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A STUDY IN THE THEORY AND MEASUREMENT OF HOUSEHOLD LABOR SUPPLY

---- PROVISIONAL REPORT ----

by

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§VI A Synthetic Model of Labor Supply for type A Household

--Refinement of precision of the estimates of the parameters using the synthetic model--

So far, the analysis was focused on the wives' (non-principal potential earners') acceptance and rejection of employment opportunity offered by firms. There are, however, earning opportunities without being employed by the employers. As a matter of fact, a number of working wives other than employee (self-employed wives) are found in the FIES data. Hence, a comprehensive theory of labor supply should, originally, be able to treat the earning behavior of self-employed wives, too. From this point of view the theory of wives'(non principal potential earners') labor supply behavior developed so far is of the first approximation in the sense that it describes wives' acceptance or rejection of employee status only. (I shall hereafter distinguish those two kinds of working wives(members) of households by using phrases employee wives and self employed wives).

We used the labor supply theory of first approximation of this kind in order to estimate preference parameters. The results of the examination in section (V) seems to show that we need a more precise theory of labor supply, that is, the theory of second approximation. As is mentioned above the second approximation theory should be able to clarify the behavior of self employed as well as employee wives.

In this context a more precise model of wives' labor supply is developed in this section.

[6.1] Labor supply model of ~~the~~ type A households constructed by taking into account wives' self-employed earning opportunities

The synthetic model of wives' labor supply for type A households should clarify the conditions by which participation status of wife (non principal potential earner) in a given household belongs to either of the following four patterns:

(1) She (or non principal potential earner) is neither an employee nor self-employed.

(2) She is not an employee but self-employed.

(3) She is an employee but is not self-employed.

(4) She is both an employee and self-employed.

Taking into account the results so far, let (1) the income leisure preference function be quadratic and (2) wife's income generating function (production function) be linear, i.e., the marginal earning rate (marginal value productivity) with respect to wife's labor hour being a constant. Proposition (2) is introduced for the sake of brevity without impairing substantial characteristics of the model.

§.1-1. The determinants of wife's pattern of labor participation

Let us consider a group of type A households with a common level of principal earner's income, I (Fig.Ⅶ-1). Let the marginal earning rate (marginal value productivity of wife's self-employed work) be v which is supposed to be common to all the households considered.

The wage rate offered by firms to the wives of the households and assigned hours of work are denoted by w and $\bar{h}$ respectively which are supposed to be common to all the households considered.

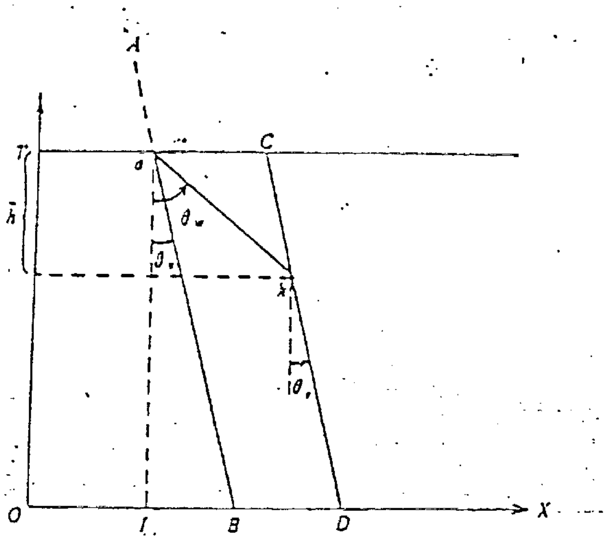
In Fig Ⅶ-1 tan θ_w^s and tan θ_v^s stand for w and v respectively. When the wife accepts an employee opportunity, her income leisure position is given by point k. CD is the line passing through point and parallel to $\hat{AB}$, $\hat{AB}$ being a line of generating self-employed income. If the wife accepts the employee opportunity and further works as self-employed, household income will be augmented along with the line k D.

Now consider a contour passing through point a. The gradient of the contour at point a, $|dx/d\lambda|_a$, will vary among the households considered due to the difference of income-leisure preference among them.

Let us call the sub group of households i with

1) $|dx/d\lambda|_a^i > v$

Fig. VI - 1



ne group I, and the sub group of households j with

$$2)\qquad \left| dx/d\lambda \right|_a^j < v$$

ne group II.

It will be clearly seen that household i included in group is a household for which there is no tangency point of contour ı the line a B, while there is the tangency point of contour on ne line a B for a household included in group II. It will be eedless to say that household with $|dX/d\lambda|_a = v$ is the one in which ne tangency point lies just on point a.

As to the households of group II, tangency point of contour ies below point a on the line aB. On the other hand, for the ouseholds of group I, there is no tangency point between the points and B. For those households the tangency point will be situated some point on the dotted line Aa which is in an ineffective zone f the indifference map.

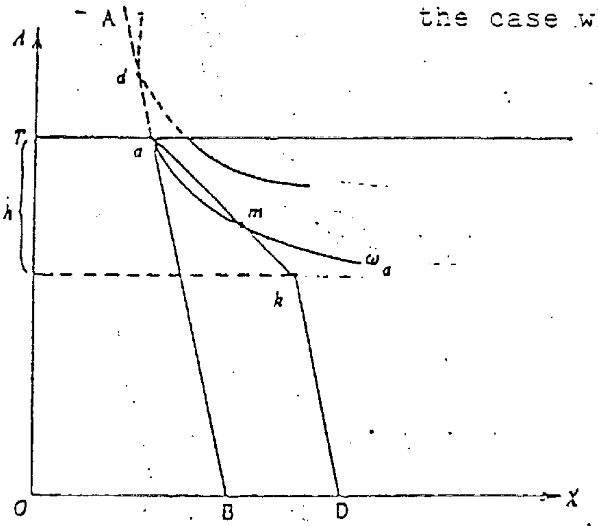
.1.1 Wives' participation behavior in the households of group I.

1.1.1.-1-

In Fig VI/2 a contour ω_a of household in group I is depicted. angency point of aB and contour is shown by point d in the ineffective one of the indifference map.

Let the intersection point of ω_a and ak be m. In Fig VI/2 point is situated above point k on the line ak. First we shall examine he behavior of a wife of household with such a contour ω_a as is hown in Fig VI/2. When the wife accepts an employee opportunity er income-leisure situation is given by point k. Her situation s shown by point a if she neither accepts the opportunity nor works

Fig VI-2



the case where a is selected

to earn her self-employed income. When the wife participates both in the employee opportunity and the work for earning self-employed income her situation is shown by some point between k and D on the line kD. (By the definition of group I, a household wife does not participate in the work for earning self-employed income only that is, does not situate between a and B).

Among those three situations point a is clearly the best one because point a lies on the contour with highest utility indicator compared to point k and any points between k and D. Hence, point a is chosen by this kind of household (wife).

6-1.1.1.-2-

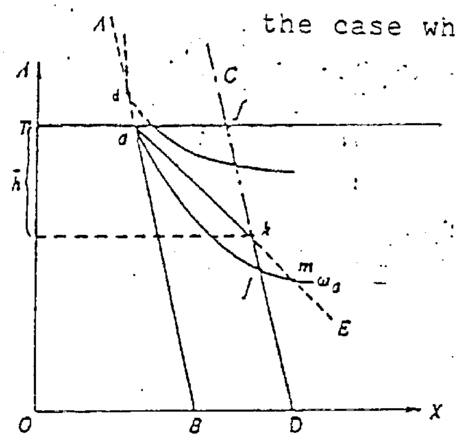
In Fig Ⅱ-3 the indifference curve of a household in which the intersection point, m, of contour passing through point a, ω_a, and the line aB lies below point k.

If the wife in a household of this kind participated both in an employee opportunity and in self employed work her income-leisure situation would be given by the tangency point of contour and line kD somewhere between k and J, as the hours of work for earning self-employed income can be adjusted as the supplier(wife) desires contrary to the case in which a wife is an employee.

It should be noted, however, that there does not exist any tangency point on the indifference curve and line between the points k and J on the line cD. If there were a tangency point, g, which is not shown in Fig Ⅱ-(3), it would be said that, when the principal earners income is Tf (in Fig Ⅱ-3), the non principal earner's (wife's) optimal hours of work for the wage rate v ($\equiv \tan\sigma_v$) is given by

Fig VI-3

the case where k is selected



the ordinate difference of point f and g. If such a case occurs it would be clear that, by comparing points d and g, the larger the principal earners income the longer the nonprincipal earner's (wife's) optimal hours of work, the nonprincipal earners wage rate v being given. This means, under the assumption of quadratic preference function, that the locus of MHLS (see section ⟨3.3.4⟩) on the X~Λ plane is downward sloping. However, the downward sloping locus is evidently inconsistent with the observed facts, that is, downward sloping participation ratio curve, as has been disussed in section ⟨3.2.5⟩.

Hence, it was proved that, under the assumption of quadratic preference function, there should be no tangency point between points k and J for the consistency between the model and the observation.

By the examination mentioned above, any points between k and J lie on the indifference curves with inferior values of utility indicator compared to the indifference curve passing through point k. It is clearly seen that point k is preferable to point a. Hence point k is prefered, that is, the wife of the household with such a indifference map as is shown in Fig VI-3. accepts the employee opportunity and does not earn an additional self-employed income.

6.1.1.2 Wives' participation behavior in the households of group II.

In a household of group II there exists a tangency point of line aB and indifference curve, d, as shown in Fig VI-4.

6-1.1.2-1- Household in which tangency point, d, lies between points a and P.

Let the intersection point of line aB and horizontal line passing through point k be denoted by P as shown in Fig VI-4. Consider a household in which the tangency point, d, lies somewhere between points a and P. For this type of household, let the crossing point of ω_a and ak be denoted by m'.

6-1.1.2-1-1- In the first place consider a household in which point m' lies above point k as is shown in Fig VI-4.

The wife (non-principal potential earner) of this kind of household prefers point d, because d is situated on the indifference curve with the highest indicator among the points k, a, and all the points between k and D. Hence, she works for earning self-employed income only and does not accept employee opportunities.

6-1.1.2-1-2-

Let the extention of line ak be kF(dotted line) in Fig VI-5. An intersection point of kF and contour ωd is denoted by m'. Consider a household in which point m' lies below point k as shown in Fig VI-5.

The wife (non-principal potential earner) in this type of household will never choose any points between k and j. If she chooses those points it would mean that she is both an employee and self-employed. But a closer look will show such a case can not occur. If she chooses any point between k an J, this point would have to be a tangency point. However there could not be any tangency point between k and J because of the requirement of the upward sloping MHLS locus which must be fulfilled on account of the consistency between the model and the observation, as was shown in section ⟨3.2.5⟩. Hence, d is preferred to a, and any points between k and J are preferred to d, therefore k is prefered to the points between k and J. That is, the wife will be an employee and will not work for a self-employed income.

Fig VI-4

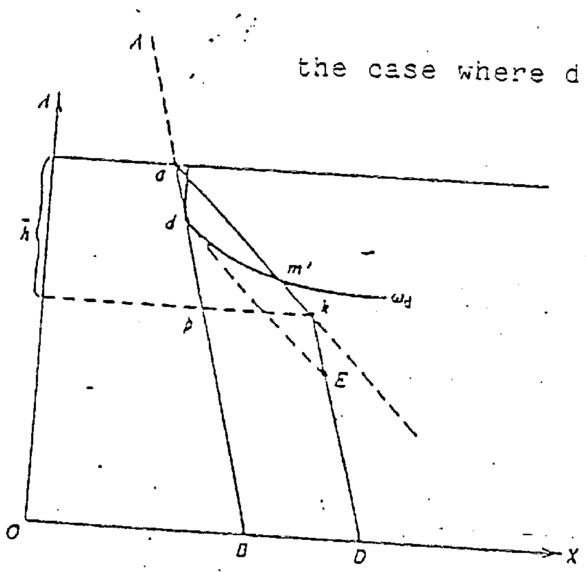
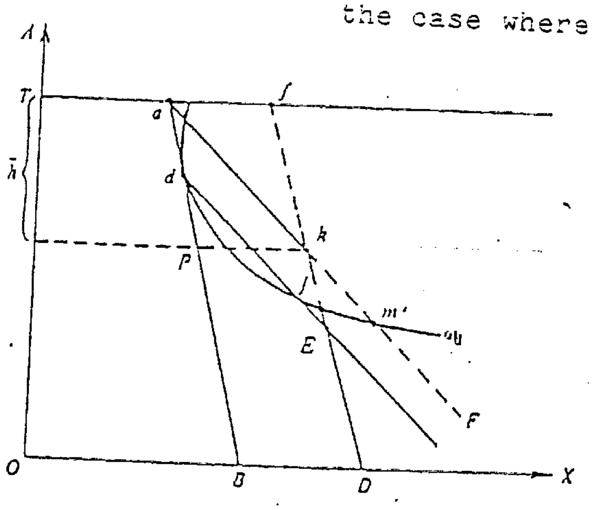


Fig VI-5



6-1.1.2-2- Household in which point d lies between points p and B.

An indifference map of this kind of household is depicted in Fig VI-6. For this kind of household two types of households are further discriminated from each other.

6-1.1.2-2-1 In Fig VI-6 point e is a tangency point of the indifference curve and the line fk which is the extention of line kD. Consider a household indifference map which has such characteristics that there exists a tangency point between the indifference curve and the line fk.

For this kind of household all the points between k and D on the line k D are situated on the indifference curves with minor indicators compared to the indifference curve on which point k lies, because the gradient of contour at point k, $|dx/d\Lambda|_k$, to the vertical axis is larger than that of line k D to the vertical axis.

Hence, among points a, d, k and all the points between k and D, d is preferred to a and all the points between k and D is preferred to d, and k is preferred to all the points between k and D; thus k is preferred. This means that the wife (non principal potential earner) of this household accepts the employee opportunity only and has no self-employed income.

6-1.1.2-2-2- Consider a household in which point e lies below point k. The indifference map of this kind of household is depicted in Fig VI-7.

It will clearly be seen e is preferred to a, d and k. Hence, the wife (non principal potential earner) will accept the employee opportunity and at the same time she will work for the self-employed income.

Fig VI - 6

the case where k is selected

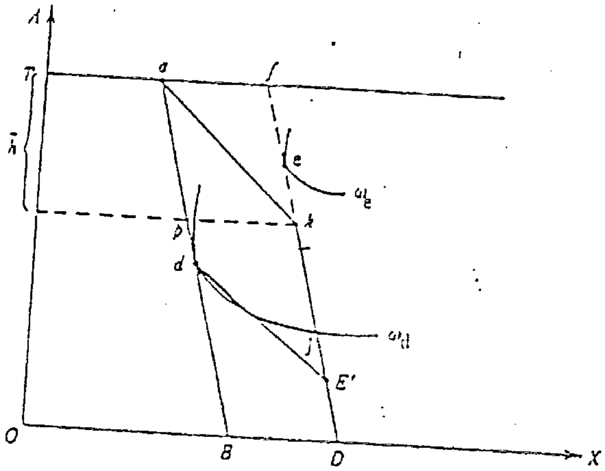
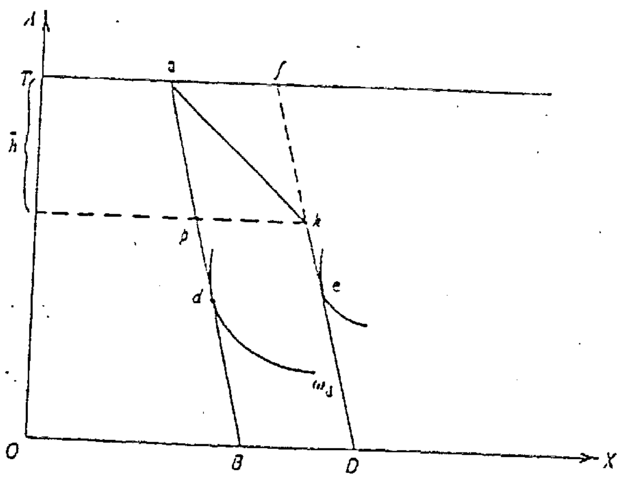


Fig VI - 7

the case where e is selected



6.1.2 Labor participation Model of type A Household.

6.1.2.1 Summary on the patterns of wives' labor participation.

The patterns of wives' labor participation behavior discussed in the previous section, 6.1.1 is summarized in TabⅥ-1.

Tab.Ⅵ-1 -

1. group I Households with point d above a	1-1 Households with m above k	→	point a is prefered (no participation both in employee opportunity and self employed work)
	1-2 Households with m below k	→	point k is prefered (accepts employee opportunity but no participation in self employed work)
2. group II Households with point d below a	2-1 Households with point d between a and p	→	2-1-1 households with point m' above k → point d is prefered (earning of self-employed income only) 2-1-2 households with point m' below k → point k is prefered (earning from employee opportunity only.)
	2-2 Households with point d between p and B	→	2-2-1 househods with point e above k → k is prefered (earning from being employee opportunity only) 2-2-2 households with point e below k → e is prefered (earning from both employee opportunity and self-employed)

Now, let us denote the coordinates of point d, m, m' and e with regard to hours of work by H(d), H(m), H(m') and H(e) respectively. The coordinates of both points k and d with respect to hours of work are $\bar{h}$ (hours of work assigned by firms). The coordinates of both points B and D with respect to hours of work are T which stands for the wife's (non principal potential earner's) total disposable time (composed of leisure and hours of work if any). Hours of work for earning self-employed income and that for earning from employee opportunity are denoted by x^{Hd} and x^{He} respectively. The coordinates of point a with respect to hours of work is zero.

Making use of these notations, the conditions in Tab. IV-1 is rewritten as shown in Tab. IV-2.

Tab. IV-2

(1) Households with H(d)<0	(1-1) households with H(m)<$\bar{h}$	$x^{He}=0$, $x^{Hd}=0$	case [illegible]
	(1-2) households with H(m)>$\bar{h}$	$x^{He}=h$, $x^{Hd}=0$	[illegible]
(2) Households with H(d)>0	(2-1) households with H(h)<$\bar{h}$	(2.1.1) households with H(m')<$\bar{h}$ $x^{He}=0$, $x^{Hd}>0$	[illegible]
		(2.1.2) households with H(m')>$\bar{h}$ $x^{He}=h$, $x^{Hd}=0$	[illegible]
	(2-2) households with H(d)>$\bar{h}$	(2.2.1) households with H(e)<h $x^{He}=h$, $x^{Hd}=0$	[illegible]
		(2.2.2) households with H(e)>h $x^{He}=h$, $x^{Hd}>0$	[illegible]

6.1.2.2 The Relation between H(m) and H(d) for the households with $H(d) \leq 0$.

In order to construct the synthetic model for type A households, we shall first consider a group of households with $H(d) < 0$.

With regard to the determinants of participation behavior of this kind of household the position of point m in relation to the position of point d in Fig VI 2 is fundamentally important. Let the relation of H(m) to H(d) be

1) $H(m) = \phi[H(d)]$

where

2) $H(d) < 0$.

A concrete analytical form of ϕ is given in the subsequent section.

6.1.2.3 The Relation between H(m') and H(d) for the households with $\bar{h} > H(d) > 0$.

For the households where $H(d) > 0$ holds the position of point m' in Fig VI-4 and 5 is important. Let the relation between H(m') and H(d) be denoted by

3) $H(m') = f[H(d)]$

where

4) $\bar{h} > H(d) > 0$.

An analytical form of f is given in the subsequent section.

6.1.2.4 The Relation betweeen H(d) and H(e) for the Households with $H(d) > \bar{h}$.

For this kind of household the position of e is also important. Let the relation between H(e) and H(d) be

5) $H(e) = \psi[H(d)]$

where

6) $H(d) > \bar{h}$.

The analytical form of ψ is given in the subsequent section.

6.1.2.5 On the graphs of functions ϕ, f and ψ.

The functions Ψ, f and Φ assumed to be monotonic are depicted by the curves $\alpha\alpha'$, $\alpha'\beta$ and $\gamma\gamma'$ respectively in Fig VI-8 and VI-9. It should be noted that curve $\alpha\alpha'$ standing for Ψ and $\alpha'\beta$ standing for f have a point of conjunction, α', because when $H(d) = 0$, $f[H(d)] = \Psi[H(d)]$ holds, as can be seen in Fig VI-3 and 4. Fig VI-8 differs from Fig VI-9 in that point α' lies above point $\bar{H}$ on the vertical axis in the former while point α' lies below point $\bar{H}$ in the latter.

We shall begin by examining Fig VI-8. The numbers attached to the curves correspond to those in the column of Tab. VI-2. It should be remarked that the participation pattern denoted by ③ does not occur when functions Ψ and Φ are of the shape shown in Fig VI-8. Pattern ③ is for self-employed wives only, not wives who are employees. However, according to the observation, a pattern such as ③ does exist. Hence, since the shapes of the curves shown in Fig VI-8 are not consistent with observation, they should be excluded.

Another possible shape of the curves is shown in Fig VI-9. In this figure it can be seen that pattern ③ exists. Although pattern ② does not appear in this figure patterns ④ and ⑤ are quite the same as ②. Hence all the participation patterns observed for type A household appear in Fig VI-9. In this sense the shapes of functions (curves) of Ψ, f and Φ in Fig VI-9 are consistent with observation.

Fig VI-8

the case where the shapes of the curves are not consistent with observation

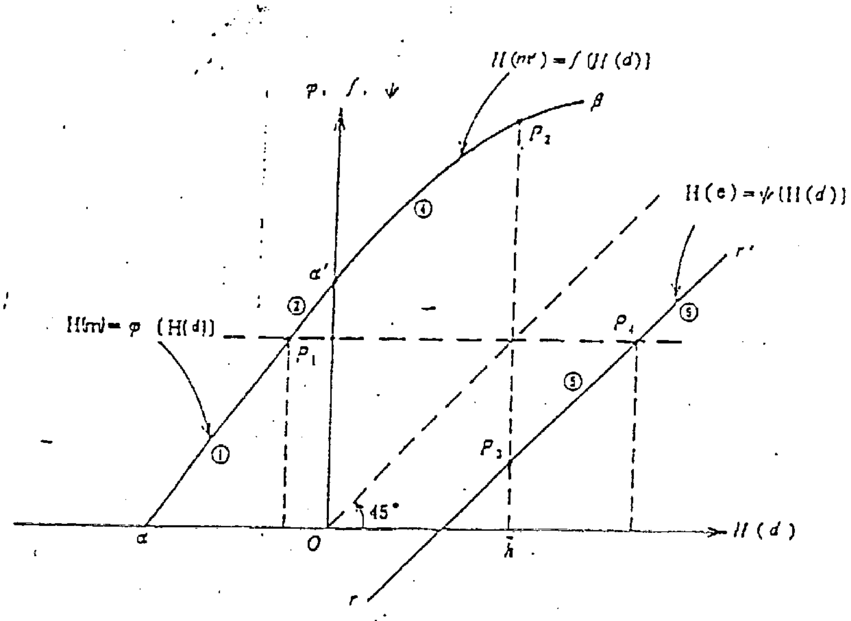


Fig VI-9

the case where the shapes of the curves are consistent with observation

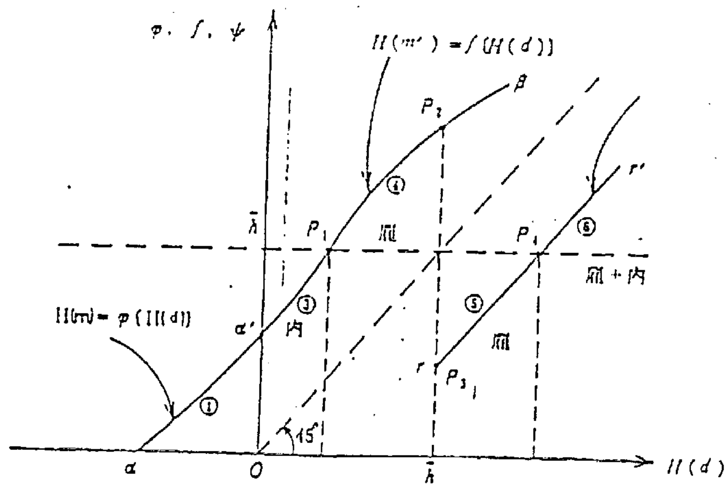


Fig VI—10 Ⓐ

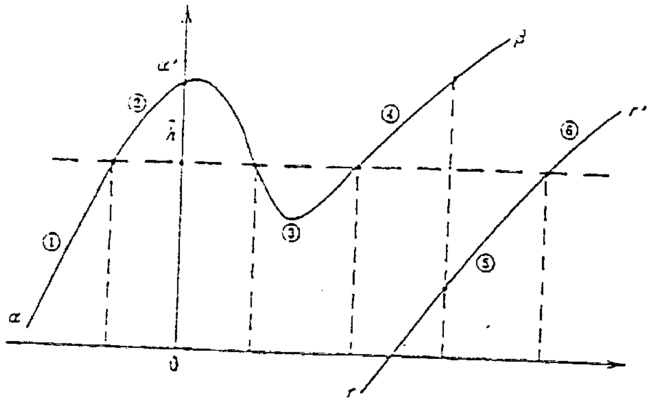


Fig VI—10 Ⓑ

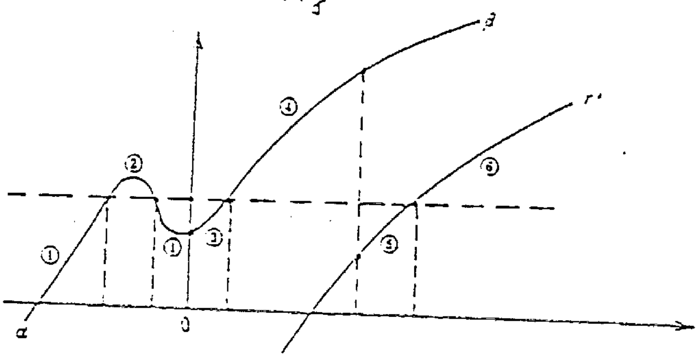
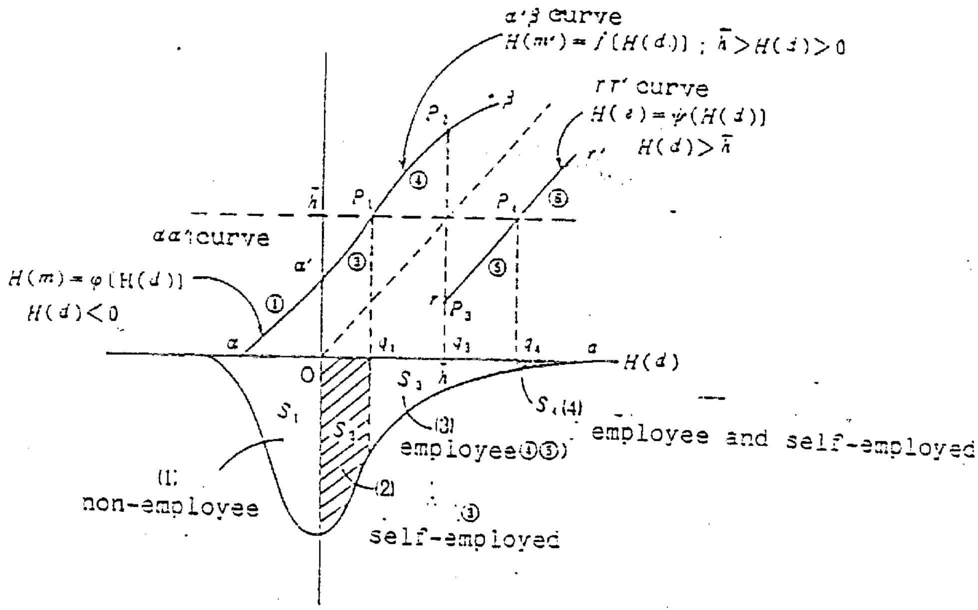


Fig VI—11



Taking into account the results in section 6.1.2.5, it will be seen that the participation patterns generayed from Fig VI-3 and VI-4 a exclusive of each other. This is due to that we employed the curves $\alpha\alpha'$ and $\alpha'\beta$ are upward sloping monotonic curves. This specific characteristics of the curves stems from the postulate that the preference function is approximated by quadratic function.

Contrary to the upward sloping monotonic curves, the shape of curve $\alpha\alpha'\beta$, as shown in Fig. 10Ⓐ or 10Ⓑ might be conceivable. In Fig. 10Ⓐ, function f is not monotonic. In Fig. 10Ⓑ, function g is not monotonic. In these figures, it can be seen, both cases ② and ③ in Tab. VI-2 (or the cases shown in Fig. VI-3 and VI-4) can coexist.

However, quadratic preference function does not yield such cases as shown in 10Ⓐ or Ⓑ.

6.1.2.6 Probabilities of generating various participation patterns in type A Households.

In this section the determination of the probabilities of generating four patterns of participation in type A household will be clarified when principal earner's (husband's) income, I, wage rate, w, and hours of work assigned by firms, $\bar{h}$, and earning rate of self-employed work, v, for non-principal potential earner (wife) are given.

The first and second quadrants of Fig VI-11 depict the same curves shown in Fig VI 9. The density distribution curve of H(d) is depicted in the third and the forth quadrants. This distribution reflects the differences in magnitudes of preference parameters among households where the common values of I, w, v and $\bar{h}$ are given respectively.

