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Abstract
In this paper, we will discuss the details of the compilation process of asset-liability matrix from flow-of-funds account (financial balance sheets) taking the example of Japan 1954-1999. Asset-liability matrix is a sector-by-sector square matrix, so the advantage is that we can apply the tremendous asset that the input-output analysis has accumulated since the early days of its development. However, input-output and asset-liability matrices are not necessarily identical twins. One of the leading peculiarities of the asset-liability matrix is that two distinct sector-by-sector matrices are derived from a set of balance sheets. The first one describes the propagation process of fund-raising while the other one depicts that of fund-employment. When there are discrepancies in the valuation of assets and liabilities, the magnitude of the dispersion could be different in one system from another. This will give us a clue to the generation mechanism of financial bubbles.

Key Words
Asset-liability matrix, Flow-of-funds analysis, Dispersion index, Financial bubbles, Japanese economy

JEL Classification Numbers
E50; C67; O11
1. Introduction

Since the early days of its development, the flow-of-funds accounts were in the form of a display rack of the balance sheets of various institutional sectors. A merit of this type of tabulation is that we can use all the basic principles of modern accounting system. The quadruple entry system proposed by Copeland (1952) is a logical evolution of the double entry system commonly practiced in business accounting. Another merit is that it is not too difficult to collect the balance sheets because most of the institutional sectors have some sort of balance sheets whatsoever. Especially in case of financial institutions, they are obliged to make detailed balance sheets. So the coverage is quite high, if not one hundred per cent. In addition to that, detailed statements attached to them often disclose the particulars of the counter parties of the transactions. That is why we can reconstruct the balance sheet of the household sector, for example, even though it does not make one by itself. Some other merits include the fact that the advances in the information technology allow the financial institutions to collect the data real-time. These merits are rarely found in other fields of economic statistics.

Although Stone (1966), Brainard and Tobin (1968) and Klein (1983) among others proposed alternative flow-of-funds accounts in the form of asset-liability matrices (i.e. sector-by-sector square matrix), this kind of tabulation practice was never realized except in few experimental project like that of the Economic Planning Agency of Japan in the 1950s. The advantage of asset-liability matrix is that you can make good use of the wealth the input-output analysis has accumulated for more than half century. Between 1950 and 1980, the input-output analysis was one of the most frequently employed techniques in economic forecasting. Even today, it is often used in China and other rapidly developing countries. Actually, it is demonstrated in Tsujimura and Mizoshita (2003) that the combination of asset-liability matrix and the Leontief inverse could be a powerful weapon to analyse the effect of money-market operations of the central banks in details. The only problem is that it is an enormous work to compile an asset-liability matrix from scratch.

At Keio Economic Observatory, we have compiled asset-liabilities matrices of Japan for 1954-1999. The fundamental technique we have employed is the

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1 Paragraph 11.103 of the 1993 SNA states that the flow-of-funds accounts record the ‘net acquisition’ of financial assets and ‘net incurrence’ of liabilities for all institutional sectors by type of financial assets. This terminology is in contradiction to the U.S. Flow of Funds Accounts that include both ‘flow’ and ‘levels’ since its inauguration back in 1955. U.S. Guide to Flow of Funds Accounts (p.31 of the 2000 edition) state that “in many cases, data collected from reports or other sources for use in the accounts are in levels form; staff members of the Flow of Funds Section calculate the flows from these series”. 1993 SNA does not elaborate in this respect.
supply-and-use method originally proposed by Stone and Klein cited above. However, supply-and-use method is too mechanical to include all the detailed information available in this country, so that we have explored dummy-instrument method in addition to that. Although the asset-liability matrix is based on the 1968 SNA flow-of-funds accounts originally prepared by the Bank of Japan, the number of the institutional sectors is almost doubled to 35. (An aggregated 11-sector matrix is also available.) The list of the institutional sectors is given in Table 1. The financial institutions, especially the banking sector is divided into as many sub-groups as possible to examine their role in the development of the Japanese economy. The non-financial private corporations have been divided into four sub-groups while the non-corporate enterprises have been separated from the households. The asset-liability matrix is based on the 1968 SNA, unless otherwise stated.

In the latter half of this treatise, the results of the overall analysis of the Japanese economy in transition from poverty to prosperity will be discussed. For reasons of space, only the indices obtainable directly from the Leontief inverse will be examined in this tract. One of the leading peculiarities of the asset-liability matrix is that two distinct sector-by-sector matrices are derived from a set of balance sheets. That means there are two Leontief inverses as well. The first one describes the propagation process of fund-raising while the other one depicts that of fund-employment. We call them liability-oriented system and asset-oriented system respectively. In this regard, the valuation of the assets and liabilities plays important role. When there are discrepancies in the valuation of assets and liabilities, the magnitude of the dispersion could be different in one system from another. This will give us a clue to the generation mechanism of financial bubbles.

2. Compilation of Asset-Liability Matrix
The first step to create an asset-liability-matrix is to collect balance sheets of the institutional sectors and put them side-by-side into a display case called flow-of-funds accounts. In this section, we will discuss the details of the compilation process of asset-liability matrix, a sector-by-sector square matrix, from flow-of-funds accounts. The fundamental approach employed here is the supply-and-use method widely used in the compilation of input-output tables. Supply-and-use method is a tool to tabulate the transactions of each financial instrument by particular institutional sector in asset-liability matrix under an assumption that every supplier of funds put them into one reservoir representing the market of the instrument and every user of funds draw them from that reservoir. Although supply-and-use method portrays the transactions of
negotiable instruments like stocks and bonds on organized markets (stock exchange etc.) very well, it is a poor tool to depict the direct transactions of non-negotiable instruments including deposits and loans. In this regard, we have introduced dummy-instrument method as a remedy.

2. Supply-and-Use Method
2.1. Asset and Liability Matrices
As mentioned above, flow-of-funds accounts are in form of a display rack of balance sheets of various institutional sectors. By picking out each row of assets and liabilities of the balance sheet of a sector separately, we can form two matrices \( E \) (Asset Matrix) and \( R \) (Liability Matrix). As depicted in Figure 1, \( E \)-table consists of a matrix \( E \) and vectors \( s^E, z \). Each element \( e_{ij} \) of matrix \( E \) represents the amount of funds allocated to the \( i \)'th financial instrument by the \( j \)'th institutional sector. Vectors \( s^E \) is the excess liabilities, \( s^E \) is the total amount of the financial instrument in terms of assets, \( z \) is either the sum of assets or liabilities of particular sector whichever is greater; where, \( n \) is the number of financial instruments while \( m \) is the number of the institutional sectors.

\[
E = \begin{bmatrix}
  e_{11} & e_{12} & \cdots & e_{1n} \\
  e_{21} & e_{22} & \cdots & e_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  e_{m1} & e_{m2} & \cdots & e_{mn}
\end{bmatrix}
\quad
E^E = \begin{bmatrix}
  s^E \\
  s^E \\
  \vdots \\
  s^E
\end{bmatrix}
\quad
z = \begin{bmatrix}
  z_1 \\
  z_2 \\
  \vdots \\
  z_m
\end{bmatrix}
\]

Likewise, \( R \)-table consists of a matrix \( R \) and vectors \( p, s^R, z \). Each element \( r_{ij} \) of matrix \( R \) represents the amount of funds raised in the form of \( i \)'th financial instrument by the \( j \)'th institutional sector. Vectors \( p \) is the excess financial assets, \( s^R \) is the total amount of the financial instruments in terms of liabilities.

\[
R = \begin{bmatrix}
  r_{11} & r_{12} & \cdots & r_{1m} \\
  r_{21} & r_{22} & \cdots & r_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{n1} & r_{n2} & \cdots & r_{nm}
\end{bmatrix}
\quad
p = \begin{bmatrix}
  \rho_1 \\
  \rho_2 \\
  \vdots \\
  \rho_m
\end{bmatrix}
\quad
s^R = \begin{bmatrix}
  s^R \\
  s^R \\
  \vdots \\
  s^R
\end{bmatrix}
\quad
z = \begin{bmatrix}
  z_1 \\
  z_2 \\
  \vdots \\
  z_m
\end{bmatrix}
\]

In this expression, the column sum \( z_j \) is either the sum of assets or liabilities of the \( j \)'th institutional sector whichever is larger.
(1) \( z_j = \max \left( \sum_{i=1}^{n} e_{ij}, \sum_{i=1}^{n} r_{ij} \right) \)

In case

(2) \( \sum_{i=1}^{n} e_{ij} > \sum_{i=1}^{n} r_{ij} \),

the excess financial assets \( \rho_j \) and the excess liabilities \( \varepsilon_j \) are defined as follows.

