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Abstract	Since the pioneering work by Stone(1954), a large body of literaturehas developed concerning the estimation of complete demand systems. Becausemost demand systems are nonlinear in either or both parameters and variables,estimation of the parameters of the system is usually carried out usingiterative methods. Here, it is crucial that initial estimates be selectedwith desirable characteristics, both statistically and theoretically.In this paper, we propose an alternative approach that is useful whenthere are many parameters to be estimated in the system. (1) We specify notthe indirect utility function but the direct utility function with a budgetconstraint. As the first order condition for utility maximization, we obtainthe marginal rates of substitution (MRS) equations which serve as our initialestimates of the parameters. These allow us to calculate the unobservablemarginal utility of total expenditure, and also to reestimate all the initialparameters in the system. The appeal of this approach is that it is relativelyeasy to obtain initial estimates whose values satisfy restrictions imposed bytheory. This is because MRS equations measure the relationship between onlytwo commodities, so that the number of parameters to be estimated is smallcompared to that of the whole system.In Section II we provide a detailed discussion of the theoretical model.This is followed by a discussion of the data and the estimation procedure inSection III. In Section IV, we highlight the empirical results by comparingour estimates of marginal budget shares, and elasticities of demand with thoseof previous researchers. Also, we calculate the correlation matrix of thedisturbances to examine the plausibility of an additive utility function.These are followed in Section V by some concluding remarks.
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June 1985

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I. Introduction

Since the pioneering work by Stone(1954), a large body of literature has developed concerning the estimation of complete demand systems. Because most demand systems are nonlinear in either or both parameters and variables, estimation of the parameters of the system is usually carried out using iterative methods. Here, it is crucial that initial estimates be selected with desirable characteristics, both statistically and theoretically.

Traditional approaches typically begin by specifying the indirect utility function. Demand equations, derived from applying Roy's identity to the indirect utility function, are first estimated separately by OLS to obtain initial estimates of the nonlinear system. Based on these estimates, the feasible parameters of the system are obtained using seemingly unrelated regressions (SUR) or nonlinear simultaneous equation methods which impose cross equation and/or parameter restrictions.

A stumbling block in using this approach is obtaining theoretically plausible parameters when the number of parameters in the system becomes large. This might arise, for example, when total expenditure is divided into a large number of commodities and services, or when the theory includes not only total expenditure and prices but also other demographic factors or shift variables which affect the determination of consumer expenditure allocation. This is one of the main reasons why traditional approaches typically analyse only relatively broad categories of commodities and services. The risk of not doing so is that parameters estimates are often obtained that violate restrictions imposed by theory.

In this paper, we propose an alternative approach that is useful when there are many parameters to be estimated in the system.[1] We specify not the indirect utility function but the direct utility function with a budget constraint. As the first order condition for utility maximization, we obtain the marginal rates of substitution (MRS) equations which serve as our initial estimates of the parameters. These allow us to calculate the unobservable marginal utility of total expenditure, and also to reestimate all the initial parameters in the system. The appeal of this approach is that it is relatively easy to obtain initial estimates whose values satisfy restrictions imposed by theory. This is because MRS equations measure the relationship between only two commodities, so that the number of parameters to be estimated is small compared to that of the whole system.

In Section II we provide a detailed discussion of the theoretical model. This is followed by a discussion of the data and the estimation procedure in Section III. In Section IV, we highlight the empirical results by comparing our estimates of marginal budget shares, and elasticities of demand with those of previous researchers. Also, we calculate the correlation matrix of the disturbances to examine the plausibility of an additive utility function. These are followed in Section V by some concluding remarks.

II. Theoretical model

In this section, we first contrast the alternative approaches based either on the Stone-Geary direct or indirect utility function. We then extend the direct utility function approach to include the effects of demographic factors, habit formation, and stock adjustment of durable goods.

Indirect utility function approach

Most traditional approaches start with the Stone-Geary indirect utility function:

$$(2.1)\qquad \Psi = (M - \Sigma p_j \gamma_j)\Pi p_j^{-\beta_j}, \qquad \Sigma\beta_j = 1,$$

where M is total expenditure, p_j is the price of the jth item, γ_j's and β_j's are parameters to be estimated, and both M and p_j are assumed to be exogenous. By Roy's identity, the linear expenditure system (LES) demand functions for N commodities and services may be derived as:

$$(2.2)\qquad q_i = -(\partial\Psi/\partial p_i)/(\partial\Psi/\partial M) = (\gamma_i \Pi p_j^{-\beta_j} + (M - \Sigma p_j\gamma_j)(\beta_i/p_i)\Pi p_j^{-\beta_j})/\Pi p_j^{-\beta_j}$$
$$= \gamma_i + (M - \Sigma p_j \gamma_j)(\beta_i/p_i).$$
$$(i = 1,\ 2,\ \ldots, N)$$

Econometric specifications typically add a random disturbance to (2.2), transform the demand functions into share functions to avoid heteroscedasticity of the disturbance, and drop one equation from the system because of singularity, such that

$$(2.3)\qquad s_i = p_i q_i/M = \gamma_i(p_i/M) + \beta_i(1 - \Sigma\gamma_j(p_j/M)) + u_i,$$
$$(i = 1,\ 2,\ \ldots,\ N-1)$$

where u_i is assumed to be i.i.d. normal. OLS regressions of (2.3) for each ith equation are used to obtain initial estimates for the demand system.