Taking into account the results summarized in Tab VI 2, it will clearly be seen area S, under the distribution curve, gives the probability that the wife (non-principal potential earner) is neither an employee nor self-employed. This is the probability that pattern ① in Tab VI 2 occurs. Let us call S_p, the probability of non-participation.

Area S_2 in Fig ~~IV~~ IV-11 gives the probability that participation pattern ③ in Tab.IV-2 occurs. This is the probability that the wife engages in self-employed work only without accepting employee opportunity. Let us call this probability the probability of self-employment participation, μ_d , where ~~where~~

$$\mu_d \equiv \text{number of self employed wives without accepting employee opportunity} \Big/ \text{number of wives}$$

Area S_3 gives the probability that either participation pattern ④ or ⑤ in Tab.IV-2 occurs. Here it should be noted ④ and ⑤ are the same pattern. Let us call this probability the probability of accepting employee opportunity without self-employed work, or in short probability of employee, μ_e,

where

$$\mu_e \equiv \text{number of wives accepting employee opportunity without self-employed work} \Big/ \text{number of wives.}$$

Area S_4 stands for the probability that participation pattern ⑥ in Tab.IV 2 occurs. Let us call this probability the probability of double participation, μ_{ed},

where

$$\mu_{ed} \equiv \text{number of wives participating both self-employed work and employee opportunity} \Big/ \text{number of wives.}$$

It will be needless to say that

$$\text{non-participation probability} + \mu_d + \mu_e + \mu_{ed} = 1.$$

Prior to drawing the curves in Fig. 11 the values of I, w, $\bar{h}$ and v have to be given. That is, when these conditions change the shape of all the curves change simultaneously and, in effect, the areas si (i=1.2.3.4) or magnitude of μ_d; μ_e and μ_{ed} change. Hence analytical forms of the function Ψ, f, Φ and the size distribution function of H(d) have to be known in order to describe the changes in participation probabilities corresponding to the changes in I, w, h and v. This will be discussed in the following section.

6.1.2.7 Analytical Forms of Functions ϕ, f, and ψ

In this section analytical forms of ϕ, f, and ψ are determined making use of quadratic preference function. All the available information including plausibility of variational Y-model, previously obtained in the analysis of the model of first approximation, are taken into account in the process of determining analytical forms of those functions.

6.1.2.7.1 Analytical Form of ϕ

6.1.2.7.-1-1. In order to obtain concrete form of ϕ it is necessary to calculate the coordinates of point d in Fig.2 or 3. The equation of line aB is given by

1) $X = I + vh$

where h and X stand for hours of work (for employee opportunity and/or self-employed work) and household's income respectively. v stands for the earning rate of self-employed work.

The preference function ω is given by

$$2)\quad \omega = \frac{1}{2}\gamma_1 X^2 + \gamma_2 X + \gamma_3 X\Lambda + \gamma_4 \Lambda + \frac{1}{2}\gamma_5 \Lambda^2$$

where

$$\Lambda \equiv T - h \ .$$

Under the constraint of (1), (2) is maximized with respect to h. When the value of h maximizing ω is negative that value of h stands for H(d) in the function ϕ. This stems from the fact that the indifference maps shown in Fig. VI-2 or 3 are the maps of households with such γ_4 that makes tangency point, d, on AB situate in ineffective range.

Hence, we obtain

$$3)\quad H(d) = \frac{-(\gamma_1 v - \gamma_3) I - v(\gamma_2 + \gamma_3 T) + \gamma_4 + \gamma_5 T}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}$$

where

$$H(d) < 0 \ .$$

The value of H(d) varies among households with given I, w, $\bar{T}$ and v owing to the difference in γ_4 of each household. Hence the size distribution of γ_4 can be easily transformed to that of H(d) by using equation (3).

6.1.2.7.1-2. The equation of indifference curve ω_a in Fig. VI-2 and 3 can be obtained as follows. By inserting the values of ordinates of point a in Fig. VI-2 and 3,

4) $X = I$

5) $\Lambda = T$,

into the left hand side of preference function (2), we obtain the value of indicator ω_a at point a,

$$6)\quad \omega_a = \frac{1}{2}\gamma_1 I^2 + \gamma_2 I + \gamma_3 I T + \gamma_4 T + \frac{1}{2}\gamma_5 T^2 ,$$

I and T being given. Hence, the equation of indifference curve ω_a can be written as

$$7)\quad \omega_a = \frac{1}{2}\gamma_1 X^2 + \gamma_2 X + \gamma_3 X\Lambda + \gamma_4 \Lambda + \frac{1}{2}\gamma_5 \Lambda^2 ,$$

where the $\bar{\omega}_a$ is given by (6).

6.1.2.7.-1-3 Finally let us obtain the crdinate of point m in Fig.Ⅱ 2 and 3.

The equation of line ak is given by

$$8)\quad X = I + wh.$$

We can solve (8) together with (7) for h. The solution is the coordinate of point m with respect to hours of work, H(m), that is,

$$9)\quad H(m) = \frac{(-\gamma_1 w + \gamma_3) I - w(\gamma_2 + \gamma_3 T) + \gamma_4 + \gamma_5 T}{\frac{1}{2}(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)} .$$

It will be seen that magnitude of H(m) varies among households considered owing to differences in γ_4 of each household.

6.1.2.7.-1-4 Now, we are ready to get a concrete form of funcution ϕ.

The parameter γ_4, the magnitude of which is supposed to vary among households, is included both in equations (9) and (3). Hence, by elimianting common parameter γ_4 both in (9) and (3) we obtain a relation between H(m) and H(d),

$$(10)\quad H(m) = \frac{2(\gamma_1 v^2 - 2\gamma_3 v + \gamma_5)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} H(d) + \frac{2(v - w)(\gamma_1 I + \gamma_2 + \gamma_3 T)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} ,$$

where $H(d) < 0$.

This is the function ϕ when the preference function is quadratic.

6.1.2.7.2 Analytical Form of function f

Function f stands for a relation between point m' and d in Fig. 4 and 5.

Coordinate of point d, H(d), is previously given by (3),

$$3)\quad H(d) = \frac{-(\gamma_1 V - \gamma_3) I - V(\gamma_2 + \gamma_3 T) + \gamma_4 + \gamma_5 T}{\gamma_1 V^2 - 2\gamma_3 V + \gamma_5}$$

However, with regard to the case shown in Fig. 4 and 5, it should be noted that contrary to the previous case,

$$3')\quad H(d) > 0 .$$

That is, the magnitudes of parameter of preference function γ_4, which generates the indifference curve as shown Fig. 4 and 5 must be of a value which makes the right hand side of equation (3) positive.

6.1.2.7.2-1 We shall obtain the equation of ω_d in Fig. 4 and 5.

The coordinates of point d are given by

$$11)\quad X = I + vH(d)$$

$$12)\quad \Lambda = T - H(d)$$

where H(d) is given by (3). Inserting (11) and (12) in to (2) we have

$$13)\quad \omega_d = \frac{1}{2}\gamma_1 [I + VH(d)]^2 + \gamma_2 [I + VH(d)] + \gamma_3 [I + VH(d)][T - H(d)] + \gamma_4 [T - H(d)] + \frac{1}{2}\gamma_5 [T - H(d)]^2 .$$

Given I and v, the value of ω_d in (13) is specific to each household with specific value of γ_4 .

The equation of contour ω_d in Fig.4 and 5 is given by

14) $$\omega_d = \frac{1}{2}\gamma_1 X^2 + \gamma_2 X + \gamma_3 X\Lambda + \gamma_4 \Lambda + \frac{1}{2}\gamma_5 \Lambda^2 ,$$

where ω_d is given by (13).

The equation of segment ak or that of extention of the segment is given by

15) $$X = I + wh \qquad ; \qquad T - \Lambda \equiv h.$$

Hence, we can obtain the ordinate of point m' by solving (14) and (15) simultaneously with respect to h. By denoting this solution H(m') we have

16) $$H(m') = \frac{-1}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5}\left[I(\gamma_1 w - \gamma_3) + (\gamma_2 + \gamma_3 T)w - \gamma_4 - \gamma_5 T\right]$$
$$\pm\Big[\left[I(\gamma_1 w - \gamma_3) + (\gamma_2 + \gamma_3 T)w - \gamma_4 - \gamma_5 T\right]^2 - 2(\gamma_1 w^2 - 2\gamma_3 + \gamma_5)$$
$$\times\Big\{\frac{1}{2}\gamma_1 I^2 + (\gamma_2 + \gamma_3 I)I + \gamma_4 T + \frac{1}{2}\gamma_5 T^2 - \Big(\frac{1}{2}\gamma_1(I + vH(d))^2$$
$$+ \gamma_2(I + vH(d)) + \gamma_3(I + vH(d))(T - H(d)) + \gamma_4(T - H(d)) + \frac{1}{2}\gamma_5(T - H(d))^2\Big]$$
$$\frac{1}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} ,$$

where H(d) is given by (3).

By examining Fig. VI 4 and 5, the algebraically larger root among the two given by (16) is adopted as the value of H(m').

2.2-2 Finally we shall deduce function f.

By eliminating the common parameter γ_4 included in both (16) [illegible]) and (3), we have

17) $H(m') = \dfrac{-k - \sqrt{D}}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5}$,

and,

$$K \equiv (w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T) - (\gamma_1 v^2 - 2\gamma_3 v + \gamma_5) h_d^*$$

$$D \equiv (w-v)\{(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)^2 - 2(\gamma_1 I + \gamma_2 + \gamma_3 T)(\gamma_1 v^2 - 2\gamma_3 v + \gamma_5) h_d^* + (\gamma_1 v^2 - 2\gamma_3 v + \gamma_5)[2\gamma_3 - \gamma_1 (w-v)](h_d^*)^2\},$$

where h_d^* is the abbreviation of $\bar{H}(d)$ given by (3). Equation (17) is the function f when the preference function ω is quadratic.

6.1.2.7.3 Analytical Form of function ψ

Function ψ stands for the relation between point d and e in Fig. 6 and 7.

6.1.2.7.3-1 Firstly the coordinate of H(d) is given by

(3) $H(d) = \dfrac{-(\gamma_1 v - \gamma_2)I - v(\gamma_2 + \gamma_3 T) + \gamma_4 + \gamma_5 T}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}$

as previously shown in 6.1.2.7.1-1 However,

(3') $H(d) > \bar{h}$

must be held here, in order that point d lies below p in Fig. 6 and 7.

6.1.2.7.3-2 In the second place we shall obtain the coordinate of point e. Taking into account that the coordinates of point k is given by

(17) $X = I + w\bar{h}$

and

(18) $\Lambda = T - \bar{h}$,

the equation of line fD passing through point k is written as

(19) $X = I + (w - v)\bar{h} + vh$,

where $\bar{h}$ stands for the coordinate of hours of work on the line fD.

Under the constraint of (19), we shall obtain the value of h maximizing ω in (2). This value of h is H(e). Hence we have

$$(20) \quad H(e) = \frac{-(\gamma_1 \nu - \gamma_3)[T + (w - \nu)\bar{h}] - \nu(\gamma_2 + \gamma_3 T) + \gamma_4 + \gamma_5 T}{\gamma_1 \nu^2 - 2\gamma_3 \nu + \gamma_5}$$

6.127.3-3 We are ready to get analytical form of Ψ:

That is, by elimination of γ_4 included in both (3) and (20), the relation between H(d) and H(e).

$$(2\underset{2}{1}) \quad He = H(d) - \frac{(\gamma_1 \nu - \gamma_3)(w - \nu)\bar{h}}{\gamma_1 \nu^2 - 2\gamma_3 \nu + \gamma_5} \qquad \text{--- function } \Psi$$

where

$$H(d) > \bar{h}, \qquad \text{and} \qquad w > \nu,$$

is obtained. This is the function Ψ when the preference function ω is quadratic.

6.1.3. Calculation of supply (participation) probability.

In this section the calculation of μ_e, μ_d and μ_{ed} is discussed.

6.1.3.1 The coordinates of points q_1 and q_4

It can be seen that function f contains preference parameters, r_1, r_2, r_3 and r_5, and exogenous variables, v, w, $\bar{h}$ and I, respectively; that is, f is rewritten as

23) $H(m') = f[H(d), r_1, r_2, r_3, r_5 | v, w, \bar{h}, I]$

where $H(d) > 0$.

In the same fashion function ψ can be rewritten as

24) $H(e) = \psi [H(d), r_1, r_3, r_5 | v, w, \bar{h}]$

where $H(d) > \bar{h}$.

Applying $H(m') = \bar{h}$ to the left hand side of equation (20), we have

25) $\bar{h} = f[H(d), r_1, r_2, r_3, r_5 / v, w, \bar{h}, I]$.

This equation can be solved for H(d). Let us denote the solution for H(d) by $H(d)_{q_1}$. Hence

26) $H(d)_{q_1} = f^{-1} [r_1, r_2, r_3, r_5 / v, w, \bar{h}, I]$,

where f^{-1} stand for the inverse function of f. $H(d)_{q_1}$ given by (26) is the coordinate of point q_1 on H(d) axis in Fig 11.

Now we shall obtain the coordinate of point q_4 in Fig 10.

Replacing H(e) on the left hand side of equation (24) by $\bar{h}$ we have

(27) $\bar{h} = \psi [H(d), r_1, r_3, r_5 | v, w, h]$.

We can solve (27) with respect to H(d) and let us denote the solution by $H(d)_{q_4}$. Hence we have

(28) $H(d)_{q_4} = \psi^{-1}[r_1, r_3, r_5 / v, w, \bar{h}]$

where ψ^{-1} is the inverse function of ψ. Equation (28) gives the coordinate of point q_4 in Fig VI-11.

It can be seen that $H(d)_{q_4}$ is invariant with the principal earner's income level, I, because ψ and ψ^{-1} does not contain I as an argument. This stems from the characteristics of quadratic function ω.

6.1.3.2 Density distribution function of H(d)

Finally we shall discuss the density distribution function of H(d). H(d) has been given by (See 6.1.2.7(-1)

(3) $$H(d) = \frac{-(\gamma_1 V - \gamma_3) I - V(\gamma_2 + \gamma_3 T) + \gamma_4 + \gamma_5 T}{\gamma_1 V^2 - 2\gamma_3 V + \gamma_5},$$

where the magnitude of γ_4 varies among households considered. With respect to a household i, the value of γ_4^i is given by

(29) $$\gamma_4^i = \bar{\gamma}_4 \cdot u_i$$

where $\bar{\gamma}_4$ is a constant which is common to all the households considered and u_i is a random variable distribution of which is long-normal with mean $E(u_i)$, and variance σ^2, where

$E(u_i) = 1,$

σ_u^2 being a constant. Let the density distribution of u_i be

(30) $I(u, \sigma_u^2)$

where suffix i is deleted. By considering (29), (3) can be reduced to

$$H(d) = \frac{-(\gamma_1 V - \gamma_3) I - V(\gamma_2 + \gamma_3 T) + \bar{\gamma}_4 u + \gamma_5 T}{\gamma_1 V^2 - 2\gamma_3 V + \gamma_5}.$$

Solving this equation with respect to u, we have

(31) $$u = \frac{1}{\bar{\gamma}_4}\{(\gamma_1 V^2 - 2\gamma_3 V + \gamma_5) H(d) + (\gamma_1 V - \gamma_3) I + (\gamma_2 + \gamma_3 T) V - \gamma_5 T\}$$

or in short,

(32) $u = u(H(d), \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 | v, I)$

From (31) we have

(33) $$du = \frac{1}{\bar{\gamma}_4}(\gamma_1 V^2 - 2\gamma_3 V + \gamma_5) \cdot dH(d).$$

From (32), (30) and (33), we have

(34) $\ell(u)\cdot du = \ell\left[v(H(d)),\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5 \mid V, I, \sigma\right] \left|\frac{du}{dH(d)}\right| \cdot dH(d)$

$$= \ell H(d)\left[H(d),\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5 \mid V, I, \sigma\right] \left|\frac{\gamma_1 V^2 - 2\gamma_2 V + \gamma_5}{\gamma_4}\right| \cdot dH(d),$$

This is the function which transforms the distribution function of u, $\ell(u)$, to that of H(d), $\ell[H(d)]$. The right hand side of equation (34) (except for dH(d)) is the density distribution function of H(d) depicted in Fig VI-11.

For the sake of brevity, let us denote distribution function, the right hand side of (34), (except for dH(d)) by

(35) $\ell^*(H(d), \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 \mid V, I, \sigma)$.

It can be seen from (34) that the distribution of H(d) is invariant with respect to changes in w.

6.1.3.3 Participation Probability

By using (35), μ_d shown by area S_2 in Fig VI-11 is given by the definite integration of ℓ^*, i, e.

(36) $\mu_d = \int_0^{H(d)q_1} \ell^*(H(d),\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5 \mid V, I, \sigma)\, dH(d)$,

where $H(d)q_1$ is given by (26).

In the same manner, μ_e shown by area S_3 in Fig VI-11 is given by

(37) $\mu_e = \int_{H(d)q_1}^{H(d)q_4} \ell^*(H(d),\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5 \mid V, I, \sigma)\, dH(d)$;

where $H(d)q_4$ is given by (28).

The value of μ_{ed} shown by area S_4 in Fig VI-11 is given by

(38) $\mu_{ed} = \int_{H(d)q_4}^{\infty} \ell^*(H(d),\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5 \mid V, I, \sigma)\, dH(d)$.

6.1.3.4 It can be seen from (36), (37) and (38) that the values of three kinds of probability of participation, μe, μ^d and μed, are respectively determined by the values of $\{\gamma_i\}$ $(i=1,\cdots\cdots,5)$, σ v, w, and I. It should be noted that the magnitude of w affects the probabilities _via_ limits of integration, H(d)q/ and H(d)q4, as well, because these are functions of w respectively.

Employing an abridged formulation, (36), (37) and (38) can be rewritten as

(39) $\mu d = \mu d(\{\gamma_i\}, \sigma, v, w, I)$,

(40) $\mu e = \mu e(\{\gamma_i\}, \sigma, v, w, I)$,

and

(41) $\mu ed = \mu ed(\{\gamma_i\}, \sigma, v, w, I)$.

Making use of these relation we can proceed to obtain the estimates of preference parameters, $\{\gamma_i\}$ and σ, of secondary order precision. This procedure is shown in the following section.

[6.2.] Augmenting the Precision of Estimates of Preference Parameters

(6.2.1)

Let us denote the values of preference parameters obtained previously in < 5.2 > by $\{\gamma_i^{(1)}\}$ and $\sigma^{(1)}$ $(i=1,\cdots\cdots,5)$. These values can be considered to be the first approximation for the true values of preference parameters, $\{\gamma_i\}$ and σ. For, the model used to estimate those parameters was the first approximation in the sense that the model took into account the wife's earning by employee opportunity only among two, that is earning by self-employed work and that by accepting employee opportunity.

By inserting the observed values for w, v and I, w_j^o, v^o and I_j^o respectively, together with $\{\gamma_i^{(1)}\}$ and $\sigma^{(1)}$, into 39), 40) and 41), we have theoretical (or estimated) values for μ^e, μ^d and μ^{ed}, that is,

42) $\mu_j^e(1) = \mu^e(\{\gamma_i^{(1)}\}, \sigma^{(1)}, v^o, w^o, I_j^o)$

43) $\mu_j^d(1) = \mu^d(\{\gamma_i^{(1)}\}, \sigma^{(1)}, v^o, w^o, I_j^o)$

44) $\mu_j^{ed}(1) = \mu^{ed}(\{\gamma_i^{(1)}\}, \sigma^{(1)}, v^o, w^o, I_j^o)$

Now, let us denote the observed values for participation probabilities of jth principal earner's income class by μ_{j0}^{d}, μ_{j0}^{e} and μ_{j0}^{ed}.

Let the differences between observed values and first approximation values be

(a) $u_j^d \equiv (\mu_{j0}^d - \mu_j^d(1))$, $u_j^e \equiv (\mu_{j0}^e - \mu_j^e(1))$, $u_j^{ed} \equiv (\mu_{j0}^{ed} - \mu_j^{e}(1))$

where $\mu_j^d(1)$, $\mu_j^e(1)$ and $\mu_j^{ed}(1)$ are first approximation values.

Let

(b) $\delta(\{u_j^d\}, \{u_j^e\}, \{u_j^{ed}\})$

be an objective function properly defined, where $\{\ \}$ stands for a row vector. Initial values for the preference parameters, $\{\gamma_i^{(1)}\}$ and $\sigma^{(1)}$, are allowed to vary so as to minimize δ.

Hence, we have to choose proper functional form for the objective function δ. In relation to this, it should be noted that equations 39) through 41) are exact relations that do not include any shocks, or disturbances in relations, in them. In contrast to those relations, ordinal consumption functions, for example, include shocks which, it is assumed, reflect random movements in consumer's preference parameters and so on. In other words, as far as we treat shock model at least, discrepancies between observed values and theoretical (estimated) values for consumptions are allowed. However, present model for supply probability, or the system or equations 39) through 41), has been deduced by definit integration of distribution function of $H(d^*)$, and the distribution function reflects the distribution function of the preference parameters. And upper and lower limits of the definit integration are not random variables. Therefore, also the values for definit integrals are not random variables. Hence, as far as present model is concerned there is no room for allowing shocks for the equation system, 39) through 41).

Contrary to probability functions 39) through 41), each household's supply function with respect to wives' optimal hours of work for employment opportunities or an aggregation of them do include shocks reflecting the differences in the preference parameters among households, as consumption functions do.

However, as explaned above, the probability equations are conceived of exact relations, and the differences between theoretical values μ^d, μ^e and μ^{ed}, and observals μ_0^d, μ_0^e and μ_0^{ed}, respectively, are considered to reflect sampling or observational errors (disturbances in variables) caused from limited size of samples.

Hence, denoting sampling or observational errors by additive random variable u_d, u_e and u_{ed}, we have

$$2\text{-}45)\quad \mu_0^d = \mu^d(\{\gamma_i\}, \sigma, v, w, I) + u_d$$

$$46)\quad \mu_0^e = \mu^e(\{\gamma_i\}, \sigma, v, w, I) + u_e$$

$$47)\quad \mu_0^{ed} = \mu^{ed}(\{\gamma_i\}, \sigma, v, w, I) + u_{ed} ,$$

which constitute an error model, not a shock model.

Multiply n on the both sides of equations 45) through 47), respectively, we have

$$2\text{-}48)\quad n\mu_0^d = n\mu^d(\{\gamma_i\}, \sigma, v, w, I) + nu_d$$

$$2\text{-}49)\quad n\mu_0^e = n\mu^e(\{\gamma_i\}, \sigma, v, w, I) + nu_e$$

$$50)\quad n\mu_0^{ed} = n\mu^{ed}(\{\gamma_i\}, \sigma, v, w, I) + nu_{ed} .$$

where n stands for sample size (number of households or number of wives) for each principal earner's income class.

Rewriting 2-48), we have

$$2\text{-}51)\quad n\mu_0^d = n\mu^d(\{\gamma_i\}, \sigma, v, w, I) + \varepsilon_d$$

$$52)\quad n\mu_0^e = n\mu^e(\{\gamma_i\}, \sigma, v, w, I) + \varepsilon_e$$

$$53)\quad n\mu_0^{ed} = n\mu^{ed}(\{\gamma_i\}, \sigma, v, w, I) + \varepsilon_{ed}$$

where

$$54)\quad \varepsilon_d \equiv n\cdot u_d ,\quad 55)\quad \varepsilon_e \equiv n\cdot u_e \quad \text{and} \quad 53)\quad \varepsilon_{ed} \equiv n\cdot u_{ed} .$$

ε_d, ε_e and ε_{ed} are , respectively, differences between observed and theoretical values, and they have a joint binomial distribution. As n is large enough, the joint distribution can be fully approximated by normal distribution,

$$55) \quad N(0, 0, 0, \sigma_d, \sigma_e, \sigma_{ed}, \sigma^2_{e \cdot d}, \sigma^2_{d \cdot ed}, \sigma^2_{e \cdot ed})\,.$$

where, $0's$, σ_d, σ_e, σ_{ed} stand for, respectively, means and standard deviations with respect to ε_d, ε_e and ε_{ed}, and $\sigma^2_{e \cdot d}$, $\sigma^2_{d \cdot ed}$ and $\sigma^2_{e \cdot ed}$ stand for their covariances.

From 45), 46) and 47), we have

$$u_d = \frac{1}{n}\varepsilon_d, \quad u_e = \frac{1}{n}\varepsilon_e, \quad \text{and} \quad u_{ed} = \frac{1}{n}\varepsilon_{ed},$$

hence, u_d, u_e and u_{ed} have joint probability distribution

$$N\left(0, 0, 0, \frac{1}{n}\sigma_d, \frac{1}{n}\sigma_e, \frac{1}{n}\sigma_{ed}, \frac{1}{n^2}\sigma^2_{d \cdot e}, \frac{1}{n^2}\sigma^2_{d \cdot ed}, \frac{1}{n^2}\sigma^2_{e \cdot ed}\right)$$

as approximation.

Now, under the constraint that ε_d, ε_e and ε_{ed} have joint distribution 55), we shall obtain maximum likelihood estimates of $\{\delta_i\}$ and σ . Because v , w and I , in 48) and 49), are fixed in repeated samples, equations having fixed variate on the right hand sides of the equations, although 48) and 49) can be treated as regression equations they constitute an error model.

Let n_e , n_d and n_{ed} be, respectively, numbers of persons of employed by employers, self-employed and of those who participate both opportunities among n persons. Taking into that those variates have binomial distributions, we have

56-1) $\quad E(n_e) = n\mu_e$

56-2) $\quad E(n_d) = n\mu_d$

56-3) $\quad E(n_{ed}) = n\mu_{ed}$

57-1) $\quad var(n_e) = n\mu_e(1-\mu_e)$

57-2) $\quad var(n_d) = n\mu_d(1-\mu_d)$

57-3) $\quad var(n_{ed}) = n\mu_{ed}(1-\mu_{ed})$

57-4) $cov(n_e, n_d) = -n\mu_e\mu_d$

57-5) $cov(n_e, n_{ed}) = -n\mu_e\mu_{ed}$

57-6) $cov(n_d, n_{ed}) = -n\mu_d\mu_{ed}$,

where var and cov, respectively, stand for variance and covariance of the variates in the parentheses.

Parameters $\{\gamma_i\}$ and σ are estimated so as to minimize

58) $\delta \equiv U'\Sigma^{-1}U$,

where

59) $U' \equiv [u_e^1\ u_d^1\ u_{ed}^1 \cdots\cdots u_e^m\ u_d^m\ u_{ed}^m]$,

m stand for number of principal earner's income classes, and Σ standing for variance-covariance matrix with respect to U_d, U_e and U_{ed}.

Population variance and covairance are estimated by sample variance and covariance.

Estimation procedure is summarized below.

Firstly, participation probabilities, μ^d, μ^e and μ^{ed}, are computed, making use of $\{\gamma_i^{(1)}\}$ and $\sigma^{(1)}$, by equations 45) through 47). Secondly, by using the computed participation probabilities, we have U' in 59). Inserting those values into 58), together with Σ, we have $\delta^{(1)}$, the value of δ corresponding to $\{\gamma_i^{(1)}\}$. We compute $\{\gamma_i^{(2)}\}$ and $\sigma^{(2)}$, respectively, using $\{\gamma_i^{(1)}\}$ and $\sigma^{(1)}$ so as to reduce the magnitude of δ. That is; let the shifts in $\{\gamma_i\}$ and σ be doneted by $\Delta\gamma_i$ (i=2, ···, 5) and $\Delta\sigma$ respectively. It will be needless to say that $\gamma_1^{(1)} \equiv -1$ and $\Delta\gamma_1 \equiv 0$. It can be seen from 42) through 44) that revised values for participation probabilities, after assigning the shifts for the parameters, $\mu^d(\Delta)$, $\mu^e(\Delta)$ and $\mu^{ed}(\Delta)$, are given by

60) $\mu_j^d(\Delta) = \mu^d(\{\gamma_i^{(1)} + \Delta\gamma_i\}, \sigma^{(1)} + \Delta\sigma, v^o, w^o, I_j^o)$

61) $\mu_j^e(\Delta) = \mu^e(\{\gamma_i^{(1)} + \Delta\gamma_i\}, \sigma^{(1)} + \Delta\sigma, v^o, w^o, I_j^o)$

62) $\mu_j^{ed}(\Delta) = \mu^{ed}(\{\gamma_i^{(1)} + \Delta\gamma_i\}, \sigma^{(1)} + \Delta\sigma, v^o, w^o, I_j^o)$.

$\delta^{(2)}$ can be computed, by employing 60) through 62), from 58).

$$\hat{\gamma}_i^{(2)} \equiv \gamma_i^{(1)} + \Delta \gamma_i \quad ; \; i = 2, 3, 4, 5$$

and

$$\hat{\sigma}^{(2)} = \sigma^{(1)} + \Delta \sigma$$

are revised estimates for the preference parameters.

ADDENDUM for the Computation Procedure

[6.2.1] Calculation of abscissa for q_1 in Fig.VI-11 for A·B Type Preference function

(6.2.1.1) In equation

$$H(m')=f[H(d)]$$

let $H(m')=\bar{h}$, and the equation can be solved for H(d). The solution is the abscissa of point q_1.