(3) \( \rho_j = z_j - \sum_{i=1}^{n} r_{ij} \)

(4) \( \varepsilon_j = 0 \).

In contrast to this, when

(5) \( \sum_{i=1}^{n} e_{ij} < \sum_{i=1}^{n} r_{ij} \)

is the case, \( \varepsilon_j \) and \( \rho_j \) are expressed in the following manner.

(6) \( \varepsilon_j = z_j - \sum_{i=1}^{n} e_{ij} \)

(7) \( \rho_j = 0 \).

In case

(8) \( \sum_{i=1}^{n} e_{ij} = \sum_{i=1}^{n} r_{ij} \),

then

(9) \( \varepsilon_j = \rho_j = 0 \).

2.1.2. Asset-Oriented System vs. Liability Oriented System

In the field of input-output analysis, certain mathematical method has been used to convert the supply-and-use matrices into one square matrix. In Fig.2, \( V \) is the supply matrix while \( U \) is the use matrix. Firstly, let us pay our attention to the flow of funds itself. From this point of view, each column of \( R \) is the vector that represents the fund raising portfolio of the institutional sector. Since the fund raising portfolio of an institutional sector in the flow-of-funds analysis corresponds to the input structure of an industry in the input-output analysis, we can put \( R \) into the place of \( U \) in Fig.2. From the same viewpoint, each row of the transposition matrix of \( E \) is the vector that represents the allocation of the funds of the institutional sector. Since the allocation of
funds of an institutional sector in the flow-of-funds analysis corresponds to the supply structure of an industry in the input-output analysis, we can put $E'$ into the place of $V$ in Fig.2.

Alternatively, let us pay our attention to the flow of financial instruments rather than the flow of funds itself. From this viewpoint, everything looks the other way round. Now, each column of $E$ is the vector that represents the demand for various financial instruments while each row of the transposition matrix of $R$ is the vector that represents the supply of each financial instrument. In that sense, we can put $E$ into the place of $U$ and $R'$ into the place of $V$ in Fig.2. This two-sided nature of the flow-of-funds analysis generates two sector-by-sector matrices rather than one. We will call the former liability-oriented system and the latter asset-oriented system. That is

(10) $U = R$

(11) $V = E'$

in the liability-oriented system, and

(12) $U^* = E$

(13) $V^* = R'$

in the asset-oriented system (denoted by superscript *).

Following the footsteps of the input-output analysis, the sector-by-sector flow-of-funds matrix or asset-liability matrix of the liability-oriented system could be derived in the succeeding formulae. Firstly, the input coefficients are defined for matrix $U$.

(14) $b_{ij} = \frac{u_{ij}}{z_j}$

Secondly, the allocation coefficients are defined for matrix $V$.

(15) $d_{ij} = \frac{v_{ij}}{s_j}$

So that,

$B = \begin{bmatrix}
      b_{11} & b_{12} & \ldots & b_{1m} \\
      b_{21} & b_{22} & \ldots & b_{2m} \\
      \vdots & \vdots & \ddots & \vdots \\
      b_{n1} & b_{n2} & \ldots & b_{nm}
\end{bmatrix}$

and

$D = \begin{bmatrix}
      d_{11} & d_{12} & \ldots & d_{1n} \\
      d_{21} & d_{22} & \ldots & d_{2m} \\
      \vdots & \vdots & \ddots & \vdots \\
      d_{m1} & d_{m2} & \ldots & d_{mn}
\end{bmatrix}$
Next, we will define the sector-by-sector flow-of-funds matrix $Y$ and its input coefficient matrix as follows.

$$Y = \begin{bmatrix}
  y_{11} & y_{12} & \cdots & y_{1m} \\
  y_{21} & y_{22} & \cdots & y_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  y_{m1} & y_{m2} & \cdots & y_{mm}
\end{bmatrix},$$

$$C = \begin{bmatrix}
  c_{11} & c_{12} & \cdots & c_{1m} \\
  c_{21} & c_{22} & \cdots & c_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  c_{m1} & c_{m2} & \cdots & c_{mm}
\end{bmatrix},$$

where

$$c_{ij} = \frac{y_{ij}}{z_j}.$$

2.1.3. Institutional Sector Portfolio Assumption vs. Financial Instrument Portfolio Assumption

In the field of input-output analysis, several methods have been proposed to convert the supply-and-use matrices into one square matrix. The mathematical methods used when transferring outputs and associated inputs hinge on two types of technology assumptions as stated in paragraph 15.144 of the 1993 SNA: (a) industry technology assumption and (b) product technology assumption. The former assumes that all products produced by an industry are produced with the same input structure while the latter assumes that a product has the same input structure in whichever industry it is produced. In flow-of-funds analysis, these two assumptions could be translated into (a') institutional sector portfolio assumption (corresponding to industry technology assumption) and (b') financial instrument portfolio assumption (corresponding to product technology assumption). The former assumes that institutional sectors allocate (raise) funds according to their own portfolio regardless of the means of raising (employing) funds while the latter assumes that they allocate (raise) the funds according to the portfolio peculiar to the financial instrument through which the funds have been raised (will be employed).

In case of input-output analysis, the input structure is considered to be commodity specific rather than industry specific. So, the industry technology assumption is considered to perform rather poorly as stated in paragraph 15.146 of the 1993 SNA. On the contrary, in case of flow-of-funds analysis, the portfolio should be institutional sector specific rather than financial instrument specific because it is common to categorize the
institutional sectors by their means of fund raising. According to this assumption, we can obtain the following formula based on the institutional sector portfolio assumption.

(17) \[ \mathbf{C} = \mathbf{DB} \]

In this formula, each element of matrix \( \mathbf{C} \) could be expressed as follows.

(18) \[ c_{ij} = \sum_{k=1}^{n} d_{ik} b_{kj} \]

Since \( d_{ik} \) is the \( i \)th institutional sector's asset-market share of financial instrument \( k \), and \( b_{kj} \) is the \( k \)th financial instrument's share in the \( j \)th institutional sector's fund-raising portfolio, \( c_{ij} \) means how much funds \( j \)th institutional sector raise from \( i \)th sector. Therefore, each element of matrix \( \mathbf{Y} \) could be obtained by the following relation.

(19) \[ y_{ij} = c_{ij} z_j \]

The composition of \( \mathbf{Y} \)-table is depicted in Fig.3.

Likewise, the sector-by-sector flow of funds matrix or asset-liability matrix of the asset-oriented system could be derived in the succeeding formulae. Firstly, the input coefficients are defined for matrix \( \mathbf{U}^* \).

(20) \[ b_{ij}^* = \frac{u_{ij}^*}{z_j} = \frac{e_{ij}^*}{z_j} \]

Secondly, the allocation coefficients are defined for matrix \( \mathbf{V}^* \).

(21) \[ d_{ij}^* = \frac{v_{ij}^*}{s_j} = \frac{r_{ij}^*}{s_j} \]

So that,

\[
\mathbf{B}^* = \begin{bmatrix}
    b_{11}^* & b_{12}^* & \cdots & b_{1m}^* \\
    b_{21}^* & b_{22}^* & \cdots & b_{2m}^* \\
    \vdots & \vdots & \ddots & \vdots \\
    b_{n1}^* & b_{n2}^* & \cdots & b_{nm}^*
\end{bmatrix}
\]

and

\[
\mathbf{D}^* = \begin{bmatrix}
    d_{11}^* & d_{12}^* & \cdots & d_{1m}^* \\
    d_{21}^* & d_{22}^* & \cdots & d_{2m}^* \\
    \vdots & \vdots & \ddots & \vdots \\
    d_{n1}^* & d_{n2}^* & \cdots & d_{nm}^*
\end{bmatrix}
\]

Next, we will define the sector-by-sector flow of funds matrix \( \mathbf{Y}^* \) and its input coefficient matrix \( \mathbf{C}^* \) as follows.
\[ Y^* = \begin{bmatrix} y_{11}^* & y_{12}^* & \cdots & y_{1n}^* \\ y_{21}^* & y_{22}^* & \cdots & y_{2m}^* \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1}^* & y_{m2}^* & \cdots & y_{mm}^* \end{bmatrix} \]

\[ C^* = \begin{bmatrix} c_{11}^* & c_{12}^* & \cdots & c_{1m}^* \\ c_{21}^* & c_{22}^* & \cdots & c_{2m}^* \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1}^* & c_{m2}^* & \cdots & c_{mm}^* \end{bmatrix} \]

where

\[ c_{ij}^* = \frac{y_{ij}^*}{z_j} \]

According to this assumption, we obtain the following formula.

\[ C^* = D^*B^* \]

In this formula, each element of matrix \( C^* \) could be expressed as follows.

\[ c_{ij}^* = \sum_{k=1}^{n} d_{ik}^* \delta_{kj}^* \]

Since \( d_{ik}^* \) is the \( i \) th institutional sector's liability market share of financial instrument \( k \), and \( \delta_{kj}^* \) is the \( k \) th financial instrument's share in the \( j \) th institutional sector's fund-employment portfolio, \( c_{ij}^* \) means how much funds \( j \) th institutional sector employ to \( i \) th sector. Therefore, each element of matrix \( Y^* \) could be obtained by the following relation.

\[ y_{ij}^* = c_{ij}^* z_j \]

The composition of \( Y^* \) table is depicted in Fig.4.

