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CONDITIONAL GMM ESTIMATION FOR GRAVITY MODELS

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Abstract. This paper studies finite sample performances of the conditional GMM estimators for a particular conditional moment restriction model, which is commonly applied in economic analysis using gravity models of international trade. We consider the GMM estimator with growing moments and Dominguez and Lobato's (2004) process-based GMM estimator. Under the simulation designs by Santos Silva and Tenreyro (2006), we find that Dominguez and Lobato's (2004) estimator is favorably comparable with the Poisson pseudo maximum likelihood estimator, and outperforms other estimators.

1. Setup and estimators

This note is concerned with estimation of the conditional moment restriction model

$$\mathrm{E}[Y|X] = \exp(X'\beta), \tag{1}$$

almost surely, where Y is a scalar dependent variable, X is a k-dimentional vector of covariates, and β is a k-dimensional vector of parameters. This model can be considered as an example of the nonlinear regression model for continuous Y or Poisson regression model for non-negative integer Y. This particular model has been extensively applied and studied in economic analysis using gravity models of international trade. See, e.g., Eaton and Kortum (2002), Anderson and van Wincoop (2003), Santos Silva and Tenreyro (2006), among others.

Based on a random sample $\{Y_i, X_i\}_{i=1}^n$, popular estimation methods are the nonlinear least squares (NLS) $\hat{\beta}_{NLS} = \arg\min_\beta n^{-1}\sum_{i=1}^n \{Y_i - \exp(X_i'\beta)\}^2$ whose first-order condition is

$$\frac{1}{n}\sum_{i=1}^n \{Y_i - \exp(X_i'\hat{\beta}_{NLS})\}\exp(X_i'\hat{\beta}_{NLS})X_i = 0, \tag{2}$$

and the Poisson pseudo maximum likelihood (PPML) method whose first-order condition is

$$\frac{1}{n}\sum_{i=1}^n \{Y_i - \exp(X_i'\hat{\beta}_{PPML})\}X_i = 0. \tag{3}$$

In an influential paper, Santos Silva and Tenreyro (2006) argued the inconsistency problem of the OLS estimator for the log-linear model under heteroskedasticity with normal errors, and investigated the NLS and PPML estimators. In particular, Santos Silva and Tenreyro (2006) advocated the use of the PPML estimator under heteroskedastic errors rather than the NLS estimator. Their argument is that the NLS estimator tends to give more weights on the observations where $\exp(X_i'\hat{\beta}_{NLS})$ is large and is generally noisier, and the NLS estimator tends to be less efficient than the PPML estimator. A simulation study by Santos Silva and Tenreyro (2006) endorsed the excellent performance of the PPML estimator.

In this note, we examine the finite sample performance of the conditional GMM estimator for the particular model in (1). By the law of iterated expectations, the conditional moment restriction (1) implies unconditional moment restrictions

$$\mathrm{E}[\{Y - \exp(X'\beta)\}h(X)] = 0, \tag{4}$$

for any function $h(\cdot)$ (as far as the above expectation is well-defined). Thus, both the NLS estimator (which specifies $h(X) = \exp(X'\beta)X$) and PPML estimator (which specifies $h(X) = X$) are consistent and also asymptotically normal under suitable regularity conditions.

In the context of estimation of the conditional moment restriction models, there are two substantial issues for the choice of $h(\cdot)$. First, the conditional moment restriction in (1) implies infinitely many unconditional moment restrictions in the form of (4). Thus, generally neither the NLS nor PPML estimator achieves the semiparametric efficiency bound to estimate β in the model (1). Currently several efficient estimation methods are available, such as the optimal instrumental variable estimator, and growing moment-based estimator (see, Chapter 7 of Hall (2005) for a survey). In our simulation study below, we consider the GMM estimator with growing moments (Donald, Imbens and Newey, 2003):

$$\hat{\beta}_{GMM} = \arg\min_{\beta} \left(\frac{1}{n}\sum_{i=1}^{n} g_{ni}(\beta) \right)' \left[\frac{1}{n}\sum_{i=1}^{n} g_{ni}(\hat{\beta}) g_{ni}(\hat{\beta})' \right]^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} g_{ni}(\beta) \right),$$

where $\hat{\beta}$ is a preliminary estimator, and $g_i(\beta) = \{Y_i - \exp(X_i'\beta)\}h_{ni}$ with a vector of basis functions $h_{ni} = (p_1(X_i), \ldots p_{k_n}(X_i))'$ for $k_n \to \infty$ as $n \to \infty$. A common drawback of efficient estimation methods for the conditional moment restrictions is that they typically involve some tuning parameters, such as the series lengths and bandwidths, to be chosen by the researcher.