Concrete form of function f has been given by 17) in (6.1.2.7.2). From this and $H(m')=\bar{h}$, we have

$$1)\quad \bar{h}=\frac{-K-\sqrt{D}}{\gamma_1 w^2-2\gamma_3 w+\gamma_5}$$

where

$$K\equiv(w-v)(\gamma_1 I+\gamma_2+\gamma_3 T)-(\gamma_1 v^2-2\gamma_3 v+\gamma_5)h^*$$

$$D\equiv(w-v)\{(w-v)(\gamma_1 I+\gamma_2+\gamma_3 T)^2-2(\gamma_1 I+\gamma_2+\gamma_3 T)(\gamma_1 v^2-2\gamma_3 v+\gamma_5)h^* + (\gamma_1 v^2-2\gamma_3 v+\gamma_5)[2\gamma_3-\gamma_1(w+v)](h^*)^2\}$$

and notation h^* is used in place of H(d) for the sake of abbreviation.

1) can be solved for h^*, that is, 1) can be rewritten as

$$2)\quad (\gamma_1 v^2-2\gamma_3 v+\gamma_5)(h^*)^2-2(\gamma_1 v^2-2\gamma_3 v+\gamma_5)\bar{h}\cdot h^*+\bar{h}[(\gamma_1 w^2-2\gamma_3 w+\gamma_5)\bar{h} + 2(w-v)(\gamma_1 I+\gamma_2+\gamma_3 T)]=0$$

Among two roots of equation 2), we adopt the root h^* satisfying

$$0<h^*<\bar{h}$$

as plausible solution.

We rewrite 2) as

$$2')\quad A_*(h^*)^2-2A_*\bar{h}\cdot h+B_*=0,$$

where

$$3\text{-}1)\quad A_*\equiv\gamma_1 v^2-2\gamma_3 v+\gamma_5$$

and

$$3\text{-}2)\quad B_*\equiv\bar{h}[(\gamma_1 w^2-2\gamma_3 w+\gamma_5)\bar{h}+2(w-v)(\gamma_1 I+\gamma_2+\gamma_3 T)]$$

We have solution h^* as

$$4)\quad h^* = \frac{A_* \bar{h} \pm \sqrt{A_*^2 \bar{h}^2 - A_* B_*}}{A_*} = \bar{h} \pm \sqrt{\bar{h}^2 - \frac{B_*}{A_*}}$$

Taking into account the requirement $0 < h^* < \bar{h}$, we have

$$5)\quad H(d)_{g_1} = \bar{h} - \sqrt{\bar{h}^2 - \frac{\bar{h}[(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)\bar{h} + 2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)]}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}}$$

(6.2.1.2) Calculation of abscissa for q_4 in Fig. VI-11

In equation

$$H(e) = \psi[H(d)]$$

we put left hand side equal to $\bar{h}$, that is,

$$\bar{h} = \psi[H(d)]\,.$$

By solving this equation for H(d), we have abscissa for point q_4.

Concrete form of ψ is given by 22) in 6.1.2.7-3.

$$H(e) = H(d) - \frac{(\gamma_1 v - \gamma_3)(w - v)\bar{h}}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}.$$

Applying $H(e) = \bar{h}$ for the equation, and solving it for H(d), we have

$$6)\quad H(d)_{q_4} = \bar{h} + \frac{(\gamma_1 v - \gamma_3)(w - v)\bar{h}}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5} = \bar{h}\left[1 + \frac{(\gamma_1 v - \gamma_3)(w - v)}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}\right],$$

where $H(d)_{q_4}$ stands for abscissa of point q_4.

(6.2.1.3) Calculation of abscissa of Point a in Fig. VI-11

In the equation 3) (6.1.2.7),

$$H(d) = \frac{-(\gamma_1 w - \gamma_3)I - v(\gamma_2 + \gamma_3 T) + \gamma_4 + \gamma_5 T}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5},$$

we set $\gamma_4 = 0$. Hence, we have

$$7)\quad H(d)_{max} = \frac{\gamma_5 T - (\gamma_1 v - \gamma_3)I - v(\gamma_2 + \gamma_3 T)}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5},$$

where $H(d)_{max}$ stands for the value of H(d) for household with largest value of H(d) among the households considered. Accordingly, $H(d)_{max}$ represents the abscissa of point a in Fig. VI-11.

[6.2.2] Some other constraints for the Parameters to be Estimated

From the generalized labor supply model for type A household, in which self-employed opportunities are taken into account as well as employment opportunities, we can derive some additional theoretical restrictions for the parameters to be estimated. They are in order:

(1) The derivative of function φ be positive

This constraint can be stated, by using 1-10) in (6.1.2.7.4), as

$$1)\quad \frac{2(\gamma_1 v^2 - 2\gamma_3 v + \gamma_5)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} > 0 .$$

Hence we have

$$1')\quad (\gamma_1 v^2 - 2\gamma_3 v + \gamma_5)(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5) > 0$$ ---- constraint [1]

(2) $0 < \varphi[H(d)=0] = f[H(d)=0] < \bar{h}$ must be held

This restriction means that point α', in Fig. V-11, must lie between 0 and $\bar{h}$. By applying

H(d)=0

to equation 10) in (6.1.2.7.1-4) we have

$$2)\quad \varphi[H(d)=0] = \frac{2(v-w)(\gamma_1 I + \gamma_2 + \gamma_3 T)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5},$$

which stands for the ordinate of point α' on curve $d\alpha'$, or function φ.

While, by applying H(d)=0, or $h^*=0$, to equation 2-17) in § VI-1.2.2, we have,

$$f(H(d)=0) = \frac{-K' - \sqrt{D'}}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5},$$

where,

$$K' \equiv (w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T) \quad \text{and} \quad D' \equiv (w-v)^2(\gamma_1 I + \gamma_3 + \gamma_5 T)^2 = (K')^2$$

Hence, we have

$$3)\quad f[H(d)=0] = \frac{-2K'}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} = \frac{2(v-w)(\gamma_1 I + \gamma_2 + \gamma_3 T)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5},$$

which stands for the ordinate of α' on curve $\alpha'\beta$, or function f.

Comparing 2) and 3) each other, curve $\alpha\alpha'$ and $\alpha'\beta$ in Fig. VI-8, 9 and 10, respectively, join each other at point α'.

From 2) and 3), the constraint

$$0 < g[H(d)=0] = f[H(d)=0] < \bar{h}$$

can be written as

$$4)\quad 0 < \frac{2(v-w)(\gamma_1 I + \gamma_2 + \gamma_3 T)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} < \bar{h}$$

From the first and second terms in this inequalities, we have

$$5)\quad 0 < \frac{-2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5}$$

We have had restriction

$w > v$, ---- restriction [2]-0

hence, we have

$$5.1)\quad (\gamma_1 I + \gamma_2 + \gamma_3 T)(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5) < 0 .$$

On the other hand, from second and third terms in inequalities 4), we have

$$\frac{-2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} < \bar{h}$$

or,

$$\frac{(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} > \frac{-\bar{h}}{2} .$$

Hence, according to positive or negative for the denominater in left hand side of the last inequality, if

a) $\gamma_1 w^2 - 2\gamma_3 w + \gamma_5 > 0$

we have constraint,

$$5.2)\quad (w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T) > -\frac{\bar{h}}{2}(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5),$$

and if,

b) $\gamma_1 w^2 - 2\gamma_3 w + \gamma_5 < 0$

we have constraint,

$$5.3) \quad (w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T) < -\frac{\bar{h}}{2}(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5).$$

The discussion below equation 5.1) can be alternatively restated as follows: firstly, in equation 5.1), we have

$$6.1) \quad \gamma_1 I + \gamma_2 + \gamma_3 T > 0, \qquad \text{.... restriction [2]-1}$$

because left hand side of the inequality stands for the marginal utility of household income when its non-principal potential earner does not work at all. Hence, from 5.1) we have

$$6.2) \quad \gamma_1 w^2 - 2\gamma_3 w + \gamma_5 < 0 \qquad \text{.... restriction [2]-2}$$

Taking into account these, we look into inequality

$$\frac{2(v-w)(\gamma_1 I + \gamma_2 + \gamma_3 T)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} < \bar{h}.$$

in 4). This can be rewritten as

$$\frac{(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)}{\gamma_1 w^2 - 2\gamma_3 w + \gamma_5} > \frac{\bar{h}}{2}$$

By considering 6.2), we can see left hand side of this inequality is negative. Hence, we have

$$6.3) \quad (w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T) < -\frac{\bar{h}}{2}(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)$$

.... restriction [2]-3

And from 6.2) and 1), we have

$$\gamma_1 v^2 - 2\gamma_3 v + \gamma_5 < 0 \qquad \text{.... restriction [1']}$$

This constraint is an alternative to restriction [1].

(3) Constraint that inequality $0 < H(d)_{q_1} < \bar{h}$ must be held.

The abscissa of point q_1 is given by equation 5) in (6.2.1.1); that is,

$$H(d)_{q_1} = \bar{h} - \sqrt{(\bar{h})^2 - \frac{\bar{h}\{(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)\bar{h} + 2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)\}}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}}$$

In order that the terms in the root is positive and that

$$0 < H(d)_{q_1} < \bar{h}$$

can be held, we have to have

$$7) \quad -(\bar{h})^2 < \frac{\bar{h}\{(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)\bar{h} + 2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)\}}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5} < 0$$

constraint [3]

Now, from the requirement 6.2) and 1′)

1″) $\gamma_1 v^2 - 2\gamma_3 v + \gamma_5 < 0$

should have been satisfied. Hence, from 7) we have

$$7') \quad -(\bar{h})^2(\gamma_1 v^2 - 2\gamma_3 v + \gamma_5) > -\bar{h}\{(\gamma_1 w^2 - 2\gamma_3 w + \gamma_5)\bar{h} + 2(w-v)(\gamma_1 I + \gamma_2 + \gamma_3 T)\} > 0,$$

---- constraint [3]′

which is an alternative presentation for the requirement that inequality $0 < H(d)_{e_{j1}} < \bar{h}$ be held.

(4) Constraint that $\bar{h} < H(d)_{e_{j+}} < a$ must be held

From Fig. VI-11, it can be seen that point q_+ must lie between points $\bar{h}$ and a on the horizontal axis.

The abscessa of point q_+, $H(d)_{e_{j+}}$, is given by equation (6) in (6.2.1.2), that is,

$$H(d)_{e_{j+}} = \bar{h} + \frac{(\gamma_1 v - \gamma_3)(w - v)\bar{h}}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}$$

Hence, in order that the part of the constraint, $\bar{h} < H(d)_{e_{j+}}$ be satisfied,

$$8) \quad \bar{h} < \bar{h} + \frac{(\gamma_1 v - \gamma_3)(w - v)\bar{h}}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}$$ ---- constraint [4]-1

must be held.

The abscissa of point a in Fig. V-11 can be written

$$H(d)_{max} = \frac{\gamma_5 T - (\gamma_1 v - \gamma_3) I - v(\gamma_2 - \gamma_3 T)}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5},$$

which is shown in 7) in (6.2.1.3). Hence the part of constraint $H(d)_{e_{j+}} < a$ can be written that

$$9) \quad \bar{h} + \frac{(w - v)(\gamma_1 v - \gamma_3)}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5} < \frac{\gamma_5 T - (\gamma_1 v - \gamma_3) I - v(\gamma_2 - \gamma_3 T)}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5}.$$

constraint [4]-2

From 8) it can be seen that

$$8') \quad \frac{(\gamma_1 v - \gamma_3)(w - v)\bar{h}}{\gamma_1 v^2 - 2\gamma_3 v + \gamma_5} > 0, \quad \text{where } w - v > 0,$$

---- constraint [4]-1′

must be held. From 6.2), at the same time,

$$\gamma_1 V^2 - 2\gamma_3 V + \gamma_5 < 0$$

must be held, so that, it can be seen, alternative presentation of what is required by 8′) is that

10) $\gamma_1 V - \gamma_3 < 0$ ... constraint [4]-2′

Requirement 10) can be fullfiled whenever $\gamma_3 > 0$ holds. Hence, constraints [4]-1′ and 2′ is needed in order that

$$\bar{h} < H(d)_{14} < a$$

holds.

[6.3] Improving exactness of estimated parameters by using employee-self employed model (A synthetic model for Type A household)

(6.3.1) Search for refined estimates

This is the preliminary section for obtaining refined estimates of the preference parameters. To improve the precision of estimated parameters, we employ the synthetic model described in the previous section.

In the synthetic model a new kind of exogenous variable, ν standing for the earning rate (or marginal productivity) of self employed workers was introduced. However, ν cannot be directly observed because the Family Income and Expenditure Survey in Japan does not survey hours of work. Hence, we are compelled to estimate ν as a parameter, the value of which is assumed to vary from year to year.

a) Postulates for estimating plausible values of ν

The following define what constitutes a plausible value for ν.

(1) If we have a set of reasonably good estimates of parameters, making use of the general model, we should be able to compute theoretical values for the self-employed participation rate μ_d and μ_{ed} (rate of participation both for employee and self-employed) as well as the employee participation rate μ_e, which should reasonably fit the observables μ_d^o, μ_{ed}^c and μ_e^c.

(2) In order to compute theoretical values μ_d, μ_e and μ_{ed} we, before hand, have to have numerical values for principal earner's incomes I_i of the group of households, with w and ν both assumed to be common to all groups of households. I_i and w are directly observable but ν is not, as mentioned above. Hence, owing to (1), if we try a tentative value for ν and if the value is a fairly good approximation to the true value of ν, the computed

theoretical values μ_d, μ_{ed} and μ_e will reasonably fit the observed values.

In addition to the above assumptions, the following restrictions must be satisfied by ν together with observed values for I_i and ϖ.

1 The slope of the curve standing for function ϕ should be positive.

$$2(r_1\nu^2-2r_3\nu+r_5)/(r_1\varpi^2-2r_3\varpi+r_5)>0$$

2 $0<\varphi(H(d)=0)=f(H(d)=0)<\bar{h}$

(We have this in order to attain consistency between the observation and the model.)

3 $0<q_1<\bar{h}$; where,

$$q_1=\bar{h}-\frac{\sqrt{\bar{h}^2-\bar{h}(r_1\varpi^2-r_3\varpi+r_5)\bar{h}+2(\varpi-\nu)(r_1I+r_2+I_3T)}}{r_1\nu^2-2r_3\nu+r_5}$$

4 $\bar{h}<q_4<a$; where,

$$q_4=\bar{h}+\frac{(r_1\nu-r_3)(\varpi-\nu)\bar{h}}{r_1\nu^2-2r_3\nu+r_5}$$

$$a=\frac{r_5T-(r_1\nu-r_3)I-\nu(r_2+r_3T)}{r_1\nu^2-2r_3\nu+r_5}$$

(These restrictions stem from $\varphi(H(d)=\bar{h})<\bar{h}$ and $\varphi(H(d)=a)>\bar{h}$

5 $\bar{h}<f(\bar{h})$, 6. $\varphi(\bar{h})<\bar{h}$, 7. $\bar{h}<\varphi(a)$

Condition 1 yields $(r_1\nu^2-2r_3\nu+r_5)(r_1\varpi^2-2r_3\varpi+r_5)>0$

Condition 2 can be rewritten as

$$0<-\frac{2(\varpi-\nu)(r_1I+r_2+r_3T)}{r_1\varpi^2-2r_3\varpi+r_5}<\bar{h}\,.$$

So that we have

$$(\varpi-\sigma)(\gamma_1 I+\gamma_2+\gamma_3 T)/(\gamma_1\varpi^2-2\gamma_3\varpi+\gamma_5)\geq-\bar{h}/2,$$ where $\gamma_1\varpi^2-2\gamma_3\varpi+\gamma_5<0$,

hence

$$(\varpi-\sigma)(\gamma_1 I+\gamma_2 I+\gamma_3)<-\bar{h}(\gamma_1\varpi^2-2\gamma_3\varpi+\gamma_5)/2 .$$

With respect condition 4, in order that value in the root notation be positive and $0<\gamma_1<\bar{h}$.

$$-(\bar{h})^2<-\frac{\bar{h}\{(\gamma_1\varpi^2-2\gamma_3\varpi+\gamma_5)\bar{h}+2(\varpi-\sigma)(\gamma_1 I+\gamma_2+\gamma_3 T)\}}{\gamma_1\sigma^2-2\gamma_3\sigma+\gamma_5}<0,$$ must hold.

Condition 5 can be rewritten as follows.

$$\bar{h}<\bar{h}-\frac{(\varpi-\sigma)(\gamma_1\sigma-\gamma_3)\bar{h}}{\gamma_1\sigma^2-2\gamma_3\sigma+\gamma_5}<\frac{\gamma_3 T-(\gamma_1\sigma-\gamma_3)I-\sigma(\gamma_2+\gamma_3 T)}{\gamma_1\sigma^2-2\gamma_3\sigma+\gamma_5}$$

(b) Search for plausible values of v. To compute the theoretical values of participation rates, we first need numerical values for the preference parameters γ_i $(i=1,\ldots,5)$ and σ.

Values of γ_i and σ obtained in the previous section,

$\gamma_1=-1.0$ $\gamma_2=100.003$ $\gamma_3=0.0797$,

$\gamma_4=6000.4$ $\gamma_5=-1399.3$, and $\sigma=0.3839$, were used.

Those values are of first approximations because they are obtained making use of the employment opportunity model where nonprincipal earner's (wife's) / income from self-employed work was ignored.

Secondly, we need observed value for I_i. We used three levels of I, $I_1=12.5018$, $I_2=35.0735$, and $I_3=57.5504$ for 1964.

Finally the observed value for w: We have $w\bar{h}=15.8$ for 1964, where $\bar{h}$ is estimated as $\bar{h}=0.3501$ for 1964, and hence $W=45.13$.

Inserting the values above, together with tentative values for V, into restrictions 1 through 5, we checked to see these restrictions are satisfied. Tentative values of v ranged from 1 to 90 so as to include the observed value for W, 45.13.

The intervals of the tentative values for V are 5. It was found that the values of $\underline{v}$ satisfying the restrictions were 40 and 45 as shown in Tab.VI-1.(*)

Next we refined the process of the search, that is; (1) various sets of values for γ_i and σ are tentatively adopted, (numerical values of which were tentatively given in the vicinities of the values cited on the previous page) (2) the values of I_i are adopted from the complete 19 range of classes of principal earners' incomes in 1964, (3) the intervals of the tentative values for V were narrowed down to 1, and (4) the range of tentative values of V was also narrowed down to 35 through 50 instead of 1 through 90.

The results satisfying the conditions are shown in Tab.VI-2, where U stand for Theil's U with respect to the fit of theoretical values to the observed ones, and Φ stands for the value of the objective function(‡) which, we expect, is to be minimized for the best set of parameters.

It can be shown from the table that the values of V satisfying the restrictions and with smaller Φ are 45 and 46.

The sets of parameters, together with V's, which were found to satisfy restrictions 1 through 7 are summarized in the table below.

γ_2	100	<u>150</u>	200	
γ_3	<u>0</u>	<u>10</u>	<u>20</u>	30
γ_4	6000	7000	8000	<u>9000</u>
γ_5	-400	-1400	-2400	<u>-3400</u>
σ	0.188	0.227	0.268 (+)	
V	45	46 (‡‡)		

(*) Checking whether restrictions were fulfilled or not was carried out using values of I mentioned in the text. Values of $\bar{h}$ and Λ used for the check were as follows.

Λ max=1.0, Λ min=0.25
$\bar{h}$ min=0.25, $\bar{h}$ max=0.50

(‡) We use objective junction, $\Phi = \sum \left\{ \left(\frac{\mu_i - \hat{\mu}_i}{\hat{\mu}_i}\right)^2 + \left(\frac{\mu_e - \hat{\mu}_e}{\hat{\mu}_e}\right)^2 + \left(\frac{\mu_{ie} - \mu_e}{\hat{\mu}_{ed}}\right)^2 \right\} n$,
Hence we are minimizing χ^2.
where n stands for the number of households.

(‡‡) For computation, the value of $\bar{h}$ was assumed to be 1/3

(+) The values for σ are computed using given values of w, h, γ_2, γ_3 and γ_5.

Tab. VI-1 The values for q_1, q_2, a, φ, f ;
The values which do not satisfy the conditions 1 through 7 are shown for the reference

σ	f	q_1	r_1	c	$\varphi(-\bar{\lambda})$	$\varphi(0)-f(0)$	$f(\bar{\lambda})$	$\varphi(\bar{\lambda})$	$\varphi(\alpha)$
10	f_{min}	−0.730	0.433	1.515	1.492	1.737	2.056	0.253	1.433
	f_{med}	−0.502	0.433	1.364	1.021	1.327	1.596	0.253	1.231
	f_{max}	−0.234	0.433	1.213	0.563	0.868	1.126	0.253	1.130
30	f_{min}	−0.225	0.419	1.747	0.301	0.770	1.156	0.231	1.578
	f_{med}	−0.123		1.453	0.103	0.571	0.954		1.333
	f_{max}	−0.0173		1.159	−0.0948	0.374	0.730		1.090
35	f_{min}	−0.0952	0.397	1.597	−0.0195	0.515	0.952	0.303	1.550
	f_{med}	−0.0205		1.396	−0.152	0.383	0.800		1.349
	f_{max}	0.0730		1.096	−0.295	0.250	0.533		1.049
40	f_{min}	0.0534	0.374	1.531	−0.350	0.251	0.725	0.325	1.507
	f_{med}	0.105		1.230	−0.418	0.194	0.537		1.306
	f_{max}	0.153		1.030	−0.484	0.127	0.540		1.006
45	f_{min}	0.306	0.351	1.556	−0.691	0.00661	~~0.395~~	0.350	1.555
	f_{med}	0.314		1.259	−0.593	0.00491	0.387		1.259
	f_{max}	0.323		0.964	−0.695	0.00321	0.377		0.964
50	f_{min}	√‾ 而ご負	0.303	1.479	−1.042	−0.248	√‾	0.372	1.500
	f_{med}	√‾		1.189	−0.978	−0.184	√‾		1.211
	f_{max}	√‾		0.901	−0.915	−0.120	√‾		0.923
	* $\bar{\lambda}$ = 0.2501								
	f_{min} = 12.5018, f_{med} = 35.0735, f_{max} = 57.5504								

√‾ indicates that the value in the root notation is negative.

Tab. VI-2 The sets of parameters fulfilling condition 1 through 7

Case	r_2	r_3	r_1	r_4	a	z	R^2	R^2	R^2	TU	TU	TU	σ		σ	TU	TU	TU
1	100	0	6000	−400	.498	43	.556	.564	.470	.381	.372	.305	.262	6				
						44	.440	.564	.463	.329	.415	.317	.300	5				
						45	.316	.564	.457	.771	.478	.332	.257	5				
						46	.239	.566	.466	.706	.571	.348	.310	5				
						47	.181	.573	.454	.526	.747	.364	.353	6				
2	100	0	6000	−1400	.399	41	.425	.568	.475	.315	.206	.298	.287	5			○	
						42	.550	.567	.474	.760	.225	.311	.244	5			○	
						43	.470	.567	.472	.701	.254	.326	.216	5			○	
						44	.374	.568	.471	.540	.297	.343	.203	5			○	
						45	.204	.569	.470	.577	.362	.361	.213	5				
						46	.249	.573	.463	.509	.456	.330	.204	5				
						47	.198	.580	.457	.431	.573	.399	.125	5				
3	100	0	7000	−400	.498	43	.532	.705	.480	.922	.511	.330	.339	5				
						44	.311	.705	.473	.583	.550	.313	.436	6				
						45	.201	.706	.477	.339	.502	.301	715	6				
						46	.145	.708	.476	.738	.520	.292	.349	5			○	
						47	.105	.712	.475	.725	.316	.298	.117	7			○	
4	100	0	7000	−1400	.399	41	.550	.717	.495	.388	.351	.383	.557	5				
						42	.430	.717	.485	.347	.336	.353	.504	5				
						43	.296	.717	.485	.801	.420	.336	.432	5				
						44	.207	.717	.483	.754	.464	.318	.390	6				
						45	.153	.719	.482	.703	.525	.304	.385	5				
						46	.127	.721	.482	.648	.516	.294	.464	5			○	
						47	.097	.725	.481	.532	.780	.299	.127	7			○	
5	100	0	3000	−2400	.321	39	.508	.739	.495	.595	.292	.533	.288	6				
						40	.438	.740	.494	.850	.407	.502	.250	5				
						41	.220	.740	.494	.523	.429	.57	.225	5				

No. 2

Case	r_2	r_3	r_4	r_5	σ	n	$R^2\ x^4$	$R^2\ x^4$	$R^2\ x^{4d}$	$TU\ x$	$TU\ x^4$	$TU\ x^{4d}$	p	ρ	$TU\ x^4$	$TU\ x^4$	$TU\ x^{4d}$
						42	.194	.740	.493	.764	.454	.539	.148 7				
						43	.145	.741	.493	.743	.487	.508	.130 7				
						44	.115	.742	.492	.700	.531	.477	.120 7				
						45	.093	.743	.492	.656	.590	.448	.117 7				
						46	.076	.744	.491	.510	.575	.420	.143 7				
						47	.059	.747	.491	.557	.320	.294	.090 7				
6	100	0	9000	−3400	.273	37	.520	.751	.499	.903	.448	.203	.950 5				
						38	.449	.75	.499	.873	.453	.307	.504 5				
						39	.231	.752	.499	.342	.471	.785	.597 5				
						40	.129	.752	.499	.309	.487	.751	.514 5				
						41	.103	.753	.498	.774	.508	.725	.572 5				
						42	.106	.753	.498	.739	.534	.710	.102 7				
						43	.085	.753	.498	.702	.557	.582	.101 7				
						44	.070	.754	.498	.565	.509	.554	.112 7				
						45	.059	.755	.497	.527	.564	.525	.148 7				
						46	.048	.756	.497	.588	.740	.596	.256 7				
						47	.037	.757	.497	.545	.563	.567	.388 7				
7	150	0	5000	−400	.272	45	.215	.447	.441	.751	.211	.546	.769 3	○		○	
						46	.523	.438	.437	.597	.245	.566	.201 3	○		○	
						47	.495	.444	.433	.415	.457	.567	.141 3	○			
8	150	0	6000	−1400	.235	43	.039	.225	.428	.753	.235	.711	.142 5				
						44	.006	.173	.424	.528	.249	.728	.453 5				
						45	.202	.159	.419	.496	.205	.743	.130 3	○			
						46	.024	.151	.414	.263	.257	.756	.515 2	◐		○	
						47	.105	.174	.409	.212	.277	.763	.420 2	◐	○		
9	150	0	7000	−400	.272	45	.443	.537	.456	.231	.245	.427	.218 4			○	
						46	.259	.534	.455	.703	.232	.462	.140 4				

No. 3

Case	r_2	r_3	r_4	r_5	σ	n	$R^2\ x^4$	$R^2\ x^{4d}$	$R^2\ x^{4d}$	$TU\ x^4$	$TU\ x^4$	$TU\ x^{4d}$	p	ρ	$TU\ x^4$	$TU\ x^4$	$TU\ x^{4d}$
						47	.199	.533	.452	.552	.559	.495	.171 4				
10	150	0	7000	−1400	.2351	43	.450	.551	.453	.335	.254	.502	.527 4			○	
						44	.420	.552	.459	.703	.224	.536	.525 3	○		○	
						45	.539	.545	.455	.559	.196	.567	.196 3	○		◐	
						46	.465	.543	.452	.433	.212	.595	.104 3	○		○	
						47	.405	.553	.448	.282	.425	.621	.173 3	○			
11	150	0	7000	−2400	.2091	42	.014	.383	.449	.558	.422	.321	.195 4				
						43	.000	.269	.444	.573	.391	.547	.259 3	○			
						44	.015	.252	.440	.453	.251	.570	.307 2	◐			
						45	.359	.241	.435	.339	.236	.590	.415 2	◐		○	
						46	.137	.240	.431	.225	.240	.709	.222 2	◐		○	
						47	.203	.256	.427	.108	.232	.725	.219 2	◐	◐		
12	150	0	3000	−400	.2721	45	.216	.712	.484	.393	.402	.287	.362 5				○
						46	.109	.711	.482	.308	.501	.293	.225 5				○
						47	.075	.714	.480	.695	.711	.203	.293 5			◐	○
13	150	0	3000	−1400	.2371	43	.415	.364	.483	.306	.179	.233	.192 5			◐	○
						44	.405	.531	.491	.303	.197	.299	.305 4			○	○
						45	.247	.579	.478	.593	.241	.325	.239 4				
						46	.183	.580	.475	.573	.287	.357	.204 4				
						47	.148	.586	.473	.451	.573	.391	.359 4			○	
14	150	0	3000	−2400	.2091	42	.145	.525	.476	.787	.277	.253	.227 4			○	
						43	.309	.519	.474	.565	.250	.396	.760 3	○		○	
						44	.323	.515	.471	.345	.217	.433	.408 3	○		○	
						45	.454	.513	.468	.428	.185	.471	.248 3	○		◐	
						46	.391	.514	.455	.313	.200	.505	.183 3	○		○	
						47	.320	.525	.451	.197	.424	.538	.401 3	○	◐		
15	150	0	3000	−3400	.1881	41	.202	.503	.455	.543	.445	.497	.735 3	○			