2.1.4. Issue Value vs. Current Market Value

Only when the following relation is maintained, the succeeding relations are proved. If both sides of the balance sheets in the original flow-of-funds accounts are measured in common value (either issue value or current market value), that is

\[ s^F = s^R \]

then

\[ p_j^F = z_j - \sum_{i=1}^{m} y_{ij} = p_j^R \]
In this special case, the liability-oriented system and the asset-oriented system produce a unique matrix; i.e. the transposition matrix of $Y$ is $Y^*$ and vice versa (see Appendix 1 at the end of this tract in addition to the appendix to Tsujimura and Mizoshita (2003)).

Paragraph 2.87 of the 1968 SNA as well as paragraph 10.14 of the 1993 SNA clearly states that both assets and liabilities should be expressed in current market value. The principle of SNA is that a financial claim should be measured by the amount that a debtor must pay to the creditor to extinguish the claim. In this case, there is no discrepancy in the measurement of assets and liabilities. However, when we simply sum up the existing balance sheets of the institutions, some sort of discordance is inevitable. Indeed, paragraph 101 of the IASB Framework, recognizes historical cost (the amount of cash or cash equivalent paid) as the basis for the financial statements. Although paragraphs 69 and 93 of IAS 39 stipulate that the financial assets as well as financial liabilities held for trading should be measured at fair value (i.e. current market value), equity instruments of their own issuance are exempted. The principle of IAS is that equity instruments must be recorded at the amount of proceeds received in exchange at the time of issuance.

It is no wonder that the issuer of the corporate stocks enters it on their own book in issue value while the holder of the stocks enters it on their book in the current market value. If we simply sum up those figures, we will have flow of funds accounts that have discrepancy in the total value of assets and liabilities. It is rather awkward to have discordance in book keeping values, but this is the reality we face everyday. It is proved if,

$$(30) \quad s^R \neq s^R$$

then, equations (27), (28) and (29) are no longer maintained. (See Appendix 1.) The liability-oriented system and the asset-oriented system produce somewhat different matrices. Isn't it awful? But, let us face the reality. As we discuss later in this treatise, there will be a rewards if we could overcome this problem. In case of discrepancy, $Y$ and $Y^*$ tables are presented in manners depicted in Fig.A1-2 and Fig.A1-3 of Appendix 1. It should be noted that the columns containing the differences between the issue and current market values are accommodated in these tables.
2.2. Dummy Instrument Method

Supply-and-use method is a convenient way to transform the balance sheets of flow-of-funds accounts into sector-by-sector asset-liability matrix. As far as those openly traded securitized financial instruments (e.g. bonds, stocks etc.) are concerned, there is no alternative but allocate them according to the market share. However, as for those financial instruments directly traded between the parties concerned (e.g. deposits, loans etc.), it is not uncommon that some additional information is available. In case of Japan, all deposit and loan transactions are earmarked by creditors and debtors. Whenever such additional information is attainable, dummy instrument method should be used together with supply-and-use method.

The idea is quite simple. If we know that the bank made a loan (say amounting to 100) to the local government, we add a dummy financial instrument just to record this single transaction. As depicted in Fig.5, we enter 100 on the asset side of the balance sheet of the bank while registering the same amount to the liability side of the local government. Since these transactions are entered in the dummy instrument row, no other transactions will be made entry in this particular row. When we apply supply-and-use method to these balance sheets, we will have an asset-liability-matrix depicted in Fig.6. The result of the trick is that the transaction is entered in the row of the bank and in the column of the local government on the sector-by-sector matrix in the liability-oriented system. (Of course, this transaction will be registered in the row of the local government and in the column of the bank in the asset-oriented system.)

3. Two Alternative Matrices and their Leontief Inverse

3.1. Power of Dispersion and Sensitivity of Dispersion Indices

3.1.1. Four Indices Defined

In the previous section, we have derived two distinct sector-by-sector square matrices from a set of balance sheets. However we did not elaborate in details. Why that is necessary is the question to be answered in this section. The fundamental equations of both liability-oriented system and asset-oriented system are written as follows. (See Appendix 1 for the case $s^F \neq s^A$.)

\[ \sum_{j=1}^{m} y_{ij} + \varepsilon_i = z_i \]  
\[ \sum_{j=1}^{m} y_{ji}^* + \rho_i = z_i \]

These equations could be expressed in the matrix formula.

\[ Cz + \varepsilon = z \]
Solving each equations for z yields

\[ z = (I - C)^{-1} \varepsilon. \]

\[ z = (I - C^*)^{-1} \rho. \]

We will denote \((I - C)^{-1}\) and \((I - C^*)^{-1}\) as \(\Gamma\) and \(\Gamma^*\) respectively.

\[
\Gamma = (I - C)^{-1} = \begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1m} \\
Y_{21} & Y_{22} & \cdots & Y_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{m1} & Y_{m2} & \cdots & Y_{mm}
\end{bmatrix}
\]

\[
\Gamma^* = (I - C^*)^{-1} = \begin{bmatrix}
Y^*_{11} & Y^*_{12} & \cdots & Y^*_{1m} \\
Y^*_{21} & Y^*_{22} & \cdots & Y^*_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
Y^*_{m1} & Y^*_{m2} & \cdots & Y^*_{mm}
\end{bmatrix}
\]

The elements \(y_{ij}\) indicate direct as well as indirect demand for funds in \(i\)'th institutional sector induced by the increment in demand for funds \(\varepsilon_j\) (excess-investments in terms of objective economy) by \(j\)'th sector. On the other hand, \(y^*_{ij}\) indicate the supply of funds in \(i\)'th sector induced by the increment in supply of funds \(\rho_j\) (excess-savings in terms of objective economy) by \(j\)'th sector. Since demand and supply of funds are propagated through different systems, there is an asymmetry in induced demand and supply of funds. This is one of the most prominent properties of flow-of-funds analysis.

On the analogy to input-output analysis, the indices of the power of dispersion and the indices of the sensitivity of dispersion could be calculated in the following manner.

As for the liability-oriented system, the two indices are defined as follow.

\[
wp_{j}^T = -\frac{\sum_{i=1}^{m} y_{ij}}{\frac{1}{m} \sum_{j=1}^{m} \sum_{i=1}^{m} y_{ij}}
\]

\[
ws_{j}^T = -\frac{\sum_{j=1}^{m} y_{ij}}{\frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{m} y_{ij}}
\]
In this system, the power-of-dispersion index indicate the direct as well as indirect demand for funds in total induced by the increment in demand for funds (excess-investments in terms of objective economy) by j'th institutional sector. The sensitivity-of-dispersion index in the liability-oriented system indicate the direct as well as indirect demand for funds in i'th institutional sector induced by the increment in demand for funds by each institutional sector.

The two indices are defined for the asset-oriented system as well.

\[ w_{ij}^{*} = \frac{\sum_{j=1}^{m} y_{ij}^{*}}{1 \sum_{j=1}^{m} \sum_{i=1}^{m} y_{ij}^{*}} \]

\[ w_{ji}^{*} = \frac{\sum_{i=1}^{m} y_{ij}^{*}}{1 \sum_{i=1}^{m} \sum_{j=1}^{m} y_{ij}^{*}} \]

In this system, the power-of-dispersion index indicate direct as well as indirect supply of funds in total induced by the increment in supply of funds (excess-savings in terms of objective economy) by j'th institutional sector. The sensitivity-of-dispersion index in the asset-oriented system indicate the direct as well as indirect supply of funds in i'th institutional sector induced by the increment in supply of funds by each institutional sector. While the indices represent the chain reaction originated in the demand for funds (excess-investments in terms of objective economy) in the liability-oriented system, the indices represent that originated in the supply of funds (excess-savings in terms of objective economy) in the asset-oriented system.

