Flow of Funds Analysis:
BOJ Quantitative Monetary Policy Examined

Kazusuke Tsujimura   Masako Mizoshita

October 15.  2001
Ver. 1.0

KEO DISCUSSION PAPER  No.67
1. Introduction

Flow of funds (FOF) analysis has been stem from “Social Accounting for Money flows” authored by Morris Copeland in 1949. Since then it has developed as an accounting system describing the inter-sectoral financial transactions between the economic actors. FOF accounts were included in the System of National Accounts in 1968 along with National Income Accounts, National Balance Sheet, Balance of International Payments Accounts and Input-Output Tables. FOF Accounts consist of balance sheets of the economic actors, in their traditional tabulation practice. This format of FOF is widely employed because of the easiness of compilation based on the corporate accounting system. The analysis of FOF based on the balance sheet format has been concentrated on the investigation of the difference between financial assets and liabilities of each sector. However this kind of study only describes the final balance of object-economy and the portfolio selection activity of the economic actors. It is unavoidable to build an asset-liability-matrix (ALM) to examine the inter-sectoral financial transactions on the whole. Then the first purpose of this paper is to present the compilation procedure of ALM from the FOF accounts of the balance sheet format widely available. The second purpose of this article is to demonstrate the application of ALM in the examination of the quantitative monetary policy introduced by the Bank of Japan in March 2001 and revised accordingly in line with other central banks of the world after the terrorist attack on the World Trade Center building in New York on September 11.

The introduction of ALM into the framework of FOF analysis enables us to utilize the affluent assets of Input-Output (I-O) analysis notably the concept of Leontief-Inverse. However there are not a few fundamental differences between the ALM analysis and the I-O analysis. In the Y-table of the ALM, which is the sector-by-sector matrix, the column denotes the portfolio of fund-raising, while the row indicates the portfolio of fund-employment of a particular sector. In this sense, the ALM is an I-O matrix that is descriptive of the flow of funds. However it is also possible to construct an I-O matrix that gives an account of the flow of financial instruments instead of the flow of funds. The column of this type of ALM (Y*-table) denotes the portfolio of fund-employment, while the row indicates the portfolio of fund-raising. In this context, the Y* matrix is a transposition of Y matrix.

This two-sided nature of ALM gives a significant theoretical importance to the FOF analysis. The simultaneous equations symbolised by Y portray the relations between the surplus in liabilities and the controlled total, while those represented by Y* depict the correspondence between the surplus in financial assets and the controlled total. The coexistence of these two systems of equations gives two alternative Leontief-Inverse to
the structure of ALM. This distinctive feature of ALM is best demonstrated in the investigation of the quantitative monetary policy recently adopted by the Bank of Japan (BOJ).

In the spring of 2001, BOJ announced that it will shift the target of money market operation from the inter bank interest rate to the balance of current accounts at the bank held by the financial institutions. At the same time BOJ proclaimed to increase the balance of current accounts by one trillion yen, while adding same amount of Japanese Government Bonds (JGB) in its assets. There is an asymmetry in the propagation of the supply and demand of the funds in the financial system. The demand for funds should be eventually financed by the gross induced savings (GIS), while the supply of funds brings gross induced investments (GII) in due course. The quantitative monetary policy requires the central bank to choose two items simultaneously, one in assets and another in liabilities. This action will bring GII on one hand and GIS on the other. The asymmetry in the propagation process gives net induced investments (NII) as a difference between GII and GIS. The sign and the amount of the NII is nothing but the indicator of the effect monetary policy made on the objects-economy. A policy that brings a positive number of NII gives an expansion in the economy so that it will be welcomed especially in the course of recession. In contrast with this, a money market operation that delivers negative NII, i.e. net induced savings (NIS), weakens the economy so that it should be avoided at any cost while the depression prevails. In the final part of this tract, we will present a numerical evaluation of the lately introduced BOJ quantitative monetary policy in terms of the ALM analysis.

2. The Asset Liability Matrix
2.1 E- and R-tables

In Japan, Flow of Funds Accounts are compiled on the basis of 1993 IMF Manual by BOJ every three month. The ALM used in this paper has been compiled from the Financial Assets and Liabilities tables of FOF of 1st quarter 2001(Preliminary). The first step to draw up ALM is to pick out the assets and liabilities vectors separately from the balance sheets of FOF to make out two matrices E and R. E is a matrix to show the portfolio of fund-employment of each sector, a and TE are vectors that represent excess liabilities, sum of each row. Likewise, R is a matrix to show the portfolio of fund-raising of each sector, p and TR are vectors that represent excess assets, sum of each row respectively. T is the vector that consist of either the sum of assets or liabilities whichever are greater. n and m denote the number of financial instruments and the number of institutional sectors. The structure of E and R tables in terms of their
components are depicted below.