Direct utility function approach

The corresponding direct utility function is

$$(2.4)\qquad u = \Sigma\beta_j \log(q_j - \gamma_j).$$

Maximising (2.4) under the budget constraint $M = \Sigma p_j q_j$, we obtain the first order condition:

$$(2.5)\qquad \begin{cases} (\partial u/\partial q_i)/p_i = (\partial u/\partial q_j)p_j = \lambda', \\ \qquad (i \neq j;\ i,j = 1,\ 2,\ \ldots,\ N) \\ \Sigma p_j q_j = M \end{cases}$$

where λ' is the marginal utility of total expenditure. The LES specification of the first equation of (2.5) is:

$$(2.6)\qquad \beta_i/p_i(q_i - \gamma_i) = \beta_j/p_j(q_j - \gamma_j) = \lambda'.$$

Since (2.6) is nonlinear in parameters, we rely instead on the reciprocal of the first order condition,

(2.7) $p_i(q_i - \gamma_i)/\beta_i = p_j(q_j - \gamma_j)/\beta_j = \lambda,$

which is linear in parameters $1/\beta_i$ and γ_i/β_i.[2] Solving (2.7) with the budget constraint, we obtain the LES share function as

(2.8) $s_i = p_i q_i/M = \gamma_i(p_i/M) + (\beta_i/\Sigma\beta_j)(1 - \Sigma\gamma_j(p_j/M)).$

A stochastic specification of equation (2.7) may be written as

(2.9) $p_i(a_i + \alpha_i q_i) = \lambda + v_i$

where $a_i = -\gamma_i/\beta_i$ and $\alpha_i = 1/\beta_i$.

Comparing (2.9) and (2.3), note that u_i in (2.3) is equivalent to $v_i/\alpha_i M$, and that u_i and v_i are subject to the following constraint

(2.10) $\begin{cases} \Sigma u_i = 0 \\ \Sigma v_i = 0. \end{cases}$

Further, recall that one equation in (2.3) is usually dropped to avoid singularity of the variance-covariance matrix of the multivariate $\{u_i\}$. This is mathematically identical to writing (2.9) as

(2.11) $p_i(a_i + \alpha_i q_i) = p_j(a_j + \alpha_j q_j) + v_j, \quad (j \neq i)$

because both N-1 estimating equations of (2.3) and (2.11) contain N-1 disturbances in the system.

Extensions to include demographic factors, habit formation and stock adjustment of durable goods

In addition to total expenditure and prices, a number of studies argue that demographic changes, habit formation, and stock adjustment are important factors which affect expenditure allocation. Studies by Pollak and Wales(1978, 1980, 1981) have empirically confirmed the importance of family size - the demographic effect. In their studies of U.S. consumer demand, Houthakker and Taylor(1966, 1970) considered two kinds of state adjustment: habit formation and stock adjustment, and found both to be important factors in determining expenditure allocation.[3]

In this paper, we will explicitly specify habit formation and stock adjustment of durable goods in a Stone-Geary utility function.[4] Considering the following Stone-Geary utility function:

$$(2.12)\quad \begin{cases} u = \Sigma\beta_j \log(x_j - \gamma_j) \\ x_j = d'_j S_j + q_j \\ \gamma_j = -(a'_{0j} + b'_j m + c'_j H_j), \end{cases}$$

where m is family size, H_j is habit formation calculated as the cumulative sum of constant price expenditure over the past year, S_j is the current period net stock of the jth commodity evaluated in constant dollars,[5] and q_j is the quantity purchased.

Figure 1 clarifies the implications of introducing demographic changes, habit formation and stock adjustment into the model. In terms of an indifference map, for given $q_i - q_j$, the stock adjustment effect is indicated by shifts in the $x_i - x_j$ axis to reflect changes in stock between periods. The shift in the preference field due to habit formation (H_i and H_j) or to family size (m) is described by the change in the shape of the indifference map itself.

Before turning to the estimation procedure, consider the restrictions on the parameters of the demand system suggested by theory. The restrictions are:

$$(2.13)\quad \begin{cases} x_j - \gamma_j > 0, \\ \lambda > 0, \\ H \text{ is negative definite,}^{(6)} \\ \partial(\partial u/\partial q_j)/\partial m > 0, \\ \partial(\partial u/\partial q_j)/\partial H_j > 0, \\ \partial(\partial u/\partial q_j)/\partial S_j < 0, \end{cases}$$

$$(j = 1, 2, \ldots, N)$$

where H in (2.13) is the bordered Hessian matrix. The first restriction follows from (2.12), and the next two from utility maximization. The last three restrictions refer to the effects of changes in family size, habit formation or stock adjustment on the marginal utility curve. An increase in family size from period t to period t+1 would, for example, shift the marginal utility curve upward, which is equivalent to an increase in the committed expanditure on that item. This is shown in Figure 2. Similarly, we would expect next period committed expenditure to increase with habit formation and decrease with previous stock adjustment.