3.1.2. The Principal Institutional Sectors

Figure 7 displays the power-of-dispersion indices for households, non-financial corporations and banks picked out of the eleven-sector aggregated asset-liability matrix. Each plot indicate the combination of the year referring to asset (vertical axis) and liability (horizontal axis) oriented system. Both indices are normalized so that the diagram is divided into quadrants by vertical and horizontal lines indicating unity. The households, with primary savings, are located in the midst of the second quadrant while the non-financial corporations, with primary investments, are situated in the fourth quadrant. The government stays in the fourth quadrant as well, probing that the role of government in the financial market is not much different from the non-finance corporations. The banks, intermediaries by their nature, are placed in the middle of the
chart neighbouring to the intersecting point. The most prominent thing is that, the location of the plots on the diagram show minimal change despite the laps of time.

The power-of-dispersion index in the liability-oriented system and the sensitivity-of-dispersion index in the asset-oriented system is something like the two side of the same coin. The former is an index that exhibits how far the influence spreads when the institutional sector raises new money from the market. The latter is an indicator to demonstrate how much effect the institutional sector gains when the fund-raising is activated in general. The relations between the two indices are depicted in Figure 8. In this diagram, the households are located in the third quadrant while the non-financial corporations are situated in the first quadrant. The government stays mainly in the first quadrant though some scatters are belonging to the fourth quadrant suggesting that the government does not immediately react to an increment in the savings as the non-financial corporations do. However, the plots of the non-financial corporations are wide spreading. Especially in the latter half of the observation period, the sensitivity-of-dispersion index is getting smaller with the years. When Japan was growing fast to get rid of the poverty brought by the defeat in the World War II, the corporate sector absorbed whatever funds made available for them. But, after reaching maturity, the Japanese corporations have some difficulty to find investment opportunity just as their counterparts in other so-called advanced countries. In 1999, the position of the corporate sector is much more like that of the government. Rather, the government is taking over the role of excess-funds absorber in the aftermath of the financial bubble of 1980s.

Figure 9 displays the relations between the power of dispersion index of the asset-oriented system and the sensitivity of dispersion index of the liability-oriented system. In this diagram, the households are located in the middle of the first quadrant while the non-financial corporations are situated in the lower part of the second quadrant. Most probably this is because they are acting as financial intermediaries for their affiliated companies. Unlike the corporate sector, the government is situated in the third quadrant suggesting that it plays no role as financial mediator what so ever.

3.1.3. The Financial Mediators
Talking about financial mediator, the dispersion indices are rigorous device to identify the role of each category of financial institutions. The power-of-dispersion index of the asset-oriented system and the sensitivity-of-dispersion index of the liability-oriented system are depicted in Figure 10. In this diagram, banks are scattered around the vertical axis between 1.5 and 2.5 on its scale that is upper left of the figure extending
over the first and second quadrant. The public financial corporations are situated about middle of the figure in the first quadrant while non-bank financial companies are placed in the fourth quadrant. This means that banks supply funds to only limited number of customers who spend them directly on capital investments. On the other hand, the non-bank financial companies make loans to wide-ranging customers who spend them as working-funds. The other implication is that people tend to tap banks for money first, then public financial corporation, then non-bank financial companies in that order (in the order of the sensitivity-of-dispersion index).

3.1.4 The Column Sums

It is well known that the dispersion indices are obtained by normalizing either the column sum (in case of power of dispersion index) or the row sum of the Leontief inverse matrix. Then what about those sums themselves? Figures 11 and 12 display the fluctuations in the column sum of the liability-oriented system and the asset-oriented system respectively. In Figure 11, all the lines but that of the households moves as if they are interlocked. The only exception is that the line of the government between 1962 and 1967. In Figure 12, all the lines but that of the government are synchronized. This fact suggest that we may be able to throw light upon the mechanism of the business cycle from the viewpoint of the financial market structure.

3.2. The Sum Total of the Leontief Inverse

3.2.1 Asset and Liability Dispersion Indices Defined

If there is synchronization among the column sums of each institutional sector, the total sum of them must be a useful indicator. In this sub-section, we are to examine the asset-liability matrix in terms of the sum total of the elements of its Leontief Inverse. Let us denote the sum of elements of $\Gamma$ as $w^F$ and the sum of elements of $\Gamma^*$ as $w^{F*}$. 

$$w^F = \sum_{i=1}^{m} \sum_{j=1}^{n} \gamma_{ij}$$

$$w^{F*} = \sum_{i=1}^{m} \sum_{j=1}^{n} \gamma_{ij}$$

We will call them liability dispersion index ($w^F$) and asset dispersion index ($w^{F*}$) respectively. The fluctuations in the two indices are depicted in Figure 13. These indices are obtained from the asset-liability matrix (with 35 institutional sectors) based on the balance sheets, in which the corporate stocks are registered in issue value on both asset and liability sides. In this case, the two indices move hand in hand. The two lines cut each other around 1984.
Figure 14 displays the fluctuations in the same two indices. The only difference is that both indices are obtained from the asset-liability matrix based on the balance sheets, in which the corporate stocks are expressed in current market value on the asset side and in issue value on the liability side. The dissimilarity between the two figures is plain for everyone to see. In the latter case, the asset dispersion index and the liability dispersion index go apart from one another. The subtraction of the liability dispersion index from the asset dispersion index gives the dispersion discrepancy index.

\[ w^{**} - w^r = w^{**} - w^r \]

In Figure 15 that corresponds to Figure 14, the dispersion discrepancy index, the difference between the two indices, shows sudden increase between 1973 and 1975, then between 1987 and 1992. Is it just a casual coincidence that Japan was experiencing the first oil crisis and the financial bubble during these periods? These two periods are distinguished for asset inflation in land, corporate stocks and so on. The dispersion indices are derived solely from the portfolio apportionment of the institutional sectors and free from the value of assets and liabilities itself. Yet there seems a close relationship between the dispersion indices and the asset prices. (See Appendix 2 for further details.) This observation suggests that the structure of the financial market represented in the asset-liability matrix could be the clue to the origin of the financial bubbles.

3.2.2. The Method of Decomposition

As we have mentioned before, the asset and liability dispersion indices are the total sum of each element of the respective asset-liability matrices. Although there are no systematic relations (see Appendix 3), it is obvious that there is a one-to-one relation between the coefficient matrix and its Leontief inverse. We can decompose the causes for the alteration in the Leontief inverse into two categories. One is the total sum of each element of the coefficient matrix, and the other is the apportionment of coefficients among them. While the latter is a purely monetary phenomenon, the former is considered to be the reflection of the object economy because the excess assets and liabilities are corresponding to excess savings and investments respectively. This kind of decomposition is useful to determine either the cause of financial bubbles lies in the structure of the financial market itself or it is merely a mirror image of the object economy i.e. lack of investments in plants and equipments and so on.

Let us take the example of the liability-oriented system first, followed by the
asset-oriented system later. In Section 2, we defined the coefficients $c_{ij}$ after the manner of input-output analysis.

$$c_{ij} = \frac{y_{ij}}{z_j}$$

You might recall that $z_j$ could be written as follows (even if the asset-liability matrix is based on the balance sheets, in which the corporate stocks are expressed in current value on the asset side and in issue value on the liability side).

$$z_j = \sum_{i=1}^{m} y_{ij} + \rho_j$$

If we omit $\rho_j$, we can redefine coefficient matrix $C$ as $C^*$, in which each element could be defined in the following manner.

$$c_{ij}^* = \frac{y_{ij}}{\sum_{i=1}^{m} y_{ij}}$$

Now, let us define the ratio of $\rho_j$ to $z_j$ as follows.

$$c_{ij}^{**} = \frac{\rho_j}{z_j} = 1 - \sum_{i=1}^{m} c_{ij}^*$$

Then the relations between $c_{ij}$ and $c_{ij}^{**}$ are explicit.

$$c_{ij} = c_{ij}^{**} \times (1 - c_{ij}^{**})$$

By introducing subscript of time $t$, the differences in $c_{ij}$ could be decomposed in the following manner. When there are two such subscripts, the former one refers to the time concerning to $c_{ij}^*$ while the latter one refers to the time concerning to $c_{ij}^{**}$. 