\[
\begin{array}{cccc}
  e_{11} & e_{12} & \cdots & e_{1m} \\
  e_{21} & e_{22} & \cdots & e_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  e_{n1} & e_{n2} & \cdots & e_{nm} \\
\end{array}
\begin{array}{cccc}
  r_{11} & r_{12} & \cdots & r_{1m} \\
  r_{21} & r_{22} & \cdots & r_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{n1} & r_{n2} & \cdots & r_{nm} \\
\end{array}
\begin{array}{cccc}
  t_{1} & t_{2} & \cdots & t_{m} \\
\end{array}
\]

Figure 1: E-table

Figure 2: R-table

2.2 Y and Y* tables

The ALM based on the fund-raising portfolio is denoted as Y while the one based on fund-employment is symbolized as Y*. We use Superscript letter * in case fund-employment assumption is concerned. To compile Y table in accordance with fund-raising portfolio, first R matrix is substituted for U matrix (commodity by sector) and transposed E matrix for V matrix (sector by commodity).

\[
U = R \\
V = E'
\]

In case of Y* table that represent fund-employment portfolio, we take E matrix as U* matrix and transposed R matrix as V* matrix.

\[
U^* = E \\
V^* = R'
\]

The coefficient matrices, B and B*, are constructed from U, U* and T by dividing the cells in each column of U and U* by the column sums T.

\[
b_j = \frac{u_j}{t_j} \\
b_j^* = \frac{u_j^*}{t_j}
\]

In the same manner, coefficient matrices D and D* corresponding to V and V* are defined as follows:

\[
d_j = \frac{v_j}{t_j} \\
d_j^* = \frac{v_j^*}{t_j}
\]
where $t_{Ej}$ is the sum of assets and $t_{Rj}$ is the sum of liabilities for financial instrument $j$. That is, $d_j^i$ is regarded as $i$ sector's share of assets for $j$ financial instruments while $d_j^r$ is $i$ sector's share of liabilities for $j$ financial instruments. The $m \times m$ coefficient matrices $C$ and $C^*$ corresponding to $Y$ and $Y^*$ are estimated using actor portfolio assumption, which is an equivalent of the industry technology assumption in the scheme of I-O analysis.

$$C = DB$$

$$C^* = D'B^*$$

Using these matrices, $C$ and $C^*$, transaction quantity matrices $Y$ and $Y^*$ are reduced in the following manner:

$$y_{ij} = c_{ij} t_{ij}$$

$$y_{ij}^* = c_{ij}^* t_{ij}$$

where $y_{ij}$ is the amount of funds provided from sector $i$ to sector $j$, and $y_{ij}^*$ is the amount of financial instruments supplied from sector $i$ to sector $j$. These tables are shown in Fig. 3 and Fig. 4.

When we compare $Y^*$ table with $Y$ table, $\eta^Y$ vector and $\rho^Y$ vector are situated symmetrically. Moreover matrix $Y^*$ is obtained by interchanging matrix $Y$'s rows and columns, i.e. matrix $Y^*$ is a transpose matrix of $Y$. As demonstrated in the Appendix, whether we adapt fund-raising assumption or fund-employment assumption, only one transaction quantity matrix is derived from a pair of matching E- and R-matrices. In that sense, the matrices $Y$ and $Y^*$ are just like two sides of the same coin.

$$y_{i1} \ y_{i2} \ \cdots \ y_{im} \ \epsilon_{1}^{Y} \ t_{1}^{Y} \ \cdots \ y_{i1}^{*} \ y_{i2}^{*} \ \cdots \ y_{im}^{*} \ \rho_{1}^{Y} \ t_{1}^{Y}$$

$$y_{m1} \ y_{m2} \ \cdots \ y_{mm} \ \epsilon_{1}^{Y^*} \ t_{m}^{Y^*} \ \cdots \ y_{m1}^{*} \ y_{m2}^{*} \ \cdots \ y_{mm}^{*} \ \rho_{m}^{Y^*} \ t_{m}^{Y^*}$$

Figure 3: $Y$-table

Figure 4: $Y^*$-table

3 Methodologies

3.1 ALM dealing with BOJ as exogenous sector
The ultimate object of this paper is to analyse the effect of monetary policy by estimating the induced amount of demand and supply of funds through the intersectoral financial transactions represented in Leontief-inverse, so it is necessary to deal with BOJ, the central bank, as an exogenous sector. We denote two vectors \( \epsilon^Y_{BOJ} \) and \( \rho^Y_{BOJ} \) as follows:

\[
\begin{align*}
\epsilon^Y_{BOJ} &= \begin{bmatrix}
\epsilon^{Y,1}_{BOJ} + \epsilon^{Y,1}_1 \\
\epsilon^{Y,2}_{BOJ} + \epsilon^{Y,2}_2 \\
\vdots \\
\epsilon^{Y,m}_{BOJ} + \epsilon^{Y,m}_m
\end{bmatrix} \\
\rho^Y_{BOJ} &= \begin{bmatrix}
\rho^{Y,1}_{BOJ} + \rho^{Y,1}_1 \\
\rho^{Y,2}_{BOJ} + \rho^{Y,2}_2 \\
\vdots \\
\rho^{Y,m}_{BOJ} + \rho^{Y,m}_m
\end{bmatrix}
\end{align*}
\]

