	0.566	0.936
36			-10	2	1	1	2				0.341	112.0	-124.1	0.473	0.0761	0.941
37			-10	2	1	2	1			-1400	0.165	196.5	-332.5	106.4	0.551	0.934
38			-10	2	1	2	2				0.274	131.2	-126.7	0.966	0.126	0.938
39			-10	2	1	3	1			-2400	0.131	238.6	-334.2	68.9	0.497	0.932
40			-10	2	1	3	2				0.230	150.4	-128.2	7.40	0.436	0.936
41			-10	2	1	4	1			-3400	0.109	280.8	-335.1	27.20	0.383	0.931
42			0	2	1	1	1			-400	0.381	103.8	-200.7	26.87	0.381	0.942
43			0	2	1	1	2				0.408	99.3	- 99.3	0.603	0.0971	0.943
44			0	2	1	2	1			-1400	0.275	130.8	-207.8	26.72	0.380	0.938
45			0	2	1	2	2				0.328	115.1	-102.2	3.12	0.252	0.940
46			0	2	1	3	1			-2400	0.216	157.8	-210.9	17.13	0.329	0.936
47			0	2	1	3	2				0.275	130.8	-103.9	10.52	0.549	0.938
48			0	2	1	4	1			-3400	0.178	184.8	-212.4	5.6.3	0.220	0.934

1963

No.	sample	weight	r_3	r_2	r_4	r_5	h	r_2	r_4	r_5	σ	H_0	H_2	$\sum u^2$	Tu	R
1	*T*	*W*	-10	2	1	1	1	100	6000	-400	0.227	148.3	-301.4	141.4	0.564	0.735
2	*T*	*W*	-10	2	1	1	2				0.361	110.0	-115.2	2.61	0.163	0.705
3	*T*	*W*	-10	2	1	2	1			-1400	0.167	187.0	-305.0	129.0	0.552	0.747
4	*T*	*W*	-10	2	1	2	2				0.283	127.7	-118.1	3.53	0.214	0.723
5	*T*	*W*	-10	2	1	3	1				0.132	225.6	-306.6	84.53	0.499	0.754
6	*T*	*W*	0	2	1	1	1			-400	0.406	102.9	-187.9	29.42	0.372	0.695
7	*T*	*W*	0	2	1	1	2				0.441	98.6	- 92.6	3.67	0.218	0.687
8	*T*	*W*	0	2	1	2	1			-1400	0.281	128.4	-196.2	34.18	0.389	0.724
9	*T*	*W*	0	2	1	2	2				0.341	113.5	- 96.3	6.49	0.317	0.710
10	*T*	*W*	0	2	1	3	1			-2400	0.216	153.9	-199.4	23.87	0.347	0.737
11	*T*	*W*	0	2	1	3	2				0.281	128.4	- 98.1	15.42	0.552	0.724
12	*T*	*W*	0	2	1	4	1			-3400	0.176	179.4	-200.9	9.42	0.252	0.745
13	*T*	*W*	10	2	1	1	1			-400	0.753	80.5	-114.7	3.21	0.178	0.618
14	*T*	*W*	10	2	1	1	2				0.530	90.7	- 75.8	6.66	0.322	0.666
15	*T*	*W*	10	2	1	2	1			-1400	0.433	99.5	-138.6	8.52	0.245	0.688
16	*T*	*W*	10	2	1	2	2				0.402	103.4	- 80.4	9.88	0.417	0.696
17	*T*		-10	2	1	1	1			-400	0.391	148.3	-286.5	22.15	0.342	0.735
18		*W*	-10	2	1	1	1				0.186	148.3	-304.0	179.7	0.628	0.945
19		*W*	-10	2	1	1	2				0.279	109.8	-118.3	1.82	0.144	0.946
20		*W*	-10	2	1	2	1			-1400	0.139	186.9	-306.3	155.4	0.610	0.945
21		*W*	-10	2	1	2	2				0.226	127.7	-119.8	0.460	0.0874	0.946
22		*W*	-10	2	1	3	1			-2400	0.111	225.6	-307.4	99.8	0.557	0.945
23		*W*	-10	2	1	3	2				0.190	145.6	-120.7	7.56	0.463	0.945
24		*W*	-10	2	1	4	1			-3400	0.0928	264.3	-307.9	39.8	0.441	0.944
25			-10	2	1	1	1			-400	0.183	148.3	-304.1	186.4	0.632	0.945
26			-10	2	1	1	2				0.275	109.8	-118.4	2.04	0.151	0.946
27			-10	2	1	2	1			-1400	0.137	187.0	-306.4	161.0	0.615	0.945
28			-10	2	1	2	2				0.223	127.7	-119.9	0.411	0.0820	0.946
29			-10	2	1	3	1			-2400	0.110	225.6	-307.4	103.5	0.561	0.945
30			-10	2	1	3	2				0.188	145.6	-120.8	7.43	0.457	0.945
31			-10	2	1	4	1			-3400	0.0917	264.3	-307.9	41.4	0.446	0.944

H_0, H_1 is calculated at γ_i.

T of sample column indicates the case of all samples, and non-mark indicates where sample size is more than 50. W of weight column indicates weight by the number of households in a class and non-mark indicates non-weight.

1, 2, etc. of γ_3, γ_4, γ_5 columns are informal marks by level.

1964

No.	sample	weight	r_3	r_2	r_4	r_5	h	r_2	r_4	r_5	σ	H_0	H_2	$\sum u^2$	Tu	R
1	*T*	*W*	-10	2	1	1	1	100	6000	-400	0.292	141.3	-266.2	70.8	0.459	0.908
2	*T*	*W*	-10	2	1	1	2			-400	0.438	107.1	-102.5	2.19	0.163	0.906
3	*T*	*W*	-10	2	1	2	1			-1400	0.219	176.0	-271.2	59.9	0.438	0.909
4	*T*	*W*	0	2	1	1	1			-400	0.477	101.6	-169.5	13.4	0.269	0.905
5	*T*	*W*	0	2	1	1	2			-400	0.509	97.6	- 83.3	5.50	0.284	0.904
6	*T*	*W*	0	2	1	2	1			-1400	0.346	125.3	-178.9	13.58	0.270	0.908
7	*T*	*W*	0	2	1	2	2			-1400	0.411	111.4	- 87.2	11.08	0.444	0.907
8	*T*	*W*	0	2	1	3	1			-2400	0.272	149.1	-183.0	7.34	0.214	0.909
9	*T*	*W*	10	2	1	1	1			-400	0.724	81.0	-110.9	1.11	0.0979	0.899
10	*T*	*W*	10	2	1	1	2			-400	0.579	90.8	- 69.3	9.31	0.397	0.903
11	*T*		-10	2	1	1	1			-400	0.299	141.3	-265.6	65.0	0.448	0.908
12	*T*		-10	2	1	1	2			-400	0.451	107.1	-101.9	2.61	0.181	0.906
13	*T*		-10	2	1	2	1			-1400	0.224	176.0	-270.9	55.5	0.429	0.909
14	*T*		0	2	1	1	1			-400	0.492	101.6	-168.2	11.2	0.252	0.905
15	*T*		0	2	1	1	2			-400	0.527	97.6	- 82.6	6.20	0.308	0.905
16	*T*		0	2	1	2	1			-1400	0.355	125.3	-178.3	11.79	0.257	0.908
17	*T*		0	2	1	2	2			-1400	0.423	111.5	-86.79	11.68	0.462	0.907
18	*T*		0	2	1	3	1			-2400	0.278	149.0	-182.7	6.34	0.202	0.909
19	*T*		10	2	1	1	1			-400	0.757	81.0	-108.3	0.811	0.0868	0.899
20	*T*		0	2	1	1	2			-400	0.601	90.8	- 68.4	10.23	0.424	0.903
21		*W*	-10	2	1	1	1				0.246	141.4	-269.5	77.14	0.524	0.884
22		*W*	-10	2	1	1	2				0.362	107.1	-105.6	0.471	0.0873	0.888
23		*W*	-10	2	1	2	1			-1400	0.186	176.0	-273.0	64.4	0.501	0.882
24		*W*	-10	2	1	2	2				0.295	123.5	-107.9	2.82	0.252	0.886
25		*W*	-10	2	1	3	1			-2400	0.149	210.8	-274.7	37.85	0.435	0.881
26			-10	2	1	1	1			-400	0.229	141.3	-270.6	95.59	0.550	0.884
27			-10	2	1	1	2			-400	0.337	107.1	-106.6	0.399	0.0769	0.888
28			-10	2	1	2	1			-1400	0.173	176.0	-273.7	80.1	0.528	0.882
29			-10	2	1	2	2				0.274	123.5	-108.6	2.12	0.210	0.886
30			-10	2	1	3	1			-2400	0.139	210.8	-275.1	47.99	0.464	0.881
31			-10	2	1	3	2				0.232	139.9	-109.8	10.33	0.584	0.884
32			-10	2	1	4	1			-3400	0.116	245.5	-275.9	15.80	0.332	0.880

"1" of h column indicates h=0.5, and "2" indicates h=0.

There is not a case that satisfies the TW case in 1961.

Tab. V－4

σ being computed making use of all samples

all samples , with weights and without weights

σ	1961		1962		1963		1964		total		
	1	2	1	2	1	2	1	2	$h=0.5$ 1	$h=0.25$ 2	計
0 ～0.1											
0.1～0.2			1		1				2	0	2
0.2～0.3			1		3	2	2		6	0	6
0.3～0.4			2	3		2	2		4	5	9
0.4～0.5			2	2	2	2	2	4	6	8	14
0.5～0.6			1	2		1		3	1	6	7
0.6～0.7				1				1	0	2	2
0.7～0.8			1		1		2		4	0	4

In the second row of tables 1 and 2 respectively indicate the cases where h＝0.5 and h＝0.25 were used.