---

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\[ \Delta c_{ij,t} = c_{ij,t} - c_{ij,t-1} \]
\[ = c_{ij,t}^\delta \times (1 - c_{ij,t}) - c_{ij,t-1}^\delta \times (1 - c_{ij,t-1}) \]
\[ = 2 \times c_{ij,t}^\delta \times (1 - c_{ij,t}) - 2 \times c_{ij,t-1}^\delta \times (1 - c_{ij,t-1}) \]
\[ = 2 \times \frac{c_{ij,t}^\delta \times (1 - c_{ij,t}) - c_{ij,t-1}^\delta \times (1 - c_{ij,t-1})}{2} \]
\[ + \frac{c_{ij,t}^\delta \times (1 - c_{ij,t}) - c_{ij,t-1}^\delta \times (1 - c_{ij,t-1})}{2} \]
\[ + \frac{c_{ij,t}^\delta \times (1 - c_{ij,t}) - c_{ij,t-1}^\delta \times (1 - c_{ij,t-1})}{2} \]
\[ + \frac{c_{ij,t}^\delta \times (1 - c_{ij,t}) - c_{ij,t-1}^\delta \times (1 - c_{ij,t-1})}{2} \]
\[ = \frac{\| c_{ij,t} \| (1 - c_{ij,t}) - c_{ij,t-1} \| (1 - c_{ij,t-1})}{2} \]
\[ + \frac{\| c_{ij,t} \| (1 - c_{ij,t}) - c_{ij,t-1} \| (1 - c_{ij,t-1})}{2} \]
\[ + \frac{\| c_{ij,t} \| (1 - c_{ij,t}) - c_{ij,t-1} \| (1 - c_{ij,t-1})}{2} \]
\[ + \frac{\| c_{ij,t} \| (1 - c_{ij,t}) - c_{ij,t-1} \| (1 - c_{ij,t-1})}{2} \]

(i) The differences in \( c_y \) caused by the transition of \( c_{yp} \) from \( t-1 \) to \( t \) while \( c_{yp}^s \) is kept at \( t \).

(ii) The differences in \( c_y \) caused by the transition of \( c_{yp} \) from \( t-1 \) to \( t \) while \( c_{yp}^s \) is kept at \( t-1 \).

(iii) The differences in \( c_y \) caused by the transition of \( c_{yp}^s \) from \( t-1 \) to \( t \) while \( c_{yp} \) is kept at \( t \).

(iv) The differences in \( c_y \) caused by the transition of \( c_{yp}^s \) from \( t-1 \) to \( t \) while \( c_{yp} \) is kept at \( t-1 \).

Therefore, the first term of (48) represents the differences in \( c_y \) caused by the transition of \( c_{yp} \) from \( t \) to \( t+1 \), equally arithmetically weighted by \( c_{yp}^s \) at \( t \) and \( t+1 \). Likewise, the second term of the equation indicates the differences in \( c_y \) caused by the transition of \( c_{yp}^s \) from \( t \) to \( t+1 \), equally arithmetically weighted by \( c_{yp} \) at \( t \) and \( t+1 \). In matrix notation, we could rewrite (48) as follows.

\[ \Delta C_{lj} = C_{lj} - C_{lj-1} \]
\[ = \begin{bmatrix} \| C_{lj} \| (1 - C_{lj}) - C_{lj-1} \| (1 - C_{lj-1}) \end{bmatrix} + \begin{bmatrix} \| C_{lj} \| (1 - C_{lj}) - C_{lj-1} \| (1 - C_{lj-1}) \end{bmatrix} \]
\[ + \begin{bmatrix} \| C_{lj} \| (1 - C_{lj}) - C_{lj-1} \| (1 - C_{lj-1}) \end{bmatrix} \]
\[ + \begin{bmatrix} \| C_{lj} \| (1 - C_{lj}) - C_{lj-1} \| (1 - C_{lj-1}) \end{bmatrix} \]

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When the equation above is retained, the following relation is also proved. (See Appendix 4.)

\[ \Delta r_{t,t} = r_{t,t} - r_{t-1,t-1} \]

Then the differences in liability dispersion index could be decomposed as follows. (See Appendix 4 also.)

\[ \Delta w_{t,t}^P = w_{t,t}^P - w_{t-1,t-1}^P \]

Therefore, the first term of the expanded right side of the above equation represents the differences in \( w_t^P \) caused by the transition of \( c_{t-1}^P \) from \( t-1 \) to \( t \), equally arithmetically weighted by \( c_{t-1}^P \) at \( t-1 \) and \( t \). Likewise, the second term of the equation indicates the differences in \( w_t^P \) caused by the transition of \( c_t^P \) from \( t-1 \) to \( t \), equally arithmetically weighted by \( c_t^P \) at \( t-1 \) and \( t \). In other words, the first term is the portion attributed to the changes in the objective economy (decline in investments etc.) while the second term is the segment referring to the changes in the structure of the financial market (alterations in portfolio allocation etc.).

There must be no use to repeat all the process for the asset-oriented system. Exactly following the above procedure, we will obtain the following relation for the asset dispersion index.

\[ \Delta w_{t,t}^* = w_{t,t}^* - w_{t-1,t-1}^* \]

If we recall (43), the differences in the dispersion discrepancy index could be written as follows.
Then, we can make decomposition of the differences in the index as well by subtracting (51) from (52).

\[
\Delta w^T_{t,t-1} = \left( w^*_{t,t-1} - w^*_{t,1,t-1} \right) - \left( w^*_{t,1,t-1} - w^*_{t,1,1,t-1} \right)
\]

(53)

\[
\Delta w_{t,t-1} = \frac{\left\{ (w^*_{t,t} - w^*_{t,1,t-1}) + (w^*_{t,1,t} - w^*_{t,1,1,t-1}) \right\} - \left\{ (w^*_{t,t} - w^*_{t,1,t-1}) + (w^*_{t,1,t} - w^*_{t,1,1,t-1}) \right\}}{2}
\]

(54)

On the right side of the equation, the first term is the portion attributed to the changes in the objective economy (i.e. excess savings or excess investments) while the second term is the segment referring to the changes in the structure of the financial market (i.e. asset or liability portfolio selection of the institutional sectors).

3.2.3. The Results of the Decomposition

The decomposition of the differences in the dispersion discrepancy index is depicted in Figure 16. The pillars are divided into two parts: the dotted portion indicates the alteration attributable to the mutation of the financial structure, and the segment with oblique lines attributable to the changes in the object economy. All the pillars exhibit that the effects of the mutation of the financial structure are not significant as those of the object economy reflected in the excess assets and liabilities, that should be a mirror image of excess savings and investments. The conclusion is that the financial bubble or asset inflation is not merely a financial phenomenon, but deeply rooted into the object economy. To examine the problem in details let us see Figure 17 and 18 that display the fluctuations in excess asset or liability of the households and the non-financial private corporations. Taking the example of the bubble era (late 1980s through early 1990s), the excess asset is shooting up between 1987 and 1989. On the other hand, excess liability of the large manufacturing corporations turned into excess asset in 1988 and reached more than 50 trillion yen in 1990. Although the large non-manufacturing corporations remain in excess liability during this period, the shape of the line is synchronizing to that of large manufacturing corporations as if a curious coincidence. By summing up
these casual observations, we may tentatively conclude that misalignment of the household savings and the corporate investments led us to the financial bubbles of this period.

4. Concluding Remarks
In this tract, we have demonstrated the detailed process of compiling asset-liability matrix from flow-of-funds accounts (financial balance sheets) readily available in most of the OECD countries. Asset-liability matrix is a sector-by-sector square matrix, so the advantage is that we can apply the tremendous asset that the input-output analysis has accumulated since the early days of its development. However, input-output and asset-liability matrices are not necessarily identical twins. One of the leading peculiarities of the asset-liability matrix is that two distinct sector-by-sector matrices are derived from a set of balance sheets. That means there are two Leontief inverses as well. The first one describes the propagation process of fund-raising while the other one depicts that of fund-employment. We called them liability-oriented system and asset-oriented system respectively. In this regard, the valuation of the assets and liabilities plays important role. When there are discrepancies in the valuation of assets and liabilities, the magnitude of the dispersion could be different in one system from another.