All the restrictions in (2.13) are satisfied by the following conditions:

$$(2.14)\quad \begin{cases} b'_j < 0,\ c'_j < 0,\ d'_j > 0,\ \beta_j > 0, \\ a'_{0j} + b'_j m + c'_j H_j + d'_j S_j + q_j > 0. \end{cases}$$

In Section III these conditions (2.14) are used to select the initial set of parameter estimates for the maximum likelihood procedure.(7)

III. The estimation procedure

We apply this methodology to the analysis of U.S. consumer behavior over the period from 1960-I to 1981-IV, i.e. an observation period of eighty-eight quarters.(8) The data used are quarterly series of prices, per capita quantities, and population obtained from the _National Income and Product Accounts of the United States_ (NIPA), stocks of three consumer durable goods from Musgrave(1979, 1982) in the _Survey of Current Business_ (SCB), and the number of households from _Current Population Report_(P-20). Commodities and services are classified into the following thirteen categories:

1. Motor vehicles and parts
2. Furniture and household equipment
3. Other durable goods
4. Food
5. Clothing
6. Gasoline and oil
7. Fuel oil and coal
8. Other nondurable goods
9. Housing

10. Electricity and gas

11. Other household operation

12. Transportation

13. Other services.

As noted in Section II, the fundamental equation on which our approach is based is the MRS equation of two commodities. Introducing family size, habit formation and stock adjustment into (2.11), we obtain the following equation:

$$(3.1) \quad p_k(a_{0k} + b_k m + c_k H_k + \alpha_k q_k) = p_j(a_{0j} + b_j m + c_j H_j + d_j S_j + \alpha_j q_j),$$

where $a_{0i} = a'_{0i}/\beta_i$, $b_i = b'_i/\beta_i$, $c_i = c'_i/\beta_i$, $d_i = d'_i/\beta_i$ and $\alpha_i = 1/\beta_i$. Note that equation (3.1) is linear in parameters. Further, because (3.1) is a homogeneous function of the parameters, we can choose any arbitrary kth commodity as the normalized item and define α_k as unity.(9) We choose Food (item 4) as the normalized item largely because it does not include a stock variable, S_4, which reduces the number of parameters to be estimated.

To obtain the initial estimates of the parameters, we propose the use of a new technique.(10) This is useful, even if the demand system and MRS equations are nonlinear in original parameters and variables, when MRS equations can be made linear.

Consider the following equation,

$$(3.2) \quad y = X\beta + \varepsilon$$

where y is a vector of order T, X is a matrix of T by K (T > K), β is a vector of order K and ε is a vector of random disturbances of order T. If we use K arbitrary raws in X, $\hat{\beta}$ is uniquely estimated such as

$$(3.3) \quad \hat{\beta} = \bar{X}^{-1}\bar{y}$$

where $\bar{X}$ is a K by K matrix of full rank, and $\bar{y}$ is a vector of order K and $\hat{\beta}$ is the linear unbiased estimator.(11) Using this method, we obtain ${}_{T}C_{K}$ sets of $\hat{\beta}$.

Let us apply this method to the present model. Rewriting (3.1) as

$$(3.4)\quad q_k = -a_{0k} - b_k m - c_k H_k + a_{0j}(p_j/p_k) + b_j m(p_j/p_k) + c_j H_j(p_j/p_k) + d_j S_j(p_j/p_k) + \alpha_j q_j(p_j/p_k) + \varepsilon_j,$$
$$(j = 1, 2, \ldots, 13;\ j \neq k)$$

where item k is the normalized item and fixed as 4, and j will change from 1 through 13 except k. In relation to (3.2), we can define y, X, ε and β as

$$(3.5)\quad y = \begin{bmatrix} q_k(1) \\ \cdots \\ q_k(T) \end{bmatrix},\ \varepsilon = \begin{bmatrix} \varepsilon_j(1) \\ \cdots \\ \varepsilon_j(T) \end{bmatrix},\ \beta = \begin{bmatrix} -a_{0k} \\ -b_k \\ -c_k \\ a_{0j} \\ b_j \\ c_j \\ d_j \\ \alpha_j \end{bmatrix},$$

$$X = \begin{bmatrix} -1 & m(1) & H_k(1) & z_j(1) & m(1)z_j(1) & H_j(1)z_j(1) & S_j(1)z_j(1) & q_j(1)z_j(1) \\ & & & & \cdots & & & \\ -1 & m(T) & H_k(T) & z_j(T) & m(T)z_j(T) & H_j(T)z_j(T) & S_j(T)Z_j(T) & q_j(T)Z_j(T) \end{bmatrix}.$$

where $z_j(t) = p_j(t)/p_k(t)$. For item 4 we can obtain $(N-1)_T C_K$ sets of a_{04}, b_4 and c_4. Using the proper sets of the parameters $\hat{\beta}$, which satisfy all the theoretical restrictions indicated in (2.14), we made the marginal distribution of $\hat{\beta}$, and found the mode of the parameters of the normalized item; $\hat{a}_{04}$, $\hat{b}_4$ and $\hat{c}_4$ as 210.0, -70.0 and -0.11, respectively. Then we calculated the reciprocal of the marginal utility of total expenditure $(\hat{\lambda}(t))$ using $\hat{a}_{04}$, $\hat{b}_4$, $\hat{c}_4$ and (2.7) for each period,

$$(3.6)\quad \hat{\lambda}(t) = p_4(t)(\hat{a}_{04} + \hat{b}_4 m(t) + \hat{c}_4 H_4(t) + q_4(t)).$$
$$(t = 1, 2, \ldots, 88)$$