Tab.V－5 The computed values of H_0, H_2 and $\sum U^2$, given γ, σ and h, for 1964

case	$TW211(r_3=10)$		$TW211(r_3=0)$		$TW212(r_3=0)$		$TW211(r_3=10)$		$T211(r_3=-10)$		$T211(r_3=0)$		$T212(r_3=0)$		$T211(r_3=10)$	
	h_{max}	h_{min}	h_{max}	h_{min}	h_{max}	h_{min}	h_{max}	h_{min}	h_{max}	h_{min}	h_{max}	h_{min}	h_{max}	h_{min}	h_{max}	h_{min}
σ	0.292	0.438	0.477	0.509	0.346	0.411	0.724	0.579	0.299	0.451	0.492	0.527	0.355	0.423	0.757	0.601
r_2	100		100		100		100		100		100		100		100	
r_3	− 10		0		0		10		− 10		0		0		10	
r_4	6000		6000		6000		6000		6000		6000		6000		6000	
r_5	− 400		− 400		−1400		− 400		− 400		− 400		−1400		− 400	
1961 H_0	164.6	115.5	105.3	100.4	134.6	117.5	79.3	90.3	164.6	115.5	105.3	100.4	134.6	117.5	79.3	90.3
H_2	−368.6	−132.3	−209.2	−102.9	−220.8	−107.7	−129.7	− 82.9	−367.8	−131.5	−207.6	−102.0	−220.1	−107.1	−126.6	− 81.8
1962 H_0	154.4	112.0	103.8	99.3	130.8	115.1	80.0	90.6	154.4	112.0	103.8	99.3	130.8	115.1	80.0	90.6
H_2	−323.0	−119.5	−192.7	− 94.8	−203.3	− 99.2	−122.1	− 77.3	−322.3	−118.8	−191.2	− 93.9	−202.7	− 98.7	−119.2	− 76.4
1963 H_0	148.3	109.8	102.9	98.6	128.4	113.5	80.5	90.7	148.3	109.8	102.9	98.6	128.4	113.5	80.5	90.7
H_2	−296.4	−111.7	−182.2	− 89.6	−192.3	− 93.8	−117.2	− 73.7	−295.8	−111.0	−180.8	− 88.8	−191.6	− 93.3	−114.3	− 72.8
1964 H_0	141.3	107.1	101.6	97.6	125.3	111.5	81.0	90.8	141.3	107.1	101.6	97.6	125.3	111.5	81.0	90.8
H_2	−266.2	−102.5	−169.5	− 83.4	−178.9	− 87.2	−111.0	− 69.3	−265.6	−101.9	−168.2	− 82.6	−178.3	− 86.8	−108.3	− 68.4
1961 Σu^2	59.46	2.00	14.10	3.61	13.55	6.00	5.28	5.78	54.68	2.08	2.02	3.89	11.91	6.37	4.35	6.20
1962 Σu^2	60.03	1.51	11.41	3.82	11.50	7.80	1.17	6.57	55.10	1.81	9.50	4.31	9.96	8.32	0.85	7.30
1963 Σu^2	65.74	2.68	14.57	5.08	14.33	8.96	3.76	8.00	60.53	2.91	12.43	5.55	12.63	9.42	3.15	8.65
1964 Σu^2	70.77	2.19	13.43	5.50	13.58	11.08	1.11	9.31	65.03	2.61	11.21	6.20	11.79	11.68	0.81	10.23

* Figures in the columns for γ_2, γ_4 and 5 correspond to those in the first row en Tab. V-2.

From the last equation the theoretical value, $\hat{u}^j_*$, for u^j which corresponds to the observed value of I_j, can be obtained.
A comparison of the observed values, $\hat{u}^{j0}_*$, with $\hat{u}^j_*$, is shown in Fig. V-6.
For calculating $H_0^{(t)}$ and $H_2^{(t)}$, two levels of h, h=0.5 and 0.25, were tried. It is clearly observed that the scatterd points standing for observed values lie between the two curves obtained by making use of h=0.5 and 0.25. Among those it can be seen that for the year 1961 we have two curves showing theoretical values between which the observations lie. This means that there definitely exists a suitable set of parameters which satisfy the theoretical requirements and are also consistent with the observations for the year 1961, even though we could not find their previously.

[5.2] Estimation of preference parameters by the Newton Method

In the last section, two sets of reduced form parameters (parameters of the PECI equation) were obtained for each alternative set of structural parameters (preference parameters). One of the sets of reduced form parameters is obtained by employing h=0.5 for calculating H_0 and H_2; the other is obtained by employing h=0.25. Two reduced form equations (PECI equations) were obtained for each set of structural parameters, making use of those two sets of reduced form parameters respectively. We observed there was a fairly good match between theoretical values of u^* obtained from those two PECI equations and observed values of u^*. (FigV-6) The curves standing for reduced form equations obtained by assigning h=0.5 and h=0.25, respectively, lie above and below the observed scattering. Consegventty we can conclude that the true value of h is larger than 0.25 and smaller than 0.5.

Taking into account that the parameters of the PECI equations are functions of $\gamma_i (i=1, \cdots\cdots, 5)$ and h, let us denote H_0 and H_2 as

$$H_0(\{\gamma_i\}, h \mid w) \text{ and } H_2(\{\gamma_i\}, h \mid w),$$

where $\{\gamma_i\}$ stands for a set of γ_i ($\gamma_1 \equiv -1$), with w, the nonprincipal potential earner's wage rate, and assigned hours of work, h, being given. Two sets of the parameters mentioned above are denoted as

$$H_j(\{\gamma_i\}, 0.5 \mid w) \text{ and } H_j(\{\gamma_i\}, 0.25 \mid w) \ ; \ (j=0, 2).$$

Theoretical values of u^*, $\hat{u}_*$, are given by

$$\hat{u}_* = \frac{1}{\sigma} \log \frac{I - H_0(\{\gamma_j\}, 0.5 \mid w)}{H_2(\{\gamma_i\}, 0.5 \mid w)}$$

and

$$\hat{u}_* = \frac{1}{\sigma} \log \frac{I - H_0(\{\gamma_i\}, 0.25 \mid w)}{H_2(\{\gamma_i\}, 0.25 \mid w)}$$

Employing the Newton method, we can tentatively change the values of γ_i, σ and h in the vicinity of the obtained values of γ_i and σ and h (=0.5) respectively so as to minimize $\Sigma (u_* - \hat{u}_*)^2$ where u_* stands for observed value. Let the values of γ_i, σ and h minimizing $\Sigma (u_* - \hat{u}_*)^2$ be $\gamma_i^{(1)}$, $\sigma^{(1)}$ and $h^{(1)}$ respectively. By the same procedure we can obtain $\gamma_i^{(2)}$, $\sigma^{(2)}$ and $h^{(2)}$

starting from h=0.25. It is desirable that

$$\gamma_i^{(1)}=\gamma_i^{(2)},\quad \sigma^{(1)}=\sigma^{(2)} \text{ and } h^{(1)}=h^{(2)}.$$

By the procedure mentioned above, we actually tried to get $\gamma_i^{(j)}$, $\sigma^{(j)}$and $h^{(j)}$ (j=1, 2). Initial values for applying the Newton method are as follows.

Tab. V-5

1-1)	TW 211	$\gamma_3=-10$	h=0.50	$\sigma=0.292$
1-2)	〃	〃	h=0.25	$\sigma=0.438$
2-1)	TW 211	$\gamma_3=0$	h=0.5	$\sigma=0.477$
2-2)	〃	〃	h=0.25	$\sigma=0.509$
3-1)	TW 212	$\gamma_3=0$	h=0.50	$\sigma=0.346$
3-2)	〃	〃	h=0.25	$\sigma=0.412$
4-1)	TW 211	$\gamma_3=10$	h=0.50	$\sigma=0.724$
4-2)	〃	〃	h=0.25	$\sigma=0.579$

However, the results were unsatisfactory; that is, we could not obtain suitable convergence values of parameters satisfying

$$\gamma_i^{(1)}=\gamma_i^{(2)},\quad \sigma^{(1)}=\sigma^{(2)} \text{ and } h^{(1)}=h^{(2)}. \ (i=2,\ 3,\ 4,\ 5)$$

Before we attain a convergence value γ, the value of the parameters obtained by the Newton method failed to satisfy theoretical constraints assigned to the preference parameters. That is, it was found that, by employing the Newton method, calculated values of $\{\gamma_i\}$ moved radically step by step and finally exceeded the limit given by theoretical requirement. It seems that the Newton Method is not suitable in this case for obtaining $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$.

As to the values of σ obtained by the Newton Method, the movement of the values are more moderate than are $\{\gamma_i\}$. The value of h shows no remarkable movement. Taking into account those results, we fixed the values of $\{\gamma_i\}$ and tentatively changed the values of σ and h alternatively without employing the Newton Method. Out of eight results obtained by using the sets of parameters shown in Tab. V-5, four cases with better fits were taken as the values for $\{\gamma^*_i\}$. The results are shown in Tab. V-6. The values in the table indicate $\Phi\equiv \sum u^2_{it} W_{it}$, where u stands for the residuals in the PECI equation (1) in

(4.7.2-1), subscripts is and t respectively, refer to the principal earners' income class and the year 1961 through 1964, and W_{it} stands for the number of households. The regions of σ and h with smaller Φ can be clearly seen from Tab. V-6. Among those sets of σ and h, we chose four sets of parameters taking into account their values of ϕ's. They are shown below. (see Tab. V-7 also)

$\{\gamma_2\ \gamma_4\ \gamma_5\}$	$-\gamma_3$	σ	h	ϕ^{**}
TW 211	-10	0.409	0.275	256.5
TW 211	0	0.500	0.325	262.7
TW 212	0	0.378	0.350	252.6
TW 211	10	0.681	0.425	292.3

foot note

(＊) Previously, 1/3 was used as a rough approximation for the assigned hours of work, h. When the workers are assigned to work 8 hours a day (8 hours a day is the model of the distribution of assigned hours of work in Japan.), h=1/3 holds by normalizing, 24, hours as unity. However, if we adopt one week as the time unit by which quantities of labor supply are measured, 1/3 might be an over estimate if holidays are taken into account. On the other hand, if the assigned hours of work means total hours necessary to be engaged in employee opportunities (e.g. total hours include the hours for commuting), then 1/3 might be the under estimate for h. Thus, taking into account those factors of over and underestimation, 1/3 was used as a rough approximation for h. Now, 0.25 is closer to 1/3 than 0.50 is. Hence, in Tab. V-5, we pay careful attention to the cases in which h=0.5 was applied to calculate H_0 and H_2. Among those cases, we adopt those with relatively smaller values for $\sum U^2$'S.