\( C_{BOJ} \) is the matrix obtained from matrix \( C \) by removing the row and column containing elements concerning BOJ. Likewise \( C^*_{BOJ} \) is the matrix reduced from matrix \( C^* \) by removal of the row and column containing BOJ elements. The fundamental equations respect to \( Y \) and \( Y^* \)-tables are expressed as follows:

\[
\begin{align*}
C_{BOJ} Y + \epsilon^Y &= T^Y \\
C^*_{BOJ} Y + \rho^Y &= T^Y
\end{align*}
\]

Solving each equation for \( T^Y \) yields

\[
\begin{align*}
T^Y &= (I - C_{BOJ})^{-1} \epsilon^Y \\
T^Y &= (I - C^*_{BOJ})^{-1} \rho^Y
\end{align*}
\]

where \( I \) denotes the \( m \times m \) unit matrix, \((I - C_{BOJ})^{-1}\) is the \( m-1 \times m-1 \) Leontief inverse matrix by which how much demand for funds would be induced to each sector can be calculated, and \((I - C^*_{BOJ})^{-1}\) is the \( m-1 \times m-1 \) Leontief inverse matrix by which we can calculate the amount of ultimately induced supply of funds. From the viewpoint of the object economy, the induced demand for funds means GIS, the amount of new savings required, while the induced supply of funds refers to GII that enables us to make still more investments.

### 3.2 Evaluation of Quantitative Monetary Policy

It is possible to calculate the effect of quantitative monetary policy carried by BOJ using Leontief Inverse in the same framework described above. \( s_{BOJ} \) is the \( n \times 1 \) vector of liability held by BOJ in the form of financial instrument \( i \), \( s_{BOJ} \). The \( n \times 1 \) vector \( p_{BOJ} \) is
the assets vector where an element \( p_{BOJ,i} \) denotes the financial instrument \( i \) held by BOJ. Vectors \( s_{BOJ} \) and \( p_{BOJ} \) then should be transformed into \( m \times 1 \) vectors \( s_{BOJ}^Y \) and \( p_{BOJ}^Y \), each of the is classified by sector in order to make it possible to use Leontief Inverse. \( s_{BOJ}^Y \) is the \( n \times 1 \) vector of liability held by BOJ owing to each of the \( n \) sectors \( s_{BOJ} \). The \( n \times 1 \) vector \( p_{BOJ}^Y \) is the assets vector where an element \( p_{BOJ,i} \) denotes the claims on BOJ held by each of the sectors. The method of transformation adopted here is as follows.

\[
\varepsilon_{BOJ}^Y = D \varepsilon_{BOJ}
\]

\[
p_{BOJ}^Y = D^* p_{BOJ}
\]

Given \( s_{BOJ} \) and \( p_{BOJ} \) exogenously, the induced savings and the induced investments are calculated using the model described in 2.2.

\[
\eta_S = (I - C_{BOJ})^{-1} \varepsilon_{BOJ}^Y
\]

\[
\eta_T = (I - C_{BOJ}^*)^{-1} p_{BOJ}^Y
\]

\( \eta_S \) is the \( m \times 1 \) vector of induced savings where element \( \eta_S \) denotes the induced saving generated in sector \( i \). \( \eta_T \) is the \( m \times 1 \) vector of induced investments where element \( \eta_T \) indicates induced investment generated in sector \( i \). Note that GIS \( [H_S = \Sigma \eta_S] \) is the row sum of \( \eta_S \), and GII \( [H_I = \Sigma \eta_I] \) is the row sum of \( \eta_I \). Subtracting GIS from GII obtain NIT as follows.

\[
\eta_{T-S} = \eta_T - \eta_S
\]

The element \( \eta_{T-S} \) denotes net induced investment generated in \( i \) sector. Note that NII \( [H_{I-S} = \Sigma \eta_{T-S}] \) gives the row sum of \( \eta_{T-S} \). We draw up in Table 1, \( s_{BOJ} \) and \( p_{BOJ} \) from December 2000 through August 2001 and calculated sum of GIS \( [H_S] \), GII \( [H_I] \), and NII \( [H_{I-S}] \).

3.3 Contributing Factor to Fluctuations in NII
The next question we have to answer is that what financial instrument might contribute to NII. Let \( s_{BOJ,k} \) be a \( n \times 1 \) vector of liabilities held by BOJ with only \( k \)th element \( s_{BOJ,k} = 1 \) and any other elements to be 0. \( p_{BOJ,h} \) is a \( n \times 1 \) vector of assets held by BOJ with only \( h \)th element \( p_{BOJ,h} = 1 \) and any other elements to be 0.
Next, let us define the $\varepsilon^\uparrow_{BOJ,k}$ vector and the $\rho^\uparrow_{BOJ,h}$ vector which are $m \times 1$ vectors transformed from $\varepsilon_{BOJ,k}$ and $\rho_{BOJ,h}$, respectively.