By utilizing $\hat{\lambda}(t)$, we estimated $\hat{a}_{0j}$, $\hat{b}_j$, $\hat{c}_j$, $\hat{d}_j$ and $\hat{\alpha}_j$ $(j \neq 4)$ by the following regression,

$$(3.7)\quad \hat{\lambda}(t)/p_j(t) = a_{0j} + b_j m(t) + c_j H_j(t) + d_j S_j(t) + \alpha_j q_j(t) + v_j(t).$$
$$(j \neq 4,\ t = 1, 2, \ldots, 88)$$

With these as initial estimates, we reestimate the model by the nonlinear full information maximum likelihood method (NFIML) proposed by Amemiya(1977), using the algorithm developed by Berndt, Hall, Hall and Hausman(1974). The concentrated log-likelihood function is:

(3.8) $$L = \Sigma\log||\partial f_t/\partial y'_t|| - (T/2)\log|(1/T)\Sigma f_t f'_t|,$$

where $f_{it} = p_i q_i/M - (1 - \Sigma(1/\alpha_j)a_i(p_i/M) + \Sigma a_j(p_j/M)/\alpha_j)/(\alpha_i\Sigma(1/\alpha_j))$ and $y = \{q_i\}$ (1 = 1, 2, ..., N-1). The first order derivatives of L with respect to g, which are one element of the parameters, and the second order derivatives of L with respect to g and h are:

(3.9) $$\partial L/\partial g = -\Sigma\Sigma\Omega^{ij}\Sigma(\partial f_{it}/\partial g)f_{jt},$$

(3.10) $$\partial^2 L/\partial g\partial h = \Sigma\Sigma(\Sigma\Sigma\Omega^{in}(T^{-1}\Sigma((\partial f_{nt}/\partial h)f_{mt} + f_{nt}(\partial f_{mt}/\partial h)))\Omega^{mj})\Sigma(\partial f_{it}/\partial g)f_{jt}$$
$$- \Sigma\Sigma\Omega^{ij}\Sigma((\partial f_{it}/\partial g)(\partial f_{jt}/\partial h) + (\partial^2 f_{it}/\partial g\partial h)f_{jt}),$$

where Ω^{ij} is the (i,j) element of the inverse of the sample variance-covariance matrix.

The solution of the NFIML is obtained by the equation $\partial L/\partial g = 0$. Given the assumptions in Amemiya(1977), we can obtain the limiting distribution of the parameters as

(3.11) $$T^{1/2}(\theta - \theta_0) \rightarrow N(0, -\text{plim}(1/T)\partial^2 L/\partial\theta\partial\theta'|_{\theta_0}{}^{-1}).$$

IV. Empirical results and evaluation

The empirical results are reported in Tables 1 and 2. Table 1 reports the NFIML estimates of the parameters and their standard errors. Note that all the preference parameters satisfy the theoretical restrictions listed in (2.14), a result which suggests that the proposed method produces good initial parameter estimates for the maximum likelihood procedure. The goodness of fit for the LES share functions and demand functions are reported in Table 2. Evaluated by the R squared of the share functions, all items exceed 0.8 with the exception of Automobiles and parts(1), Furniture and household equipment(2), Housing(9) and Other household operation(11), while evaluated by that of the demand functions all items exceed 0.8.

We applied the Wald test to explore several hypotheses about the preference parameters. The Wald test statistics (see Gallandt and Holly(1980)) is defined by

(4.1) $W = Th'(\theta_n)(H(\theta_n)\Omega H(\theta_n))^{-1}h(\theta_n)$,

where $h(\theta_n)$ is an r-vector valued function and $H(\theta_n) = (\partial/\partial\theta')h(\theta)$. In its linear hypothesis, (4.1) can be written simply as:

(4.2) $W = Th'(\theta_n)\Omega^{-1}h(\theta_n)$

where Ω is the asymptotic variance-covariance matrix of the related parameters.

We set up ten null hypotheses:

(i) $\begin{cases} H_0: a = 0 \\ H_a: a \neq 0 \end{cases}$ (ii) $\begin{cases} H_0: b = 0 \\ H_a: b \neq 0 \end{cases}$ (iii) $\begin{cases} H_0: c = 0 \\ H_a: c \neq 0 \end{cases}$

(iv) $\begin{cases} H_0: d = 0 \\ H_a: d \neq 0 \end{cases}$ (v) $\begin{cases} H_0: d = \alpha \\ H_a: d \neq \alpha \end{cases}$ (vi) $\begin{cases} H_0: a_{0i} = 0 \\ H_a: a_{0i} \neq 0 \end{cases}$

(vii) $\begin{cases} H_0: b_i = 0 \\ H_a: b_i \neq 0 \end{cases}$ (viii) $\begin{cases} H_0: c_i = 0 \\ H_a: c_i \neq 0 \end{cases}$ (xi) $\begin{cases} H_0: d_i = 0 \\ H_a: d_i \neq 0 \end{cases}$