(＊＊) ϕ stands for the values of the objective function applied to the years 1961 through 1964.

Making use of the values for those parameters listed in the upper half of Tab. V-7, the values of H_0 an H_2 were calculated respectively. It was found that the differences between those calculated values and those (the parameters of the reduced form) previously estimated by the least squares method were quite small. The values of σ in the above table are to close to those estimated from the reduced form as well. This confirms the arguement in [5.1].

Further, for the year 1964, we expanded the range of trial values of γ_3. Previously, we adopted the ranges for σ and h by consulting Tab. V-5. Here, expansion of the ranges for parameters was carried out by consulting the values in Tab. V-7. The levels of $\{\gamma_i\}$ used are shown in Tab. V-8. We checked if the restrictions are fulfilled. These results are shown in the Tab. V-9.

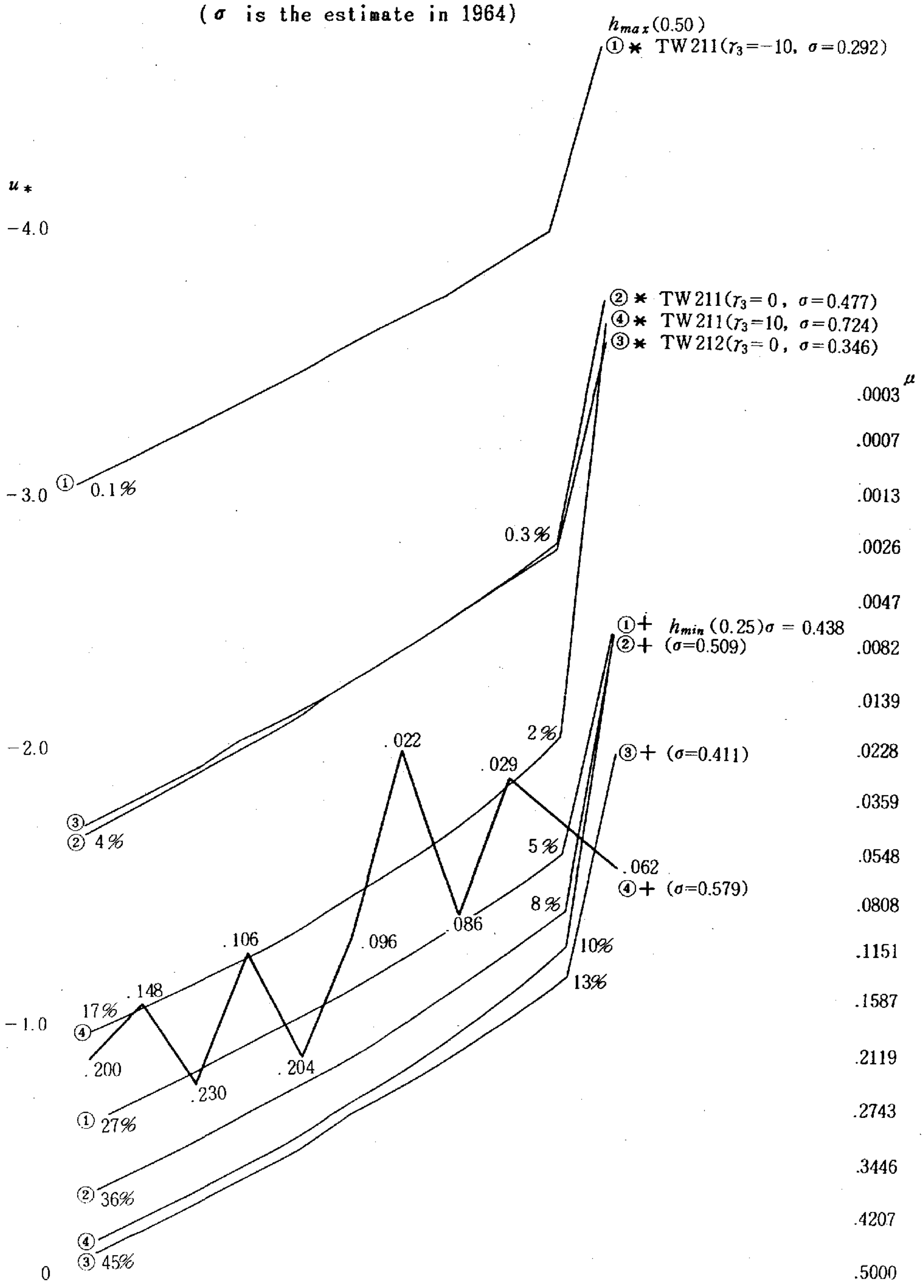
Among the cases shown in Tab. V-9, we took the cases which are the same as those in Tab. V-7. Given the values of $\{\gamma_i\}$ of those cases, levels of σ and h, respectively, were varied as shown in Tab.6 to compute the sum of squares of the difference between observed and calculated values for the participation rate, $\Sigma\ (\mu - \mu)^2 W$ (where W stands for numbers of households included in each principal earner's income class). Results are shown in Tab. V-10. The values in this table differ from the values in Tab.6 in that the former values were computed by employing the objective function $\phi \equiv \Sigma\ (u^* - \hat{u}^*)\ W$ instead of the newly defined objective function $\phi \equiv \Sigma\ (\mu - \hat{\mu})^2 W$. From Tab V-10, it can be seen the results are not significantly affected by the changing definition of ϕ. Hereafter we shall use the new objective function instead of the old one because the former is more convenient to judge the fitting of theoretical (estimated) values for $\hat{\mu}$ with observed values μ.

In the lower half of Tab. V-11, the calculated values of the parameters in the PECI equations, H_0 and H_2, are shown. These values correspond to the values $\{\gamma_i\}$ which yield a minimum ϕ for each year. Next, we tried to obtain final estimates for the preference parameters making use of the values listed in Tab. V-11. In Tab V-12, the results of estimation by steepest ascent method are listed. (The Newton Method was inadequate because the estimates for the parameters exceeded the ranges assigned by theoretical requirement.) Initial values of preference parameters used for the estimation are those shown in Tab V-12.

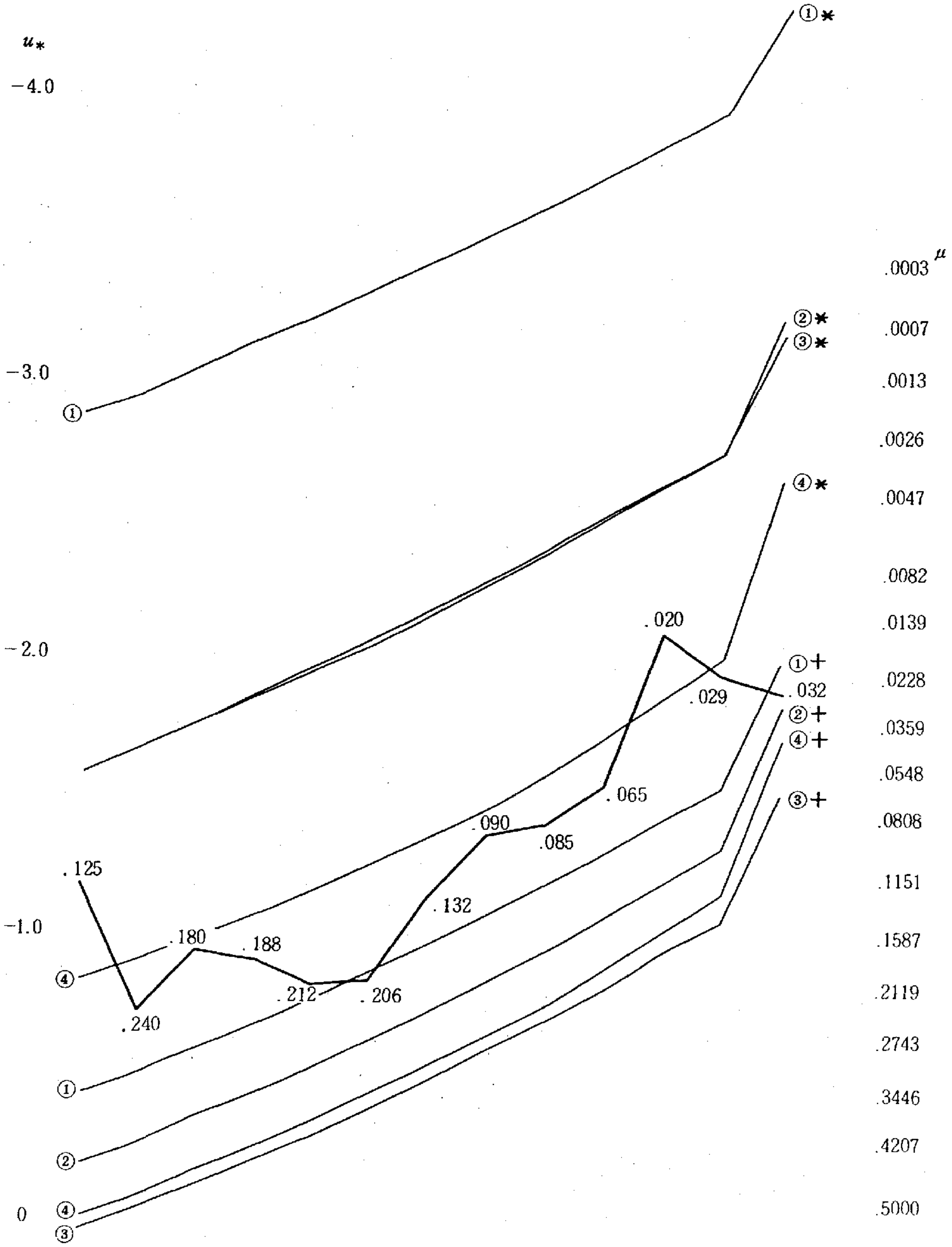
The results in Tab V-12 are not satisfactory because no remarkable improvement in the magnitudes of objective function were observed, and because there are no considerable difference between convergence values and initial values for the parameters to be estimated. This can be interpreted to mean that the objective function has local minima with respect to the parameters. Hence, it would not be possible to attain better results so long as those initial values of parameters are used for the estimation by the steepest ascent method: The initial values themselves need to be improved.