No. 4

Case	r_2	r_3	r_4	r_5	s	z	$R^2\ \mu^d$	$R^2\ \mu^s$	$R^2\ \mu^{sd}$	$TU\ \mu^d$	$TU\ \mu^s$	$TU\ \mu^{sd}$	∂	∂	$TU\ \mu^d$	$TU\ \mu^s$	$TU\ \mu^{sd}$
						42	.007	.4 3	.451	.526	.421	.533	.197 3	○			
						43	.048	.482	.453	.416	.359	.566	.375 2	⊖			
						44	.118	.474	.454	.213	.246	.556	.472 2	⊖			
						45	.213	.470	.450	.215	.239	.523	.299 2	⊖	○	○	
						46	.226	.474	.446	.124	.219	.547	.210 2	⊖	⊖	○	
						47	.499	.498	.442	.081	.310	.569	.453 2	⊖	⊖		
16	150	0	9000	− 400	.272	45	.100	.745	.494	.944	.533	.513	.533 5				
						46	.041	.744	.492	.357	.702	.461	.373 5				
						47	.025	.746	.491	.313	.320	.411	.348 5				
17	150	0	9000	−1400	.227	43	.521	.735	.49	.353	.371	.491	.257 5				
						44	.186	.733	.49	.386	.405	.436	.137 5				
						45	.098	.733	.49	.310	.460	.335	.311 5				
						46	.070	.734	.4	.725	.551	.342	.551 5				
						47	.052	.737	.48	.527	.735	.209	.373 5				
18	150	0	9000	−2400	.229	42	.330	.712	.492	.377	.173	.374	.430 5			⊖	
						43	.256	.710	.490	.733	.189	.330	.244 5			⊖	
						44	.223	.709	.488	.565	.213	.299	.147 5			○	○
						45	.170	.710	.487	.537	.254	.234	.923 4			○	○
						46	.140	.711	.485	.498	.365	.226	.565 4				○
						47	.113	.717	.483	.335	.502	.202	.112 5				
19	150	0	9000	−3400	.183	41	.275	.666	.387	.748	.232	.232	.551 4			○	○
						42	.557	.563	.484	.536	.234	.234	.214 4			○	○
						43	.499	.561	.482	.526	.203	.203	.192 4			○	
						44	.416	.559	.480	.422	.203	.232	.121 4			○	
						45	.353	.560	.473	.222	.175	.357	.798 3	○		⊖	
						46	.310	.564	.475	.231	.206	.403	.598 3	○	○	○	
						47	.253	.575	.472	.159	.445	.440	.113 4		⊖		

No. 5

Case	r_2	r_3	r_4	r_5	s	z	$R^2\ \mu^d$	$R^2\ \mu^s$	$R^2\ \mu^{sd}$	$TU\ \mu^d$	$TU\ \mu^s$	$TU\ \mu^{sd}$	∂	∂	$TU\ \mu^d$	$TU\ \mu^s$	$TU\ \mu^{sd}$
20	200	0	7000	− 400	.190	46	.023	.254	.385	.561	.246	.315	.553 3	○			
						47	.005	.254	.377	.431	.412	.325	.141 3	○			
21	200	0	9000	− 400	.186	46	.404	.254	.432	.581	.230	.704	.269 3	○		○	
						47	.543	.250	.425	.431	.369	.727	.774 2	⊖			
22	200	0	9000	−1400	.171	45	.020	.007	.412	.554	.290	.766	.504 3	○			
						46	.002	.000	.404	.465	.329	.785	.120 3	○			
						47	.003	.002	.397	.263	.342	.799	.431 2	⊖	○		
23	200	0	9000	−2400	.155	44	.154	.522	.386	.580	.525	.317	.235 4				
						45	.132	.543	.377	.552	.485	.328	.734 3	○			
						46	.116	.549	.353	.423	.431	.337	.264 3	○			
						47	.093	.524	.360	.315	.403	.345	.127 3	○			
24	200	0	9000	− 400	.183	46	.309	.518	.455	.762	.223	.497	.135 4			○	
						47	.215	.518	.450	.547	.463	.543	.518 3	○			
25	200	0	9000	−1400	.171	45	.397	.489	.453	.695	.266	.506	.425 3	○		○	
						46	.547	.475	.447	.493	.212	.542	.394 2	⊖		○	
						47	.507	.481	.441	.277	.224	.573	.455 2	⊖	○		
26	200	0	9000	−2400	.155	44	.012	.209	.436	.648	.446	.696	.562 3	○			
						45	.000	.172	.429	.473	.394	.724	.115 3	○			
						46	.008	.152	.422	.312	.318	.746	.411 2	⊖			
						47	.048	.175	.415	.155	.299	.765	.249 2	⊖	⊖		
27	200	0	9000	−3400	.144	43	.133	.193	.414	.556	.366	.769	.251 4				
						44	.114	.220	.406	.530	.533	.786	.325 3	○			
						45	.099	.234	.397	.421	.488	.300	.200 3	○			
						46	.084	.245	.389	.327	.419	.313	.507 2	⊖			
						47	.060	.255	.331	.252	.249	.324	.519 2	⊖	○		
28	100	10	5000	− 400	.527	42	.355	.561	.456	.325	.242	.414	.255 5			○	
						43	.485	.559	.454	.767	.270	.432	.122 5			○	

№ 6

Case	r_2	r_3	r_4	r_5	σ	τ	$R^2\ \mu^d$	$R^2\ \mu^a$	$R^2\ \mu^{ad}$	$TU\ \mu^d$	$TU\ \mu^a$	$TU\ \mu^{ad}$	p		δ	$TU\ \mu^d$=	$TU\ \mu^a$=	$TU\ \mu^{ad}$=
						44	.489	.557	.452	.705	.211	.449	.322	5				
						45	.450	.553	.450	.539	.374	.465	.273	5				
						46	.402	.562	.448	.567	.474	.481	.299	5				
						47	.244	.574	.445	.479	.577	.496	.410	7				
29	100	10	7000	−400	.527	42	.506	.524	.455	.268	.215	.317	.573	5				
						43	.503	.522	.464	.318	.349	.229	.920	5				
						44	.424	.522	.452	.764	.395	.342	.170	5				
						45	.355	.522	.450	.707	.453	.357	.225	5				
						46	.301	.525	.459	.544	.554	.372	.125	7				
						47	.247	.535	.457	.566	.735	.387	.579	5				
30	100	10	7000	−1400	.478	40	.455	.567	.473	.550	.222	.294	.229	5			○	○
						41	.546	.566	.477	.501	.240	.294	.153	5			○	○
						42	.475	.565	.475	.750	.254	.297	.117	5			○	○
						43	.390	.565	.474	.696	.297	.305	.933	5			○	
						44	.325	.566	.473	.541	.343	.315	.903	5				
						45	.273	.568	.471	.584	.409	.327	.123	5				
						46	.233	.571	.470	.522	.511	.341	.315	5				
						47	.195	.579	.468	.451	.703	.356	.155	7				
31	100	10	3000	400	.527	42	.552	.564	.473	.900	.405	.300	.129	5				
						43	.445	.563	.471	.358	.440	.295	.103	7				○
						44	.341	.563	.470	.313	.483	.294	.390	5				○
						45	.272	.564	.468	.764	.542	.296	.158	7				○
						46	.224	.565	.467	.709	.529	.301	.193	5				
						47	.179	.573	.465	.541	.736	.308	.405	5				
32	100	10	3000	−1400	.473	40	.575	.704	.485	.397	.233	.235	.292	5				
						41	.483	.704	.485	.358	.259	.270	.203	5				
						42	.355	.703	.484	.316	.356	.249	.252	5				

№ 7

Case	r_2	r_3	r_4	r_5	σ	τ	$R^2\ \mu^d$	$R^2\ \mu^a$	$R^2\ \mu^{ad}$	$TU\ \mu^d$	$TU\ \mu^a$	$TU\ \mu^{ad}$	p		δ	$TU\ \mu^d$=	$TU\ \mu^a$=	$TU\ \mu^{ad}$=
						43	.270	.704	.483	.772	.420	.330	.292	6				
						44	.217	.704	.481	.725	.456	.315	.320	5				
						45	.150	.706	.480	.577	.523	.304	.115	7				
						46	.151	.708	.479	.524	.618	.297	.595	5				○
						47	.121	.714	.473	.562	.782	.293	.319	7				○
33	100	10	9000	−2400	.390	38	.596	.734	.496	.315	.296	.543	.143	7				
						39	.500	.734	.495	.984	.400	.514	.105	7				
						40	.335	.734	.494	.350	.416	.535	.316	5				
						41	.237	.734	.494	.314	.438	.555	.561	5				
						42	.191	.735	.493	.776	.465	.525	.563	5				
						43	.145	.735	.492	.737	.499	.497	.582	6				
						44	.121	.736	.492	.597	.543	.469	.724	5				
						45	.102	.737	.491	.555	.501	.442	.174	7				
						46	.086	.739	.490	.511	.534	.415	.136	7				
						47	.069	.742	.489	.560	.325	.392	.496	7				
34	150	10	6000	−400	.334	44	.100	.338	.440	.757	.245	.645	.113	4				
						45	.341	.375	.43·	.534	.227	.665	.230	3				
						46	.479	.369	.431	.496	.251	.662	.112	3				
						47	.520	.331	.427	.335	.453	.598	.125	3				
35	150	10	7000	−400	.334	44	.530	.580	.462	.329	.190	.475	.302	4				
						45	.454	.574	.459	.712	.217	.505	.124	4			○	
						46	.364	.572	.456	.538	.200	.532	.712	3	○			
						47	.299	.579	.453	.444	.534	.553	.109	4				
36	150	10	7000	−1400	.293	43	.175	.504	.457	.734	.270	.550	.598	3	○		○	
						44	.427	.494	.453	.513	.238	.577	.322	3	○		○	
						45	.521	.483	.450	.493	.209	.602	.157	3	○		○	
						46	.524	.487	.445	.370	.221	.624	.959	2	○		○	

№ 8

Case	r_2	r_3	r_4	r_5	s	t	$R^2_{\mu 4}$	$R^2_{\mu *}$	$R^2_{\mu **}$	$TU_{\mu 4}$	$TU_{\mu *}$	$TU_{\mu **}$	δ	n	∂	$TU_{\mu 4}$	$TU_{\mu *}$	$TU_{\mu **}$
						47	.473	.501	.442	.232	.423	.645	.193	3	○	○		
37	150	10	7000	−2400	.254	41	.022	.259	.450	.7311	.434	.511	.975	5				
						42	.001	.249	.446	.5355	.409	.535	.525	3	○			
						43	.005	.332	.442	.5222	.378	.556	.166	3	○			
						44	.029	.318	.438	.415	.339	.576	.730	2	○			
						45	.072	.310	.434	.212	.329	.694	.392	2	○		○	
						46	.142	.312	.430	.209	.240	.710	.242	2	○	○	○	
						47	.292	.340	.425	.105	.353	.724	.405	2	○	○		
38	150	10	3000	−400	.234	44	.450	.569	.478	.382	.259	.307	.240	5			○	
						45	.253	.566	.475	.790	.325	.330	.122	5				
						46	.188	.566	.472	.683	.425	.355	.771	4	—			
						47	.149	.571	.470	.557	.541	.325	.116	5				
39	150	10	3000	−1400	.238	43	.553	.540	.476	.308	.179	.250	.510	4			○	
						44	−.421	.536	.473	.703	.181	.381	.325	4			○	
						45	.220	.534	.470	.595	.209	.412	.193	4			○	
						46	.277	.535	.468	.495	.234	.444	.134	4			○	
						47	.229	.543	.465	.363	.335	.474	.232	4				
40	150	10	3000	−3400	.254	41	.334	.558	.474	.300	.316	.397	.612	5				
						42	.257	.550	.471	.707	.293	.432	.139	4			○	
						43	.497	.574	.468	.595	.254	.465	.713	3	○		○	
						44	.537	.569	.465	.485	.229	.497	.412	3	○		○	
						45	.310	.566	.462	.379	.195	.527	.255	3	○		○	
						46	.457	.569	.458	.273	.205	.554	.195	3	○	○	○	
						47	.400	.554	.455	.164	.422	.560	.378	3	○	○		
41	150	10	3000	−3400	.227	40	.007	.481	.465	.713	.452	.503	.462	4				
						41	.001	.457	.462	.502	.432	.535	.456	3	○			
						42	.022	.455	.458	.496	.407	.565	.192	3	○			

№ 9

Case	r_2	r_3	r_4	r_5	s	t	$R^2_{\mu 4}$	$R^2_{\mu *}$	$R^2_{\mu **}$	TU_{μ}	$TU_{\mu *}$	$TU_{\mu **}$	δ	n	∂	$TU_{\mu 4}$	$TU_{\mu *}$	$TU_{\mu **}$
						43	.068	.444	.455	.296	.374	.592	.101	3	○			
						44	.131	.437	.451	.202	.332	.517	.591	2	○			
						45	.211	.434	.447	.212	.277	.539	.374	2	○	○	○	
						46	.316	.439	.443	.127	.218	.559	.275	2	○	○	○	
						47	.472	.456	.439	.081	.334	.573	.502	2	○	○		
42	150		3000	−400	.234	44	.298	.715	.488	.323	.425	.244	.235	5				
						45	.133	.713	.485	.355	.480	.314	.147	5				
						46	.093	.714	.484	.777	.569	.294	.385	5				○
						47	.074	.717	.482	.581	.746	.235	.122	6				○
43	150	10	9000	−1400	.238	43	.412	.704	.488	.372	.249	.316	.999	5		○		
						44	.224	.702	.485	.790	.235	.295	.548	5		○	○	
						45	.164	.702	.484	.703	.345	.287	.315	5			○	
						46	.133	.703	.482	.511	.446	.291	.206	5			○	
						47	.108	.703	.480	.507	.582	.304	.205	5				
44	150	10	9000	−2400	.254	41	.271	.581	.488	.585	.159	.299	.105	5			○	○
						42	.334	.578	.486	.793	.181	.268	.250	5			○	○
						43	.288	.576	.484	.696	.175	.290	.143	5			○	○
						44	.297	.573	.482	.599	.182	.304	.350	4			○	
						45	.249	.575	.480	.503	.216	.327	.525	4			○	
						46	.215	.577	.477	.408	.309	.355	.366	4				
						47	.173	.585	.475	.309	.553	.385	.336	4				
45	150	10	9000	−3400	.227	40	.082	.535	.484	.792	.322	.295	.107	5				○
						41	.253	.531	.482	.685	.304	.315	.466	4				
						42	.539	.527	.479	.580	.231	.344	.274	4			○	
						43	.529	.524	.477	.480	.251	.375	.157	4			○	
						44	.482	.522	.474	.383	.215	.409	.104	4			○	
						45	.437	.522	.471	.290	.184	.442	.575	3	○	○	○	

No.10

Case	r_2	r_3	r_4	r_5	s	t	$R^2\ u^4$	$R^2\ u^2$	$R^2\ u^{4d}$	$TU\ u^4$	$TU\ u^2$	$TU\ u^{4d}$	p		p	TU_{u^4}	TU_{u^2}	$TU_{u^{4d}}$
						45	.391	.527	.459	.202	.204	.474	.495	3	○	○	○	
						47	.327	.541	.456	.132	.433	.504	.301	3	○	◐		
46	200	10	7000	−400	.229	45	.023	.045	.399	.734	.372	.729	.150	4				
						46	.008	.075	.392	.550	.333	.501	.223	3	○			
						47	.000	.061	.334	.248	.397	.312	.873	2	◐			
47	200	10	8000	−400	.229	45	.135	.241	.436	.763	.256	.532	.323	3	○		○	
						46	.411	.320	.431	.568	.226	.706	.150	3	○		○	
						47	.523	.325	.424	.351	.274	.725	.577	2	◐			
48	200	10	8000	−1400	.220	44	.027	.059	.422	.737	.418	.740	.163	4				
						45	.003	.034	.415	.565	.371	.758	.241	3	○			
						46	.001	.023	.408	.401	.213	.773	.703	2	◐			
						47	.019	.335	.401	.227	.344	.767	.405	2	◐	○		
49	200	10	9000	−400	.229	45	.533	.578	.463	.318	.135	.510	.135	4			◐	
						46	.366	.570	.458	.543	.221	.550	.439	3	○		○	
						47	.294	.573	.453	.447	.443	.535	.231	3	○		−	
50	200	10	9000	−1400	.227	44	.118	.57	.454	.777	.313	.592	.117	4				
						45	.442	.449	449	.599	.255	.526	.196	2	○		○	
						46	.503	.438	.444	.424	.215	.555	.503	2	◐			
						47	.532	.449	.438	.234	.340	.561	.453	2	◐			
51	200	10	9000	−2400	.189	43	.021	.250	.442	.741	.459	.570	.207	4				
						44	.00021	.215	.435	.576	.422	.596	.248	3	○			
						45	.006	.188	.430	.424	.370	.719	.774	2	◐			
						46	.025	.177	.423	.280	297	.739	.332	2	◐	○	○	
						47	.087	.203	.417	.140	.301	.756	.251	2	◐	◐		
52	200	10	9000	−3400	.174	42	.123	.005	.425	.723	.563	.736	.201	5				
						43	.096	.030	.418	.593	.533	.755	.775	3.	○			
						44	.078	.064	.411	.476	.504	.771	.250	3	○			

No.11

Case	r_2	r_3	r_4	r_5	s	t	$R^2\ u^4$	$R^2\ u^2$	$R^2\ u^{4d}$	$TU\ u^4$	$TU\ u^2$	$TU\ u^{4d}$	p		p	TU_{u^4}	TU_{u^2}	$TU_{u^{4d}}$
						45	.065	.092	.404	.373	.456	.735	.111	3	○			
						46	.052	.094	.397	.335	.357	.798	.564	2	◐	○		
						47	.030	.040	.390	.222	.334	.909	.400	2	◐	○		
53	150	20	7000	−1400	.343	42	.043	.453	.455	.773	.310	.561	.243	4				
						43	.203	.441	.451	.560	.233	.535	.319	3	○		○	
						44	.364	.431	.447	.549	.251	.507	.395	3	○		○	
						45	.451	.425	.444	.439	.222	.523	.213	3	○		○	
						46	.509	.425	.430	.325	.232	.545	.132	3	○		○	
						47	.517	.444	.436	.197	432	.564	.225	3	○	◐		
54	150	20	8000	−1400	.343	42	.252	.594	.472	.929	.215	.396	.163	5			○	
						43	.535	.538	.459	.727	.199	.425	.790	4			◐	
						44	.476	.533	.456	.525	.183	.455	.415	4			◐	
						45	.417	.530	.453	.522	.201	.483	.221	4			○	
						46	.270	.532	.450	.415	.274	.509	.144	4			○	
						47	.316	.593	.457	.299	.511	.534	.203	4		○		
55	150	20	8000	−2400	.300	41	.085	.340	.468	.755	.328	.457	.404	4				
						42	.289	.532	.465	.546	.304	.497	.192	4				
						43	.450	.324	.452	.45	.274	.515	.105	4			○	
						44	.515	.519	.459	.443	.239	.542	.603	3	○		○	
						45	.528	.517	.456	.344	.205	.566	.361	3	○		○	
						46	.514	.520	.452	.244	.214	.539	.245	3	○	○	○	
						47	.457	.536	.449	.140	.424	.510	.424	3	○	◐		
56	150	20	8000	−3400	.256	40	.001	.440	.462	.575	.443	537	.136	4				
						41	.006	.425	.458	.572	.423	.554	.505	3	○			
						42	.031	.414	.455	.474	.397	.539	.251	3	○			
						43	.071	.404	.451	.382	.364	.512	148	3	○			
						44	.121	.397	.447	.294	.323	.533	.538	2	◐	○		

№ 12

Case	r_2	r_3	r_4	r_5	σ	s	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ		ϕ	TU_{μ^d}	TU_{μ^e}	$TU_{\mu^{ed}}$
						45	.136	.395	.444	.209	.271	.552	.560	2	⊖	○	○	
						46	.278	.401	.440	130	.222	.570	402	2	⊖	⊖	○	
						47	.431	.430	435	.083	.353	.535	.312	2	⊖	⊖		
57	150	20	9000	−1400	.343	42	.559	.567	.483	.377	.193	.232	.202	5			⊖	○
						43	.404	.554	.481	.792	.196	.398	.375	5			⊖	○
						44	.296	.562	.478	.705	.224	.313	.493	5			○	
						45	.245	.561	.475	615	.273	.344	.250	5			○	
						46	.211	.563	.474	.521	.350	.353	.153	5				
						47	.177	.570	.471	.416	.610	.334	.183	5				
58	150	20	9000	−3400	.300	41	.438	.544	.482	.318	.220	.303	.333	5			○	
						42	.535	.540	.480	.723	.204	.222	.273	5			○	
						43	.444	.535	.478	.525	.193	.245	.152	5			⊖	
						44	.373	.534	.475	.535	.180	.373	.351	4			⊖	
						46	.333	.534	.473	.442	.199	401	.497	4			⊖	
						45	236	.537	.470	.350	.230	.420	.321	4			○	
						47	.251	.549	.457	.254	.523	.453	.448	4		○		
59	150	20	9000	−3400	.253	40	.153	.593	.479	.740	.237	.349	.109	5				
						41	.266	.592	.477	.539	.213	.273	.364	4				
						42	.505	.557	.474	.541	.233	.409	.341	4			○	
						43	.535	.553	.471	.446	.253	.440	.201	4			○	
						44	.523	.531	.468	.256	.227	.459	.121	4			○	
						45	.437	.531	.453	.253	.193	.498	.755	3	○	○	⊖	
						46	.451	.536	.453	183	.207	.524	.524	3	○	⊖	○	
						47	.398	.503	.459	.113	.429	.549	.356	3	○	⊖		
60	200	20	5000	−1400	.245	43	.346	.102	.429	.317	.439	.715	.152	5				
						44	.306	.071	.422	.556	.404	.734	.547	3	○			
						45	.300	.350	.416	.504	.353	.751	.144	3	○			

№ 13

Case	r_2	r_3	r_4	r_5	σ	s	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ		ϕ	TU_{μ^d}	TU_{μ^e}	$TU_{\mu^{ed}}$
						45	.305	.042	.410	.357	.202	.755	.545	2	⊖			
						47	.333	.061	.414	.301	.350	.773	.331	2	⊖	○		
61	200	20	9000	−1400	.245	43	.305	.443	.455	.553	.248	.532	.105	5				
						44	.220	.423	.451	.591	.312	.513	.463	3	○			
						45	.415	.407	.4 5	.533	.255	541	.131	3	○		○	
						46	.500	.400	.440	.275	.222	.556	.501	2	⊖		○	
						47	.535	.414	.425	.204	.353	.583	.494	2	⊖	○		
62	200	20	9000	−2400	.223	43	.003	.228	.441	.572	.442	.573	.537	3	○			
						44	.003	.209	.435	.525	.404	.595	.159	3	○			
						45	.016	.189	.429	.389	.353	.716	.510	2	⊖			
						46	.040	.182	.424	.259	.295	.734	.255	2	⊖	○	○	
						47	.113	.211	.418	.131	.315	.750	.257	2	⊖	⊖		
63	150	30	5000	−1400	.402	41	.090	.541	.457	.356	.250	.435	.140	5			○	
						42	.370	.532	.454	.761	.241	.453	.532	5			○	
						43	.504	.525	.451	.662	.221	.489	.223	5				
						44	.511	.519	.455	.564	.204	.514	1.200	4				
						45	.493	.515	.455	.466	.206	.537	.477	4				
						46	.456	.518	.451	.365	.256	.559	.250	4				
						47	.407	.533	.448	.251	.497	.580	.259	4				
64	150	30	5000	−2400	.350	40	.311	.493	.466	.506	.353	.473	.253	5				
						41	.103	.487	.463	.704	.333	.505	.365	4				
						42	.248	.477	.460	.504	.314	.530	.457	4				
						43	.359	.459	.456	.507	.234	.554	.223	4			○	
						44	.447	.463	.453	.412	.250	.577	.125	4			○	
						45	.494	.461	.450	.319	.217	.597	.594	3	○		○	
						46	.518	.465	.445	.224	.225	.517	.420	3	○	○	○	
						47	.511	.486	.443	.122	.427	.634	.553	3	○	⊖		

VI－2

№ 14

Case	r_2	r_3	r_4	r_5	a	v	$R^2\ \mu^d$	$R^2\ \mu^e$	$R^2\ \mu^{ed}$	$TU\ \mu^d$	$TU\ \mu^e$	$TU\ \mu^{ed}$	ϕ	ϕ	TU_{μ^d}	TU_{μ^e}	$TU_{\mu^{ed}}$
	150	30	9000	−1400	.402	41	.277	.622	.478	.894	.191	.319	.536 5			◎	
						42	.540	.617	.475	.810	.186	.339	.541 6			◎	
						43	.456	.613	.473	.722	.186	.362	.211 6			◎	
						44	.385	.610	.470	.634	.200	.386	.905 5			○	
						45	.339	.608	.467	.544	.242	.412	.408 5			○	
						46	.303	.611	.465	.450	.338	.437	.202 5				
						47	.260	.622	.462	.346	.573	.461	.182 5				
66	150	30	9000	−2400	.350	40	.140	.605	.479	.853	.264	.336	.270 6			○	
						41	.450	.599	.476	.760	.247	.360	.117 6			○	
						42	.531	.594	.474	.667	.228	.386	.544 5			○	
						43	.497	.590	.471	.576	.207	.413	.262 5			○	
						44	.455	.587	.468	.486	.191	.440	.133 5			◎	
						45	.418	.586	.465	.396	.196	.466	.706 4			◎	
						46	.381	.590	.463	.307	.265	.491	.406 4			○	
						47	.330	.604	.460	.213	.503	.415	.455 4		○		
67	200	30	9000	−1400	.285	43	.050	.396	.451	.775	.348	.604	.139 4				
						44	.227	.377	.446	.627	.312	.631	.305 3	○			
						45	.358	.362	.441	.484	.267	.655	.110 3	○		○	
						46	.442	.357	.436	.342	.229	.676	.485 2	◎		○	
						47	.516	.375	.431	.184	.366	.695	.552 2	◎	◎		
68	200	30	9000	−2400	.258	42	.015	.247	.444	.764	.457	.654	.265 4				
						43	.000	.218	.439	.621	.429	.677	.369 3	○			
						44	.007	.194	.434	.488	.391	.698	.124 3	○			
						45	.021	.178	.428	.364	.341	.716	.540 2	◎			
						46	.046	.175	.423	.244	.278	.732	.282 2	◎	○	○	
						47	.121	.206	.417	.127	.330	.747	.295 2	◎	◎		

Figures of parameters under which bars are attached give relatively small values for Φ (the smaller Φ, the more favorable).

With regard to γ_3, γ_4 and γ_5, the values 0, 9000 and -3400 respectively are terminal values of the ranges for the parameters. Among those ranges, tentative values of parameters were given to compute theoretical values for μ_e, μ_{ed} and μ_d.

By employing those theoretical values and corresponding observed values, numerical values for objective function Φ were computed. Examining values for Φ, it was found, with respect γ_4 and γ_5, terminal values 9000 and -3400, respectively, yielded relatively smaller values of Φ.

Hence, we extended the range of trial values of the parameters γ_4 and γ_5. The range for γ_3 was not extended because the analyses in the previous sections show that positive values for γ_3 give relatively favorable results. Thus we extended as shown below ranges for the values of γ_4 and γ_5 tentatively given to compute (new) theoretical values for μ^e, μ^{ed} and μ^d.

γ_2	150			
γ_3	0	10	20	
γ_4	9000	10000	11000	12000
γ_5	-3400	-4400	-5400	-6400

The tentative value for h is fixed at 1/3. The computations were conducted for the year 1964.

Among the sets of parameters tried (See Tab. VI-3) the following sets were adopted.