In the latter half of the paper, we have examined the nature of the asset-liability matrices of Japan between 1954 and 1999. The most distinctive observation is that, the role of the principal institutional sectors of Japan exhibited minimum change in terms of dispersion indices in the transition process from the poverty in 1950s to the prosperity in the recent years. Only the sensitivity of dispersion index of the non-financial private corporations in the asset-oriented system has decreased significantly in the latter half of the twentieth century. In the past, the corporate sector was willing to absorb all the funds that the households would supply. But, after reaching maturity, the Japanese corporations no longer have inexhaustible opportunity to invest in plants and equipments they used to have. This widened the gap between the asset dispersion index and the liability dispersion index causing the financial bubbles in late 1980s. The result of the decomposition analysis exhibit that the effects of the mutation of the financial structure are not significant as those of the object economy reflected in the excess assets and liabilities, that should be a mirror image of excess savings and investments. The conclusion is that the financial bubble or asset inflation is not merely a financial phenomenon, but deeply rooted into the object economy.

The flow-of-funds analysis based on the asset-liability matrix is still in the cradle.
when we take the quantity and the quality of the input-output analysis of the past half-century into consideration. But we have no reason to be pessimistic. The Samaritans are there willing to give us helping hands. One of the most prominent observations in this treatise is that the coefficients of the asset-liability matrix are not so changeable as many people have suspected. If it is the case, asset-liability matrix should be a powerful weapon to make economic projections at least in the short run. The application to the money-market operation of the central banks is clearly demonstrated in Tsujimura and Mizoshita (2003) cited before. We sincerely hope to see remarkable developments in this field of study in the very near future.

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Appendix 1

As mentioned in section 2.1.1, components of E and R tables are expressed as Fig. A1-1.

\[
\begin{array}{cccc}
  e_{11} & e_{12} & \cdots & e_{1m} \\
  e_{21} & e_{22} & \cdots & e_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  e_{n1} & e_{n2} & \cdots & e_{nm} \\
\end{array}
\]

\[
\begin{array}{cccc}
  r_{11} & r_{12} & \cdots & r_{1m} \\
  r_{21} & r_{22} & \cdots & r_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{n1} & r_{n2} & \cdots & r_{nm} \\
\end{array}
\]

\[
\begin{array}{cccc}
  s_1 & \cdots & \cdots & s_m \\
  z_1 & \cdots & \cdots & z_m \\
\end{array}
\]

E table

R table

Fig.A1-1 Components of E-table and R-table

(1) Y table

Using the notations of E and R tables, each element of matrices B, D, C, Y can be written as follows.

(A1-1) \[ b_{ij} = \frac{r_{ij}}{z_j} \quad (i=1,\ldots,n, \; j=1,\ldots,m) \]

(A1-2) \[ d_{ij} = \frac{e_{ij}}{z_j} \quad (i=1,\ldots,m, \; j=1,\ldots,n) \]

(A1-3) \[ c_j = \sum_{k=1}^{n} d_{jk} \]

(A1-4) \[ y_{ij} = \frac{e_{ik}}{s_k} \sum_{j=1}^{m} \frac{r_{ij}}{z_j} \]

Based on equation (A1-4), column sum of matrix Y is as follows.

(A1-5) \[ \sum_{i=1}^{m} y_{ij} = \sum_{i=1}^{m} e_{ik} \times \frac{r_{ij}}{s_k} \]

\[ = \sum_{k=1}^{n} r_{kj} \sum_{i=1}^{m} e_{ik} \]

\[ = \sum_{i=1}^{m} r_{ij} \frac{e_{ik}}{s_k} \]

\[ \sum_{i=1}^{m} \left( i=1,\ldots,m, \; j=1,\ldots,m, \; k=1,\ldots,n \right) \]
Then column sum of matrix $Y$ is equal to column sum of matrix $R$.

\[(A1-6)\]
\[z_j - \sum_{i=1}^{m} y_{ij} = z_j - \sum_{k=1}^{n} r_{kj} = \rho_j \quad (i=1,\ldots,m, j=1,\ldots,m, k=1,\ldots,n)\]

Therefore, even if $s^E \neq s^R$, the subtraction of column sum of $Y$ from total financial assets or liabilities $z_j$ is equivalent to excess assets $\rho_j$.

Likewise row sum of matrix $Y$ is as follows.

\[(A1-7)\]
\[\sum_{j=1}^{m} y_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ik} \times r_{kj} = \sum_{i=1}^{m} \sum_{k=1}^{n} s^R_k \times s^E_k \quad (i=1,\ldots,m, j=1,\ldots,m, k=1,\ldots,n)\]

So the difference between $z_j$ and row sum of matrix $Y$ is

\[(A1-8)\]
\[z_j - \sum_{j=1}^{m} y_{ij} = z_j - \sum_{i=1}^{m} e_{ik} \times r_{kj} = z_j - \sum_{k=1}^{n} s^R_k \quad (i=1,\ldots,m, j=1,\ldots,m, k=1,\ldots,n)\]

On the other hand, the difference between $z_j$ and column sum of matrix $E$ is equal to excess liability $e_j$.

\[(A1-9)\]
\[z_j - \sum_{k=1}^{n} s^E_k = e_j \quad (i=1,\ldots,m, j=1,\ldots,m, k=1,\ldots,n)\]

In case $s^E \neq s^R$, we have to take account of not only excess liabilities but also the appraisal profit and loss $\phi_j$ given by subtracting equation (A1-9) from equation (A1-8).

\[(A1-10)\]
\[\phi_j = (z_j - \sum_{k=1}^{n} e_{ik} \times s^R_k) - (z_j - \sum_{k=1}^{n} e_{ik}) = \sum_{i=1}^{m} e_{ik} \sum_{k=1}^{n} s^R_k - \sum_{k=1}^{n} s^E_k = \sum_{i=1}^{m} e_{ik} (1 - \frac{s^R_k}{s^E_k}) \quad (i=1,\ldots,m, j=1,\ldots,m, k=1,\ldots,n)\]

Equation (A1-10) indicates that the appraisal profit and loss $\phi_j$ is weighted average of the column sum of matrix $E$, where total liabilities divided by total financial assets are used for weight. If $s^E = s^R$, i.e. $\frac{s^R_k}{s^E_k} = 1$, then $\phi_j = 0$, otherwise $\phi_j$ should be accommodated into $Y$ table explicitly. The component of $Y$ table is shown as follows in general form.
The same procedure is applied to $Y^*$ table. Each element of matrices $B^*$, $D^*$, $C^*$, $Y^*$ can be written as follows.

\[(A1\cdot11)\quad b_{ij}^* = \frac{e_{ij}}{s_j} \quad (i=1,\ldots,n, j=1,\ldots,m)\]

\[(A1\cdot12)\quad d_{ij}^* = \frac{r_{ik}}{s_j} \quad (i=1,\ldots,m, j=1,\ldots,n)\]

\[(A1\cdot13)\quad c_{ij}^* = \sum_{k=1}^{n} d_{ik}^* b_{kj}^* \]

\[(A1\cdot14)\quad y_{ij}^* = z_j \times \sum_{k=1}^{n} \frac{r_{ik}}{s_k} \times \frac{e_{ij}}{z_j} \quad (i=1,\ldots,m, j=1,\ldots,m, k=1,\ldots,n)\]

According to equation (A1\cdot14), column sum of matrix $Y^*$ is as follows.

\[
\sum_{i=1}^{m} s_{j} \sum_{k=1}^{n} \frac{r_{ik}}{s_k} \times \frac{e_{ij}}{s_j} = \sum_{i=1}^{n} \frac{e_{ij}}{s_j} \sum_{k=1}^{m} r_{ik} \quad (i=1,\ldots,m, j=1,\ldots,m, k=1,\ldots,n)
\]

Then, column sum of matrix $Y^*$ is equal to column sum of matrix $E$. 

**Fig.A1-2 Component of Y table**
Therefore, the subtraction of column sum of matrix \( Y^* \) from \( z_j \) is equivalent to excess liability \( e_j \), even if \( s^E \neq s^R \).