We can conceive of two factors affecting the initial values of parameters; that is, (1) the range of numerical values of parameters tentatively varied for calculating the values of the objective function, and (2) the model itself which is used to calculate the values for objective function. With respect to the former, we adopted the range as wide as possible. With respect to the latter, it seems that reconsideration of the model might be needed. Thus far we have used an employment opportunity model. In this model, hours of self employed work for non principal potential earners' is deleted from the analysis. However, some non principal potential earners are self-employed, and the data for self-employed wives in type A households are available. Hence, in order to reexamine the initial values of parameters in Tab V-11, we shall construct, in section Ⅵ, a model including the selection of self-employed work.

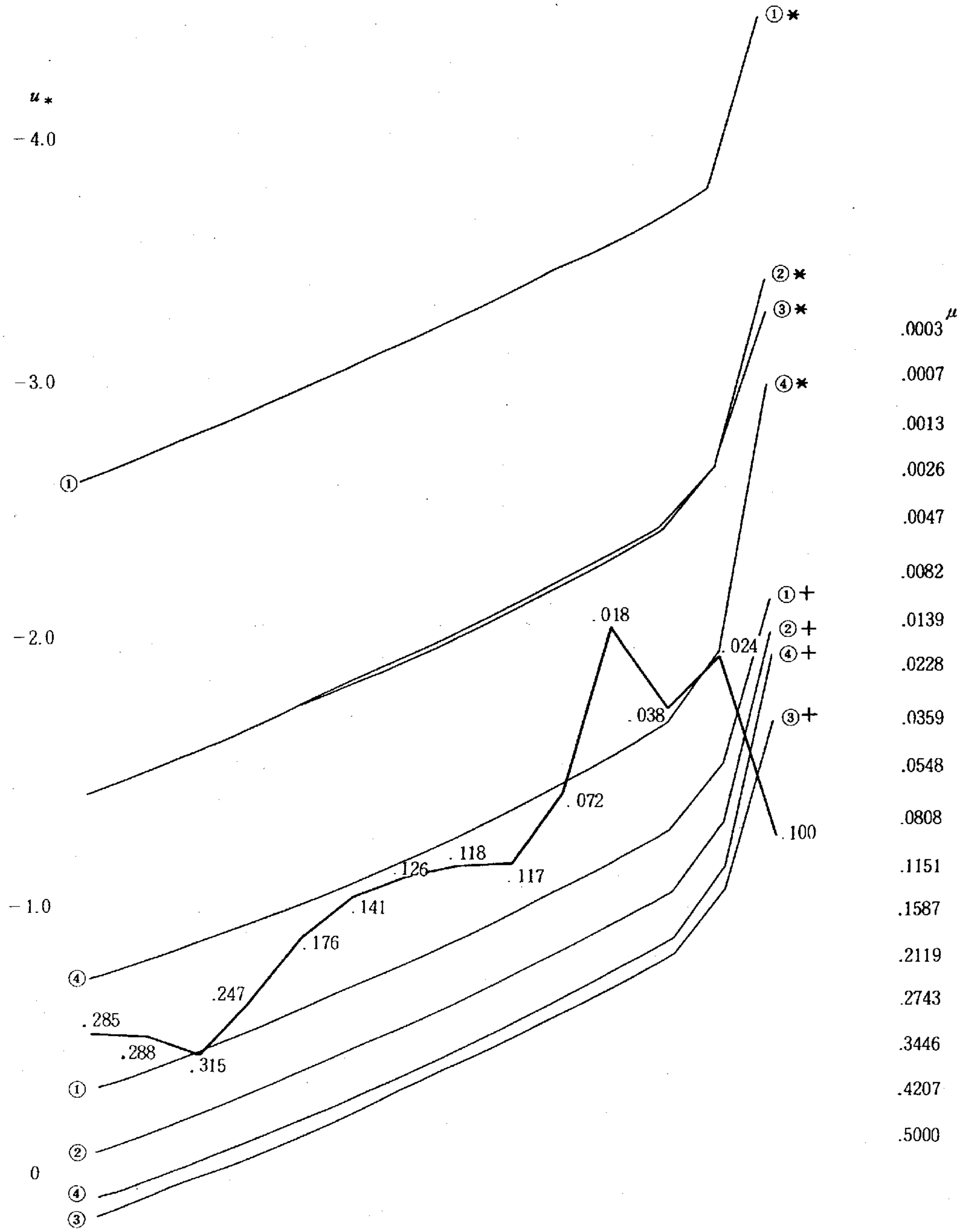
Fig. V-6 TW in 1961; "$*$" indicates h=0.5, and "+" indicates h=0.25
(σ is the estimate in 1964)



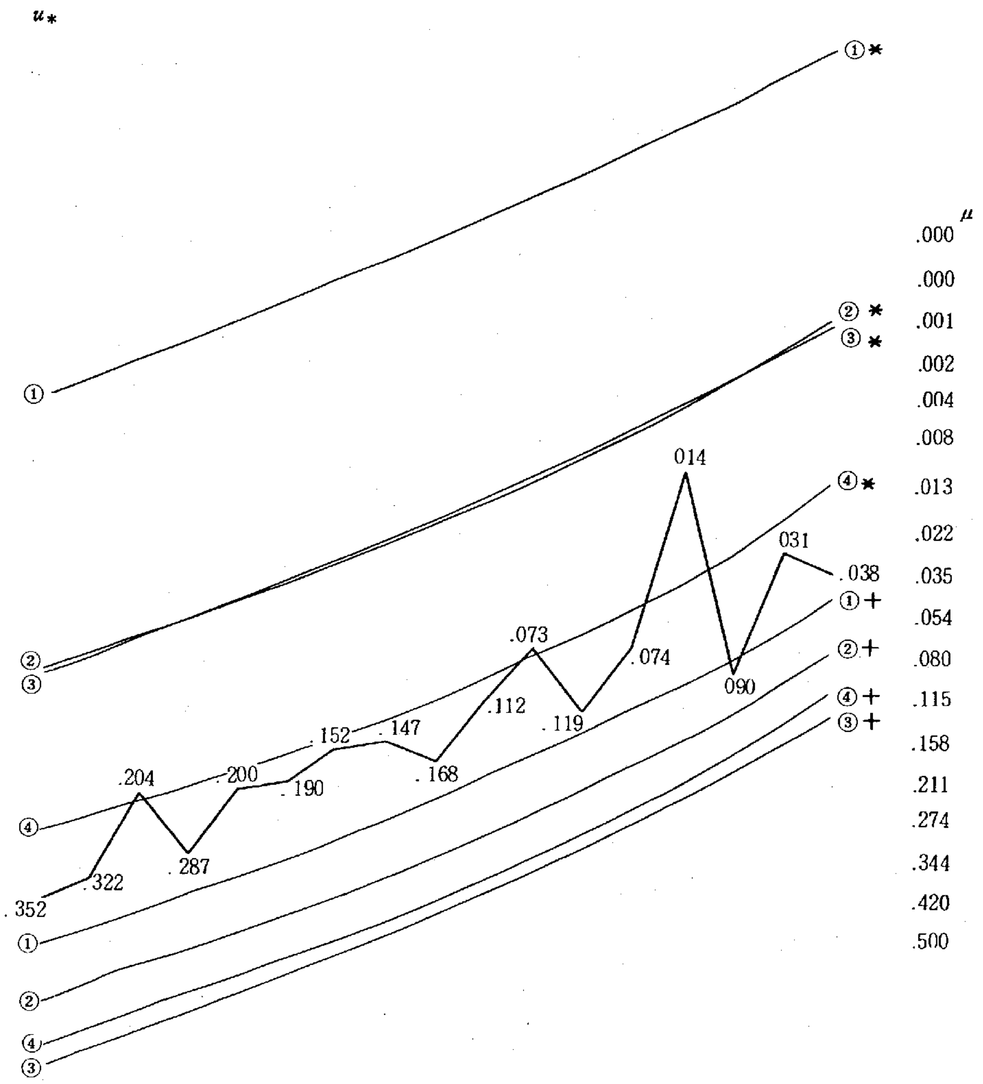
1962 *TW* (σ 1964)



1963 *TW* (σ 1964)



1964 *TW* (σ_i is the observation)



Tab. V－6 The serch for the preference parameters (second estimation making of employee－model)