(x) $\begin{cases} H_0: \alpha_i = 0 \\ H_a: \alpha_i \neq 0 \end{cases}$

where $a = (a_{01}, a_{02}, \ldots, a_{013})$, $b = (b_1, b_2, \ldots, b_{13})$, $c = (c_1, c_2, \ldots, c_{13})$, $d = (d_1, d_2, d_3)$, and α in (v) is $(\alpha_1, \alpha_2, \alpha_3)$. The first hypothesis tests for whether all intercepts are zero. The next three tests (ii) through (iv) examine the significance of total effect of family size, habit formation and stock adjustment. The fifth tests for the equality of the user price of the stock of a durable good and its purchased price. In this formulation, where

(4.3) $x_i = d'_i S_i + q_i$,

equality holds if $d'_i = i$.[12] Finally the last five tests examined the significance of each parameter.[13]

The Wald tests are reported in Table 3. In (ii) through (iv), the results suggest that consumer behavior is significantly affected by family size and habit formation, but not stock adjustment at the 0.05 level of significance. The rejection of null hypothesis (v) indicates that quantity purchased and stocks of durable goods cannot be treated as homogeneous; in the present model the difference between purchased price and user price for a durable good is indicated by the parameters d_i and α_i. (vi) through (x) suggest that family size effect is weak in Furniture and household equipment(2), Other durable goods(3), Gasoline and oil(6), Electricity and gas(10), and Other household operation(11) (case(vii)). All of the parameters of habit formation except Other household operation(11) are meaningful(case(viii)). On the other hand, none of the parameters of the stock adjustment effect are significantly different from zero(case(ix)).

Given the last finding, we reestimated the parameters of the model excluding the stock adjustment effect. These results are reported in Tables 4 and 5. This new specification produces a slightly poorer fit than the previous model, but the difference is not significant. The hypothesis tests of family size and habit formation suggest that these effects are still present in the model.

Tables 6 and 7 compare our results with those of previous researchers. For example, measured against Phlips(1972) estimates of marginal budget share (MBS) we find smaller MBS's in durable goods, smaller MBS's in nondurable goods except Clothing(5) and Other nondurable goods(8), and a considerably higher Housing(9) MBS. In Table 7, we campared income, and both uncompensated and compensated price elasticity of demand to those obtained by Houthakker and Taylor(1970), and by Phlips(1972). Most elasticities have the same property of being elastic or inelastic except the income elasticity of two durable goods ((2), (3)), Clothing(5), Other nondurable goods(8), and the price elasticity of Other nondurable goods(8). Another point to note is that we estimated slightly higher income and price elasticities of Housing(9).

Finally we examined the correlation matrix of disturbances to determine if the assumption of an additive utility function is appropriate (see Phlips(1971)).

Only if all correlations are negative (i.e. all goods are substitutes), can we confirm the existence of an additive utility function. From Table 8 it is clear signs differ, suggesting that the Stone-Geary utility function may be too restrictive and is a misspecification of the function, even though we confirmed the importance of habit formation and family size effects. It is difficult to identify complementarity of two goods intuitively when commodities and services are highly aggregated. However, in the present classification, we can say that Automobiles and parts(1) and Gasoline and oil(6) can be considered as complements. This is empirically verified by the positive correlation shown in Table 8.

V. Concluding remarks

We proposed an alternative method of obtaining the initial estimates to conduct iteration for a nonlinear complete demand system. Our approach is unique in specifying a direct utility function and using the first order condition, MRS equation, which is the result of the utility maximization. The MRS equation is the relation between two arbitrary commodities, though implicitly the relation of the whole system. Therefore, when we estimate MRS equations, the number of parameters which appear in the equation is relatively small, and it is easy to impose restrictions on the parameters. Even if initial estimates of the present method do not have the property of consistency, we can obtain asymptotically consistent estimates by the iteration procedure of NFIML method. The present approach would be useful when we have to treat a large number of parameters or commodities in a complete demand system.

The idea of estimating MRS equations presented in this paper will be useful to apply a multitemporal maximization scheme, e.g. an application to the life cycle hypothesis of labor supply or saving.

We distinguished two factors contained in the state adjustment hypothesis in a specific form; habit formation as the endogeous shift of the preference field and stock adjustment as the shift of the stock axis of durable goods.

Based on this hypothesis we specified family size, habit formation and stock adjustment of durable goods in the Stone-Geary utility function, and showed the importance of family size and habit formation in consumer demand theory. To extract stock adjustment explicitly we will have to get better data for the stock of consumer durable goods and consider a more complex model than the present one. Finally, it seems restrictive to assume an additive utility function even for the aggregate data.

Footnotes

1. In the present paper, total expenditure is divided into thirteen categories, and the model includes not only total expenditure and prices but also demographic factors, habit formation and stock adjustment. Thus the total number of the parameters in the system totals fifty-five.

2. In relation to estimation procedure in Section III, it is important to stress on the following two points concerning the MRS equation (2.7): First, it is the fundamental equation used to estimate preference parameters and, second, it is linear in parameters. We estimated MRS equations because this reduced the number of parameters to be estimated in each equation. To obtain initial estimates we developed a relatively easy method which is based on the linear model explained in Section III.