Likewise row sum of matrix \( Y^* \) is as follows.

\[
\sum_{j=1}^{m} y^*_j = \sum_{j=1}^{m} \sum_{k=1}^{n} \frac{r_{jk}}{s_k} \times e_{ij}
\]

(A1.17)

\[
= \sum_{k=1}^{n} r_{ik} \sum_{j=1}^{m} e_{ij}
\]

\[
= \sum_{k=1}^{n} \frac{r_{ik}}{s_k} \times \frac{s^E_k}{s_k} \quad (i = 1, \ldots, m, j = 1, \ldots, m, k = 1, \ldots, n)
\]

So the difference between \( z_j \) and row sum of matrix \( Y^* \) is

\[
z_j - \sum_{j=1}^{m} y^*_j = z_j - \sum_{k=1}^{n} \frac{s^E_k}{s_k} \quad (i = 1, \ldots, m, j = 1, \ldots, m, k = 1, \ldots, n).
\]

(A1.18)

On the other hand, the difference between \( z_j \) and column sum of matrix \( R \) is equal to excess assets \( p_j \).

\[
z_j - \sum_{k=1}^{n} r_{ik} = p_j \quad (i = 1, \ldots, m, j = 1, \ldots, m, k = 1, \ldots, n)
\]

(A1.19)

In case \( s^E \neq s^R \), we have to take account of not only excess assets but also the appraisal profit and loss \( e_j^* \) given by subtracting equation (A1.19) from equation (A1.18).

\[
\phi_j^* = (z_j - \sum_{k=1}^{n} \frac{r_{ik}}{s_k} \times \frac{s^E_k}{s_k}) - (z_j - \sum_{k=1}^{n} r_{ik})
\]

(A1.20)

\[
= \sum_{k=1}^{n} \frac{r_{ik}}{s_k} \times \frac{s^E_k}{s_k} \quad (i = 1, \ldots, m, j = 1, \ldots, m, k = 1, \ldots, n)
\]

Equation (A1.20) indicates that the appraisal profit and loss \( \phi_j^* \) is weighted average of
the column sum of matrix \( R \), where total financial assets divided by total liabilities are used for weight. If \( s_E = s_R \), i.e. \( \frac{S_E}{S_R} = 1 \), then \( \phi_j^* = 0 \), otherwise \( \phi_j^* \) should be accommodated into \( Y^* \) table explicitly. The component of \( Y^* \) table is shown as follows.

\[
\begin{array}{cccccc}
& y_{11}^* & y_{12}^* & \ldots & y_{1m}^* & \rho_1 & \phi_1^* & z_1 \\
& y_{21}^* & y_{22}^* & \ldots & y_{2m}^* & \rho_2 & \phi_2^* & z_2 \\
& \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
& y_{m1}^* & y_{m2}^* & \ldots & y_{mm}^* & \rho_m & \phi_m^* & z_m \\
& \epsilon_1 & \epsilon_2 & \ldots & \epsilon_m & \\
z_1 & z_2 & \ldots & z_m \\
\end{array}
\]

Fig. A1-3 Component of \( Y^* \) table

Furthermore, as shown in equations (A1-4) and (A1-14), each element of matrix \( Y \) and matrix \( Y^* \) can be written as follows, respectively.

\[
y^* = \sum_{k=1}^{m} e_{ik} \times \frac{r_k}{s_k} \\
y^* = \sum_{k=1}^{m} r_{ik} \times \frac{r_k}{s_k}
\]

If \( s_E = s_R \) then \( y^*_j = y^*_j \), i.e. matrix \( Y^* \) is transposed matrix of \( Y \). But in case of \( s_E \neq s_R \), matrix \( Y^* \) is different from transposed matrix of \( Y \).

(3) Leontief inverse

In case of \( s_E \neq s_R \), the fundamental equations of both liability-oriented system and asset-oriented system are written as follows.

\[
A1-21 \quad \sum_{j=1}^{n} y_j + \epsilon_i + \phi_i = z_i \\
A1-22 \quad \sum_{j=1}^{m} y_j^* + \rho_i + \phi_i^* = z_i
\]

These equations could be expressed in the matrix formula.

\[
A1-23 \quad Cz + \epsilon + \phi = z \\
A1-24 \quad C^* z + \rho + \phi^* = z
\]

Solving each equations for \( z \) yields

\[
A1-25 \quad z = (I - C)^{-1} (\epsilon + \phi),
\]
(A1.26) \[ z = (1 - C)\cdot (p + q^*) . \]

These are corresponding to equations mentioned in section 3.1.1.

Appendix 2

We apply the techniques of cointegration tests developed by Engle and Granger (1987) to examine the casual relationship between the rate of change in the current total value of the existing corporate stocks and the rate of change in the dispersion discrepancy index. For simplification, the former is noted as LSTOK and the latter as LDISP. Table A2-1 reports the summary of augmented Dickey-Fuller (ADF) test. (a) is the null hypothesis that a single unit root exists in LSTOK and \( \Delta \) LSTOK. Based on the ADF-t statistics, the null hypothesis of a unit root in LSTOK is not rejected at 1% significance level, while that in \( \Delta \) LSTOK is rejected. (b) is the null hypothesis that a single unit root exists in LDISP as well as \( \Delta \) LDISP. The result is that the null hypothesis of a unit root in LDISP is not rejected, while that in \( \Delta \) LDISP is rejected. Then, it suggests that both LSTOK and LDISP are characterized by I (1) process.

<table>
<thead>
<tr>
<th>LSTOK</th>
<th>with time trend</th>
<th>without time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lags</td>
<td>-3.51017</td>
<td>-3.33405</td>
</tr>
<tr>
<td>( \Delta ) LSTOK</td>
<td>-4.82551**</td>
<td>-4.84112**</td>
</tr>
<tr>
<td>Lags</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LDISP</th>
<th>with time trend</th>
<th>without time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lags</td>
<td>-3.00825</td>
<td>-2.44912</td>
</tr>
<tr>
<td>( \Delta ) LDISP</td>
<td>-4.27347**</td>
<td>-4.32816**</td>
</tr>
<tr>
<td>Lags</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

** indicate that the null hypothesis that a single unit root exists can be rejected at 1% significance level. Critical value for the ADF test can be given from Davidson and MacKinnon (1993). Optimal lag length was chosen based on the AIC2.

Table A2-2 presents the results of Engle-Granger cointegration test between LSTOK and LDISP. The asymptotic critical values for cointegration tests are -4.32 at 1%
significance level) and -3.78 (at 5 % significance level) (see Davidson and MacKinnon
(1993) table 20.2). The results indicate that the null hypothesis of no cointegration is not
being rejected at 1% significance level but is rejected at 5% significance level. The
results indicate that LSTOK and LDIP are cointegrated and have a long run
relationship.

| Table A2-2 Cointegration test |
|-------------------------------|----------------|
| ADF test statistics          | DISP=f(STOK)   |
| Lags                         | 3.90078        |

Appendix 3

According to the definition of inverse matrix, sum of elements included in Leontief
inverse $\Gamma$ is calculated as follows,

$$
\sum_{i=1}^{m} \sum_{j=1}^{m} a_{ij} = \frac{1}{|X|} \sum_{i=1}^{m} \sum_{j=1}^{m} X_{ij}
$$

where $X$ (Greek letter, $\phi$) is $(I-C)$, $X_{ij}$ is cofactor matrix of $X$ and $|X|$ is
determinant of matrix $X$. Equation (A3-1) suggests that there are three possible
causes that might cause an increment in the total sum of each element of $\Gamma$. They are

1. $|X|$ is decreasing,
2. $|X_{ij}|$ of which $i+j$ is an even number is increasing, and
3. $|X_{ij}|$ of which $i+j$ is an odd number is decreasing.

If we define permutation $\phi(s_1,s_2,\cdots,s_m)$ as $+1$ when it is given by interchanging an even number of times, or as

$-1$ when it is given by interchanging an odd number of times, $|X|$ is shown as follows,

$$
|X| = \sum \phi(s_1,s_2,\cdots,s_m)X_{s_1s_2}\cdots X_{s_ms_m}
$$

Where $\sum$ expresses the sum of every permutation, which we can consider $m!$ ways
from $(1,2,\cdots,m)$. As we have to think over every possible combination, it is difficult to
find the mathematical relation between coefficient matrix $C$ and the sum of elements of Leontief inverse $(1 - C)^{-1}$.