The second issue is on consistency of point estimators. In an insightful paper, Dominguez and Lobato (2004) argued that even though the conditional moment restriction (1) uniquely identifies the parameters β, the implied unconditional moment restriction (4) with finite dimensional $h(\cdot)$ may not fully exploit information contained in (1) and identification of β may not be guaranteed. In this case, the GMM estimator is typically inconsistent. To address this issue, Dominguez and Lobato (2004) observed that the conditional moment restriction (1) is equivalent to the continuum of the unconditional moment restrictions $\mathrm{E}[\{Y - \exp(X'\beta)\}I(X \leq x)] = 0$ for all x, and proposed the following estimator

$$\hat{\beta}_{DL} = \arg\min_{\beta} \sum_{l=1}^{n} \left[\sum_{i=1}^{n} \{Y_i - \exp(X_i'\beta)\} I(X_i \leq X_l) \right]^2. \tag{5}$$

Dominguez and Lobato (2004) showed the consistency and asymptotic normality of this estimator under mild regularity conditions. Although $\hat{\beta}_{DL}$ does not achieve the semiparametric efficiency bound, it does not involve any tuning parameters.

In the next section, we evaluate the finite sample properties of $\hat{\beta}_{GMM}$ and $\hat{\beta}_{DL}$ based on the simulation designs motivated by gravity models.

2. Simulation

We now assess the finite sample performances of the conditional GMM estimators and other estimators by Monte Carlo simulations. We adopt simulation designs by Santos Silva and Tenreyro (2006). The dependent variable is generated by

$$Y_i = \exp(\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i})\eta_i,$$

for $i = 1, \ldots, 1000$, where X_{1i} follows the standard normal distribution, X_{2i} is a dummy variable that takes 1 with probability 0.4 and 0 otherwise, η_i is a log-normal random variable with mean 1 and variance σ_i^2, and $\beta = (\beta_0, \beta_1, \beta_2)' = (0, 1, 1)'$. The covariates X_{1i} and X_{2i} are independent. As in Santos Silva and Tenreyro (2006), we consider the following specifications of the conditional variance σ_i^2:

Case 1: $\sigma_i^2 = \exp(-2X_i'\beta)$; $\mathrm{Var}(Y_i|X_i) = 1$.

Case 2: $\sigma_i^2 = \exp(-X_i'\beta)$; $\mathrm{Var}(Y_i|X_i) = \exp(X_i'\beta)$.

Case 3: $\sigma_i^2 = 1$; $\mathrm{Var}(Y_i|X_i) = \exp(2X_i'\beta)$.

Case 4: $\sigma_i^2 = \exp(-X_i'\beta) + \exp(X_{2i})$; $\mathrm{Var}(Y_i|X) = \exp(X_i'\beta) + \exp(X_{2i})\exp(2X_i'\beta)$.

In Case 1, the NLS estimator is optimal because the conditional variance of Y_i is constant. However this case is typically unrealistic for bilateral trade models. In Case 2, the conditional variance of Y_i equals its conditional mean, and the PPML estimator is optimal. In Case 3, because η_i follows the log-normal with unit variance, the OLS estimator of the log-linear model becomes consistent and the maximum likelihood estimator. In this case, the gamma pseudo-maximum-likelihood (GPML) is optimal because the conditional variance of Y_i is proportional to the square of its conditional mean. Finally Case 4 is the most complicated and perhaps realistic case. The conditional variance of Y_i is a quadratic function of its conditional mean and other variables. For each case, we conduct simulations with and without rounding errors in the dependent variable.

For this model, we consider seven estimation methods: (i) DL, (ii) GMM, (iii) PPML, (iv) GPML, (v) NLS, and (vi) OLS.[1]

Table 1 presents estimation biases and MSEs for β_1 and β_2 based on 1000 Monte Carlo replications. As shown in Santos Silva and Tenreyro (2006), OLS is very biased except for Case 3 and NLS is biased in the cases where heteroskedasticity is severe. Moreover, GPML is very sensitive to rounding errors and does not perform well in the presence of rounding errors.