TW 211 （$r_3=-10$） $\phi=\sum_{i,t} v^2_{it} w_{it}$（$w_{it}$ weight）

		1	2	3	4	5	6	7	8	9	10	11	
	h σ	0.292	0.307	0.321	0.336	0.350	0.365	0.380	0.394	0.409	0.423	0.438	
1	0.250	1119.3	779.9	544.9	392.8	306.9	274.1	284.4	329.4	402.7	499.0	614.2	
2	0.275	2366.5	1743.2	1268.0	911.5	650.8	467.8	348.3	281.0	256.5	267.6	308.2	6.6
3	0.300	4025.1	3097.1	2363.1	1785.5	1334.7	987.6	725.7	534.4	401.5	317.5	274.4	
4	0.325	5967.0	4722.4	3718.8	2910.0	2260.2	1740.6	1328.7	1006.1	758.2	572.9	440.3	
5	0.350	8103.3	6536.1	5257.3	4212 3	3358.3	2661.7	2095.3	1637.5	1270.6	980.3	754.7	
6	0.375	10371.6	8479.9	6923.7	5640.3	4580.1	3704.2	2981.2	2386.0	1898.2	1500 9	1180.4	
7	0.400	12727.0	10511.6	8678.4	7156.5	5889.9	4834.3	3954.2	3221.0	2611.4	2106.3	1690.1	
8	0.425	15137.6	12600.8	10492.6	8733.7	7261.7	6027.1	4990.3	4119.4	3388.2	2775.3	2263.2	
9	0.450	17579.7	14725.4	12345.1	10351.5	8676.1	7264.1	6072.0	5064.2	4212.2	3492.1	2884.6	
10	0.475	20036.4	16868.9	14220.3	11995.2	10118.8	8531.5	7185.7	6042.6	5070.8	4244.5	3542.4	
11	0.500	22495.1	19019.6	16106.7	13653.5	11579.0	9818.8	8321.3	7044.6	5954.6	5023.3	4227.6	

TW 211 （$r_3=0$）

		1	2	3	4	5	6	7	8	9	10	11	
	h σ	0.477	0.480	0.483	0.486	0.490	0.493	0.496	0.500	0.503	0.506	0.509	
1	0.250	1171.6	1207.8	1244.3	1281.3	1318.6	1356.3	1394.3	1432.5	1471.1	1510.0	1549.1	
2	0.275	559.5	583.2	607.6	632.8	658.6	685.1	712.3	740.0	768.4	797.3	826.7	
3	0.300	294.0	302.4	311.9	322.6	334.3	347.0	360.8	375.5	391.1	407.6	424.9	
4	0.325	293.8	284.9	277.5	271.7	267.4	264.5	262.9	262.7	263.8	266.1	269.6	7.8
5	0.350	499.9	472.2	446.5	422.8	401.0	381.0	362.9	346.4	331.7	318.5	306.9	
6	0.375	868.3	820.8	775.7	733.1	692.7	654.9	619.1	585.5	553.9	524.3	496.7	
7	0.400	1366.2	1298.0	1232.9	1170.6	1111.1	1054.4	1000.3	948.7	899.7	853.0	808.6	
8	0.425	1968.2	1878.8	1793.0	1710.5	1631.3	1555.3	1482.3	1412.4	1345.3	1281.0	1219.5	
9	0.450	2654.7	2543.8	2436.9	2333.8	2234.5	2138.8	2046.7	1957.9	1872.5	1790.2	1711.1	
10	0.475	3410.4	3277.7	3149.5	3025.6	2905.9	2790.3	2678.6	2570.8	2466.7	2366.3	2269.3	
11	0.500	4223.3	4068.6	3918.8	3773.8	3633.6	3497.9	3366.6	3239.5	3116.6	2997.8	2882.8	

TW 212 （$r_3 = 0$）

		1	2	3	4	5	6	7	8	9	10	11	
	σ h	0.346	0.352	0.359	0.365	0.372	0.378	0.385	0.392	0.398	0.405	0.411	
1	0.250	2046.3	2150.3	2254.7	2359.5	2464.5	2569.7	2674.8	2779.8	2884.7	2989.3	3093.7	
2	0.275	1007.2	1090.1	1175.5	1263.0	1352.4	1443.5	1535.9	1629.6	1724.2	1819.6	1915.8	6.6
3	0.300	451.3	501.0	555.8	615.2	678.5	745.5	815.7	888.7	964.4	1042.2	1122.2	7.2
4	0.325	260.9	268.7	284.6	307.7	337.4	373.0	413.9	459.8	510.0	564.2	622.0	7.8
5	0.350	352.3	311.7	282.5	263.5	253.8	252.6	259.2	272.9	292.9	318.9	350.2	8.4
6	0.375	664.8	571.0	492.0	426.4	373.2	331.2	299.5	277.2	263.5	257.8	259.3	9.0
7	0.400	1153.7	1002.9	870.6	755.1	655.0	569.0	496.0	434.9	384.8	344.7	313.9	9.6
8	0.425	1785.4	1574.8	1386.4	1218.3	1068.8	936.5	819.9	717.8	629.0	552.4	487.2	10.2
9	0.450	2534.7	2261.9	2015.2	1792.5	1591.6	1411.0	1248.9	1104.0	974.8	860.2	759.1	
10	0.475	3382.0	3045.3	2738.7	2459.5	2205.7	1975.2	1766.2	1577.0	1406.1	1252.1	1113.7	
11	0.500	4312.7	3910.5	3542.4	3205.5	2897.4	2615.7	2358.5	2123.9	1910.0	1715.5	1538.8	

TW 211 （$r_3 = 10$）

		1	2	3	4	5	6	7	8	9	10	11	
	σ h	0.579	0.594	0.608	0.623	0.637	0.652	0.666	0.681	0.695	0.710	0.724	
1	0.250	2662.1	1774.9	1943.0	3230.8	3421.6	3612.5	3803.4	3994.2	4184.7	4374.8	4564.5	
2	0.275	913.5	1042.0	1178.0	2114.4	2288.5	2464.8	2643.1	2822.8	3003.7	3185.6	3368.2	
3	0.300	499.5	583.4	679.0	1320.5	1468.7	1621.7	1778.9	1939.7	2103.7	2270.3	2439.3	
4	0.325	316.2	348.5	397.2	784.9	899.9	1022.7	1152.4	1288.2	1429.4	1575.3	1725.3	
5	0.350	324.2	299.1	295.4	460.2	536.1	623.1	720.2	826.1	939.8	1060.5	1187.5	
6	0.375	493.4	406.2	345.7	310.6	342.6	389.4	449.5	521.5	603.9	695.9	796.2	
7	0.400	800.6	647.3	526.0	308.7	292.8	295.6	315.3	349.9	397.9	458.0	529.0	
8	0.425	1227.9	1004.8	819.5	433.3	366.2	321.8	297.9	292.3	303.2	328.9	368.0	10.2
9	0.450	1761.2	1465.2	1212.9	668.0	546.6	452.2	382.1	333.8	305.2	294.3	299.4	
10	0.475	2389.5	2017.6	1695.6	999.6	821.4	674.5	555.9	462.7	392.5	342.9	312.2	
11	0.500	2850.8	3040.4	3230.8	1418.2	1180.7	979.1	809.8	669.7	555.9	466.0	397.6	

Tab. V－7

ケース	$TW211$ （$r_3=-10$）				$TW211$ （$r_3=0$）				$TW212$ （$r_3=0$）				$TW211$ （$r_3=10$）			
r_2	100				100				100				100			
r_3	− 10				0				0				10			
r_4	6000				6000				−6000				6000			
r_5	− 400				− 400				−1400				− 400			
σ	0.409				0.500				0.378				0.681			
h	0.275				0.325				0.350				0.425			
$\sum_{it} v_{it}^2 w$	256.5				262.7				252.6				292.3			
	1961	1962	1963	1964	1961	1962	1963	1964	1961	1962	1963	1964	1961	1962	1963	1964
H_0	119.4	115.5	113.0	110.0	102.1	100.9	100.1	99.0	125.2	122.1	120.2	117.7	82.4	83.0	83.4	83.8
H_2	−151.0	−136.1	−127.0	−116.3	−134.5	−123.8	−117.1	−108.9	−152.7	−140.6	−133.0	−123.7	−118.6	−111.4	−106.7	−100.9
H_2'	−164.2	−148.0	−138.1	−126.4	−152.3	−140.3	−132.7	−123.4	−164.1	−151.1	−142.9	−132.9	−149.6	−140.5	−134.6	−127.2
W	46.5	50.5	53.5	57.5	39.4	42.8	45.2	48.6	36.6	39.7	42.0	45.1	30.1	32.7	34.6	37.2
$\sum_{it} v_{it}^2$	2.14	0.712	2.06	0.726	2.46	0.724	2.22	0.732	2.01	0.697	1.94	0.729	3.85	0.820	2.88	0.750
R	0.613	0.860	0.709	0.907	0.591	0.859	0.690	0.905	0.618	0.860	0.717	0.907	0.529	0.851	0.636	0.901
$T^{heil}U$	0,150	0.0867	0.148	0.0842	0.160	0.0878	0.153	0.0849	0.146	0.0865	0.145	0.0850	0.196	0.0932	0.173	0.0854

Tab. V－8 level of parameter

r_2	50	100	150	200	
r_3	−10	0	10	20	30
r_4	6000	7000	8000	9000	
r_5	−400	−1400	−2400	−3400	

Tab. V－9 The sets of the prameters which fulfill the theoretica restrictions.