3. Note that while both Houthakker and Taylor(1970) and Phlips(1972) consider the effect of state variables in endogeous shifts of the preference field, they do not explicitly distinguish between habit formation and stock adjustment.

4. We define habit formation as the endogeous shift of the preference field and stock adjustment of durable goods as the shift of the stock axis.

5. Note that S_j to be positive for durable goods and difinitionally zero for nondurable goods and serivices.

6. $H = \begin{bmatrix} 0 & b \\ b' & D \end{bmatrix}$

where $b = (p_1, \ldots, p_N)$, and D is the Nth order square matrix of second derivatives. The determinant of the matrix H is:

$$|H| = |D - bD^{-1}b'|$$
$$= (-1)^n \Pi(\alpha_i/(x_i - \gamma_i)^2)\Sigma(p_i^2(x_i - \gamma_i)^2/\alpha_i).$$
$$(|D| \neq 0)$$

Therefore, H is negative definite matrix, and the second order condition is fulfilled in the area where preference field exists.

7. Before entering into the next section, it is important to note the following remark. We also tried the traditional technique, the indirect utility function approach, to obtain feasible parameters of the model. Combining (2.3) and OLS procedure, we first obtained initial estimates of the model. Introducing family size, habit formation and stock adjustment into (2.3), the number of parameters appearing in each equation to be estimated totals forty-three. Though these initial estimates are consistent estimators, it was difficult for them to satisfy the theoretical restrictions indicated in (2.14). After applying SUR method to the model whose initial estimates are obtained from OLS, we could not obtain theoretically plausible parameters set of the model.

8. The observation period ends at the fourth quarter of 1981 because the stock data for the three durable goods are available only up to that date.

9. This normalization is mathematically equivalent to the typical approach of normalizing the sum of β_i's as unity.

10. To start an iteration, we need initial estimates which are as close as posible to the true value. Amemiya(1983) proposed two kinds of procedures in obtaining initial estimates; a pure guess and a method by Hartley and Booker (1965). In the present method referring to the thinking of Hartley and Booker, we consider a new method in obtaining initial estimates both theoretically and statistically plausible, and also it is easy to compute estimator.

11. $\bar{X}$ and $\bar{\varepsilon}$ are not mutually independent in the present analysis. Therefore, we obtain a biased linear estimator. However, the estimator obtained from our maximum likelihood method is asymptotically consistent.

12. If $d_i = \alpha_i$ in case(v), then $d_i = d'_i/\beta_i = 1/\beta_i = \alpha_i$, and therefore $d'_i = 1$.

13. From (i) through (iv) we calculated

$$W = \hat{\theta}'\Omega(\hat{\theta})^{-1}\hat{\theta},$$

where $\hat{\theta}$ indicates $\hat{a}$, $\hat{b}$, $\hat{c}$ or $\hat{d}$. For (v) we calculated

$$W = (\hat{d} - \hat{\alpha})'\Omega(\hat{d} - \hat{\alpha})^{-1}(\hat{d} - \hat{\alpha})$$

where $\Omega(\hat{d} - \hat{\alpha})^{-1}$ is equal to $\Omega_{\hat{d}\hat{d}}^{-1} + \Omega_{\hat{\alpha}\hat{\alpha}}^{-1} - \Omega_{\hat{\alpha}\hat{d}}^{-1} - \Omega_{\hat{d}\hat{\alpha}}^{-1}$. For (vi) to (x) we calculated

$$W = \hat{\theta}_i^{\,2}/\hat{s}_{ii},$$

where $\hat{s}_{ii}$ is the variance of θ_i.

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Figure 1. Effects of family size, habit formation and stock adjustment