Appendix 4

Considering that $\Gamma_{i,t} = I + C_{i,t} + C_{i,t}^2 + C_{i,t}^3 + \ldots$, the Leontief inverse matrix can be expressed as follows.

\[
\Delta \Gamma_{i,t} = \Gamma_{i,t} - \Gamma_{i-1,t-1} = \left( I + C_{i,t} + C_{i,t}^2 + C_{i,t}^3 + \ldots \right) - \left( I + C_{i-1,t-1} + C_{i-1,t-1}^2 + C_{i-1,t-1}^3 + \ldots \right)
\]

\[
= \left( C_{i,t} - C_{i-1,t-1} \right) + \left( C_{i,t}^2 - C_{i-1,t-1}^2 \right) + \left( C_{i,t}^3 - C_{i-1,t-1}^3 \right) + \ldots
\]

\[
= \left( C_{i,t} - C_{i-1,t-1} \right) + \left( C_{i,t}^2 - C_{i-1,t-1}^2 \right) + \left( C_{i,t}^3 - C_{i-1,t-1}^3 \right) + \ldots
\]

Collecting matrices with same subscript and adding $I$ to each term, we obtain

\[
\Delta \Gamma_{i,t} = \left( I + C_{i,t} + C_{i,t}^2 + C_{i,t}^3 + \ldots \right) - \left( I + C_{i-1,t-1} + C_{i-1,t-1}^2 + C_{i-1,t-1}^3 + \ldots \right)
\]

\[
= \left( C_{i,t} - C_{i-1,t-1} \right) + \left( C_{i,t}^2 - C_{i-1,t-1}^2 \right) + \left( C_{i,t}^3 - C_{i-1,t-1}^3 \right) + \ldots
\]

In Leontief inverse matrix notation, equation (A4-2) can be rewritten as

\[
\Delta \Gamma_{i,t} = \left( \Gamma_{i,t} - \Gamma_{i-1,t-1} \right) + \left( \Gamma_{i,t}^2 - \Gamma_{i-1,t-1}^2 \right) + \left( \Gamma_{i,t}^3 - \Gamma_{i-1,t-1}^3 \right) + \ldots
\]

Dispersion index $w_{i,t}$ is sum of elements of Leontief Inverse, then

\[
w_{i,t}^\Gamma = \Gamma_{i,t} \Gamma_{i,t}^i
\]
where \( i' = (1, 1, \ldots, 1) \) is unit vector whose whole elements are one. Then the changes in the dispersion discrepancy index are

\[
\Delta w_{ij} = i' \left( \Delta \Gamma_{lj} \right) i = \frac{1}{2} \left( \frac{\| i' \Gamma_{lj} - i' \Gamma_{lj-1,i} \|}{\| i' \Gamma_{lj} - i' \Gamma_{lj-1,i} \|} + \frac{\| i' \Gamma_{lj} - i' \Gamma_{l-1,j}i \|}{\| i' \Gamma_{lj} - i' \Gamma_{l-1,j}i \|} \right)
\]

Using equation (A4-4), equation (A4-5) can be expressed as follows.

\[
(A4-6) \quad \Delta w_{ij} = \frac{1}{2} \left( \frac{(w_{ij}^T - w_{ij-1}^T) + (w_{ij-1}^T - w_{ij-2,i}^T)}{4} + \frac{(w_{ij}^T - w_{i-1,j}^T) + (w_{i-1,j}^T - w_{i-2,j-1}^T)}{4} \right)
\]
Figure 1. E table and R table

Figure 2. Transaction Matrices (left) and Corresponding Coefficient Matrices (right)
Figure 3. Y table

Figure 4. Y* table
Figure 5. Dummy instrument on the balance sheet

Figure 6. Dummy instrument on the Asset-Liability-Matrix
Figure 7. The power of dispersion indices

The power of dispersion index (asset oriented system)

The power of dispersion index (liability oriented system)
Figure 8. The power of dispersion indices (liability-oriented system) and the sensitivity of dispersion indices (asset-oriented system)
Figure 9. The power of dispersion indices (asset-oriented system) and the sensitivity of dispersion indices (liability-oriented system)
Figure 10. The power of dispersion indices (asset-oriented system) and the sensitivity of dispersion indices (liability-oriented system)
Figure 11. The column sum of the liability-oriented system
Figure 12. The column sum of the asset-oriented system

- x – banks
- ● – non-financial corporations
- □ – households
- ○ – government
Figure 14. Liability dispersion index and asset dispersion index
(expressed in current market value on the asset side and in issue value on the liability side)
Figure 15. Dispersion discrepancy index

[Graph showing a line plot with fiscal years on the x-axis and a range from 0 to 140 on the y-axis, with data points indicating trends over time.]
Figure 16. Result of decomposition
Figure 18. Excess asset or liability of the non-financial private corporations
<table>
<thead>
<tr>
<th>35 sectors</th>
<th>11 sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Bank of Japan</td>
<td>Bank of Japan</td>
</tr>
<tr>
<td>2 Long term credit banks</td>
<td>Banks</td>
</tr>
<tr>
<td>3 Trust banks</td>
<td>Association</td>
</tr>
<tr>
<td>4 City banks</td>
<td>financial</td>
</tr>
<tr>
<td>5 Regional banks</td>
<td>institutions</td>
</tr>
<tr>
<td>6 Regional banks II (Mutual loans and savings banks)</td>
<td></td>
</tr>
<tr>
<td>7 Foreign banks in Japan</td>
<td>Securities</td>
</tr>
<tr>
<td>8 Credit associations</td>
<td>companies</td>
</tr>
<tr>
<td>9 Credit cooperatives</td>
<td></td>
</tr>
<tr>
<td>10 Labor credit associations</td>
<td></td>
</tr>
<tr>
<td>11 Financial institutions for agriculture, forestry and fisheries</td>
<td>Securities institutions</td>
</tr>
<tr>
<td>12 Investment trust</td>
<td></td>
</tr>
<tr>
<td>13 Securities companies</td>
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</tr>
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<td>14 Securities finance corporations</td>
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<tr>
<td>15 Money market broker</td>
<td>Non-banks</td>
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<tr>
<td>16 Non-bank financial companies</td>
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</tr>
<tr>
<td>17 Life insurance</td>
<td>Insurance and pension institutions</td>
</tr>
<tr>
<td>18 Non-life insurance</td>
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</tr>
<tr>
<td>19 Pension funds</td>
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</tr>
<tr>
<td>20 Postal savings and postal life insurance</td>
<td>Public financial institutions</td>
</tr>
<tr>
<td>21 Fiscal loan fund</td>
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<tr>
<td>22 Public financial corporations</td>
<td>Government</td>
</tr>
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<td>23 Central government</td>
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</tr>
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<td>24 Non-financial public corporations</td>
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<td>25 Local governments</td>
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</tr>
<tr>
<td>26 Non-financial private corporations (manufacturing, large)</td>
<td>Non-financial private corporations</td>
</tr>
<tr>
<td>27 Non-financial private corporations (manufacturing, small and medium)</td>
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</tr>
<tr>
<td>28 Non-financial private corporations (non-manufacturing, large)</td>
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<tr>
<td>29 Non-financial private corporations (non-manufacturing, small and medium)</td>
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<td>30 Non-corporate enterprises (manufacturing)</td>
<td>Personal</td>
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<td>31 Non-corporate enterprises (non-manufacturing)</td>
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<td>32 Farming households</td>
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<td>33 Households</td>
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<td>34 Private non-profit institutions serving households</td>
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<td>35 Overseas</td>
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<td>Financial instruments</td>
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</tr>
<tr>
<td>1</td>
<td>Deposits with the Bank of Japan</td>
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<td>2</td>
<td>Government deposits</td>
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<td>Currency</td>
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<td>Foreign currency deposits</td>
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<td>Postal savings</td>
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<td>Trust beneficiary rights</td>
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<td>Non-life insurance reserves</td>
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<td>Local government securities</td>
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<td>Public corporation securities</td>
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<td>19</td>
<td>Bank debentures</td>
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<td>Corporate bonds</td>
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<td>Investment trust beneficiary certificates</td>
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<td>Foreign denominated securities</td>
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<td>Bank of Japan loans</td>
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<td>Call loan and money</td>
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<td>Bills purchased and sold</td>
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<tr>
<td>30</td>
<td>Loans by private financial institutions, (to small corporate enterprises)</td>
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<td>Loans by private financial institutions, (to public organizations)</td>
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<td>Loans by private financial institutions, (to non-corporate enterprises)</td>
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<tr>
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<td>Loans by private financial institutions, (housing loans)</td>
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<tr>
<td>34</td>
<td>Loans by private financial institutions, (consumer credit)</td>
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<tr>
<td>35</td>
<td>Loans by private financial institutions, (to overseas)</td>
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<td>Loans by public financial institutions, (not classified below)</td>
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<td>Loans by public financial institutions, (to small corporate enterprises)</td>
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<td>Loans by public financial institutions, (to public organizations)</td>
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<td>Loans by public financial institutions, (to non-corporate enterprises)</td>
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<td>Loans by public financial institutions, (housing loans)</td>
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<td>41</td>
<td>Loans by public financial institutions, (consumer credit)</td>
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<td>Loans by public financial institutions, (to overseas)</td>
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<td>43</td>
<td>Trade credits and foreign trade credits</td>
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<td>Deposits with the Government Fiscal Loan Fund</td>
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<td>Foreign exchange reserves</td>
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<td>External claims and debts</td>
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<td>Others</td>
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<td>Difference between financial assets and liabilities</td>
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<td>Total</td>
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