$\varepsilon^\uparrow_{BOJ,k} = D \cdot \varepsilon_{BOJ,k}$

$\rho^\uparrow_{BOJ,h} = D \cdot \rho_{BOJ,h}$

By multiplying the vectors above by the Leontief Inverse, we can estimate the amount of GIS $[\eta^\uparrow_{m,k}]$ generated by a unit increment in the financial assets held by BOJ. Likewise, we can calculate the amount of GII $[\eta^\uparrow_{m,h}]$ generated by a unit increment in the liabilities held by BOJ.

$\eta^\uparrow_{Sk} = (I - C_{BOJ}^{-1})^{-1} \varepsilon^\uparrow_{BOJ,k}$

$\eta^\uparrow_{Th} = (I - C_{BOJ}^{-1})^{-1} \rho^\uparrow_{BOJ,h}$

Note that $H^s_{Sk}$ gives the rowsum of $\eta^\uparrow_{m,k}$, $H^t_{Th}$ gives the rowsum of $\eta^\uparrow_{m,h}$, and $H^l_{l-S,Sk}$ is obtained by subtracting $H^s_{Sk}$ from $H^t_{Th}$.

$H^s_{Sk} = \sum_{i=1}^{m} \eta^\uparrow_{m,i,Sk}$

$H^t_{Th} = \sum_{i=1}^{m} \eta^\uparrow_{m,i,Th}$

$H^s_{l-S,Sk} = H^t_{Th} - H^s_{Sk}$

The results of the estimation are presented in 4.2.1. The relations between GII, GIS and NII are presented in components as follows using the formulae mentioned above.

$H_{l-S} = H_{l} - H_{S}$

$= \sum_{k=1}^{n} H^s_{sk} \cdot \rho_{BOJ,k} - \sum_{k=1}^{n} H^s_{sk} \cdot \varepsilon_{BOJ,k}$

$= \sum_{k=1}^{n} (H^s_{sk} \cdot \rho_{BOJ,k} - H^s_{sk} \cdot \varepsilon_{BOJ,k})$
In the equations above, each term enclosed in parenthesis identifies which financial instrument causes what amount of NIT. This relationship leads us to a new indicator of measurement of contribution $\kappa$. 

$$\kappa_k = H^{\ast}_k \cdot PBOJ,k - H^{\ast}_k \cdot \varepsilon_{\ast,BOJ}.$$ 

The decomposition of the changes in NII $[\Delta H_{t-s}]$ over a period and the decomposition of changes in $\kappa$ $[\Delta \kappa_t]$ over a period can then be calculated as the difference of preceding term and the present term. The latter is an indicator widely known as degree of contribution.

$$\Delta H_{t-s} = H_{t-s}^{\ast} - H_{t-s}^{\ast-1}$$

$$\Delta \kappa_t = \kappa_t - \kappa_t^{\ast-1}$$

4. The Results
4.1 Monthly Fluctuations and Degree of Contribution to NII

The fluctuations in the three indices namely GII $[H_t]$, GIS $[H_s]$ and NII $[H_{t-s}]$ between December 2000 and September 2001 are depicted in Fig.6. During this period, that is before and after the introduction by BOJ of quantitative monetary policy in March, the NII not only stayed in the negative region despite Japan was suffering from severe depression for the first time in more than 40 years, but the magnitude was increasing. This fact suggests that the money market operation of this period that reflects newly adopted quantitative monetary policy was a total failure.

The monthly change in the net induced investments and the degree of contribution to it are shown in Fig.7. NII are decreasing every month except for February and September. In March during which the alteration of the target of the BOJ monetary policy took place, the balance in the current account held by commercial banks and other institutions increased by 1.1 trillion yen, while the central bank added more than 1.7 trillion yen of JGB to its asset portfolio. As the consequence of this shift in the monetary target, the NII decreased by almost 10 trillion yen. While the increment of JGB as an asset pushed up the net induced investments 2.8 trillion yen, the increase in the balance of current account slashed the figure by as much as 5.4 trillion yen. As far as the degree of contribution is concerned, the combined effect of the two was not significant anyway. The main positive contributing factors of the month were BOJ accounts at the agent commercial banks and the increment of the financial bill held by the central bank. Two prime elements that made excess negative contributions were the
rise in the government deposits with BOJ and the addition in the bank notes issued. In conclusion the installation of the quantitative monetary target failed to give favourable effects to the then exhausted Japanese economy.