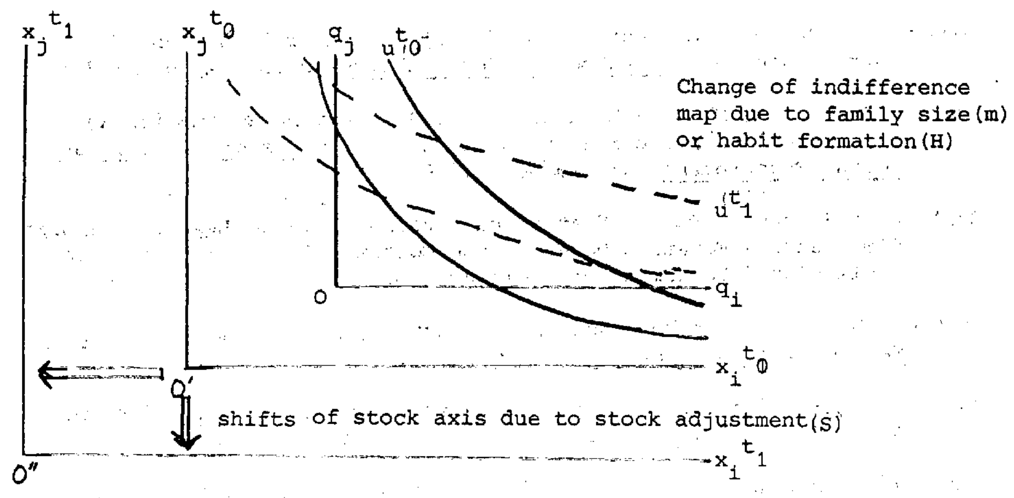

Figure 2. Shift of the marginal utility curve due to the increase in family size

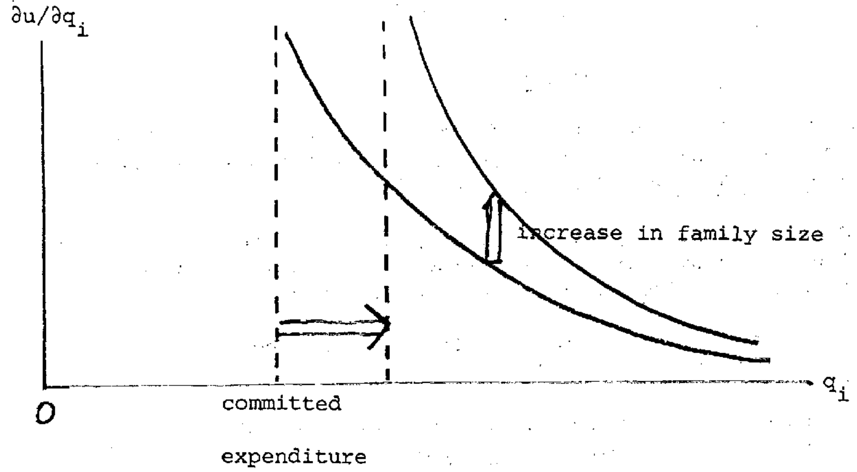

Table 1. Preference parameters including the stock adjustment

item	a_{0i}	b_i	c_i	d_i	α_i
1	488.17	-131.92	-.2429	.01117	1.2091
	(126.89)	(33.90)	(.1235)	(.03743)	(.4090)
2	41.32	-25.80	-2.2134	.24632	6.9455
	(568.86)	(144.16)	(.7635)	(.1757)	(2.3830)
3	101.69	-29.34	-5.2696	.42801	18.2115
	(651.31)	(172.35)	(2.5379)	(.46425)	(8.8885)
4	209.67	-69.87	-.1100		1.0000
	(67.57)	(21.20)	(.02379)		
5	262.20	-82.83	-.6279		3.9782
	(112.20)	(31.32)	(.2605)		(.9704)
6	29.09	-16.29	-5.3302		24.9022
	(109.12)	(30.69)	(2.6684)		(11.2417)
7	154.50	-52.95	-3.2959		26.7961
	(65.79)	(26.01)	(1.0722)		(5.7152)
8	427.86	-126.34	-.1896		1.0262
	(78.94)	(23.95)	(.0333)		(.0988)
9	290.26	-81.32	-.2139		1.3035
	(101.76)	(28.71)	(.0521)		(.2034)
10	52.71	-16.64	-2.8005		14.9068
	(148.43)	(39.18)	(1.0985)		(4.3533)
11	-2.55	-16.32	-4.1095		21.0468
	(230.35)	(54.47)	(2.9138)		(12.2261)
12	816.08	-197.15	-3.6835		11.4256
	(160.43)	(38.53)	(.9239)		(2.6949)
13	388.18	-94.81	-.2503		1.0167
	(121.76)	(33.26)	(.0789)		(.2205)

Note: The pharenthesis in the table indicates the standard error.

Table 2. Goodness of fit for LES share functions and LES demand functions

item	R^2 (LES share)	R^2 (LES demand)
1	.5754	.8355
2	.7112	.9891
3	.8978	.9858
4	.9577	.8254
5	.9181	.9598
6	.9679	.9204
7	.8651	.9068
8	.9458	.9677
9	.7273	.9887
10	.9563	.9410
11	.7184	.9788
12	.9523	.9847
13	.9774	.9936

Table 3. Hypothesis testing using the Wald test

case	χ^2	d.f.
(i)	76.34	13
(ii)	6248.94	13
(iii)	276769.92	13
(iv)	2.06	3
(v)	18.50	3

Note: $\chi^2(13)_{0.05} = 22.4$

$\chi^2(13)_{0.05} = 7.81$

$\chi^2(1)_{0.05} = 3.84$

case	(vi)	(vii)	(viii)	(ix)	(x)
item	$\chi^2_{a_{0i}}$	$\chi^2_{b_i}$	$\chi^2_{c_i}$	$\chi^2_{d_i}$	$\chi^2_{\alpha_i}$
1	14.79	15.13	3.86	.08	8.73
2	.005	.03	8.40	1.96	8.43
3	.02	.02	4.31	.84	4.19
4	9.62	10.85	21.38		
5	5.46	6.98	5.80		16.80
6	.07	.28	3.98		4.90
7	5.51	4.14	9.44		21.98
8	35.87	27.82	32.32		207.83
9	8.13	8.02	16.83		41.04
10	.12	.18	6.19		11.72
11	.0001	.089	1.98		2.96
12	25.87	26.17	15.89		17.97
13	10.16	8.12.	10.06		21.25