Tab.VI-3 The sets of parameters satisfying the conditions for the year 1964

Case	r_2	r_3	r_1	r_5	ε	σ	TU_{μ}	TU_{μ^d}	$TU_{\mu^{ed}}$	φ		φ	TU_{μ^d}	TU_{μ^e}	$TU_{\mu^{ed}}$	判定
1	150	0	9000	−3400	.133	44	.422	.208	.332	.121	4			○		
						45	323	.176	.367	.798	3	○		●		
						46	.231	.206	.403	.598	3	○	○	○		
2	150	0	9000	−3400	.227	44	.442	.202	.430	.325	2	●		○		
						45	.345	.173	.463	.503	2	●		●		
						46	.248	.213	.494	.401	2	●	○	○		
3	150	0	9000	−3400	.253	44	.470	.210	.516	.597	2	●		○		
						45	.377	.194	.541	.379	2	●		●		
						46	.290	.234	.564	.248	2	●	○	○		
4	150	0	9000	−4400	.158	44	.231	.321	.540	.317	2	●	○			
						45	.151	.263	.569	.221	2	●	●	○		
						46	.087	.198	.596	.182	2	●	●	●		
5	150	0	9000	−4400	227	44	.293	301	.602	.294	2	●	○			
						45	.213	.250	.523	.200	2	●	○	○		
						46	.137	210	.643	.156	2	●	●	○		
6	150	0	9000	−4400	③											
7	150	0	9000	−5400	①											
8	150	0	9000	−5400	③											
9	150	0	9000	−5400	③											
10	150	0	9000	−6400	①											
11	150	0	9000	−5400	②											
12	150	0	9000	−5400	③											
13	150	0	10000	−3400	.158	44	.594	.249	.411	.652	5			○		
						45	.508	.308	.262	.403	5					
						46	.425	.415	.323	.275	5					
						44	.570	.198	.267	.362	3	○		●	○	

No 2

Case	r_2	r_3	r_1	r_5	ε	σ	TU_{μ^e}	TU_{μ^e}	$TU_{\mu^{ed}}$	φ		φ	TU_{μ^d}	TU_{μ^e}	$TU_{\mu^{ed}}$	判定
14	150	0	10000	−3400	.227	45	.483	.251	.271	.621	3	○		○	○	
						46	395	.356	.286	.548	3	○			○	
15	150	0	10000	−3400	.253	.44	.562	.185	.322	.159	3	○		●		
						.45	.475	.220	.350	.294	2	●		○		
						.46	386	.320	.378	.311	2	●				
16	150	0	10000	−4400	.158	.44	.334	.185	.295	.630	3	○		●		
						.45	.252	.170	.322	.442	3	○	○	●		
						.46	.178	.233	.353	.360	3	○	●	○		
17	150	0	10000	−4400	.227	.44	.359	.191	.395	.465	2	●		●		
						.45	.276	.175	.426	.311	2	●	○	●		
						.46	.193	.224	.456	.234	2	●	●	○		
18	150	0	10000	−4400	③											
19	150	0	10000	−5400	①											
20	150	0	10000	−5400	②											
21	150	0	10000	−5400	②											
22	150	0	10000	−6400	①											
23					②											
24					③											
25	150	0	11000	−3400	.138	44	.755	.523	.738	.553	6					
						45	.691	.579	.692	.404	6					
						46	.624	.564	.542	.138	7					
26	150	0	11000	−3400	.227	.44	.703	.390	.470	.133	5					
						.45	.632	.454	.423	.953	4					
						.46	.558	.553	.379	.792	4					
27	150	0	11000	−3400	.253	.44	.567	.307	.275	.711	3	○			○	
						.45	.592	.373	.250	.519	3	○			○	
	150	0	11000	−3400	.253	46	.514	.479	.255	.489	3	○			○	

No. 3

Case	r_2	r_3	r_1	r_5	a	a	TU μ^4	TU μ^{4a}	TU μ^{4d}	a		○	TU μ^4	TU μ^4	TU μ^{4a}	判定
28	150	0	11000	−4400	.188	44	.506	.259	.431	.210	5					
						45	.431	.323	.282	.142	5					
						45	.260	.424	.340	.106	5					
29	150	0	11000	−4400	.227	44	.483	.201	.251	.391	3	○		○	○	
						45	.40	.259	.250	.300	3	○		○	○	
						45	.329	.258	.270	.291	3	○			○	
30	150	0	11000	−4400	.253	44	.479	.183	.310	.328	2	◎		◎		
						45	.402	.225	.335	.357	2	◎		○		
						45	.323	.238	.352	.325	2	◎				
31	150	0	11000	−5400	.188	44	.258	.171	.271	.212	3	○	○	◎	○	☆
						45	.199	.171	.289	.230	3	○	◎	◎	○	☆
						45	.142	.245	.313	.245	3	○	◎	○		
32	150	0	11000	−5400	②											
33					③											
34	150	0	11000	−5400	①											
35					③											
36					②											
37	150	0	12000	−3400	.188	44	.373	.762	.313	.253	7					
						45	.333	.795	.392	.241	7					
						45	.789	.344	.567	.256	7					
38	150	0	12000	−3400	.227	44	.313	.507	.725	.223	5					
						45	.761	.556	.535	.152	5					
						45	.706	.729	.543	.117	5					
39	150	0	12000	−3400	.258	44	.763	.481	.476	.425	4					
						45	.704	.541	.424	.328	4					
	150	0	12000	−3400	.253	45	.640	.630	.094	.313	4					
40	150	0	12000	−4400	.188	44	.573	.517	.725	.750	5					

No. 4

Case	r_2	r_3	r_1	r_5	a	a	TU μ^4	TU μ^4	TU μ^{4a}	a		○	TU μ^4	TU μ^4		判定
						45	.517	.377	.592	.459	5					
						45	.555	.556	.546	.327	5					
41	150	0	12000	−4400	.227	44	.520	.332	.460	.448	4					
						45	.554	.449	.416	.343	4					
						45	.495	.552	.375	.214	4					
42	150	0	12000	−4400	.258	44	.554	.300	.254	.320	3	○			○	
						45	.315	.368	.252	.250	3	○			○	
						45	.443	.476	.248	.251	3	○			○	
43	150	0	12000	−5400	.188	44	.432	.271	.448	.384	4			○		
						45	.367	.339	.400	.332	4					
						45	.305	.453	.357	.511	4					
44	150	0	12000	−5400	.227	44	.412	.208	.257	.210	3	○		○	○	
						45	.342	.259	.252	.170	3	○		○	○	
						45	.276	.360	.253	.182	3	○	○		○	
45	150	0	12000	−5400	③											
46	150	0	12000	−6400	.188	44	.217	.164	.250	.254	3	○	○	◎	○	☆
						45	.153	.120	.257	.195	3	○	◎	◎	○	☆
						45	.118	.271	.283	.184	3	○	◎	○	○	☆
47	150	0	12000	−6400	③											
48					③											
49	150	10	9000	−3400	.188	44	.270	.227	.228	.127	5			○		
						45	.231	.193	.357	.373	5		○	◎		
						46	.207	.206	.391	.380	5		○	○		
50	150	10	9000	−3400	.227	44	.382	.215	.409	.104	4			○		
50	150	10	9000	−3400	.227	45	.290	.184	.442	.575	3	○	○	◎		
						46	.202	.204	.474	.495	3	○	○	○		
51	150	10	9000	−3400	.258	44	.406	.215	.489	.301	2	◎		○		

No. 5

Case	r_2	r_3	r_1	r_1	σ	s	TU x^d	TU x^s	TU x^{sd}	ϕ	ϕ	TU x^d	TU x^s	TU x^{sd}	判定
						45	.315	.189	.516	.511 2	○		○		
						46	.222	.216	.541	.413 2	○	○	○		
52	150	10	9000	−4400	.188	44	.183	.203	.515	.606 3	○	○			
						45	.110	.273	.547	.382 3	○	○	○		
						46	.074	.202	.576	.252 3	○	○	○		
53	150	10	9000	−4400	.227	44	.228	.209	.576	... 2	○	○			
						45	.161	.254	.600	... 2	○	○	○		
						46	.096	.202	.621	. 2	○	○	○		
54	150	10	9000	−4400	③										
55	150	10	9000	−5400	①										
56					③										
57					③										
58	150	10	9000	−5400	③										
59					③										
60					③										
61	150	10	10000	−3400	.188	44	.541	.242	.456	.394 5			○		
						45	.450	.226	.403	.570 5			○		
						46	.357	.299	.353	.262 6					
62	150	10	10000	−3400	.227	44	.517	.194	.293	.270 5			○	○	
						45	.423	240	.287	.188 5			○	○	
						46	.252	.241	.293	.114 5				○	
63	150	10	10000	−3400	.258	44	.506	.177	.210	.633 3	○		○		
						45	.421	.215	.325	.451 3	○		○		
.	150	10	10000	−3400	.253	46	.326	.211	.262	.385 3	○				
64	150	10	10000	−4400	.188	44	.297	.204	.301	.314 5		○	○		
						45	.224	.182	.319	.157 5		○	○		
						46	.159	.221	.345	.118 5		○	○		

No. 6

Case	r	r_3	r_1	r_1	σ	s	TU x^d	TU x^s	TU x^{sd}	ϕ	ϕ	TU x^d	TU x^s	TU x^{sd}	判定
65	150	10	10000	−4400	.227	44	.312	.200	.370	.396 3	○		○		
						45	.232	.175	.405	.276 3	○	○	○		
						46	.159	.214	.435	225 3	○	○	○		
66	150	10	.0000	−4400	③										
67	150	10	10000	−5400	.188	44	.133	.304	.462	.337 3	○	○			
						45	.080	.245	.494	227 3	○	○	○		
						46	.084	.186	.524	.169 3	○	○	○		
68	150	10	10000	−5400	③										
69	150	10	10000	−5400	③										
70	150	10	10000	−5400	①										
71	150	10	10000	−5400	③										
72	150	10	10000	−5400	③										
73	150	10	11000	−3400	.188	44	706	.498	.764	.207 7					
						45	.540	.556	.718	.211 7					
						46	.375	.547	.566	.245 7					
74	150	10	11000	−3400	.227	44	.555	.370	.513	.714 6					
						45	.534	.434	.464	.412 6					
						46	.512	.506	.417	.253 6					
75	150	10	11000	−3400	.258	44	.518	.293	.314	.554 4					
						45	.544	.354	.291	.458 4				○	
						46	.453	.461	.277	.272 4				○	
76	150	10	11000	−4400	.163	44	.457	.252	.476	.213 6			○		
	150	10	11000	−4400	.188	45	.297	.312	.424	.371 6					
						46	.335	.422	.378	571 6					
77	150	10	11000	−4400	.227	44	.441	.195	.291	.577 4			○	○	
						45	.367	.248	.280	.461 4			○	○	
						46	.297	.255	.281	.252 4		○		○	

№7

Case	r_2	r_2	r_1	r_3	s	σ_c	TU μ^2	TU μ^2	TU μ^{2d}	φ		φ	TU μ^2	TU μ^2	TU μ^{2d}	判定
78	150	10	11000	−4400	.258	44	.424	.178	.298	.241	3	○		○	○	
						45	.353	.222	.220	.131	3	○		○		
						46	.232	.321	.245	.173	3	○	○			
79	150	10	11000	−5400	.188	44	.241	.155	.237	[illegible]	3		○	○	○	△
						45	.131	.173	.234	.709	4		○	○	○	△
						46	.142	.242	.311	.492	4		○	○		
80	150	10	11000	−5400	.227	44	.257	.195	.345	.189	3	○	○	○		
						45	.158	.172	.374	.140	3	○	○	○		
						46	.127	.223	.403	.125	3	○	○	○		
81	150	10	11000	−5400	③											
82	150	10	11000	−6400	①											
83					②											
84					③											
85	150	10	12000	−3400	.188	44	.836	.740	.922	.377	7					
						45	791	.777	.902	.421	7					
						46	.743	.832	.378	.327	7					
86	150	10	12000	−3400	.227	44	.774	.555	.752	.102	7					
						45	.719	.537	.712	.557	6					
						46	.562	.714	.570	.192	7					
87	150	10	12000	−3400	.258	44	.722	.450	.319	.579	5					
						45	.561	.521	.475	.456	5					
	150	10	12000	−3400	.258	46	.596	.514	.402	.346	5					
88	150	10	12000	−4400	.188	44	.535	.437	.761	.443	5					
						45	.575	.559	.713	.418	5					
						46	.515	.562	.572	.185	7					
89	150	10	12000	−4400	.227	44	.550	.256	.504	.115	6					
						45	.515	.422	.458	.743	5					

№8

Case	r_2	r_2	r_1	r_3	s	s	TU μ^2	TU μ^2	TU μ^{2d}	φ		φ	TU μ^2	TU μ^2	TU μ^{2d}	判定
						46	450	.533	.414	.525	5					
90	150	10	12000	−4400	.258	44	.543	.284	.301	.184	4			○		
						45	.475	.351	.221	.140	4				○	
						46	.405	.461	.259	.123	4				○	
91	150	10	12000	−5400	.188	44	.404	.254	.492	.272	6			○		
						45	.343	.330	.442	.223	6					
						46	.290	.443	.396	.146	6		○			
92	150	10	12000	−5400	.227	44	.373	.201	.289	.233	4			○	○	
						45	.313	.259	.275	.174	4			○	○	
						46	.251	.370	.271	.146	4		○		○	
93	150	10	12000	−5400	①											
94	150	10	12000	−5400	.188	44	.197	.173	.284	.485	4		○	○	○	△
						45	.149	.182	.220	.224	4		○	○	○	△
						46	.129	.255	.228	.250	4		○	○	○	△
95	150	10	12000	−5400	②											
96					③											
97	150	20	99000	−3400	.188	44	.234	.252	.230	.550	6			○		
						45	.253	.214	.354	.255	6		○	○		
						46	205	.213	.385	.517	5		○	○		
98	150	20	9000	−3400	.227	44	.209	.225	.395	.348	5			○		
	150	20	99000	−3400	.227	45	.254	.198	.427	.442	5		○	○		
						46	.181	.203	.460	.243	5		○	○		
99	150	20	99000	−3400	.258	44	.255	.227	.469	.121	4			○		
						45	.253	.193	.498	.755	3	○	○	○		
						46	.183	.207	.524	.524	3	○	○	○		
100	150	20	9000	−4400	.188	44	.155	.247	.496	.531	5		○			
						45	.095	.258	.529	.297	5		○	○		

No. 9

Case	r_2	r_3	r_4	r_5	a	p	$TU\ \mu^4$	$TU\ \mu^6$	$TU\ \mu^{4d}$	φ	φ	$TU\ \mu^4$	$TU\ \mu^6$	$TU\ \mu^{4d}$	判定
						45	.094	.214	.560	.159 5		◎	○		
101	150	20	9000	−400	.227	44	.196	.219	.556	.562 3	○	◎			
						45	.124	.252	.531	.350 3	○	◎	○		
						45	.077	.204	.505	.228 3		◎	○		
102	150	20	9000	−4400	③										
103	150	20	9000	−5400	①										
104					③										
105					③										
106	150	20	9000	−6400	①										
107					③										
108					③										
109	150	20	10000	−3400	.188	44	.497	.241	.492	.175 7			○		
						45	.424	.237	.436	.206 7			○		
						45	.352	.386	.387	.252 7					
110	150	20	10000	−3400	.227	44	.474	.198	.321	.246 6			◎		
						45	.395	.224	.306	−.131 6			○		
						45	.324	.229	.305	−1.905 6					
111	150	20	10000	−3400	.253	44	.451	.179	.309	.192 5			◎		
						45	.379	.208	.309	.115 5			○		
	150	20	10000	−3400	.253	45	.201	.298	.354	−.752 4			○		
						44	.274	.225	.312	.332 6		○	○		
						45	.213	.198	.223	.159 6		○	◎		
						45	.179	.224	.345	.131 7		◎	○		
113	150	20	10000	−4400	.227	44	.273	.214	.352	.141 5		○	○		
						45	.205	.184	.292	.342 4		○	◎		
						45	.145	.242	.422	.536 4		◎	○		
114	150	20	10000	−4400	③										

No. 10

Case	r_2	r_3	r_4	r_5	a	p	$TU\ \mu^4$	$TU\ \mu^6$	$TU\ \mu^{4d}$	φ	φ	$TU\ \mu^4$	$TU\ \mu^6$	$TU\ \mu^{4d}$	判定
115	150	20	10000	−5400	.188	44	.111	.317	.444	.160 5		◎			
						45	.077	.253	.477	.919 4		◎	○		
						45	.105	.195	.509	.552 4		◎	◎		
116	150	20	10000	−5400	③										
117					③										
118	150	20	10000	−6400	①										
119					③										
120					③										
121	150	20	1100	−3400	.188	44	.558	.473	.734	178 7					
						45	.593	.544	.733	220 7					
						45	532	.529	.539	173 7					
122	150	20	11000	−3400	.227	44	.511	.253	.548	.191 6					
						45	.342	.417	.497	.135 7					
						45	.475	.521	.448	.113 7					
123	150	20	11000	−3400	.253	44	.376	.275	.349	.297 6			○		
						45	.503	.323	.320	.167 6					
						45	.401	.446	.201	.102 6					
124	150	20	11000	−4400	.188	44	.405	.249	.511	.321 6			○		
	150	20	11000	−4400	.188	45	.372	.204	.457	.329 6					
						45	.221	.411	.408	.379 6					
125	150	20	11000	−4400	.227	44	.408	.198	.220	.353 6			◎		
						45	.239	.243	.302	.201 6			○		
						45	.278	.245	.295	.117 6		○		○	
126	150	20	11000	−4400	.253	44	.297	.177	.297	.353 4			◎	○	
						45	.223	.212	.315	.240 4			○		
						45	.254	.209	.337	.181 4		○			
127	150	20	11000	−5400	.188	44	.227	.204	.306	.555 5		○	○		

No. 11

Case	r	r	r	r	σ	θ	TU x^d	TU x^s	TU x^sd	φ	φ	TU x^d	TU x^s	TU x^sd	判定
						45	.179	.20	.205	.447 5		○	○		
						45	.160	.42	.316	.249 6		○	○		
128	150	20	11000	−5400	.227	44	[illegible]	.197	.235	.282 4		○	○		
						45	.156	.177	.252	.250 4		○	○		
						45	.121	.222	.230	.178 4		○	○		
129	150	20	11000	−5400	①										
130	150	20	11000	−5400	①										
131					②										
132					②										
133	150	20	12000	−5400	.155	44	.795	.716	.930	.282 7					
						45	.749	.755	.910	.545 7					
						45	.558	.318	.835	.229 7					
134	150	20	12000	−5400	.227	44	.725	.563	.772	.167 7					
						45	.573	.518	.732	.279 7					
						46	.521	.700	.591	.535 7					
135	150	20	12000	−5400	.253	44	.534	.441	.553	.431 5					
						45	.522	.524	.508	.259 5					
[illegible]	150	20	12000	−5400	.256	45	.553	.500	.465	.139 7					
136	150	20	12000	−4400	.153	44	.557	.475	.781	.195 7					
						45	.533	.541	.738	.234 7					
						45	.482	.533	.592	.115 7					
137	150	20	12000	−4400	.227	44	.545	.252	.539	.309 5					
						45	.492	.419	.491	.454 5					
						46	.421	.525	.445	.322 6					
138	150	20	12000	−4400	.253	44	.509	.272	.275	.271 5			○		
						45	.442	.233	.210	.240 5					
						46	.376	.448	.232	.170 5				○	

No. 12

Case	r	r	r	r	x	z	TU x^d	TU x^s	TU x^sd	φ	φ	TU x^d	TU x^s	TU x^sd	判定
139	150	20	1200	−5400	.153	44	.232	.250	.527	.116 7					
						45	.223	.322	.486	.541 6					
						46	.235	.474	.427	.485 6		○			
140	150	20	12000	−5400	.227	44	.253	.200	.320	.529 5			○		
						45	.232	.253	.300	.392 5		○	○		
						46	.229	.261	.299	.253 5		○		○	
141	150	20	12000	−5400	③										
142	150	20	12000	−5400	.235	44	.190	.193	.310	.128 6	⑤	○	○		
						45	.154	.190	.298	.215 5	⑥	○	○	○	△
						45	.148	.254	.299	.105 5		○	○	○	△
143	150	20	12000	−5400	③										
144	150	20	12000	−5400	③										

Fig VI-14

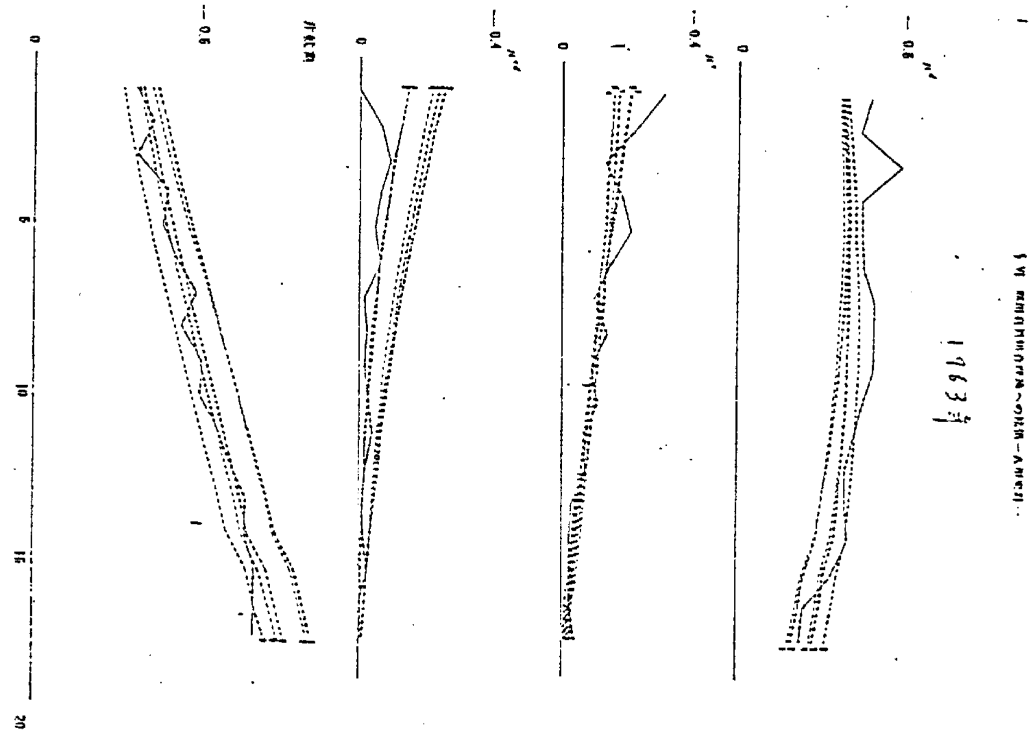


Fig VI-13

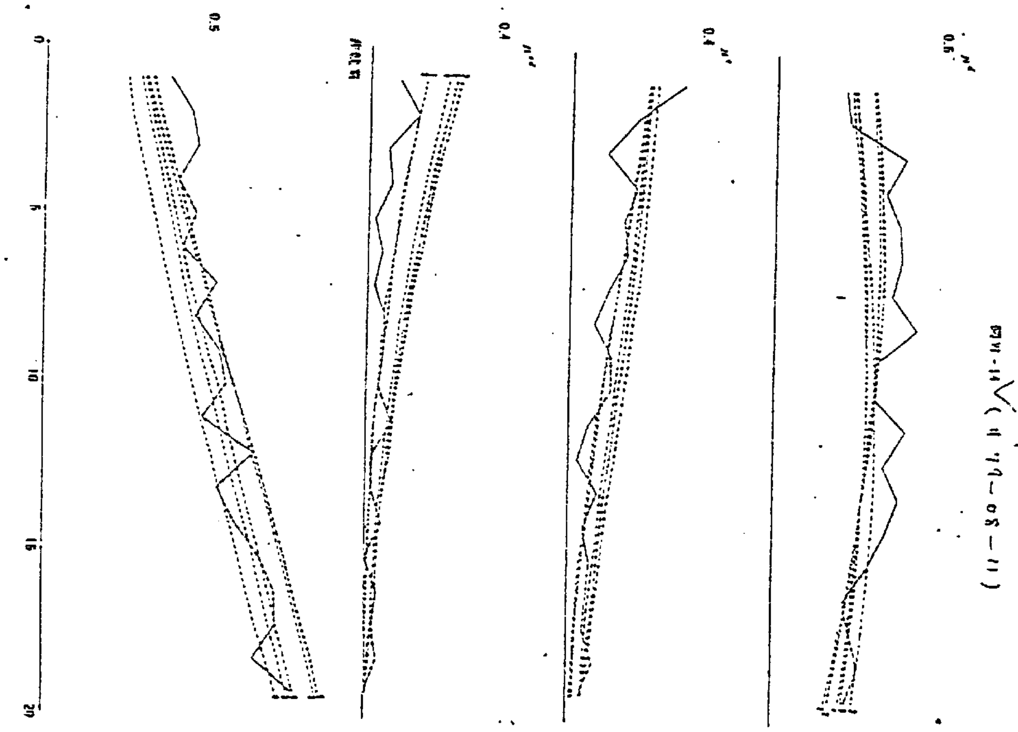


Fig VI-13

Fig VI-14 continued

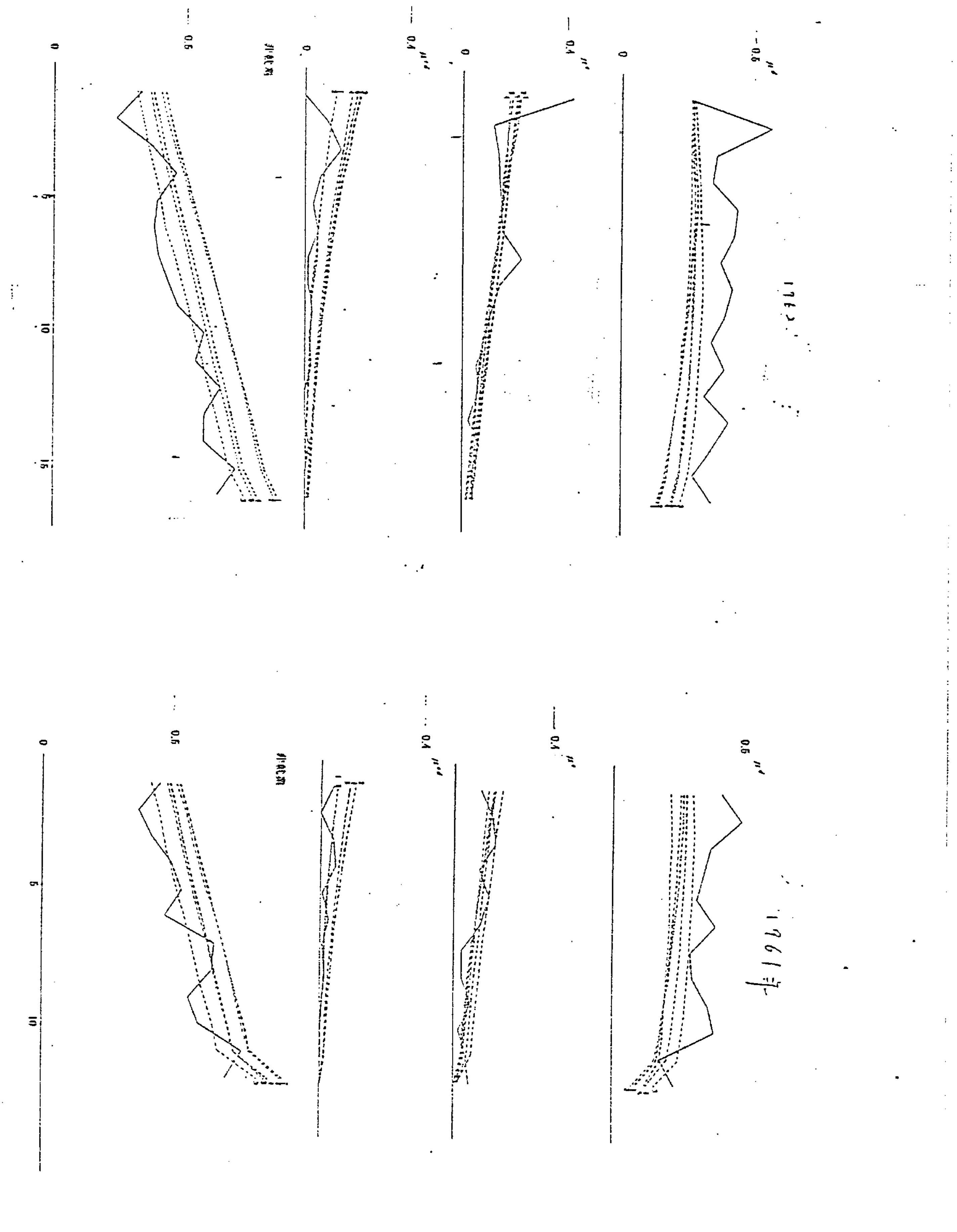


Fig VI-13(a)