PPML performs very well for all cases. In each case, PPML has small bias and is relatively robust to rounding errors in the dependent variable. GMM is more robust to the rounding errors than PPML. Similar to NLS, however, GMM is somewhat biased in the cases where heteroskedasticity is severe. Among the methods we consider, the performance of DL is the best. The biases of DL are small in various situations and outperforms PPML in terms of MSE in the cases where heteroskedasticity is severe (Cases 3 and 4). This outperformance of DL is maintained even when the rounding errors are present. It is also remarkable that DL performs as well as PPML even for Case 2, where PPML is optimal.

[1] For GMM, we set $\hat{\beta}$ as the PPML estimator and $h_{ni} = (1, X_{1i}, X_{2i}, X_{1i}^2, X_{1i}X_{2i})'$. Our preliminary simulation suggests that the results are less sensitive to the choice of h_{ni}.

Overall, our simulation results suggest that DL is favorably comparable to PPML and is better than other estimation methods.

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Table 1. Simulation Results

	Without rounding error				With rounding error			
	β_1		β_2		β_1		β_2	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
				Case 1: $\mathrm{Var}(Y_i\|X_i) = 1$				
DL	0.00012	0.00058	0.00221	0.00211	0.02338	0.00120	0.04365	0.00450
GMM	-0.00064	0.00012	-0.00167	0.00206	0.00278	0.00014	0.02381	0.00260
PPML	0.00048	0.00025	0.00122	0.00075	0.01959	0.00068	0.02144	0.00135
GPML	0.01376	0.00449	0.00903	0.00682	0.11126	0.02101	0.09514	0.02094
NLS	0.00015	0.00006	0.00051	0.00030	0.00228	0.00007	0.00314	0.00034
OLS	0.38977	0.15339	0.35705	0.13061	-	-	-	-
				Case 2: $\mathrm{Var}(Y_i\|X_i) = \exp(X_i'\beta)$				
DL	0.00041	0.00071	-0.00042	0.00224	0.02723	0.00154	0.04820	0.00502
GMM	-0.00041	0.00051	0.00058	0.00331	0.00167	0.00052	0.02877	0.00438
PPML	0.00043	0.00034	-0.00049	0.00157	0.02279	0.00090	0.02312	0.00225
GPML	0.00612	0.00180	0.00132	0.00350	0.13728	0.02402	0.11457	0.02023
NLS	0.00082	0.00108	0.00089	0.00363	0.00307	0.00108	0.00389	0.00367
OLS	0.21111	0.04540	0.19923	0.04201	-	-	-	-
				Case 3: $\mathrm{Var}(Y_i\|X_i) = \exp(2X_i'\beta)$				
DL	-0.00113	0.00286	0.00358	0.00488	0.02962	0.00386	0.06148	0.00924
GMM	-0.00987	0.01192	0.02943	0.03336	-0.00969	0.01174	0.06344	0.03969
PPML	-0.00420	0.00480	0.00826	0.00973	0.02280	0.00536	0.03649	0.01144
GPML	-0.00016	0.00096	0.00367	0.00398	0.19904	0.04305	0.16872	0.03557
NLS	0.10079	1.11170	0.07932	0.80596	0.10326	1.11281	0.08753	0.96348
OLS	-0.00014	0.00070	0.00226	0.00272	-	-	-	-
				Case 4: $\mathrm{Var}(Y_i\|X_i) = \exp(X_i'\beta) + \exp(X_{2i})\exp(2X_i'\beta)$				
DL	0.00146	0.00809	-0.00549	0.01298	0.03703	0.00971	0.04405	0.01602
GMM	-0.01351	0.02014	0.00570	0.06553	-0.00407	0.02040	0.04061	0.07003
PPML	-0.00174	0.01066	-0.01108	0.02275	0.02559	0.01133	0.01385	0.02352
GPML	0.00505	0.00341	-0.00467	0.01256	0.12917	0.02413	0.09926	0.02772
NLS	0.31399	8.84744	0.03984	3.43802	0.31984	8.92830	0.04349	2.97709
OLS	0.13356	0.01927	-0.12998	0.02318	-	-	-	-

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