Table 4. Preference parameters obtained excluding the stock adjustment variable

item	a_{0i}	b_i	c_i	α_i
1	502.75 (69.83)	-136.24 (19.89)	-.2866 (.1573)	1.4971 (.5936)
2	661.26 (138.27)	-183.03 (37.29)	-1.7547 (.6924)	7.1005 (2.6000)
3	525.08 (214.84)	-142.27 (56.28)	-4.3882 (1.9780)	18.0902 (7.3948)
4	209.75 (66.32)	-69.93 (21.70)	-.1099 (.0223)	1.0000
5	263.16 (103.94)	-83.07 (29.37)	-.6295 (.2527)	3.9814 (.9078)
6	30.51 (1111.35)	-16.67 (31.54)	-5.3439 (2.6115)	24.9382 (10.8317)
7	154.89 (76.84)	-53.12 (29.37)	-3.2720 (1.0154)	26.7173 (5.1814)
8	473.60 (77.01)	-126.53 (23.96)	-.1899 (.0314)	1.0255 (.0919)
9	290.06 (96.56)	-81.29 (27.77)	-.2133 (.0535)	1.3017 (.2132)
10	53.93 (150.28)	-16.95 (40.17)	-2.7958 (.9682)	14.8713 (3.8469)
11	-.35 (212.32)	-16.84 (49.98)	-4.1292 (2.5229)	21.1009 (10.6374)
12	819.43 (152.47)	-197.97 (36.66)	-3.6939 (.9587)	11.4394 (2.7657)
13	387.82 (100.35)	-94.71 (28.09)	-.2499 (.0802)	1.0162 (.2251)

Note:The pharenthesis in the table indicates the standard error.

Table 5. Hypothesis testing without stock adjustment

item	$\chi^2_{a_{0i}}$	$\chi^2_{b_i}$	$\chi^2_{c_i}$	$\chi^2_{\alpha_i}$
1	51.82	46.89	3.31	6.35
2	22.86	24.08	6.42	7.45
3	5.97	6.38	4.92	5.89
4	9.99	10.38	24.18	
5	6.40	7.94	6.20	19.23
6	.07	.27	4.18	5.30
7	4.06	3.27	10.38	26.58
8	37.81	27.87	36.57	124.48
9	9.02	8.56	15.88	37.25
10	.12	.17	8.33	14.94
11	.000002	.11	2.67	3.93
12	23.88	29.15	14.84	17.10
13	14.93	11.36	9.71	20.37

Note: $\chi^2(1)_{0.05} = 3.84$.

Table 6. Marginal budget shares of Phlips(1972) and ours

item	Phlips	Ours	
1	.2680	.1304	
2	.1141	.0275	
3	.0256	.0108	
4	.1829	.1952	
5	.1072	.0490	
6	.0178	.0078	.0151
7	-	.0073	
8	.0869	.1904	
9	.0214	.1499	
10	.0417	.0131	.0224
11	-	.0093	
12	.0284	.0170	
13	.1060	.1921	

Table 7. Income, uncompensated and compensated price elasticities of demand

	Houthakker and Taylor(1970)			Phlips(1972)			Ours		
item	income	uncompensated	compensated	income	uncomp.	comp.	income	uncomp.	comp.
1	6.34	-1.87	-1.54	5.48	-1.00	-.73	2.55	-1.39	-1.21
2	2.43	-.93	-.76	2.04	-.45	-.33	.48	-.28	-.25
3	1.24	-.49	-.47	1.50	-.29	-.27	.47	-.26	-.25
4	.71	-.47	-.21	.74	-.29	-.11	.87	-.56	-.37
5	1.24	-.57	-.41	1.18	-.30	-.19	.64	-.37	-.33
6	.41	-.16	-.15	.58	-.12	-.11	.21	-.12	-.11
7	-	-	-	-	-	-	1.07	-.58	-.57
8	.79	-.38	-.28	.97	-.25	-.16	2.20	-1.13	-.94
9	.06	-.03	-.02	.17	-.05	-.03	.98	-.59	-.44
10	.88	-.37	-.31	.80	-.18	-.14	.52	-.29	-.27
11	-	-	-	-	-	-	.25	-.14	-.13
12	.76	-.32	-.29	.96	-.20	-.17	.47	-.26	-.24
13	.78	-.41	-.26	.74	-.23	-.12	1.01	-.62	-.43

Table 8. The correlation matrix of the disturbances

item													
1	1.00												
2	.38	1.00											
3	.23	.71	1.00										
4	-.72	-.52	-.39	1.00									
5	.18	.68	.58	-.37	1.00								
6	.24	-.08	.08	-.13	-.03	1.00							
7	-.13	-.32	-.24	.19	-.55	-.14	1.00						
8	-.64	-.61	-.41	.55	-.38	-.23	-.04	1.00					
9	-.61	-.30	-.39	.25	-.16	-.36	.00	.39	1.00				
10	-.12	-.40	-.24	.07	-.53	.29	.60	.00	-.04	1.00			
11	.07	.42	.38	-.18	.37	-.12	.03	-.31	-.11	-.16	1.00		
12	.31	.17	.04	-.44	-.08	-.21	.11	-.09	-.19	.17	.06	1.00	
13	-.62	-.66	-.54	.37	-.53	-.20	.22	.54	.52	.22	-.41	-.11	1.00