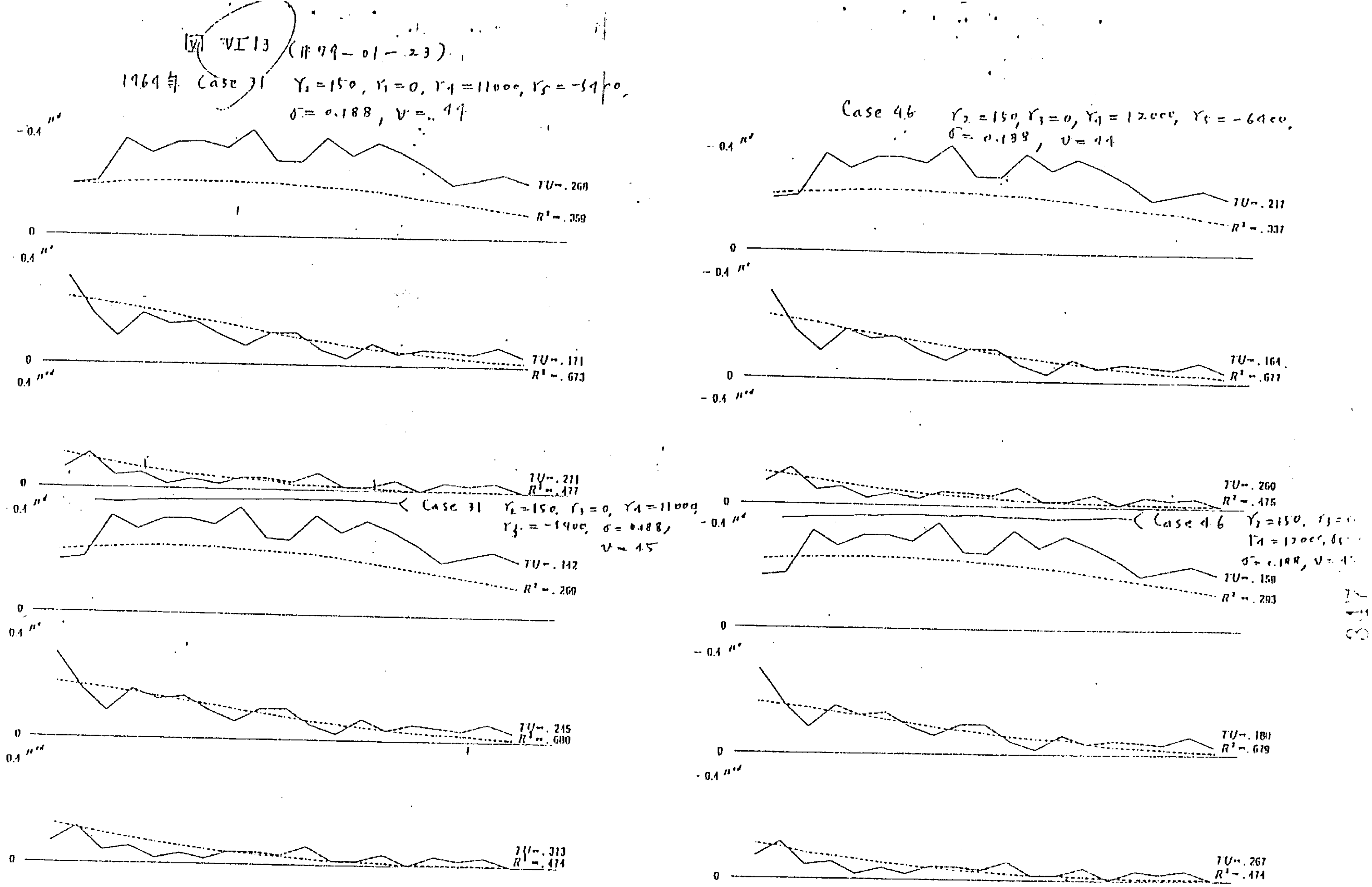


図 VI-13 (#79-01-23)
1964年 Case 31 Y_2=150, Y_3=0, Y_4=11000, Y_5=-5400, σ=0.188, ν=.49
Case 46 Y_2=150, Y_3=0, Y_4=12000, Y_5=-6400, σ=0.188, ν=44
TU=.260
R²=.350
TU=.171
R²=.673
TU=.271
R²=.477
TU=.142
R²=.260
TU=.245
R²=.600
TU=.313
R²=.474
TU=.217
R²=.337
TU=.164
R²=.677
TU=.260
R²=.475
TU=.150
R²=.293
TU=.180
R²=.679
TU=.267
R²=.474
-0.4
0.4
0

Fig VI-13(a)

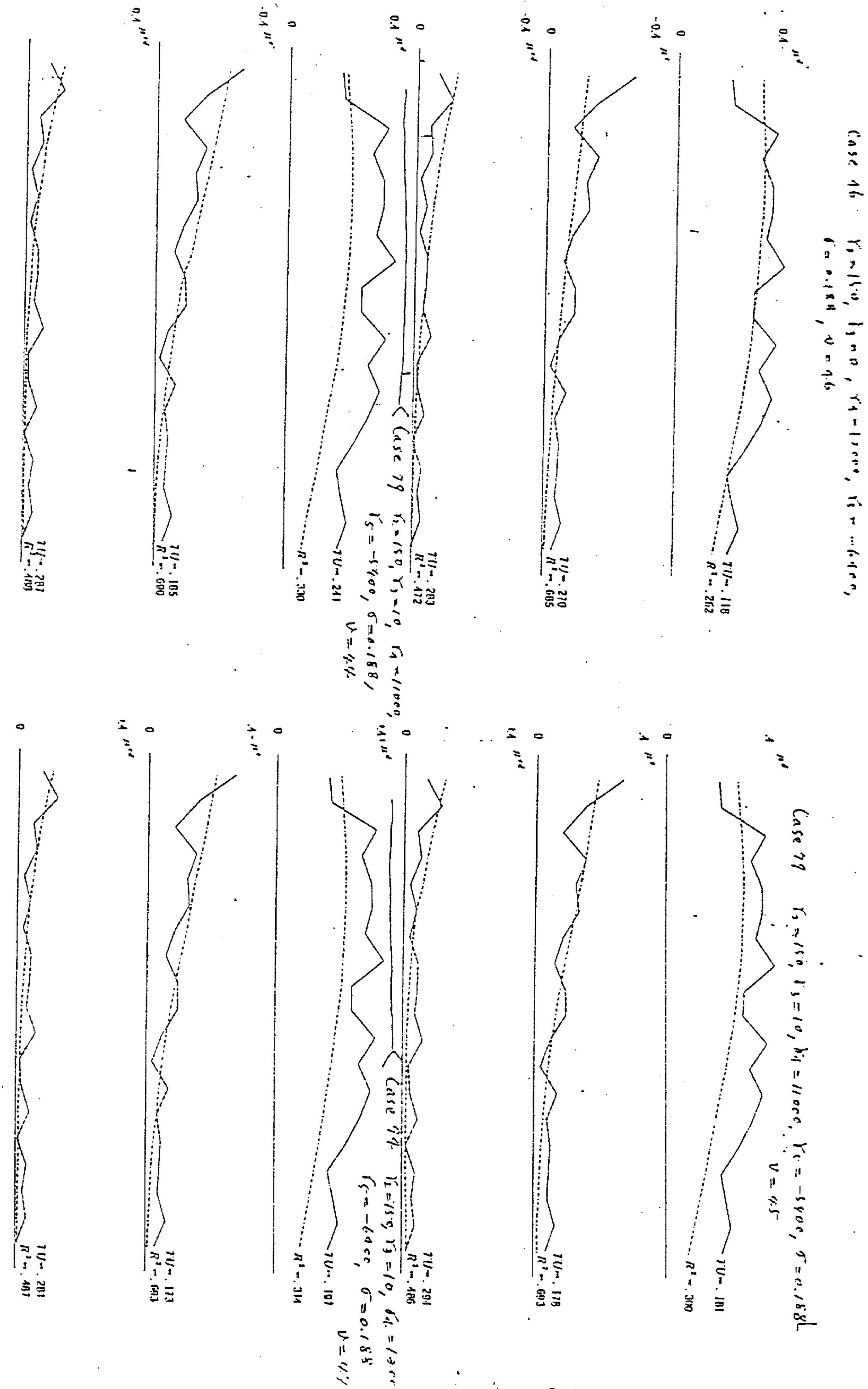


Fig VI-13(a)

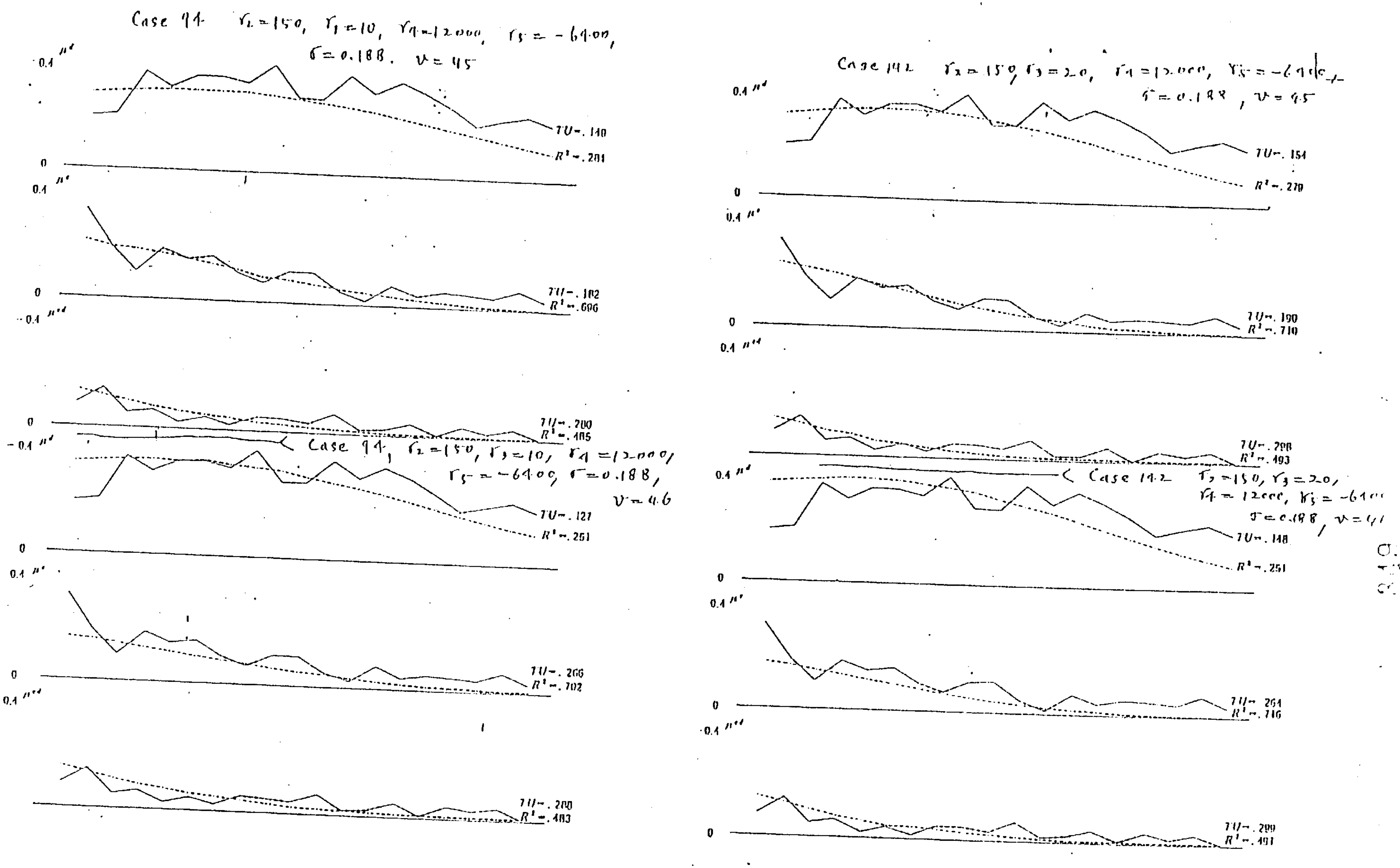
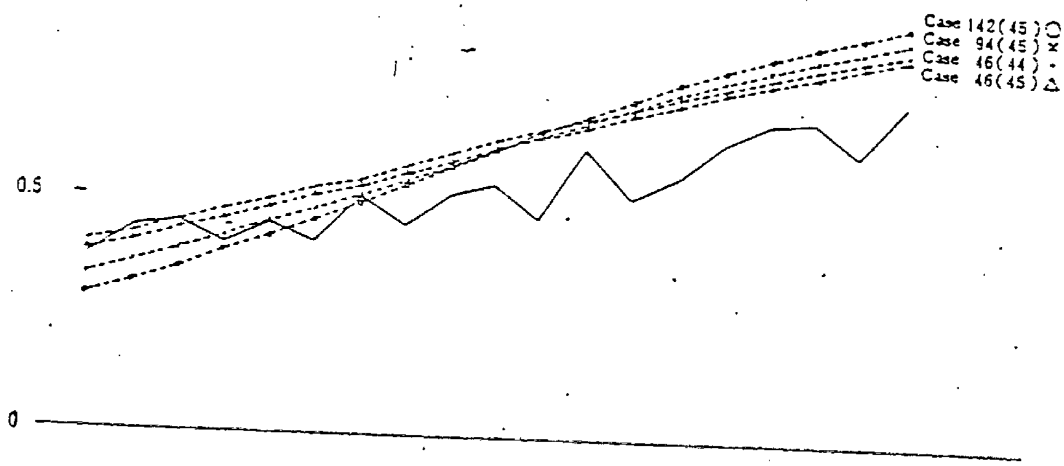

Case 14 r_2=150, r_3=10, r_4=12000, r_5=-6400, σ=0.188, ν=45
Case 14, r_2=150, r_3=10, r_4=12000, r_5=-6400, σ=0.188, ν=4.6
Case 14.2 r_2=150, r_3=20, r_4=12000, r_5=-6400, σ=0.188, ν=45
Case 14.2 r_2=150, r_3=20, r_4=12000, r_5=-6400, σ=0.188, ν=47
TU=.140
R²=.201
TU=.182
R²=.606
TU=.200
R²=.485
TU=.127
R²=.251
TU=.266
R²=.702
TU=.200
R²=.483
TU=.154
R²=.270
TU=.190
R²=.710
TU=.298
R²=.493
TU=.148
R²=.251
TU=.264
R²=.716
TU=.299
R²=.491

Fig VI-13(b)

μ^{α}, μ^{e}, $\mu^{e\alpha}$ のfit のよい4ケースにおける
非就業のグラフ



Tab. VI-4

表VI-4 (e 4—25)

(L) $r_1=150$, $r_3=0$, $r_4=11000$, $r_5=-5400$, $e=0.188$

年	e	r^2_{NON}	r^2_{d}	r^2_{e}	r^2_{ed}	TU_{NON}	TU_{d}	TU_{e}	TU_{ed}	AP_{NON}	AP_{d}	AP_{e}	AP_{ed}	TU_{d+e+ed}	$TU_{合}$	AP_{d+e+ed}	$AP_{合}$	OBF
29	44	0.830	0.259	0.673	0.477	0.109	0.258	0.171	0.271	16.6	59.7	52.9	220.3	0.711	0.820	242.9	259.4	290.0
	45	0.823	0.212	0.675	0.475	0.099	0.199	0.171	0.229	14.9	48.5	58.9	186.8	0.659	0.758	204.2	319.1	290.3
	46	0.835	0.259	0.680	0.474	0.090	0.142	0.245	0.313	12.7	21.6	117.4	161.0	0.700	0.791	210.0	222.7	245.5
38	36	0.960	0.148	0.340	0.254	0.208	1.762	0.223	0.402	25.7	705.2	37.2	252.4	1.358	1.576	994.3	1030.5	1410.9
	37	0.960	0.011	0.329	0.258	0.196	0.674	0.189	0.263	24.1	416.3	25.2	199.3	1.225	1.421	551.0	585.1	554.9
	38	0.961	0.224	0.329	0.272	0.184	0.586	0.159	0.225	32.3	223.9	22.5	157.0	1.091	1.273	472.5	505.3	301.3
	39	0.961	0.530	0.339	0.275	0.171	0.500	0.147	0.222	30.4	203.1	29.5	125.6	0.970	1.141	258.2	288.5	172.7
	40	0.961	0.501	0.339	0.231	0.153	0.417	0.124	0.225	28.4	147.5	27.8	102.5	0.866	1.024	277.9	306.2	111.1
	41	0.961	0.527	0.340	0.225	0.144	0.325	0.118	0.341	25.1	106.5	20.1	37.1	0.734	0.938	222.6	249.3	72.5
	42	0.961	0.633	0.342	0.239	0.130	0.256	0.166	0.258	23.7	74.5	45.9	75.7	0.787	0.917	195.0	219.7	51.6
	43	0.961	0.642	0.345	0.234	0.116	0.173	0.205	0.295	21.0	48.1	101.6	59.3	0.879	0.995	219.0	240.0	56.5
	44	0.960	0.637	0.353	0.298	0.102	0.108	0.741	0.425	18.1	27.7	722.1	55.7	1.273	1.390	316.3	334.5	1292.5
37	24	0.836	0.025	0.554	0.337	0.242	1.725	0.215	0.437	29.9	724.5	24.8	172.2	1.438	1.740	981.4	1021.3	1255.9
	25	0.837	0.044	0.553	0.340	0.221	0.701	0.211	0.445	28.4	460.0	24.5	129.2	1.257	1.568	531.6	572.0	434.2
	26	0.838	0.143	0.553	0.243	0.219	0.616	0.208	0.401	26.9	313.9	25.3	114.0	1.224	1.443	463.1	500.0	214.9
	37	0.838	0.156	0.553	0.246	0.207	0.551	0.208	0.257	25.2	224.8	27.1	94.9	1.106	1.313	356.3	392.0	120.9
	38	0.839	0.152	0.553	0.249	0.194	0.449	0.220	0.246	22.2	163.7	42.2	81.4	1.014	1.229	287.2	320.6	74.4
	39	0.839	0.146	0.553	0.231	0.181	0.359	0.254	0.223	21.3	118.5	58.5	71.6	0.951	1.142	246.7	273.0	52.6
	40	0.840	0.141	0.554	0.254	0.168	0.220	0.221	0.246	20.0	54.0	91.3	67.5	0.957	1.125	242.5	271.3	52.2
	41	0.840	0.124	0.555	0.257	0.154	0.217	0.506	0.253	25.6	57.3	218.3	54.8	1.084	1.237	340.4	357.0	144.7
25	30	0.726	0.204	0.549	0.240	0.215	0.889	0.224	0.620	25.4	4627.3	78.6	256.6	1.744	1.959	4972.0	5008.4	127496.8
	31	0.736	0.020	0.550	0.242	0.206	0.807	0.225	0.556	34.2	343.7	31.4	205.3	1.589	1.795	1120.4	1164.6	1091.4
	32	0.735	0.153	0.552	0.244	0.197	0.720	0.214	0.491	32.9	310.5	36.2	153.4	1.425	1.622	755.2	728.1	425.5
	33	0.734	0.241	0.553	0.246	0.187	0.633	0.200	0.430	31.5	248.9	94.5	125.0	1.253	1.449	568.3	599.8	256.4
	34	0.733	0.413	0.555	0.249	0.175	0.548	0.184	0.378	29.9	250.0	108.7	99.1	1.106	1.282	457.3	487.7	199.4
	35	0.731	0.445	0.557	0.251	0.165	0.460	0.175	0.225	28.2	182.5	131.1	31.1	0.970	1.135	394.6	422.8	225.3
	36	0.730	0.464	0.559	0.253	0.154	0.375	0.194	0.212	26.4	133.0	173.3	59.3	0.882	1.035	375.5	402.0	297.4
	37	0.728	0.477	0.561	0.256	0.142	0.231	0.284	0.209	24.3	94.7	271.0	50.6	0.885	1.027	425.2	450.7	556.2
	38	0.725	0.499	0.563	0.258	0.129	0.225	0.548	0.225	22.1	52.5	745.0	55.4	1.078	1.208	562.9	585.0	418.9

r^2 は理論値と観測値の間の決定係数。TUは Theil's U、APは平均絶対誤差。添字NONは[illegible]、dは[illegible]、eは[illegible]。
edは[illegible]を示す。d+e+edはdとeとedを[illegible]合計。合はdとeとedおよびNONの合計。

2. $r_1=150$, $r_3=0$, $r_4=12000$, $r_5=-6400$, $e=0.188$

年	e	r^2_{NON}	r^2_{d}	r^2_{e}	r^2_{ed}	TU_{NON}	TU_{d}	TU_{e}	TU_{ed}	AP_{NON}	AP_{d}	AP_{e}	AP_{ed}	TU_{d+e+ed}	$TU_{合}$	AP_{d+e+ed}	$AP_{合}$	OBF
29	44	0.823	0.227	0.676	0.475	0.097	0.217	0.154	0.260	15.0	51.2	48.1	189.9	0.640	0.737	289.2	304.2	254.2
	45	0.826	0.293	0.679	0.474	0.087	0.159	0.180	0.257	13.3	25.6	55.4	163.9	0.606	0.693	254.3	278.1	196.1
	46	0.828	0.252	0.685	0.472	0.079	0.118	0.270	0.283	12.0	22.8	115.4	142.2	0.672	0.750	280.4	292.3	184.6
28	24	0.959	0.273	0.329	0.270	0.217	1.318	0.212	0.507	26.7	28464.9	28.4	232.4	1.537	1.754	28835.7	28872.4	401207590
	25	0.960	0.229	0.329	0.272	0.206	0.729	0.223	0.455	25.4	820.1	37.2	256.6	1.397	1.603	923.9	959.2	1138.1
	26	0.960	0.022	0.329	0.276	0.195	0.659	0.190	0.408	33.9	329.4	25.5	213.3	1.257	1.452	532.2	672.1	302.7
	27	0.960	0.128	0.329	0.279	0.184	0.579	0.174	0.257	22.3	273.4	33.3	171.5	1.120	1.304	473.3	510.5	234.4
	38	0.960	0.432	0.329	0.282	0.172	0.501	0.155	0.225	30.5	200.2	30.8	137.5	0.992	1.164	258.4	298.9	174.1
	39	0.960	0.551	0.329	0.285	0.159	0.425	0.134	0.318	28.7	148.8	27.4	111.0	0.877	1.036	287.3	316.0	110.6
	40	0.960	0.610	0.340	0.289	0.146	0.351	0.112	0.313	25.7	110.4	25.5	91.2	0.782	0.928	227.1	253.7	72.2
	41	0.960	0.629	0.341	0.293	0.133	0.279	0.127	0.220	24.5	30.3	29.1	78.1	0.727	0.850	187.4	211.9	49.2
	42	0.960	0.638	0.343	0.296	0.119	0.209	0.190	0.238	22.1	55.8	46.2	68.8	0.737	0.857	170.3	192.9	38.3
	43	0.959	0.640	0.347	0.300	0.106	0.141	0.336	0.253	19.6	34.2	104.2	52.9	0.840	0.945	201.3	221.4	54.3
	44	0.958	0.622	0.354	0.303	0.092	0.089	0.759	0.292	16.3	20.7	725.2	60.1	1.240	1.332	509.0	525.3	1255.4
27	33	0.837	0.017	0.554	0.243	0.240	0.785	0.216	0.557	29.5	585.2	23.8	191.9	1.537	1.777	912.1	961.6	1027.4
	34	0.837	0.005	0.554	0.245	0.229	0.667	0.212	0.505	28.1	431.2	23.4	158.4	1.404	1.633	621.1	659.2	402.5
	35	0.838	0.104	0.554	0.247	0.218	0.610	0.208	0.456	26.7	302.9	22.2	128.9	1.274	1.492	465.0	501.7	209.2
	36	0.838	0.152	0.553	0.249	0.207	0.533	0.207	0.413	25.1	222.0	33.1	107.2	1.153	1.360	362.3	397.5	121.0
	37	0.839	0.154	0.553	0.251	0.195	0.459	0.212	0.377	33.4	165.4	24.3	91.2	1.047	1.242	291.4	324.3	74.7
	38	0.839	0.149	0.554	0.253	0.182	0.386	0.229	0.252	31.6	122.3	40.5	79.6	0.966	1.149	241.4	275.0	49.6
	39	0.839	0.144	0.554	0.255	0.170	0.315	0.270	0.228	29.6	90.4	55.4	70.1	0.923	1.093	215.9	245.5	38.2
	40	0.839	0.139	0.555	0.257	0.157	0.247	0.353	0.237	27.4	55.0	92.3	64.6	0.937	1.094	221.9	249.3	43.7
	41	0.839	0.132	0.557	0.259	0.143	0.182	0.530	0.347	25.1	43.8	221.2	62.4	1.059	1.202	327.4	352.5	131.3
25	30	0.730	0.092	0.542	0.252	0.202	0.785	0.222	0.606	32.5	720.9	56.1	234.2	1.623	1.825	1031.2	1064.3	850.6
	31	0.729	0.054	0.543	0.253	0.193	0.705	0.221	0.547	32.2	465.1	67.7	184.6	1.473	1.667	717.4	749.7	365.1
	32	0.728	0.253	0.545	0.254	0.184	0.626	0.208	0.487	31.0	326.2	70.1	146.0	1.320	1.504	542.3	573.3	204.0
	33	0.727	0.269	0.547	0.256	0.174	0.546	0.192	0.430	29.5	228.5	76.0	117.8	1.169	1.342	432.3	461.8	136.4
	34	0.725	0.418	0.548	0.257	0.163	0.468	0.178	0.379	27.3	177.4	87.5	95.5	1.025	1.189	360.4	388.3	111.6
	35	0.724	0.444	0.550	0.259	0.153	0.392	0.174	0.325	26.2	132.1	106.5	80.1	0.904	1.057	318.7	344.9	118.4
	36	0.722	0.461	0.553	0.261	0.141	0.317	0.203	0.312	24.3	96.9	141.9	68.4	0.832	0.974	307.2	331.5	172.3
	37	0.720	0.474	0.555	0.263	0.130	0.243	0.303	0.304	22.3	68.4	223.5	60.5	0.850	0.979	352.4	374.7	283.8
	38	0.718	0.486	0.558	0.265	0.118	0.160	0.567	0.312	20.1	43.5	519.3	56.3	1.048	1.166	719.2	739.3	2289.5

3. $\gamma_2=150,\ \gamma_3=10,\ \gamma_4=11000,\ \gamma_5=-5400,\ \varepsilon=0.188$

[illegible] | e	r^2_{NON}	$r^2_{\mu^d}$	$r^2_{\mu^c}$	$r^2_{\mu^{cd}}$	TU_{NON}	TU_{μ^d}	TU_{μ^c}	$TU_{\mu^{cd}}$	AP_{NON}	AP_{μ^d}	AP_{μ^c}	$AP_{\mu^{cd}}$	$TU_{\mu^{\pm d}}$	$TU_{\pm}$	$AP_{\mu^{\pm d}}$	$AP_{\pm}$	OBF
29|44	0.826	0.220	0.690	0.488	0.120	0.241	0.185	0.257	18.0	69.0	120.1	977.2	0.713	0.832	1166.4	1184.4	10082.3
45	0.830	0.209	0.692	0.486	0.111	0.181	0.178	0.294	16.9	49.5	163.4	783.1	0.653	0.764	996.0	1012.9	7087.9
46	0.834	0.266	0.699	0.425	0.105	0.143	0.242	0.312	17.0	25.1	273.0	630.8	0.696	0.801	939.9	955.9	4919.7
28|35	0.956	0.072	0.850	0.200	0.219	0.827	0.229	0.526	37.2	1416.4	44.3	1362.4	1.582	1.801	2823.1	2560.3	27454.1
36	0.958	0.077	0.849	0.206	0.207	0.735	0.218	0.474	35.6	580.3	44.2	1027.6	1.427	1.634	1652.1	1637.7	12011.0
37	0.959	0.404	0.849	0.213	0.195	0.642	0.203	0.429	33.8	376.2	44.3	782.1	1.273	1.468	1202.6	1236.5	6862.9
38	0.961	0.556	0.848	0.220	0.183	0.552	0.184	0.392	31.9	255.4	45.0	597.4	1.127	1.310	907.8	929.7	4017.3
39	0.962	0.609	0.848	0.227	0.170	0.465	0.159	0.353	29.7	193.4	47.8	459.9	0.993	1.163	701.0	730.8	2294.4
40	0.962	0.631	0.848	0.223	0.157	0.382	0.132	0.360	27.4	142.2	53.6	363.5	0.874	1.031	559.2	586.5	1543.3
41	0.963	0.641	0.849	0.240	0.144	0.303	0.117	0.365	24.8	103.7	67.4	278.1	0.784	0.928	449.2	474.0	916.3
42	0.963	0.647	0.850	0.247	0.130	0.228	0.154	0.381	22.0	73.4	104.5	221.3	0.763	0.893	399.3	421.3	629.9
43	0.963	0.648	0.853	0.254	0.117	0.182	0.292	0.404	18.9	48.5	215.6	180.9	0.858	0.975	445.1	464.0	640.4
44	0.962	0.641	0.857	0.261	0.104	0.127	0.736	0.433	15.5	35.9	1408.5	149.9	1.296	1.401	1594.3	1609.8	11013.2
27|33	0.831	0.001	0.556	0.291	0.251	0.851	0.222	0.502	41.1	1610.9	48.0	520.3	1.584	1.936	2279.2	2300.3	10627.7
34	0.833	0.068	0.555	0.297	0.240	0.762	0.228	0.545	39.7	651.0	49.7	467.6	1.524	1.775	1162.3	1208.0	2458.[illegible]
35	0.835	0.152	0.554	0.307	0.229	0.672	0.222	0.490	38.1	420.4	52.4	354.5	1.384	1.613	827.4	853.5	1292.1
36	0.837	0.151	0.554	0.308	0.217	0.584	0.216	0.440	25.4	295.5	57.4	272.0	1.241	1.457	625.3	661.7	725.2
37	0.838	0.157	0.553	0.313	0.205	0.499	0.213	0.400	34.5	215.4	66.0	212.0	1.112	1.316	493.4	527.9	438.9
38	0.839	0.153	0.553	0.219	0.192	0.417	0.220	0.371	32.3	158.6	80.5	168.8	1.008	1.199	407.9	440.3	221.4
39	0.840	0.149	0.553	0.324	0.179	0.339	0.248	0.357	30.0	116.1	108.7	125.0	0.943	1.122	360.6	390.6	213.4
40	0.841	0.144	0.553	0.329	0.165	0.266	0.321	0.357	27.5	84.7	173.9	113.0	0.943	1.108	371.5	399.0	248.3
41	0.842	0.138	0.554	0.334	0.152	0.202	0.495	0.369	24.7	60.5	396.7	98.2	1.065	1.217	555.5	580.1	792.1
26|30	0.744	0.034	0.568	0.299	0.215	0.875	0.245	0.896	35.6	1662.6	221.0	548.7	1.819	2.034	2432.3	2467.9	5795.0
31	0.744	0.161	0.569	0.293	0.206	0.785	0.241	0.634	34.4	796.5	241.4	406.9	1.660	1.866	1444.8	1479.2	1942.3
32	0.744	0.255	0.570	0.298	0.196	0.694	0.230	0.569	33.0	522.1	270.2	303.3	1.493	1.689	1095.6	1128.5	1471.4
33	0.744	0.424	0.571	0.202	0.186	0.603	0.216	0.504	31.5	371.2	311.3	226.7	1.323	1.506	909.3	940.7	1484.1
34	0.743	0.453	0.573	0.306	0.175	0.514	0.200	0.443	29.8	272.5	372.8	169.5	1.156	1.231	816.8	846.5	1870.1
35	0.743	0.468	0.574	0.310	0.163	0.427	0.190	0.390	27.9	204.6	475.7	129.5	1.007	1.170	829.7	837.5	2370.0
36	0.741	0.479	0.574	0.315	0.151	0.343	0.206	0.352	25.8	153.2	661.8	100.2	0.901	1.062	915.1	940.9	5476.0
37	0.740	0.456	0.574	0.319	0.139	0.253	0.292	0.322	23.5	113.1	1166.5	87.9	0.887	1.025	1367.5	1391.0	16843.1
38	0.728	0.494	0.570	0.223	0.127	0.185	0.553	0.233	21.0	79.9	3545.7	74.6	1.075	1.201	3700.2	3721.2	152432.2