In August 2001, BOJ announced that the outstanding balance of current accounts held at the central bank should be raised from around 5 trillion yen to around 6 trillion yen. However, the net induced investment kept declining even further. Although the decline in the amount of the banknotes issue made a positive contribution of 13 trillion yen to the net induced investments, security-lending transactions gave 23 trillion yen negative effects on it. Meanwhile the commercial financial institutions added 0.9 trillion yen to their balance of current accounts with BOJ to make it 5.9 trillion yen, to make the degree of contribution to the net induced investments -4 trillion yen. JGB holdings by BOJ grew 2 trillion yen to 65 trillion yen and the degree of contribution was +2 trillion yen. The net contribution of these two items combined was negative setting aside the smallness in the magnitude of the contribution itself.

On September 18, BOJ announced a further relaxation package of its monetary policy after the terrorist attack in U.S. on September 11. In cooperation with fellow central banks, BOJ promised to keep the balance of the current account held by the commercial banks and other institutions well above the previous level. Actually the balance doubled within a month to reach 12.5 trillion yen at the end of September. The fundamental difference of this package from the previous ones was that BOJ carefully avoided to specify what kind of asset allocation to take to match the increment in its liabilities. Actually the 6.5 trillion yen increase in the current accounts, which was partially offset by the decrease in the bills sold to the open market by BOJ, produced 1.8 trillion yen of bills bought from the commercial banks and 2.2 trillion yen of securities on loan against cash while the JGB held by BOJ increased by 1.5 trillion yen. The result of this revision in the policy was favorable as the net induced investments improved from -133 at the end of August to -128 by the close of September. The negative contribution by the increase in the current accounts with BOJ was almost completely offset by the decrease in the bills sold by the central bank, so the increase in the bills purchased through the open market operation and in the securities on loan against cash could give a significant boost in the net induced investments.

By concluding above survey, it can be said that the shift in the target for money market operations from the current uncollateralized overnight call rate to the outstanding balance of the current accounts at BOJ has only limited significance so far. The fact that the net induced investments were hovering in the negative region throughout this period suggests that the newly introduced quantitative monetary policy
was a total failure. However a detailed examination of the degree of contribution gives a clue to a successful quantitative monetary policy. For example, in those months in which improvements in NIT took place, the contribution of bills purchased was apparent. In the next section, further study in this direction will be performed.

4.2 The Examination of Money Market Operation Methods

Since the object of monetary policy is to adjust the amount of funds to the trend of market economy, daily monetary market operation must vary according to business fluctuations. The purpose of this paper is to examine what monetary policies should be taken when the market is dull or brisk too far. In this paper NIT is viewed as an indicator to review the policy from the view point of the effects of monetary policy on the object-economy. We have estimated the monthly alterations in NIT caused by the changes of BOJ's balance sheet from December 2000 to September 2001 (see 3.2.1 above). But only with this information it is not sufficient to learn what monetary policy is desirable in response to fluctuations in object economy, so that we have taken up combinations of assets and liabilities within the scope of money market operations and evaluated the effects in terms of NIT (see 3.2.2). These results are provided in Table 2 according to size. Combinations which are placed upper side of the table have positive effects on the object-economy, that is the policy to be adopted in the course of depression. Conversely, in times of overheating economy it is desirable to choose combinations listed on the lower side of the table.

This analysis showed that the best combination of liability and asset to stimulate object-economy is to raise funds by mean of bank note issuing and to employ funds in the bill-purchasing operation. It is confirmed that the method of monetary control traditionally adopted by the central banks of the western countries have marvellous effects on expanding the economy in terms of NIT. The combination of bank note issuing and securities lending transactions is in the second place, while that of bank note issuing and BOJ loan facilities is in the third. These options of money market operations have been universally adopted by the central banks worldwide. Judging in terms of NIT, as mentioned above, raising funds by bank note issuing and employing funds by purchasing bills, lending securities against cash or tendering BOJ loans are good for getting over the recession. As neutral monetary operation, of which NIT is nearly zero, there are combination of government deposits and securities lending transactions and that of deposits with the BOJ and BOJ loans and so on. Besides, the
option of money market operation actually taken by BOJ, that is to raise funds by mean of current deposits held by commercial banks etc. and to employ it as JGB, which poses negative NII, is ranked in the second place from the bottom. It is due to that increase in outright purchase of JGB leads to less GII than any other monetary operations. Therefore we could assert that the policy holding more long-term government bonds is an effectual method in cooling the boom, but is not at all fit in the depression days.

5. Policy implications
It can be said as a conclusion that there was a tactical error in the quantitative monetary policy adopted by BOJ in 2001. Especially the combination of fund raising through current account held by commercial banks etc. and fund employment in JGB is disastrous. However this does not mean that BOJ should abandon the quantitative monetary policy all together. The exploration above suggest that a good combination of fund raising and employment produce positive NII so that it is an effective remedy for recession particularly when other policy options are not readily available. The best mixture of money market operation is to raise funds through bank notes and employ them in the form of bill buying operation. This is just the way the central banks of the world have followed from the very early stage of their history. It is easy to depart from this tradition, but it is rather difficult to find an alternative money market operation practice that gives equally good performance.