1964 TW ($r_3 = -10$)

No.	Case	h	r_2	r_3	r_4	r_5	σ	H_0	H_2	$\sum u^2$
1	211	1	100	-10	6000	-400	0.292	141	-266	70.8
2	211	2					0.438	107	-102	2.2
3	212	1				-1400	0.219	176	-271	60
4	213	1				-2400	0.175	211	-274	34
5	221	1			7000	-400	0.292	141	-311	112
6	221	2					0.438	107	-120	0.77
7	222	1	100	-10	7000	-1400	0.219	176	-316	113
8	222	2					0.353	124	-124	1.13
9	223	1				-2400	0.175	211	-319	89.0
10	224	1				-3400	0.146	245	-321	52.9
11	231	1			8000	-400	0.292	141	-355	155
12	231	2					0.438	107	-137	2.94
13	232	1				-1400	0.219	176	-362	173
14	232	2					0.353	124	-141	1.51
15	233	1	100	-10	8000	-2400	0.175	211	-365	158
16	233	2					0.296	140	-144	0.73
17	234	1				-3400	0.146	245	-366	121
18	241	1			9000	-400	0.292	141	-399	199
19	241	2					0.438	107	-154	7.47
20	242	1				-1400	0.219	176	-407	235
21	242	2					0.353	124	-159	5.89
22	243	1				-2400	0.175	211	-410	235
23	243	2					0.296	140	-162	2.81
24	244	1				-3400	0.146	245	-412	206
25	244	2					0.255	156	-164	0.748
26	311	1	150	-10	6000	-400	0.172	214	-274	31.0
27	311	2					0.235	166	-109	61.3
28	312	1				-1400	0.144	249	-275	7.04
29	312	2					0.209	183	-110	109
30	313	1				-2400	0.124	284	-276	2.23
31	313	2					0.188	199	-111	178
32	314	1				-3400	0.108	319	-276	32.9
33	321	1			7000	-400	0.172	214	-319	85.5
34	321	2					0.235	166	-128	26.5
35	322	1				-1400	0.144	249	-321	48.9
36	322	2					0.209	183	-129	54.9
37	323	1				-2400	0.124	284	-322	15.9
38	323	2					0.188	199	-129	98.9
39	324	1				-3400	0.108	319	-322	0.718
40	324	2					0.170	216	-130	162
41	331	1			8000	-400	0.172	214	-365	155
42	331	2					0.235	166	-146	8.19

43	332	1	150	-10	8000	-1400	0.144	249	-367	117
44	332	2					0.209	183	-147	23.0
45	333	1				-2400	0.124	284	-368	70.5
46	333	2					0.188	199	-148	49.3
47	334	1				-3400	0.108	319	-368	28.4
48	334	2					0.170	216	-148	90.0
49	341	1			9000	-400	0.172	214	-411	233
50	341	2					0.235	166	-165	1.14
51	342	1				-1400	0.144	249	-412	201
52	342	2					0.209	183	-166	6.43
53	343	1				-2400	0.124	284	-414	152
54	343	2					0.188	199	-166	19.9
55	344	1				-3400	0.108	319	-414	95.8
56	344	2					0.170	216	-167	44.2
57	411	1	-200	-10	6000	-400	0.122	288	-276	3.54
58	411	2					0.161	226	-111	340
59	412	1				-1400	0.107	322	-276	38.9
60	412	2					0.148	242	-111	479
61	413	1				-2400	0.095	357	-277	127
62	413	2					0.137	259	-111	650
63	414	1				-3400	0.086	392	-277	284
64	414	2					0.128	275	-112	857
65	421	1			7000	-400	0.122	288	-322	13.2
66	421	2					0.162	226	-130	210
67	422	1				-1400	0.107	322	-322	0.742
68	422	1					0.148	242	-130	310
69	423	1				-2400	0.095	357	-323	21.7
70	423	2					0.137	259	-130	435
71	424	1				-3400	0.086	392	-323	90.0
72	424	2					0.128	275	-130	592
73	431	1			8000	-400	0.122	287	-368	65.6
74	431	2					0.161	226	-148	123
75	432	1				-1400	0.107	322	-368	24.7
76	432	2					0.148	242	-149	193
77	433	1				-2400	0.095	357	-369	2.07
78	433	2					0.137	259	-149	285
79	434	1				-3400	0.086	392	-369	10.0
80	434	2					0.128	275	-149	401
81	441	1			9000	-400	0.122	288	-414	146
82	441	2					0.161	226	-167	65.6
83	442	1				-1400	0.107	322	-414	90.0
84	442	2					0.148	242	-167	113
85	443	1				-2400	0.095	357	-414	40.0
86	443	2					0.137	258	-168	178
87	444	1				-3400	0.086	392	-415	7.36
88	444	2					0.128	275	-168	263

1964 *TW* ($r_3 = 0$)

№	Case	h	r_2	r_3	r_4	r_5	σ	H_0	H_2	$\sum u^2$
1	211	1	100	0	6000	-400	0.477	101	-169	13.4
2	211	2					0.509	97	- 83	5.5
3	212	1				-1400	0.346	125	-179	13.6
4	212	2					0.411	111	- 87	11.1
5	213	1				-2400	0.272	149	-183	7.34
6	221	1			7000	-400	0.477	101	-198	24.7
7	221	2					0.509	97	- 97	1.60
8	222	1				-1400	0.346	125	-209	30.1
9	222	2					0.411	111	-102	3.51
10	223	1				-2400	0.272	149	-213	24.9
11	224	1				-3400	0.224	173	-216	14.0
12	231	1			8000	-400	0.477	102	-226	37.4
13	231	2					0.509	98	-111	0.74
14	232	1				-1400	0.346	125	-238	50.0
15	232	2					0.411	111	-116	0.820
16	233	1				-2400	0.272	149	-244	48.9
17	233	2					0.346	125	-119	2.26
18	234	1				-3400	0.224	172	-247	38.1
19	241	1			9000	-400	0.477	102	-254	50.7
20	241	2					0.509	97	-125	1.92
21	242	1				-1400	0.346	125	-268	71.7
22	242	2					0.411	111	-131	1.42
23	243	1				-2400	0.272	149	-274	76.9
24	243	2					0.346	125	-137	0.724
25	244	1				-3400	0.224	172	-277	69.2
26	244	2					0.298	139	-136	1.46
27	311	1	150	0	6000	-400	0.266	152	-183	6.56
28	311	2					0.275	147	- 91	60.3
29	312	1				-1400	0.220	175	-185	1.04
30	312	2					0.245	161	- 92	99.6
31	313	1				-2400	0.188	199	-187	3.96
32	313	2					0.220	174	- 92	154
33	314	1				-3400	0.164	223	-187	22.8
34	321	1			7000	-400	0.266	152	-214	23.9
35	321	2					0.275	148	-107	30.0
36	322	1				-1400	0.220	175	-216	12.8
37	322	2					0.245	161	-107	54.7
38	323	1				-2400	0.188	199	-218	3.16
39	323	2					0.220	175	-218	90.9
40	324	1				-3400	0.164	223	-108	1.36
41	324	2					0.200	189	-109	141
42	331	1			8000	-400	0.266	152	-244	48.1
43	331	2					0.275	148	-122	12.3

44	332	1	150	0	8000	-1400	0.220	175	-247	36.5
45	332	2					0.245	161	-123	26.7
46	333	1				-2400	0.188	199	-249	21.0
47	333	2					0.220	175	-123	49.6
48	334	1				-3400	0.164	223	-250	7.11
49	334	2					0.200	189	-124	83.5
50	341	1			9000	-400	0.266	152	-275	76.6
51	341	2					0.275	148	-137	3.41
52	342	1				-1400	0.220	175	-278	67.8
53	342	2					0.245	161	-138	10.3
54	343	1				-2400	0.188	199	-280	50.9
55	343	2					0.220	175	-139	23.7
56	344	1				-3400	0.164	223	-281	30.9
57	344	2					0.200	189	-140	45.2
58	411	1	200	0	6000	-400	0.185	202	-187	5.08
59	411	2					0.190	198	- 93	281
60	412	1				-1400	0.162	225	-187	26.0
61	412	2					0.175	211	- 94	386
62	413	1				-2400	0.144	249	-188	71.0
63	413	2					0.162	225	- 94	516
64	414	1				-3400	0.129	273	-188	147
65	414	2					0.151	239	- 94	618
66	421	1			7000	-400	0.189	202	-218	2.48
67	421	2					0.190	198	-109	180
68	422	1				-1400	0.162	225	-219	1.90
69	422	2					0.175	211	-109	257
70	423	1				-2400	0.144	249	-219	16.4
71	423	2					0.162	225	-109	353
72	424	1				-3400	0.129	273	-220	52.2
73	424	2					0.151	239	-109	471
74	431	1			8000	-400	0.185	202	-249	19.3
75	431	2					0.190	198	-124	110
76	432	1				-1400	0.162	225	-250	5.97
77	432	2					0.175	211	-125	166
78	433	1				-2400	0.144	249	-251	0.702
79	433	2					0.162	225	-125	237
80	434	1				-3400	0.129	273	-251	9.22
81	434	2					0.151	239	-125	326
82	441	1			9000	-400	0.185	202	-280	48.9
83	441	2					0.189	198	-140	63.2
84	442	1				-1400	0.162	225	-281	28.8
85	442	2					0.175	211	-140	102
86	443	1				-2400	0.144	249	-282	11.2
87	443	2					0.162	225	-141	154
88	444	1				-3400	0.129	273	-282	1.38
89	444	2					0.151	239	-141	220

1964 *TW* ($r_3 = 10$)

№	Case	h	r_2	r_3	r_4	r_5	σ	H_0	H_2	$\sum u^2$
1	211	1	100	10	6000	-400	0.724	81	-111	1.11
2	211	2					0.579	91	- 69	9.31
3	221	1			7000	-400	0.724	81	-129	2.91
4	221	2					0.579	91	- 81	4.09
5	222	1				-1400	0.498	99	-149	6.86
6	222	2					0.468	103	- 86	6.68
7	231	1				-400	0.724	81	-148	5.72
8	231	2					0.579	91	- 92	1.5
9	232	1				-1400	0.498	99	-170	13.6
10	232	2					0.468	103	- 98	2.3
11	233	1					0.382	117	-179	15.6
12	241	1			9000	-400	0.724	81	-166	9.15
13	241	2					0.579	91	-104	0.734
14	242	1				-1400	0.498	99	-191	21.5
15	242	2					0.468	103	-110	0.768
16	243	1				-2400	0.382	117	-201	27.0
17	243	2					0.394	115	-114	1.37
18	244	1				-3400	0.311	135	-206	25.8
19	311	1	150	10	6000	-400	0.373	119	-135	1.14
20	311	2					0.314	134	- 78	59.9
21	312	1				-1400	0.305	137	-138	0.967
22	312	2					0.280	146	- 79	93.3
23	313	1				-2400	0.258	155	-140	5.50
24	313	2					0.252	158	- 79	138
25	321	1			7000	-400	0.373	119	-157	6.30
26	321	2					0.314	134	- 91	32.9
27	322	1				-1400	0.305	137	-161	3.14
28	322	2					0.280	146	- 92	54.7
29	323	1				-2400	0.258	155	-163	0.765
30	323	2					0.252	158	- 93	85.7
31	324	1				-3400	0.224	173	-164	2.72
32	324	2					0.229	170	- 93	127
33	331	1			8000	-400	0.373	119	179	15.5
34	331	2					0.314	134	-104	16.1
35	332	1				-1400	0.305	137	-184	12.1
36	332	2					0.280	146	-105	29.7
37	333	1				-2400	0.258	155	-186	6.47
38	333	2					0.252	158	-106	50.2
39	334	1				-3400	0.224	173	-188	1.76
40	334	2					0.229	170	-106	79.2