4. $\gamma_2=150,\ \gamma_3=10,\ \gamma_4=12000,\ \gamma_5=-6400,\ \varepsilon=0.188$

[illegible] | e	r^2_{NON}	$r^2_{\mu^d}$	$r^2_{\mu^c}$	$r^2_{\mu^{cd}}$	TU_{NON}	TU_{μ^d}	TU_{μ^c}	$TU_{\mu^{cd}}$	AP_{NON}	AP_{μ^d}	AP_{μ^c}	$AP_{\mu^{cd}}$	$TU_{\mu^{\pm d}}$	$TU_{\pm}$	$AP_{\mu^{\pm d}}$	$AP_{\pm}$	OBF
29|44	0.530	0.314	0.693	0.457	0.107	0.197	0.173	0.294	16.2	50.9	100.6	694.1	0.654	0.761	845.6	861.6	4861.6
45	0.834	0.284	0.696	0.425	0.099	0.149	0.182	0.280	15.1	35.7	138.7	571.4	0.612	0.711	745.8	760.9	3440.0
46	0.836	0.251	0.702	0.453	0.093	0.127	0.256	0.298	15.0	26.4	225.8	472.2	0.681	0.773	734.4	749.4	2505.1
28|34	0.958	0.129	0.849	0.212	0.217	0.807	0.222	0.581	37.0	1118.3	39.8	1159.9	1.609	1.827	2318.0	2265.0	15418.0
35	0.959	0.007	0.849	0.217	0.206	0.722	0.212	0.528	35.5	529.0	39.4	903.1	1.463	1.669	1481.6	1517.1	7652.5
36	0.960	0.273	0.849	0.222	0.196	0.628	0.200	0.477	33.9	358.6	38.7	705.5	1.315	1.510	1102.8	1136.7	4584.8
37	0.961	0.492	0.848	0.228	0.183	0.555	0.184	0.422	32.1	257.4	37.8	553.0	1.170	1.353	848.1	880.2	2819.9
38	0.962	0.579	0.848	0.234	0.171	0.475	0.163	0.394	30.2	190.8	36.4	434.4	1.031	1.202	661.3	691.5	1762.4
39	0.962	0.615	0.849	0.239	0.158	0.398	0.140	0.366	28.1	142.6	38.2	342.8	0.903	1.062	523.6	551.8	1116.9
40	0.962	0.632	0.849	0.243	0.145	0.324	0.119	0.350	25.9	106.2	43.9	272.5	0.793	0.938	422.6	448.5	719.4
41	0.962	0.640	0.850	0.250	0.132	0.254	0.119	0.348	23.4	77.5	55.5	217.0	0.721	0.853	350.0	373.4	478.5
42	0.962	0.645	0.852	0.256	0.119	0.182	0.177	0.357	20.7	54.1	90.0	176.4	0.722	0.841	320.5	341.2	349.6
43	0.961	0.645	0.854	0.262	0.106	0.132	0.324	0.375	17.8	35.0	191.2	146.9	0.830	0.936	373.1	390.9	382.1
44	0.960	0.639	0.859	0.258	0.092	0.113	0.755	0.399	14.6	27.0	1228.7	124.4	1.257	1.360	1290.1	1404.7	6657.6
27|32	0.833	0.006	0.557	0.300	0.249	0.832	0.228	0.657	40.9	1254.5	40.7	589.2	1.718	1.966	1914.4	1955.3	6397.4
33	0.835	0.026	0.557	0.305	0.239	0.751	0.224	0.604	39.5	605.8	41.6	456.5	1.578	1.817	1103.9	1143.4	1935.2
34	0.836	0.128	0.556	0.309	0.228	0.668	0.219	0.551	38.1	401.1	43.0	353.9	1.438	1.666	798.1	838.1	1040.7
35	0.837	0.158	0.556	0.314	0.217	0.587	0.214	0.499	36.5	297.3	45.1	277.7	1.300	1.517	610.1	646.6	617.9
36	0.838	0.158	0.555	0.318	0.205	0.509	0.211	0.451	34.8	212.6	48.9	219.6	1.170	1.375	481.2	515.9	377.0
37	0.839	0.154	0.555	0.322	0.193	0.433	0.212	0.410	32.9	159.3	56.4	176.3	1.055	1.247	392.0	424.9	229.0
38	0.840	0.150	0.555	0.327	0.180	0.360	0.225	0.378	30.8	119.2	69.9	144.4	0.963	1.143	333.5	364.2	162.1
39	0.840	0.146	0.555	0.331	0.167	0.292	0.262	0.357	28.5	83.2	95.9	119.2	0.911	1.078	303.2	331.8	130.1
40	0.841	0.142	0.556	0.335	0.154	0.228	0.343	0.349	26.1	64.6	154.9	99.7	0.920	1.074	319.2	345.2	159.5
41	0.841	0.136	0.557	0.339	0.141	0.175	0.520	0.353	23.[illegible]	46.6	358.1	88.6	1.049	1.189	493.3	516.6	518.3
26|30	0.741	0.077	0.563	0.306	0.203	0.773	0.242	0.680	33.9	698.4	146.6	436.5	1.695	1.898	1281.4	1315.3	1264.1
31	0.741	0.288	0.564	0.309	0.194	0.689	0.232	0.622	32.6	465.7	159.5	333.4	1.542	1.736	958.6	991.1	783.4
32	0.740	0.385	0.565	0.312	0.184	0.605	0.219	0.561	31.1	334.6	177.9	255.4	1.385	1.569	767.9	799.1	620.1
33	0.739	0.428	0.567	0.316	0.173	0.523	0.204	0.500	29.6	248.7	203.9	195.9	1.228	1.401	648.5	678.0	611.7
34	0.738	0.450	0.568	0.319	0.162	0.443	0.190	0.443	27.8	187.6	246.7	159.2	1.076	1.238	593.5	621.3	766.2
35	0.736	0.464	0.569	0.322	0.151	0.366	0.185	0.392	25.9	141.8	313.2	126.4	0.943	1.094	581.3	607.2	1161.4
36	0.734	0.473	0.570	0.325	0.140	0.292	0.212	0.353	23.9	106.0	437.9	100.7	0.857	0.906	644.7	668.5	2218.4
37	0.732	0.482	0.570	0.329	0.128	0.222	0.309	0.329	21.6	77.1	733.6	84.1	0.860	0.968	894.8	916.4	6105.2
38	0.730	0.492	0.568	0.333	0.116	0.158	0.574	0.322	19.2	52.2	2170.3	72.6	1.054	1.170	2295.1	2314.2	51074.0

5. $T_2=150$, $T_3=20$, $T_4=12000$, $T_5=-6400$, $\sigma=0.188$

群 ρ	r^2_{NOM}	$r^2_{\mu}{}^{d}$	$r^2_{\mu}{}^{t}$	$r^2_{\mu}{}^{td}$	TU_{NOM}	$TU_{\mu}{}^{d}$	$TU_{\mu}{}^{t}$	$TU_{\mu}{}^{td}$	AP_{NOM}	$AP_{\mu}{}^{d}$	$AP_{\mu}{}^{t}$	$AP_{\mu}{}^{td}$	$TU_{\mu}{}^{comd}$	$TU_{全}$	$AP_{\mu}{}^{comd}$	$AP_{全}$	OBF
35·44	0.827	0.302	0.707	0.494	0.119	0.190	0.188	0.310	18.2	56.3	242.4	3077.3	0.687	0.807	3975.9	3994.1	157702.3
45	0.831	0.279	0.710	0.493	0.113	0.154	0.190	0.209	18.5	42.0	348.5	2218.3	0.642	0.755	2208.7	2227.2	114794.7
46	0.835	0.251	0.716	0.491	0.108	0.143	0.254	0.299	19.2	36.0	607.6	2191.0	0.711	0.819	2834.6	2854.0	270888.9
36·34	0.954	0.010	0.855	0.156	0.218	0.756	0.229	0.639	37.0	366.7	57.3	1745.1	1.574	1.892	2589.1	2725.2	17843.3
35	0.956	0.300	0.854	0.163	0.207	0.706	0.230	0.587	35.4	516.1	59.7	4667.0	1.522	1.730	5242.8	5278.2	333556.0
36	0.959	0.513	0.854	0.171	0.195	0.618	0.218	0.535	33.7	357.2	62.8	3371.7	1.371	1.566	3791.7	3825.4	172931.6
37	0.960	0.591	0.854	0.178	0.184	0.533	0.202	0.487	31.7	251.3	67.5	2496.0	1.222	1.406	2824.9	2856.6	93296.1
38	0.962	0.622	0.853	0.185	0.172	0.452	0.182	0.445	29.6	195.8	74.0	1851.5	1.079	1.250	2121.0	2150.9	50685.0
39	0.963	0.637	0.853	0.193	0.159	0.376	0.158	0.411	27.3	147.8	83.9	1387.1	0.945	1.104	1618.8	1646.1	28271.0
40	0.963	0.644	0.853	0.200	0.147	0.302	0.133	0.389	24.7	111.1	104.3	1050.3	0.826	0.973	1255.7	1280.4	15591.3
41	0.964	0.648	0.854	0.208	0.134	0.227	0.125	0.379	21.9	82.1	144.4	800.4	0.742	0.876	1025.3	1048.7	9345.7
42	0.963	0.651	0.854	0.215	0.122	0.179	0.172	0.381	18.8	58.4	230.0	613.9	0.722	0.854	902.3	921.1	5838.1
43	0.963	0.650	0.855	0.222	0.111	0.138	0.314	0.393	15.5	43.5	463.1	475.5	0.845	0.956	982.0	997.8	5025.9
44	0.962	0.641	0.855	0.231	0.100	0.143	0.751	0.413	14.1	37.9	3154.3	373.0	1.306	1.406	2565.2	2579.3	83802.5
37·22	0.826	0.021	0.557	0.253	0.249	0.823	0.244	0.696	40.8	1025.0	64.7	2152.2	1.782	2.011	2271.9	2312.7	61981.1
23	0.831	0.130	0.556	0.260	0.229	0.736	0.240	0.644	39.4	523.5	68.5	1565.9	1.519	1.858	2217.9	2257.3	30653.4
24	0.834	0.162	0.555	0.267	0.228	0.651	0.224	0.590	37.8	400.7	72.9	1153.0	1.475	1.702	1627.5	1665.4	16210.1
25	0.836	0.163	0.554	0.274	0.216	0.563	0.228	0.537	36.0	291.8	82.2	850.9	1.322	1.545	1225.0	1251.0	8671.6
26	0.838	0.159	0.554	0.281	0.204	0.488	0.222	0.486	34.1	218.2	94.2	632.5	1.196	1.401	944.9	979.0	4745.7
27	0.839	0.155	0.553	0.287	0.192	0.412	0.220	0.442	32.0	164.7	113.0	480.8	1.074	1.266	758.5	790.5	2597.9
28	0.841	0.152	0.552	0.294	0.180	0.341	0.229	0.406	29.6	124.2	144.8	366.1	0.975	1.155	635.0	664.6	1626.2
29	0.842	0.149	0.552	0.301	0.167	0.275	0.260	0.380	27.1	93.8	202.9	283.9	0.916	1.083	581.6	608.6	1144.4
40	0.842	0.145	0.552	0.307	0.154	0.219	0.336	0.367	24.3	70.2	321.8	222.5	0.922	1.075	625.5	649.8	1273.2
41	0.842	0.139	0.552	0.314	0.142	0.179	0.511	0.367	21.7	52.8	770.4	180.0	1.057	1.199	1004.2	1025.9	4460.4
38·20	0.745	0.270	0.572	0.250	0.223	0.760	0.259	0.732	34.0	775.4	486.8	957.5	1.757	1.960	2219.6	2253.7	5537.2
21	-0.746	0.291	0.573	0.254	0.194	0.672	0.250	0.683	32.7	541.1	552.5	702.5	1.504	1.798	1797.1	1829.7	5783.1
22	0.746	0.436	0.574	0.259	0.184	0.585	0.228	0.624	31.1	399.4	645.4	516.6	1.446	1.620	1561.3	1592.4	6394.1
23	0.746	0.457	0.574	0.274	0.173	0.501	0.222	0.562	29.4	302.9	769.2	388.7	1.287	1.459	1460.8	1490.2	8291.7
24	0.746	0.469	0.575	0.279	0.162	0.420	0.229	0.502	27.5	222.5	999.6	291.3	1.131	1.293	1523.4	1550.9	12529.5
25	0.745	0.476	0.573	0.284	0.150	0.344	0.313	0.446	25.4	179.3	1371.4	218.8	0.993	1.143	1759.5	1795.0	25214.5
26	0.744	0.482	0.573	0.289	0.129	0.272	0.225	0.399	23.1	137.6	2099.0	166.2	0.897	1.036	2042.8	2426.0	59266.2
27	0.742	0.488	0.570	0.294	0.127	0.208	0.318	0.364	20.6	103.9	3937.5	129.6	0.890	1.017	4171.1	4191.7	208664.5
28	0.740	0.496	0.561	0.299	0.116	0.158	0.579	0.346	17.9	78.8	12552.2	102.2	1.083	1.199	12733.1	12751.0	2116374.1

表VI−5（ρ 4−27）

		1		ρ	2		ρ	3		ρ	4		ρ	5		ρ
	T_2		150			150			150			150			150	
	T_3		0			0			10			10			20	
	T_4		11000			12000			11000			12000			12000	
	T_5		−5400			−6400			−5400			−6400			−6400	
	σ		0.188			0.188			0.188			0.188			0.188	
35	$TU_{\mu}{}^{comd}$	⑤	0.659	45	①	0.606	45	④	0.653	45	②	0.612	45	③	0.642	45
	$TU_{全}$	④	0.758	45	①	0.693	45	⑤	0.764	45	②	0.711	45	③	0.755	45
	$AP_{\mu}{}^{comd}$	②	304.2	45	①	254.8	45	④	938.9	46	③	734.4	46	⑤	2834.6	46
	$AP_{全}$	②	319.1	45	①	278.1	45	④	955.9	46	③	749.4	46	⑤	2854.0	45
	OBF	②	246.5	46	①	184.6	46	④	4919.7	46	③	2505.1	45	⑤	70583.9	45
36	$TU_{\mu}{}^{comd}$	⑤	0.787	42	②	0.727	41	④	0.763	42	①	0.721	41	③	0.732	42
	$TU_{全}$	⑤	0.917	42	③	0.857	42	④	0.893	42	①	0.841	42	②	0.854	42
	$AP_{\mu}{}^{comd}$	②	196.0	42	①	170.8	42	④	399.3	42	③	320.5	42	⑤	902.3	42
	$AP_{全}$	②	219.7	42	①	192.9	42	④	421.3	42	③	341.2	42	⑤	921.1	42
	OBF	②	53.6	42	①	38.8	42	④	629.8	42	③	349.6	42	⑤	5035.9	43
37	$TU_{\mu}{}^{comd}$	⑤	0.951	39	③	0.922	39	④	0.943	39 40	①	0.911	39	②	0.916	39
	$TU_{全}$	⑤	1.135	40	③	1.093	39	④	1.108	40	①	1.074	40	②	1.076	40
	$AP_{\mu}{}^{comd}$	②	242.8	40	①	215.9	39	④	360.6	39	③	303.2	39	⑤	581.6	39
	$AP_{全}$	②	271.8	40	①	245.5	39	④	390.6	39	③	331.8	39	⑤	608.6	39
	OBF	②	52.6	39	①	38.2	39	④	213.4	39	③	130.1	39	⑤	1144.4	39
38	$TU_{\mu}{}^{comd}$	③	0.882	35	①	0.832	36	④	0.887	37	②	0.857	36	⑤	0.890	37
	$TU_{全}$	⑤	1.027	37	①	0.974	36	④	1.026	37	②	0.988	37	③	1.017	37
	$AP_{\mu}{}^{comd}$	②	375.6	35	①	307.2	35	④	809.7	35	③	581.3	35	⑤	1460.8	31
	$AP_{全}$	②	402.0	36	①	331.5	36	④	837.6	35	③	607.2	35	⑤	1490.2	33
	OBF	②	198.4	34	①	111.6	34	④	1471.4	32	③	511.7	33	⑤	5783.1	33

〇内の数は順位、表題の番号はP.（ ）所載の表の番号、記号については（表　）参照。

Among the sets of parameters, those with relatively small values for Φ are listed in Tab. VI-5.

It can be seen that the ranges of V found to be consistent with the theory (i.e. satisfying the theoretical restrictions) are fairly stable among the various sets of preference parameters, ① through ⑤. (see previous table) Secondly, minimum and maximum values for the ranges for each year slightly increase from 1961 through 1964. This seems to be consistent with the experience in Japanese economic growth during those years.

We can see that plausible values for V appear to be 45 and 46 for 1964, 41, 42 and 43 for 1963, 39 and 40 for 1962, and, 32, 33, 34, 36 and 37 for 1961. The underlined figures are those which appear most frequently among groups of the parameters (γ_i, σ) 1 through 5 for each year.

Fig. VI-13(b) indicates that the sums of μ^e, μ^{ed}, and μ^d are underestimated. Hence, it appeared necessary to augument the intercept of the marginal utility curve of income γ_2 and to reduce that of leisure γ_4. Before doing so, a preliminary test was conducted to, making use of data for 1964, to examine if restrictions 1 through 7 were violated by slight shifts in parameters γ_4, γ_5 and γ_3. The results were;

(a) Shifting γ_2 from 150 to 195 (intervals are 5) does not violate the restrictions and

(b) Shifting γ_4 from -6400 to -6700 (intervals are 100) does not violate the restrictions

(c) Shifting γ_3 from 0 to 10 (intervals are 2) does not violate the restrictions.

By taking advantage of results (a), (b) and (c), we set the trial level of parameters as follows, where intervals between testing levels are narrowed down in comparison to previous ones.

γ_2	150 155 160 165 170 175
γ_3	0 2 4 6 8 10
γ_4	12000 11900 11800 11700 11600 11500
γ_5	-6400 -6500 -6600 -6700
σ	0.188 0.193 0.198

By making combinations of numerical values of the parameters listed in the table, we obtained parameter sets. For each of these sets, we computed theoretical values for μ^e, μ^{ed} and μ^d. Among those results, sets of parameters with favorable Φ, TU and AAPE were selected as shown in ~~Tab.~~ the following table.

Results for the year 1964-69

	TU_1	TU_2	$AAPE^1$	$AAPE^2$	Φ	γ_2	γ_3	γ_4	γ_5	σ
1	.573	.546	297.5	310.3	359.7	150	6	12000	-6700	.188
2	.577	.643	207.3	218.9	125.3	150	2	12000	-6700	.188
3	.813	.912	115.3	141.0	15.0	155	0	12000	-6700	.188
4	.727	.792	119.9	131.0	15.4	160	0	11900	-6400	.198
5	.797	.360	118.8	122.7	13.9	165	0	12000	-6400	.198

	[illegible]	[illegible]	[illegible]	[illegible]
1	45	42	39	36
2	45	42	39	35
3	45	43	40	37
4	45	43	40	37
5	45	43	40	35

(*) Suffixes 1 and 2, respectively indicate the values when non participation probabilities are excluded and included.

The estimated values for μ^e, μ^{ed} and μ^d are depicted in Fig. VI-14.

It can be seen that the fitting for μ^e is, to some extent, improved but considerable systematic discrepancies between theoretical and observed values for μ^d remained. Hence, in order to reduce these discrepancies, the Newton method was used to estimate a better set of values of preference parameters making use of the values shown in Tab.(the above table) as initial values for the computation. However, the results of appling the Newton method did not seem to be successful, because at the point where the objective function attained its local maximum, the initial values of parameters did not change sufficiently, so that estimated μ^e, μ^{ed} and μ^d did not closely approach to the observed values.

Hence, we might suspect that the discrepancy between the estimated and the observed values did not stem from the estimation method employed but from some inadequacy in the model itself. However, it seems that we should not discard the basic characteristics of the model under consideration, because we have succeeded, at least to some extent, in following the basic characteristics of the observed data; that is, the upward convexity of the μ^d curve and downward sloping μ^{ed} and μ^e curves.

Neverthless, it seemed that we would not be able to proceed further without altering some part of the present model because the ranges of the parameters satisfying the theoretical restirictions are fairly narrow and we cannot expect any sets of parameters left contribute to the reduction of discrepancies between estimated and observed values of μ^e, μ^{ed} and μ^d.

In fact the model seems to have one point, at least, that needs to be rewritten; that is, $\bar{\gamma}_4$, the intercept of marginal utility line of leisure, has been assumed to be written as $\gamma_4 u$, with the (density) distribution function of u being log-normal. This is equivalent to assuming that the minimum value of $\bar{\gamma}_4$ is zero, an assumption which connot be expected to result in favorable approximation. Hence, we shall rewrite the model taking into account this point.

[6.3.2] Introduction of γ_4° and the estimation of the parameters

We rewrite the model replacing $\gamma_4 u$ by

1) $\gamma_4^\circ + \gamma_4 u$, $\gamma_4^\circ > 0$

where γ_4° stands for the minimum value of $\bar{\gamma}_4$ distributed among households. Hence, γ_4's in the previous model are replaced by (1). Making use of this rewritten model, we shall ~~restrict~~ reestimate the parameters of the preference function.

Parameters other than γ_4° have been estimated in the previous section. We use those estimates as initial values for obtaining second approximation estimates of the parameters together with the newly introduced γ_4° .

First we must determine the plausible range for γ_4° satisfying restrictions 1 through 7. We tentatively set this range from 0 to 1920. Computation results indicated plausible values for γ_4° were from 0 to 800.

Next, we narrowed down the range of tentative values for γ_4° . The values 0, 10, 40, 120, 320 and 800 were adopted and, together with the values for γ_4° , the numerical values for γ_5 were simultaneously varied from -6000 through -6800, the intervals being 100. The values for the other parameters to be tentatively assigned are shown in the table (c).

Making use of combinations of the values for γ_4° and γ_5 mentioned above, estimates or theoretical values for μ^e, μ^{ed} and μ^d and values for objective function Φ were computed. Among those results, the cases satisfying the restrictions are shown in Tab. VI-6 (*). However, it should be noted that plausible sets of parameters might have been excluded because of the wideness of intervals for tentatively assigned values of the parameters. In order to check this point, we alternatively took 0, 2, 4, 6, 8 and 10 for γ_4° and 0.178, 0.180, 0.182, 0.184, 0.186 and 0.188 for σ .

(*) Other parameters are given in the table below.

(c)

case	v_{64}	v_{63}	v_{62}	v_{61}	r_2	r_3	r_4	σ	h
1	45	42	39	36	150	6	12000	0.188	1/3
2	45	42	39	36	150	2	12000	0.188	1/3
3	45	43	40	37	165	0	12000	0.188	1/3
4	45	43	40	37	160	0	11900	0.198	1/3
5	45	43	40	36	165	0	12000	0.198	1/3

These values are reproduced from the table (B) on page270.

Assigned values for γ_5 and γ_4^0 are as folows.

γ_5 -6000, -6100, -6200, -6300, -6400, -6500, -6600, -6700, -6800

γ_4^0 0, 10, 40, 120, 320, 800

Tab. 81-4/6

r_2 r_3 r_4 σ
(1) 150、6、12000、 .188

n	r_4^0	r_5	ϕ_{39}	ϕ_{38}	ϕ_{37}	ϕ_{36}
1	0	−6300	147.2	184.1	352.9	127.9
2		−6400	132.5	152.5	310.6	112.1
3		−6500	122.7	125.8	272.6	97.9
4		−6600	118.1	104.2	239.5	85.6
5		−6700	119.0	88.1	211.6	75.3
6		−6800				
7	10	−6300	148.7	187.3	357.1	129.5
8		−6400	133.4	155.3	314.3	113.5
9		−6500	123.0	128.0	275.8	99.1
10		−6600	117.9	105.8	242.1	86.5
11		−6700	118.1	89.0	213.5	76.1
12		−6800	124.1			67.7
13	40	−6300	153.5	197.4	370.1	134.4
14		−6400	136.6	163.7	325.8	117.9
15		−6500	124.5	134.9	285.8	102.9
16		−6600	117.6	110.9	250.4	89.7
17		−6700	115.9	92.3	220.0	78.5
18		−6300	119.8	79.2	195.1	69.4
19	120	−6300	169.0	225.6	406.6	148.0
20		−6400	148.0	189.3	359.0	130.3
21		−6500	131.6	156.2	315.2	114.0
22		−6600	120.0	127.8	275.5	99.2
23		−6700	113.4	104.3	240.5	86.2
24		−6800	112.2	86.2	210.5	75.2
25	320	−6300	222.3	312.9	509.7	186.1
26		−6400	192.5	267.5	455.3	166.1
27	320	−6500	166.5	225.6	403.7	147.0
28		−6600	144.7	187.5	355.3	129.0
29		−6700	127.3	153.7	310.5	112.4
30		−6800	114.5	124.3	269.8	97.3
31	800	−6300	413.1	572.3	797.1	289.7
32		−6400	368.9	514.9	734.9	267.6
33		−6500	326.9	459.3	673.4	245.5
34		−6600	287.5	405.7	613.0	223.6
35		−6700	250.8	354.6	554.1	202.2
36		−6800	217.8	306.2	497.1	181.3

(2) 150、2、12000、 .188

n	r_4^0	r_5	ϕ_{39}	ϕ_{38}	ϕ_{37}	ϕ_{36}
1	0	−6300	148	192	354	130
2		−6400	134	161	323	115
3		−6500	126	135	286	101
4		−6600	123	115	254	89
5		−6700	125	100	228	80
6		−6800				
7	10	−6300	150	195	368	132
8		−6400	135	164	326	116
9		−6500	126	137	289	102
10		−6600	122	116	256	90
11		−6700	124	101	230	80
12		−6800				
13	40	−6300	154	205	281	137
14		−6400	138	172	337	120
15		−6500	127	143	298	106
16		−6600	121^5	120	264	93
17		−6700	121^4	103	235	82
18		−6800				
19	120	−6300	169	233	417	150
20		−6400	148	196	369	132
21		−6500	132	163	326	116
22		−6600	121	139	287	102
23		−6700	116	113	253	89
24		−6800	117	96	225	79
25	320	−6300	221	319	519	188
26		−6400	191	273	464	168
27	320	−6500	165	231	412	149
28		−6600	143	193	364	131
29		−6700	126	159	319	114
30		−6800	114	130	279	99
31	800	−6300	413	561	807	291
32		−6400	368	523	744	269
33		−6500	325	466	682	247
34		−6600	285	411	621	225
35		−6700	247	359	562	203
36		−6800	213	310	504	182

(3) 165、0、12000、 . .188

	r_4^0	r_5	ϕ_{39}	ϕ_{38}	ϕ_{37}	ϕ_{36}
1	0	− 6300	310	204	282	76
2.		− 6400	355	235	298	78
3		− 6500	408	276	324	83
4		− 6600	470	326	361	92
5		− 6700	540	337	408	105
6		− 6800				
7	10	− 6300	304	199	278	75
8		− 6400	343	229	294	77
9		− 6500	400	269	319	82
10		− 6600	460	318	354	90
11		− 6700	530	377	400	103
12		− 6800				
13	40	− 6300	287	187	270	74^{3}
14		− 6400	327	213	282	74^{5}
15		− 6500	376	249	303	78
16		− 6600	433	294	334	85
17		− 6700	499	350	376	96
18		− 6800				
19	120	− 6300	247	160	255	73
20		− 6400	279	178	257	70
21		− 6500	319	201	269	71
22		− 6600	368	239	290	74
23		− 6700	425	284	320	82
24		− 6800	491	339	361	93
25	320	− 6300	180	129	253	82
26		− 6400	194	128	236	73
27	320'	− 6500	216	134	227	66
28		− 6600	244	147	225	62^{9}
29		− 6700	281	169	232	62
30		− 6800	325	199	247	64
31	800	− 6300	178	215	401	149
32		− 6400	159	181	354	131
33		− 6500	145	152	311	114
34		− 6600	136	128	273	98
35		− 6700	133	109	240	84
36		− 6800	136	96	212	72

(4) 160、0、11900、 .198

	r_4^0	r_5	ϕ_{39}	ϕ_{38}	ϕ_{37}	ϕ_{36}
1	0	− 6300	285	214	308	84
2		− 6400	322	241	323	86
3		− 6500				
4		− 6600				
5		− 6700				
6		− 6800				
7	10	− 6300	280	210	305	83
8		− 6400	316	236	319	85
9		− 6500				
10		− 6600				
11		− 6700				
12		− 6800				
13	40	− 6300	265	199	297	82^{1}
14		− 6400	298	222	307	82^{7}
15		− 6500				
16		− 6600				
17		− 6700				
18		− 6800				
19	120	− 6300	231	175	282	80
20		− 6400	257	189	284	78^{2}
21		− 6500	290	212	294	78^{8}
22		− 6600				
23		− 6700				
24		− 6800				
25	320	− 6300	177	150	279	87
26		− 6400	186	146	253	79
27	320	− 6500	202	149	254	73
28		− 6600	225	159	251	69^{8}
29		− 6700	254	177	256	69^{7}
30		− 6800				
31	800	− 6300	191	240	422	151
32		− 6400	172	206	376	134
33		− 6500	155	175	334	117
34		− 6600	143	150	297	102
35		− 6700	136	130	264	88
36		− 6800	135	115	236	77

. (5) 165、0、12000、 .198

	r_4^0	r_5	ϕ_{39}	ϕ_{38}	ϕ_{37}	ϕ_{36}
1	0	− 6300	386	290	368	129
2		− 6400	438	332	398	135
3		− 6500				
4		− 6600				
5		− 6700				
6		− 6800				
7	10	− 6300	379	284	363	128
8		− 6400	430	325	392	133
9		− 6500	490	376	430	143
10		− 6600				
11		− 6700				
12		− 6800				
13	40	− 6300	359	266	349	125
14		− 6400	407	304	374	129
15		− 6500	463	351	409	137
16		− 6600				
17		− 6700				
18		− 6800				
19	120	− 6300	310	225	319	119
20		− 6400	349	255	334	120
21		− 6500	397	292	358	125
22		− 6600	453	339	393	133
23		− 6700				
24		− 6800				
25	320	− 6300	222	167	285	117
26		− 6400	244	175	279	111
27	320	− 6500	272	190	281	108^{5}
28		− 6600	307	214	291	103^{5}
29		− 6700	351	246	310	111
30		− 6800	402	286	338	117
31	800	− 6300	178	202	377	165
32		− 6400	165	174	338	149
33		− 6500	156	152	302	134
34		− 6600	154	135	272	120
35		− 6700	157	123	247	109
36		− 6800	166	118	228	99

Using combinations of the given values for γ_4° and σ , we examined if the theoretical restrictions 1 through 5 were violated. It was found that there was no case violating the restrictions. To further substantiate this conclusion, we extended the range for γ_4° from 0 through 40. The intervals of tentative values were 8. The trial levels for σ was the same as the previous ones. Combinations of the values for γ_4° and σ were checked against restrictions 1 through 5. Again, the results showed that there were no cases violating the restrictions.