There is no doubt that the best policy BOJ should take is to go back to the good old days and concentrate on the bill buying operation. However it is rather difficult to return to the old way of life because nowadays Japanese companies do not issue as much bills as they used to do. The amount of bills exchanged at Tokyo clearing house has decreased from 2600 trillion yen in 1993 to 700 trillion in 2000 because of prolonged recession and changing business practice. Nevertheless there are not a few alternative combinations of money market operation, which give positive NII regardless of the magnitude. Table2 shows that some form of fund employment like securities on loan against cash and BOJ loans to commercial banks give relatively large amount of NII. The repurchasing transactions of securities against cash have flourished since its introduction in mid 1990’s. The balance of this kind of transaction is more than 50 trillion yen, while that with BOJ is only 18 trillion yen, so that there is an ample room for increased money market operation in this field. Although the balance of BOJ loans to commercial banks has been kept minimum recent years, the establishment in February 2001 of Lombard-rate-like Complementary Lending Facility makes it possible for the central bank lending to expand. As for the fund raising, it is difficult to control
the balance of bank note issued but still the central bank can rely on the bill selling operation to adjust the total balance as it wishes. In case of BOJ, it is more or less commonplace to have bills sold and bills purchased on its balance sheet simultaneously. Table 2 confirms that even this combination of straddling operation would let NII to be positive.

It was a constructive step for BOJ to have taken the decision to shift the target of the monetary policy from the interbank interest rate to the balance of current accounts held by commercial banks and other financial institutions. Unlike the interbank rate, which is supposed to be determined in the market, the central bank can adjust its balance sheet as it wish through various means of money market operations. The outcome of the adoption of the new policy is not promising so far. A combination of money market operation like the one BOJ has practiced that brings negative NII is ruinous in case a recession prevails the economy. In contrast with this, an operation that creates positive NII is a welcome remedy to cure an economy suffering from a slump. The most important thing in exercising this kind of policy is to make a quantitative analysis of the scheme in advance to find the best available combination of money market operations. The authors believe that ALM framework of FOF analysis is a powerful and practical device to fulfill this kind of examination.

References
Appendix

This appendix shows the relation between Y-table and Y*-table.

(1) Y matrix and Y* matrix

Each element of matrix C is a coefficient matrix of Y table as follows,

\[ c_{ij} = \sum_{k=1}^{n} d_{ik} b_{kj} \]

Considering that \( u_{ij} = r_{ij} \) and that \( v_{ik} = e_{ik} \), it follows that

\[ c_{ij} = \sum_{k=1}^{n} e_{ik} t_{kJ} \]

Then each element of Y-table is expressed as follows.

\[ y_{ij} = t_{ij} c_{ij} \]

\[ = t_{ij} \sum_{k=1}^{n} e_{ik} t_{iK} \]

On the other hand each element of matrix C is a coefficient matrix of Y table as follows,

\[ c_{ij} = \sum_{k=1}^{n} d_{ik} b_{kj} \]

Considering that \( u_{*ij} = e_{ij} \) and that \( v_{*ik} = r_{ij} \), it follows that
Then each element of $Y^*$-table is expressed as follows:

$$y^*_i = t_j C^*_j = t_j \sum_{k=1}^{n} \frac{r_{ik} e_{kj}}{t_k}$$

where $t_j \cdot t_k = 1$ is satisfied then

$$y^*_i = y_i$$

or, in matrix terms

$$Y^* = Y^t$$

Then it was demonstrated that the matrix $Y^*$ is transposed matrix of $Y$.

(2) $\psi^Y$ and $\rho^Y$ in $Y$-table

First we denote the difference between column sum of $Y$ matrix, $y^*_j$, and total of assets or liabilities, $t_j$, as $\rho^*_j$ that is typical element of $\rho^Y$ vector. Denote column sum of $C$ matrix as $c_0$, $\rho^*_j$ is given as follows

$$\rho^*_j = t_j \cdot y^*_j - y^*_j$$

where $\sum_{i=1}^{n} d_{i,j} = 1$, then

$$\rho^*_j = t_j \cdot y^*_j - t_j \cdot \sum_{i=1}^{n} b_{ij}$$

Considering that $t_j = t_j^Y$, $b_{ij} = u_{ij} / t_i$ and $u_{ij} = r_{ij}$, it follows that

$$\rho^*_j = t_j \cdot \sum_{i=1}^{n} u_{ij}$$

As stayed above $\rho^*_j = \rho_0$, or $\psi^Y = \rho$ was proved.
Second we denote the difference between row sum of \( Y \) matrix, \( y_i \), and total of assets or liabilities, \( t_i \), as \( \varepsilon_i \), that is typical element of \( \varepsilon^Y \) vector. \( \varepsilon_i \) is given as follows

\[
\varepsilon_i^Y = t_i^Y - y_i
\]

\[
= t_i^Y - \sum_{j=1}^{m} t_j^Y c_{ij}
\]

\[
= t_i^Y - \sum_{j=1}^{m} t_j^Y \left( \sum_{k=1}^{n} d_{uk} b_{jk} \right)
\]

\[
= t_i^Y - \sum_{k=1}^{n} d_{ik} \left( \sum_{j=1}^{m} b_{kj} t_j^Y \right)
\]