41	341	1	150	10	9000	-400	0.373	119	-202	27.2
42	341	2					0.314	134	-117	6.35
43	342	1				-1400	0.305	137	-207	25.3
44	342	2					0.280	146	-118	14.0
45	343	1				-2400	0.258	155	-209	19.1
46	343	2					0.252	158	-119	26.8
47	344	1				-3400	0.224	173	-211	11.0
48	344	2					0.229	170	-120	46.1
49	411	1	200	10	6000	-400	0.254	157	-140	6.44
50	411	2					0.217	117	- 80	241
51	412	1				-1400	0.221	175	-141	21.0
52	412	2					0.200	189	- 80	326
53	413	1				-2400	0.195	193	-142	49.0
54	413	2					0.186	201	- 81	429
55	414	1				-3400	0.175	211	-142	94.5
56	414	2					0.173	213	- 81	551
57	421	1			7000	-400	0.254	157	-163	0.703
58	421	2					0.217	177	- 93	159
59	422	1				-1400	0.221	175	-164	3.34
60	422	2					0.200	189	- 93	221
61	423	1				-2400	0.195	193	-165	14.3
62	423	2					0.186	201	- 94	299
63	424	1				-3400	0.175	211	-166	37.4
64	424	2					0.173	213	- 94	392
65	431	1			8000	-400	0.254	157	-186	5.87
66	431	2					0.217	177	-107	101
67	432	1				-1400	0.221	175	-188	1.44
68	432	2					0.200	189	-107	147
69	433	1				-2400	0.195	193	-189	1.45
70	433	2					0.186	201	-107	205
71	434	1				-3400	0.175	211	-189	9.15
72	434	2					0.173	213	-108	277
73	441	1			9000	-400	0.254	157	-209	18.2
74	441	2					0.217	177	-120	61.7
75	442	1				-1400	0.221	175	-211	10.1
76	442	2					0.200	189	-121	94.4
77	443	1				-2400	0.195	193	-212	3.29
78	443	2					0.186	201	-121	137
79	444	1				-3400	0.175	211	-213	0.704
80	444	2					0.173	213	-121	191

1964 TW ($r_3 = 30$)

№	Case	h	r_2	r_3	r_4	r_5	σ	H_0	H_2	$\sum u^2$
1	312	1	150	30	6000	-1400	0.509	98	- 86	4.64
2	312	2					0.348	125	- 61	85.7
3	322	1			7000		0.509	98	-100	1.26
4	322	2					0.348	125	- 70	55.4
5	332	1			8000		0.509	98	-114	0.843
6	332	2					0.348	125	- 81	34.5
7	333	1				-2400	0.421	110	-119	0.709
8	333	2					0.314	134	- 82	51.4
9	342	1			9000	-1400	0.509	98	-128	2.42
10	342	2					0.348	125	- 91	20.2
11	343	1				-2400	0.421	110	-134	2.04
12	343	2					0.314	134	- 92	31.7
13	412	1	200	30	6000	-1400	0.354	123	- 91	17.9
14	412	2					0.250	159	- 62	253
15	413	1				-2400	0.310	135	- 93	32.3
16	413	2					0.232	168	- 63	324
17	422	1			7000	-1400	0.354	123	-107	6.22
18	422	2					0.250	159	- 73	179
19	423	1				-2400	0.310	135	-108	13.5
20	423	2					0.232	168	- 73	233
21	432	1			8000	-1400	0.354	123	-122	1.33
22	432	2					0.250	159	- 83	125
23	433	1				-2400	0.310	135	-124	3.93
24	433	2					0.232	168	- 84	166
25	434	1				-3400	0.275	148	-125	9.86
26	434	2					0.217	177	- 84	217
27	442	1			9000	-1400	0.354	123	-137	1.04
28	442	2					0.250	159	- 94	85.2
29	443	1				-2400	0.310	135	-139	0.749
30	443	2					0.232	168	- 94	117
31	444	1				-3400	0.275	148	-141	2.29
32	444	2					0.217	177	- 94	156

1964 TW ($r_3 = 20$)

No.	Case	h	r_2	r_3	r_4	r_5	σ	H_0	H_2	$\sum u^2$
1	312	1	150	20	6000	-1400	0.400	113	-107	2.45
2	312	2					0.314	134	- 69	88.9
3	322	1			7000		0.400	113	-125	0.779
4	322	2					0.314	134	- 80	55.0
5	323	1				-2400	0.335	128	-128	1.08
6	323	2					0.283	145	- 81	82.1
7	332	1			8000	-1400	0.400	113	-143	3.41
8	332	2					0.314	134	- 92	32.2
9	333	1				-2400	0.335	128	-147	1.75
10	333	2					0.283	145	- 92	50.8
11	334	1				-3400	0.289	143	-149	0.696
12	334	2					0.258	155	- 93	76.2
13	342	1			9000	-1400	0.400	113	-161	8.88
14	342	2					0.314	134	-103	17.3
15	343	1				-2400	0.335	128	-165	6.82
16	343	2					0.283	145	-104	29.4
17	344	1				-3400	0.289	143	-167	3.67
18	344	2					0.258	155	-105	47.0
19	412	1	200	20	6000	-1400	0.284	144	-112	18.8
20	412	2					0.225	172	- 70	284
21	413	1				-2400	0.250	159	-113	38.3
22	413	2					0.209	183	- 71	368
23	422	1			7000	-1400	0.284	144	-130	4.77
24	422	2					0.225	172	- 82	197
25	423	1				-2400	0.250	159	-131	13.6
26	423	2					0.209	183	- 82	261
27	424	1				-3400	0.224	173	-132	30.3
28	424	2					0.195	193	- 83	338
29	432	1			8000	-1400	0.284	144	-149	0.701
30	432	2					0.225	172	- 94	134
31	433	1				-2400	0.250	159	-150	2.63
32	433	2					0.209	183	- 94	183
33	434	1				-3400	0.224	173	-151	9.40
34	434	2					0.195	193	- 94	242
35	442	1			9000	-1400	0.284	144	-168	3.34
36	442	2					0.225	172	-105	89.1
37	443	1				-2400	0.250	159	-169	0.992
38	443	2					0.209	183	-106	126
39	444	1				-3400	0.224	173	-170	1.30
40	444	2					0.195	193	-106	171

Tab. V－10 ϕ value

TW 211 $r_3 = -10$ $\phi=\sum(\mu-\hat{\mu})^2 w$ (w weight)

		1	2	3	4	5	6	7	8	9	10	11
	σ h	0.292	0.307	0.321	0.336	0.350	0.365	0.380	0.394	0.409	0.423	0.438
1	0.250	22.75	17.15	12.71	9.50	7.57	6.93	7.58	9.52	12.70	17.10	22.67
2	0.275	45.76	37.94	30.72	24.25	18.65	14.02	10.44	7.97	6.62	6.42	7.37
3	0.300	66.65	58.82	50.99	43.39	36.20	29.57	23.62	18.45	14.16	10.80	8.42
4	0.325	82.43	75.57	68.31	60.89	53.47	46.22	39.27	32.78	26.84	21.57	17.05
5	0.350	93.27	87.73	81.60	74.99	68.09	61.05	54.00	47.10	40.47	34.23	28.48
6	0.375	100.54	96.25	91.28	85.70	79.64	73.24	66.61	59.87	53.16	46.60	40.29
7	0.400	105.28	102.05	98.15	93.58	88.46	82.86	76.88	70.63	64.21	57.74	51.33
8	0.425	108.30	105.86	102.89	99.29	95.05	90.28	85.04	79.42	73.49	67.36	61.14
9	0.450	110.26	108.47	106.23	103.39	99.94	95.94	91.43	86.47	81.12	75.47	69.59
10	0.475	111.53	110.22	108.54	106.33	103.54	100.23	96.40	92.08	87.33	82.20	76.77
11	0.500	112.26	111.37	110.05	108.38	106.20	103.47	100.24	96.53	92.35	87.75	82.79

TW 211 $r_3 = 0$ $\phi=\sum(\mu-\hat{\mu})^2 w$

		1	2	3	4	5	6	7	8	9	10	11
	σ h	0.477	0.480	0.483	0.486	0.490	0.493	0.496	0.500	0.503	0.506	0.509
1	0.250	62.03	64.37	66.76	69.19	71.66	74.16	76.71	79.30	81.93	84.60	87.30
2	0.275	20.78	22.00	23.27	24.59	25.96	27.38	28.84	30.35	31.91	33.52	35.17
3	0.300	7.01	7.31	7.65	8.05	8.49	8.98	9.52	10.11	10.75	11.44	12.17
4	0.325	8.09	7.70	7.35	7.05	6.79	6.58	6.41	6.28	6.20	6.16	6.17
5	0.350	16.43	15.58	14.76	13.98	13.24	12.53	11.86	11.22	10.62	10.06	9.54
6	0.375	27.72	26.59	25.48	24.40	23.35	22.33	21.33	20.36	19.42	18.51	17.63
7	0.400	39.59	38.32	37.06	35.82	34.60	33.40	32.21	31.05	29.90	28.78	27.68
8	0.425	50.85	49.53	48.23	46.93	45.64	44.36	43.09	41.83	40.59	39.35	38.13
9	0.450	60.96	59.67	58.38	57.09	55.80	54.52	53.23	51.96	50.68	49.41	48.15
10	0.475	69.76	68.53	67.30	66.06	64.82	63.58	62.33	61.08	59.83	58.58	57.33
11	0.500	77.26	76.12	74.97	73.81	72.64	71.47	70.29	69.10	67.90	66.70	65.50