Considering \( b_{ij} t_j = t_i^Y \), and \( t_i^R = t_i^\rho \), it follows that

\[
\varepsilon_i^Y = t_i - \sum_{k=1}^{n} d_{ik} \left( \sum_{j=1}^{m} u_{ij} \right)
\]

\[
= t_i - \sum_{k=1}^{n} d_{ik} t_k^R
\]

\[
= t_i - \sum_{k=1}^{n} d_{ik} t_k^\rho
\]

where \( d_{ij} = v_{ij} / t_j \) and \( v_{ij} = c_{ij} \), then

\[
\varepsilon_i^Y = t_i - \sum_{k=1}^{n} v_{ik}
\]

\[
= t_i - \sum_{k=1}^{n} v_{ik}
\]

\[
= \varepsilon_i
\]

As stayed above \( \varepsilon_i^\rho = \varepsilon_i \) or \( \varepsilon^\rho = \varepsilon \) was proved.

(3) \( \varepsilon^Y \) and \( \rho^Y \) in \( Y^* \)-table

First we denote the difference between column sum of \( Y^* \) matrix, \( y_i^* \), and total of assets or liabilities, \( t_i^* \), as \( \varepsilon_i^* \), that is typical element of \( \varepsilon^Y \) vector. Denote column sum of \( C^* \) matrix as \( c^*_k \), \( \rho_{kj}^* \) is given as follows

\[
\varepsilon_i^{Y*} = t_i^Y - y_i^*
\]

\[
= t_i^Y - c_i^* t_j^*
\]

\[
= t_i^Y - t_j^* \sum_{i=1}^{n} b_{ij}^*
\]
\[ \varepsilon_j = t_j - \sum_{i=1}^{n} u_i \]

Considered that \( t_i = t_i^Y, b_i^* = u_i^Y / t_i, \) and \( u_i^* = e_i, \) it follows that,
\[ t_j - \sum_{i=1}^{n} \varepsilon_i = \varepsilon_j \]

Then \( v_j = \varepsilon_i, \) or \( e^* = e \) was proved.

Second denoting \( p^{Y*} \) vector with typical element \( p_i^{Y*} \) that is the difference between row sum of \( Y \) matrix, \( y_i^Y, \) and total of assets or liabilities, \( t_i^Y, \) \( p_i^{Y*} \) is expressed as follows

\[ p_i^{Y*} = t_i^Y - y_i^Y \]

Considered that \( b_i^* = u_i^Y / t_i, \) \( t_i^Y = t_i^Y, \) and \( t_i^R = t_i^R_i, \) it follows that

\[ p_i^{Y*} = t_i - \sum_{k=1}^{m} d_{ik} t_k \]

Where \( d_{ik} = v_i^* / t_i \) and \( v_i^* = v_i, \) then

\[ p_i^{Y*} = t_i - \sum_{k=1}^{m} v_k \]

As stayed above \( p^{Y*} = p^Y, \) or \( p^{Y*} = p \) was proved.
Table 1: Balance sheets of BOJ

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposits with the BOJ</td>
<td>882,700</td>
<td>890,000</td>
<td>877,900</td>
<td>514,120</td>
<td>500,710</td>
<td>500,590</td>
<td>513,585</td>
<td>503,577</td>
<td>503,923</td>
<td>524,794</td>
<td>524,794</td>
<td>524,794</td>
<td>524,794</td>
</tr>
<tr>
<td>Government deposits</td>
<td>1,037,877</td>
<td>1,132,572</td>
<td>1,157,880</td>
<td>237,770</td>
<td>227,372</td>
<td>110,937</td>
<td>113,660</td>
<td>166,314</td>
<td>167,747</td>
<td>134,786</td>
<td>134,786</td>
<td>134,786</td>
<td>134,786</td>
</tr>
<tr>
<td>BOJ loans</td>
<td>1,432,747</td>
<td>1,505,826</td>
<td>1,461,229</td>
<td>1,314,763</td>
<td>1,293,329</td>
<td>1,292,972</td>
<td>1,286,112</td>
<td>1,286,112</td>
<td>1,296,747</td>
<td>1,296,747</td>
<td>1,296,747</td>
<td>1,296,747</td>
<td>1,296,747</td>
</tr>
<tr>
<td>Bills purchased and sold</td>
<td>1,400,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
<td>1,909,000</td>
</tr>
<tr>
<td>Repurchase agreement and securities lending transactions</td>
<td>1,037,777</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
<td>1,041,786</td>
</tr>
<tr>
<td>Financing bills</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
<td>1,400,000</td>
</tr>
<tr>
<td>Shares and other equities</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
<td>1,020</td>
</tr>
<tr>
<td>Deposits money</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
<td>1,445</td>
</tr>
<tr>
<td>Outward investment in securities</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
<td>1,455,000</td>
</tr>
<tr>
<td>Gold and SDRs etc.</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
<td>4,655</td>
</tr>
<tr>
<td>Reserve</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
<td>4,520,000</td>
</tr>
<tr>
<td>Planned surplus or deficit</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
<td>1,068,000</td>
</tr>
<tr>
<td>Total</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
<td>8,812,900</td>
</tr>
</tbody>
</table>