$TW\,212 \quad r_3 = 0 \quad \phi = \sum(\mu - \hat{\mu})^2 w$

		1	2	3	4	5	6	7	8	9	10	11
	h \ σ	0.346	0.352	0.359	0.365	0.372	0.378	0.385	0.392	0.398	0.405	0.411
1	0.250	141.80	149.51	157.36	165.34	173.45	181.68	190.01	198.44	206.97	215.59	224.29
2	0.275	55.01	60.16	65.55	71.17	76.99	83.03	89.26	95.67	102.27	109.03	115.95
3	0.300	16.91	19.41	22.20	25.26	28.60	32.20	36.06	40.16	44.50	49.07	53.87
4	0.325	6.52	6.78	7.33	8.17	9.30	10.71	12.40	14.35	16.58	19.07	21.81
5	0.350	10.62	9.22	8.09	7.22	6.61	6.27	6.20	6.39	6.85	7.57	8.54
6	0.375	21.36	18.88	16.61	14.55	12.72	11.11	9.73	8.59	7.68	7.01	6.59
7	0.400	34.35	31.26	28.32	25.53	22.91	20.46	18.19	16.11	14.23	12.56	11.09
8	0.425	47.29	43.95	40.69	37.52	34.46	31.52	28.71	26.04	23.52	21.16	18.96
9	0.450	59.09	55.74	52.42	49.13	45.90	42.73	39.65	36.65	33.76	30.98	28.32
10	0.475	69.35	66.15	62.93	59.70	56.48	53.28	50.11	46.99	43.92	40.93	38.01
11	0.500	77.99	75.03	72.02	68.96	65.88	62.77	59.66	56.55	53.47	50.41	47.40

$TW\,211 \quad r_3 = 10 \quad \phi = \sum(\mu - \hat{\mu})^2 w$

		1	2	3	4	5	6	7	8	9	10	11
	h \ σ	0.579	0.594	0.608	0.623	0.637	0.652	0.666	0.681	0.695	0.710	0.724
1	0.250	183.50	199.37	215.67	232.36	249.41	266.81	284.52	302.52	320.79	339.31	358.06
2	0.275	92.94	104.88	117.44	130.58	144.27	158.49	173.20	188.38	203.99	220.01	236.42
3	0.300	41.70	49.64	58.31	67.70	77.77	88.49	99.84	111.79	124.30	137.37	150.95
4	0.325	16.12	20.48	25.61	31.50	38.14	45.50	53.56	62.30	71.70	81.74	92.39
5	0.350	6.70	8.09	10.24	13.13	16.76	21.13	26.23	32.04	38.54	45.73	53.58
6	0.375	6.99	6.10	5.88	6.34	7.50	9.36	11.92	15.17	19.13	23.77	29.09
7	0.400	12.79	10.23	8.23	6.82	6.03	5.87	6.36	7.49	9.29	11.74	14.86
8	0.425	21.34	17.69	14.43	11.66	9.40	7.68	6.53	5.96	5.99	6.63	7.89
9	0.450	31.07	26.64	22.55	18.89	15.58	12.72	10.32	8.43	7.06	6.24	5.97
10	0.475	40.92	36.11	31.53	27.20	23.19	19.57	16.28	13.39	10.95	8.98	7.49
11	0.500	50.31	45.37	40.56	35.90	31.45	27.25	23.33	19.80	16.56	13.72	11.29

Tab. V − 1 1

$min\sum(\mu-\widehat{\mu})^2 w$

	$TW211$ ($\gamma_3=-10$)				$TW211$ ($\gamma_3=0$)				$TW212$ ($\gamma_3=0$)				$TW211$ ($\gamma_3=10$)			
γ_2	100				100				100				100			
γ_3	− 10				0				0				10			
γ_4	6000				6000				6000				6000			
γ_5	− 400				− 400				−1400				− 400			
$min\Sigma(u_*-u_*)^2 w$ σ	0. 409				0. 500				0. 378				0. 681			
h	0. 275				0. 325				0. 350				0. 425			
$\Sigma v^2 w$	256. 5				262. 7				252. 6				292. 3			
$min\Sigma(\mu-\hat{\mu})^2 w$ σ	0. 423				0. 506				0. 385				0. 652			
h	0. 275				0. 325				0. 350				0. 400			
$\Sigma v^2 w$	6. 42				6. 16				6. 20				5. 87			
	1961	1962	1963	1964	1961	1962	1963	1964	1961	1962	1963	1964	1961	1962	1963	1964
H_0	119. 4	115. 5	113. 0	110. 0	102. 1	100. 9	100. 1	99. 0	125. 2	122. 1	120. 2	117. 7	83. 5	84. 1	84. 4	84. 7
H_2	−150. 1	−135. 3	−126. 2	−115. 6	−134. 0	−123. 4	−116. 7	−108. 6	−152. 3	−140. 3	−132. 7	−123. 4	−115. 5	−108. 4	−103. 8	− 98. 0
H_2'	−164. 2	−148. 0	−138. 1	−126. 4	−152. 3	−140. 3	−132. 7	−123. 4	−164. 1	−151. 1	−142. 9	−132. 9	−142. 9	−134. 1	−128. 3	−121. 2
Σv^2	0. 023	0. 028	0. 023	0. 020	0. 023	0. 027	0. 024	0. 017	0. 023	0. 028	0. 023	0. 018	0. 022	0. 028	0. 024	0. 018
R	0. 785	0. 808	0. 912	0. 948	0. 787	0. 811	0. 910	0. 947	0. 786	0. 804	0. 914	0. 949	0. 791	0. 819	0. 908	0. 945
$T.u$	0. 186	0. 158	0. 118	0. 0926	0. 187	0. 157	0. 124	0. 0877	0. 185	0. 160	0. 119	0. 0887	0. 178	0. 157	0. 122	0. 0892
W	46. 5	50. 5	53. 5	57. 5	39. 4	42. 8	45. 2	48. 6	36. 6	39. 7	42. 0	45. 1	32. 0	34. 8	36. 8	39. 5

Tab. V − 1 2 The results obtained by the steepest ascent method

	$TW211$ ($\gamma_3=-10$)		$TW211$ ($\gamma_3=0$)		$TW212$ ($\gamma_3=0$)		$TW211$ ($\gamma_3=10$)	
	initial value	convergence value	initial value	convergence value	initial value	convergence value	initial value	convergence value
γ_2	100	99. 9998	100	99. 999997	100	99. 999965	100	99. 999934
γ_3	− 10	−10. 000	0	−0.0000659	0	0. 0000132	10	10. 000038
γ_4	6000	6000	6000	6000	6000	6000	6000	6000
γ_5	− 400	− 400	− 400	− 400	− 1400	− 1400	− 400	− 400
σ	0. 42331	0. 41140	0. 50610	0. 49992	0. 38494	0. 382141	0. 65163	0. 64585
h	0. 275	0. 27877	0. 325	0. 32407	0. 350	0. 35139	0. 400	0, 402383
ϕ	6. 4185	→ 6. 3907	6. 1615	→6. 156997	6. 2002	→ 6. 19630	5. 8717	→ 5. 8541

Tab. V－13 Continued

$TW211$ ($r_3 = -10$)

	初期値	収束値(71step)
ϕ	6.4185	6.3260
r_2	100.0	99.894
r_3	−10.0	−8.5756
r_4	6000.0	6004.3
r_5	−400.0	−396.06
σ	0.4233	0.4245
h	0.275	0.2779

	1961	1962	1963	1964
H	116.4	112.9	110.8	108.1
H_2	−146.4	−132.4	−123.8	−113.6
H_2'	−160.2	−144.9	−135.5	−124.3
Σu^2	0.0240	0.0279	0.0228	0.0187
R	0.786	0.806	0.915	0.949
Tu	0.192	0.160	0.119	0.0898

$TW211$ ($r_3 = 0$)

	初期値	収束値(32step)
ϕ	6.1615	6.1186
r_2	100.0	99.459
r_3	0.0	0.1050
r_4	6000.0	5999.19
r_5	−400.0	−400.58
σ	0.5061	0.4997
h	0.325	0.3183

	1961	1962	1963	1964
H_0	101.2	100.1	99.25	98.21
H_2	−131.3	−121.0	−114.4	−106.4
H_2'	−148.8	−137.0	−129.6	−120.6
Σu^2	0.0233	0.0278	0.0233	0.0178
R	0.788	0.810	0.913	0.947
Tu	0.186	0.159	0.120	0.0883

$TW212$ ($r_3 = 0$)

	初期値	収束値(23step)
ϕ	6.2002	6.1907
r_2	100.0	100.003
r_3	0.0	0.0797
r_4	6000.0	6000.4
r_5	−1400.0	−1399.3
σ	0.3849	0.3839
h	0.350	0.3501

	1961	1962	1963	1964
H_0	125.0	121.9	120.0	117.5
H_2	−152.2	−140.1	−132.5	−123.3
H_2'	−163.8	−150.8	−142.7	−132.7
Σu^2	0.0232	0.0284	0.0229	0.0177
R	0.786	0.804	0.915	0.949
Tu	0.186	0.161	0.119	0.0879

$TW211$ ($r_3 = 10$)

	初期値	収束値(32step)
ϕ	5.8717	5.8408
r_2	100.0	99.918
r_3	10.0	10.100
r_4	6000.0	6000.0
r_5	−400.0	−398.00
σ	0.6516	0.6387
h	0.400	0.3936

	1961	1962	1963	39
H_0	83.53	84.10	84.42	84.78
H_2	−114.8	−107.7	−103.1	−97.38
H_2'	−140.8	−132.1	−126.4	−119.4
Σu^2	0.0227	0.0278	0.0239	0.0173
R	0.792	0.818	0.910	0.945
Tu	0.180	0.158	0.122	0.0884