Source: BOJ
Table 2: Monetary operation options and induced effects (1000 million yen)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
<th>GII</th>
<th>GIS</th>
<th>NII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bills purchased</td>
<td>Bank note issued</td>
<td>48102</td>
<td>35931</td>
<td>12171</td>
</tr>
<tr>
<td>Securities lending transactions</td>
<td>Bank note issued</td>
<td>47089</td>
<td>35931</td>
<td>11158</td>
</tr>
<tr>
<td>BOJ loans</td>
<td>Bank note issued</td>
<td>45878</td>
<td>35931</td>
<td>9947</td>
</tr>
<tr>
<td>Bills purchased</td>
<td>Bills sold</td>
<td>48102</td>
<td>43820</td>
<td>4282</td>
</tr>
<tr>
<td>Securities lending transactions</td>
<td>Bills sold</td>
<td>47089</td>
<td>43820</td>
<td>3269</td>
</tr>
<tr>
<td>Government financing bills</td>
<td>Bank note issued</td>
<td>38778</td>
<td>35931</td>
<td>2847</td>
</tr>
<tr>
<td>BOJ loans</td>
<td>Bills sold</td>
<td>45878</td>
<td>43820</td>
<td>2058</td>
</tr>
<tr>
<td>Bills purchased</td>
<td>Current deposits with BOJ</td>
<td>48102</td>
<td>46324</td>
<td>1778</td>
</tr>
<tr>
<td>Bills purchased</td>
<td>Government deposits</td>
<td>48102</td>
<td>47177</td>
<td>925</td>
</tr>
<tr>
<td>Securities lending transactions</td>
<td>Current deposits with BOJ</td>
<td>47089</td>
<td>46324</td>
<td>765</td>
</tr>
<tr>
<td>Securities lending transactions</td>
<td>Government deposits</td>
<td>47089</td>
<td>47177</td>
<td>88</td>
</tr>
<tr>
<td>BOJ loans</td>
<td>Current deposits with BOJ</td>
<td>45878</td>
<td>46324</td>
<td>446</td>
</tr>
<tr>
<td>BOJ loans</td>
<td>Government deposits</td>
<td>45878</td>
<td>47177</td>
<td>1299</td>
</tr>
<tr>
<td>Foreign exchange reserves</td>
<td>Bank note issued</td>
<td>32538</td>
<td>35931</td>
<td>3393</td>
</tr>
<tr>
<td>Government financing bills</td>
<td>Bills sold</td>
<td>38778</td>
<td>43820</td>
<td>5042</td>
</tr>
<tr>
<td>Government financing bills</td>
<td>Current deposits with BOJ</td>
<td>38778</td>
<td>46324</td>
<td>7546</td>
</tr>
<tr>
<td>Government financing bills</td>
<td>Government deposits</td>
<td>38778</td>
<td>47177</td>
<td>8399</td>
</tr>
<tr>
<td>Foreign exchange reserves</td>
<td>Bills sold</td>
<td>32538</td>
<td>43820</td>
<td>11282</td>
</tr>
<tr>
<td>Foreign exchange reserves</td>
<td>Current deposits with BOJ</td>
<td>32538</td>
<td>46324</td>
<td>13786</td>
</tr>
<tr>
<td>Foreign exchange reserves</td>
<td>Government deposits</td>
<td>32538</td>
<td>47177</td>
<td>14639</td>
</tr>
<tr>
<td>JGB</td>
<td>Bank note issued</td>
<td>16986</td>
<td>35931</td>
<td>18945</td>
</tr>
<tr>
<td>JGB</td>
<td>Bills sold</td>
<td>16986</td>
<td>43820</td>
<td>26834</td>
</tr>
<tr>
<td>JGB</td>
<td>Current deposits with BOJ</td>
<td>16986</td>
<td>46324</td>
<td>29338</td>
</tr>
<tr>
<td>JGB</td>
<td>Government deposits</td>
<td>16986</td>
<td>47177</td>
<td>30191</td>
</tr>
</tbody>
</table>

*Note:* The amount of GII, GIS and NII produced by increases of 1 trillion yen in asset and liability items.
Figure 6: Monthly change in NII, GI, GIS (1000 billion yen)
Figure 7: Degree of contribution to monthly increment in NII