Taking into account the results of these preliminary test, it was thought that there was little chance that any combination of parameters would violate the restrictions for the ranges checked even though combinations actually tested were limited in number. Hence, we proceeded to convergence-computation for estimating parameters making use of the steepest ascent method.

Initial values of the preference parameters were tentatively chosen as,

σ=0.188, γ_2=150, γ_3=2, γ_4= 12000, γ_5=-6700 and γ_4°=40

with other parameters given as h=0.33, w_{39}=47.4, ν_{39}=45, w_{38}=44.10 ν_{38}=42, w_{37}=41.70, ν_{37}=39 w_{36}=38.4 and ν_{36}=36.

The objective function $\tilde{x}^2$ was computed by employing the entire data set from 1961 through 1964 and corresponding estimated values for the parameters.

However, before we proceed to the estimation, one point should be made. According to the experience of previous estimation results and the preliminary estimation making use of above mentioned set of initial values of parameters, the results of which are deleted here, it was found (1) that when we allow all the parameters to vary, some parameters, sometimes, clearly do not attain their optimal (minimizing x^2) value for the ranges fulfilling the restrictions and (2) that when we allow σ and γ_4° to vary, other parameters being fixed at initial values, the speed of convergence for γ_4° is extremely slow.

These experiences show that some parameters barely attain convergence when their initial values and/or initial values for the other parameters are not appropriate. Consequently, to begin with, we shall vary numerical values of a few parameters among the parameters to which initial values are attached. In the first place, we shall allow ν only to vary because we have some information for the values to be estimated. That is, the observational period under consideration is a period of fairly steady growth as shown by the growth of w as well as the growth rate of GNP, and consequently the parameter ν to be estimated is expected to grow. At the very least, a descending values or radical random movement in ν can be ruled out. This constitutes information for estimating ν . That is, if we have estimates (or convergence values) for ν that exhibit such counter-intuitive movement, it may be considered that the values for other parameters are inadequate as initial values to be adopted. Consequently, we should allow parameters other than ν to vary in order to minimize x^2, and after that, we have to vary ν by employing the newly attained values of other parameters (which we have varied with minimum x^2 principle) as given. After this procedure, we should examine if the estimated values for ν are consitent with the other information.

As a "postulate" for the estimation, we consider (1) the parameters γ_1, γ_2, γ_3, γ_4, γ_5, γ_4° and σ to be constant for all the observational years and for all the principal earner's income classes and (2) w , ν and $\bar{n}$

are considered to vary from year to year but are considered to be constant cross-sectionally. We minimize

$$\Phi_t \qquad (t=1961,\ldots\ldots,64)$$

instead of $\sum_t \Phi_t$. After obtaining Φ_t's thus minimized we calculate $\sum \hat{\Phi}_t$, where $\hat{\Phi}_t$ stands for the minimized value of Φ_t for each year.

Experiment I

As was mentioned above, we start from the estimation of ν.

		1964	63	62	61
γ_1	-1				
γ_2	150				
γ_3	2				
γ_4	12000				
γ_5	-6700				
γ_4°	40				
σ	0.188				
h	0.333				
ω_t		47.4	44.1	41.7	38.4
ν_t		45 (45.51)*	42 (42.63)	39 (40.35)	36 (37.48)

Parameters except for ν are held constant at their initial values. ν_t's are varied so as to minimize Φ_t's. The result of estimation is shown in the table. The estimates for ν_t's seem to satisfy the postulate for estimation.

(*) values in the parentheses are the convergence values.

Experiment 2.

	1964	63	62	61	Φ
v	45 → 45.48	42 → 42.63	39 → 40.35	36 → 37.48	543.298
σ	0.188 → 0.13675	0.188 → 0.17675	0.188 → 0.17675	0.188 → 0.17675	↓
r_4^o	40 → 25.3	40 → 25.3	40 → 25.3	40 → 25.3	431.099

Initial values for γ_2, γ_3, γ_4 and γ_5 are held constant at the same levels as in experiment 1, and σ and γ_4^o together with ν, are varied. Estimates for the parameters are shown in the table below. It should be noted that the estimates for ν_t's are similar to those obtained in experiment 1. Also, the direction of changes in the theoretical (estimated) values for μ_t^e, μ_t^d and μ_t^{ed} stemming from changes in the values for σ, γ_4^o and ν_t's is same as that observed in experiment 1.

The switching algebraic sign of r_4^o and the low speed of convergence which was experienced in preliminary estimation before experiment 1, which is deleted in this paper, did not occur in this experiment. In the preliminary experiment γ_4^o, together with σ, was varied, but in this experiment ν_t's were allowed to vary together with γ_4^o and σ. Hence, allowing ν_t's to vary caused γ_4^o to have a stable sign and also eliminated the problem of convergence speed.

The fitting of the cross sectional estimates for μ^e, μ^d and μ^{ed} to the observed values in 1964 is fairly good. However, the estimates were not as good for the observed data for 1961 to 1963. In particular, μ^d and μ^e are underestimated.

obtained by making use of the parameters estimated

Experiment 3.

A

	v	w	r_4	Φ
1964	45.51	47.4		462.33
63	42.63	44.1	12000 ↓	↓
62	40.35	41.7	11887.5	447.71
61	37.48	38.4		

$\sigma = 0.188$、 $r_2 = 150$、 $r_3 = 2$、 $r_5 = -6700$、 $r_4^0 = 40$、 $h = 0.333$

B

	v	w	r_5	Φ
64	45.48	47.4		431.32
63	42.63	44.1	−6700 ↓	↓
62	40.35	41.7	−6962.5	356.12
61	37.48	38.4		

$\sigma = 0.17675$、 $r_2 = 150$、 $r_3 = 2$、 $r_4^0 = 25.3$、 $h = 0.333$

C

	v	w	r_5	Φ
64	45.51	47.4		462.33
63	42.63	44.1	−6700 ↓	↓
62	40.35	41.7	−6762.5	445.64
61	37.48	38.4		

$\sigma = 0.188$、 $r_2 = 150$、 $r_3 = 2$、 $r_4 = 12000$、 $r_4^0 = 40$、 $h = 0.333$

D

	v	w	r_4	Φ
64	45.48	47.4		431.32
63	42.63	44.1	12000 ↓	↓
62	40.35	41.7	11775	430.33
61	37.48	38.4		

$\sigma = 0.17675$、 $r_2 = 150$、 $r_3 = 2$、 $r_5 = -6700$、 $r_4^0 = 25.3$、 $h = 0.333$

This experiment examined the effect of varying γ_4 and γ_5 respectively. The results are shown for cases A through D. In cases A and C we used estimates for ν_t obtained in experiment 1, while in cases B and D those estimates obtained in experiment 2 were used. Other parameters except for γ_4 and γ_5 are held constant during the four years, 1961 through 1964, the values of which are shown in the table in experiment 1, and are common to all cases A through D (with the exception of σ).

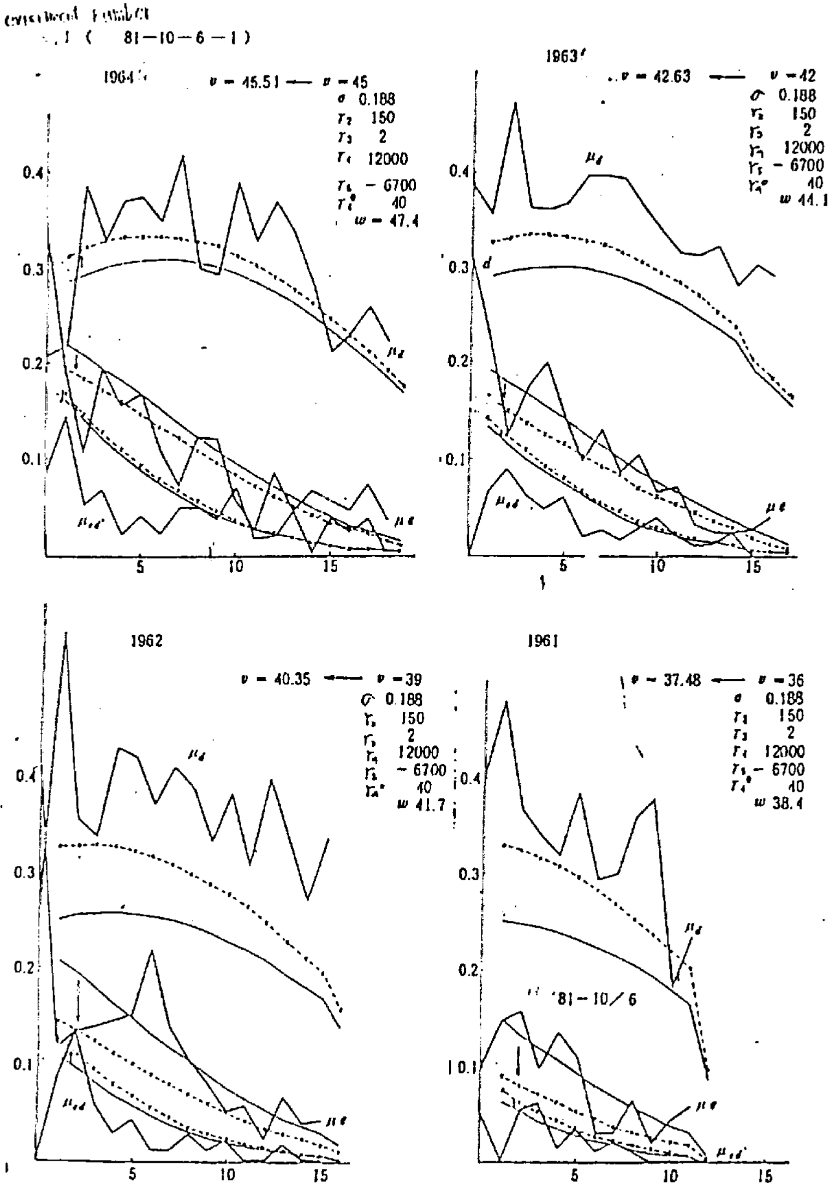
More specifically, in cases A and D γ_4 is allowed to vary, while in cases B and C γ_5 is, and in cases A and C the value of σ is the one used in experiment 1, and in cases B and D the value of σ estimated in experiment 2 is employed. The purpose of using alternative values for ν and σ is to check if the estimates for γ_4 or γ_5, respectively, are affected by slight differences in the values of parameters which are held constant for the estimation of γ_4 or γ_5.

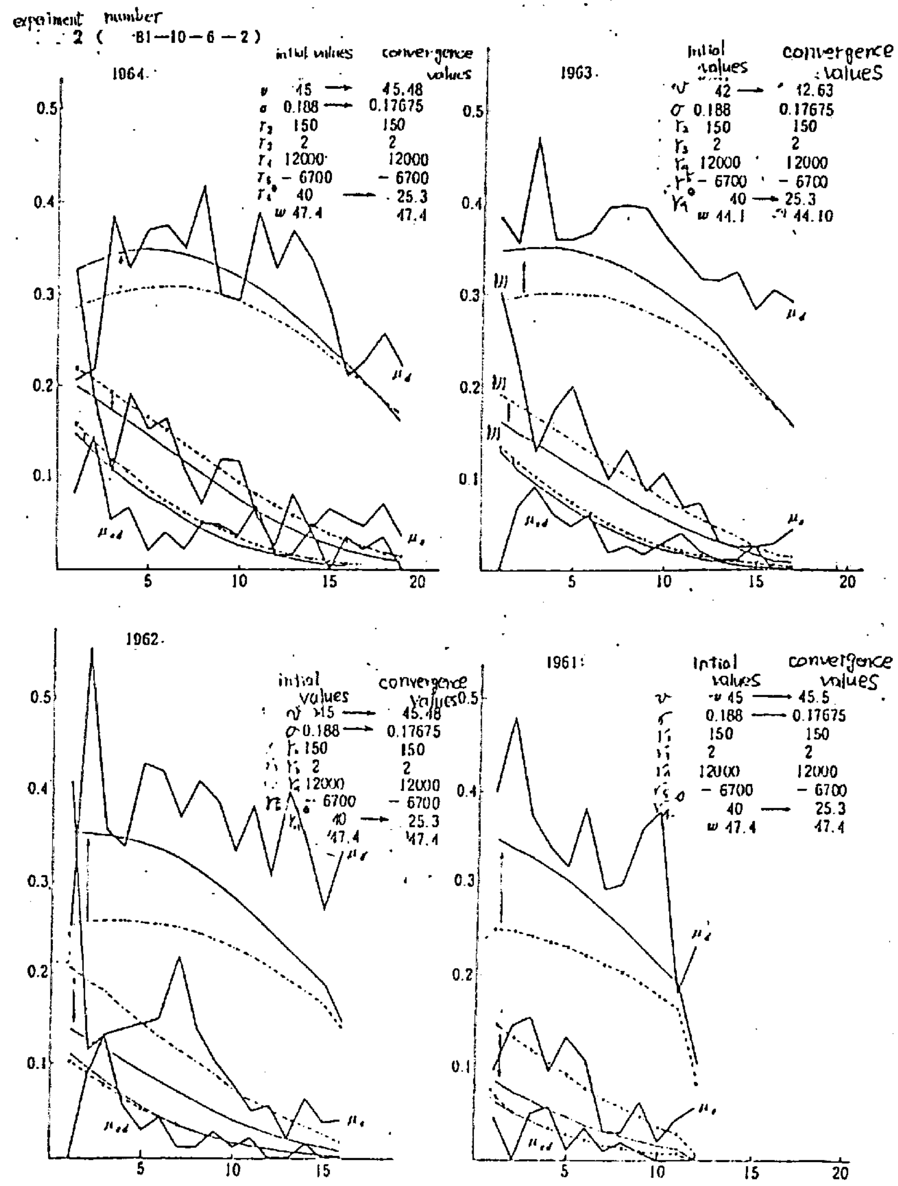
As shown in the table, the result was favorable, that is, the estimates of γ_4 and γ_5 were fairly stable for cases A and D and cases B and C respectively.

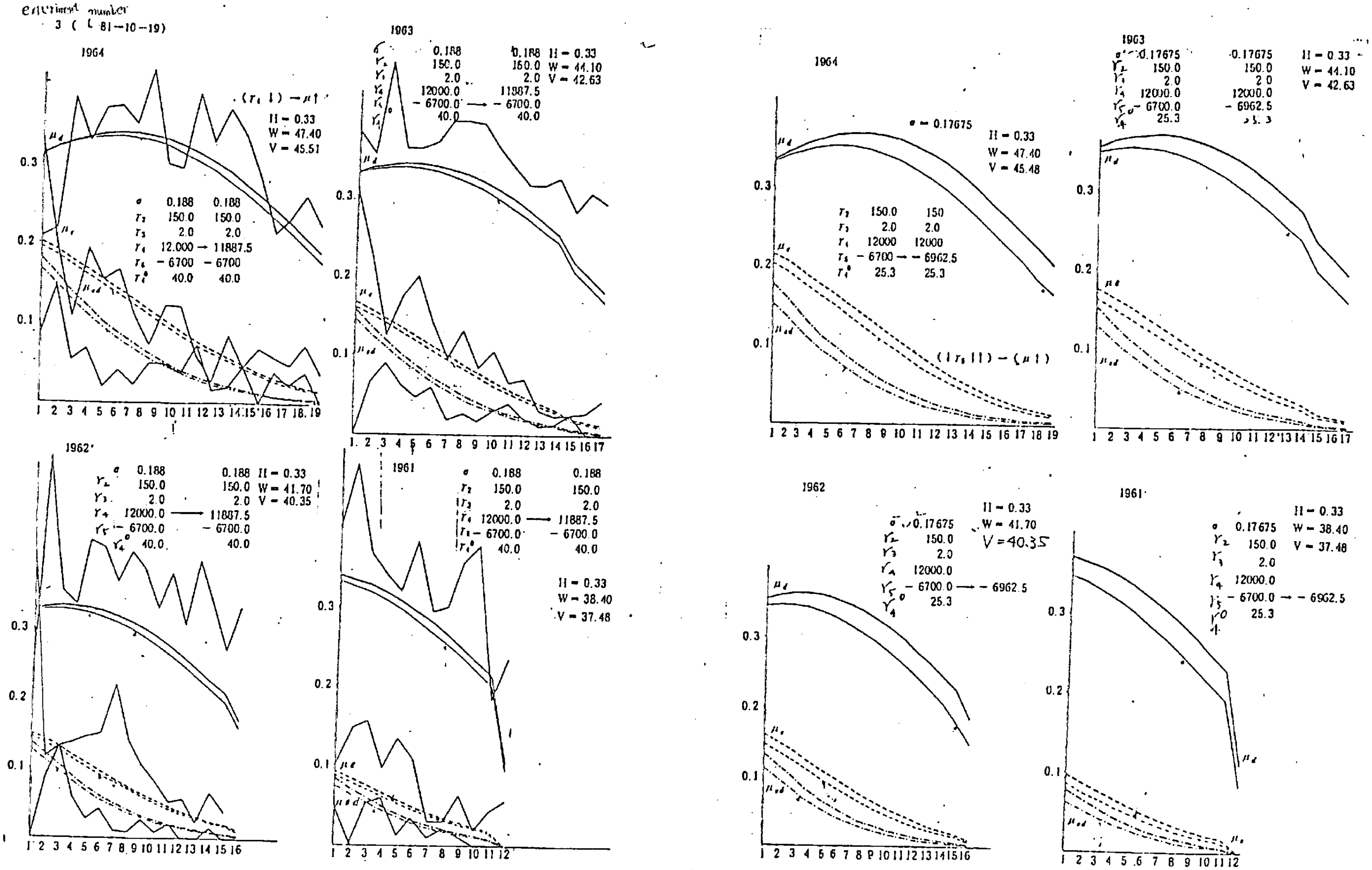
Experiment 4.

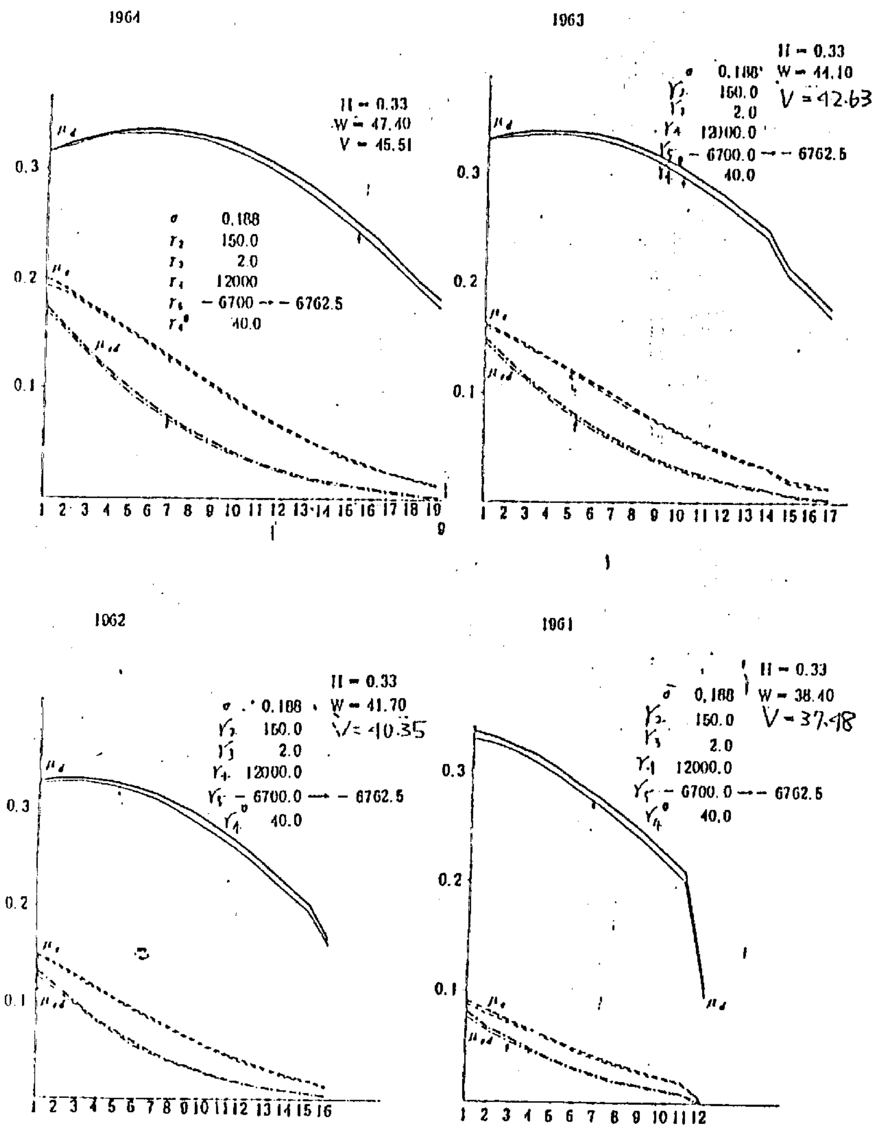
	64	63	62	61年	
v	45.51	42.63	40.35	37.48	
w	47.4	44.10	41.70	38.4	
σ	0.188	0.188	0.188	0.188	Φ
r_2	150	150	150	150	458.42
r_3	2	2	2	2	↓
r_4	12000→12112.5	12000→11875	12000→11887.5	12000→11875	430.87
r_4^0	40	40	40	40	
r_5	−6700	−6700	−6700	−6700	
h	0.333	0.333	0.333	0.333	

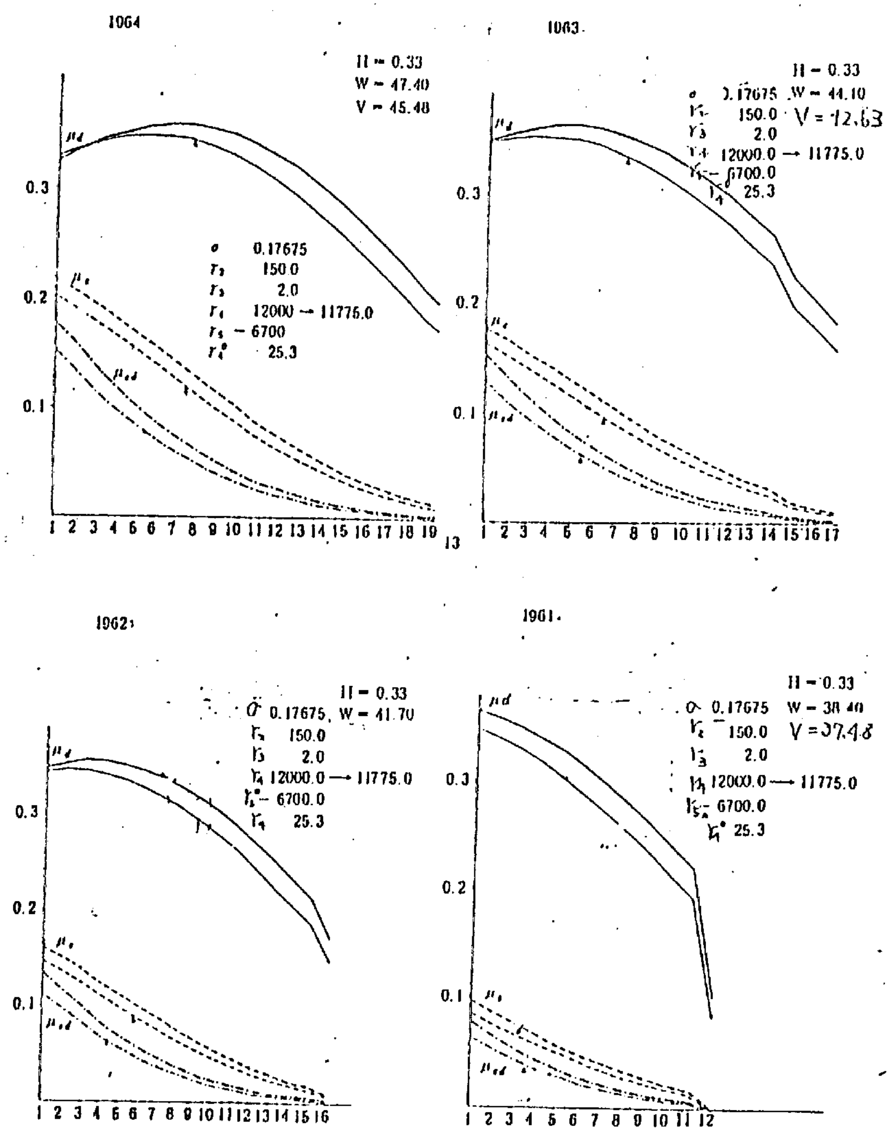
The purpose of this experiment was to check if the estimates for γ_4 are stable for the four years, 1961 through 1964. Hence we allow the estimates for γ_4 to vary from year to year in contrast to experiments 1 through 3. In those experiments estimates for γ_4, as well as other preference parameters, γ_2, γ_3, γ_5, γ_4^0 and σ, were obtained by using the postulate or a priori information that preference parameters should be stable over time.


experiment number
1 (81—10—6—1)
1964
v = 45.51 ⟵ v = 45
σ 0.188
r2 150
r3 2
r4 12000
r5 − 6700
r6° 40
w = 47.4
1963
v = 42.63 ⟵ v = 42
σ 0.188
r2 150
r3 2
r4 12000
r5 − 6700
r6° 40
w 44.1
1962
v = 40.35 ⟵ v = 39
σ 0.188
r2 150
r3 2
r4 12000
r5 − 6700
r6° 40
w 41.7
1961
v = 37.48 ⟵ v = 36
σ 0.188
r2 150
r3 2
r4 12000
r5 − 6700
r6° 40
w 38.4
81−10/6
μd
μe
μed

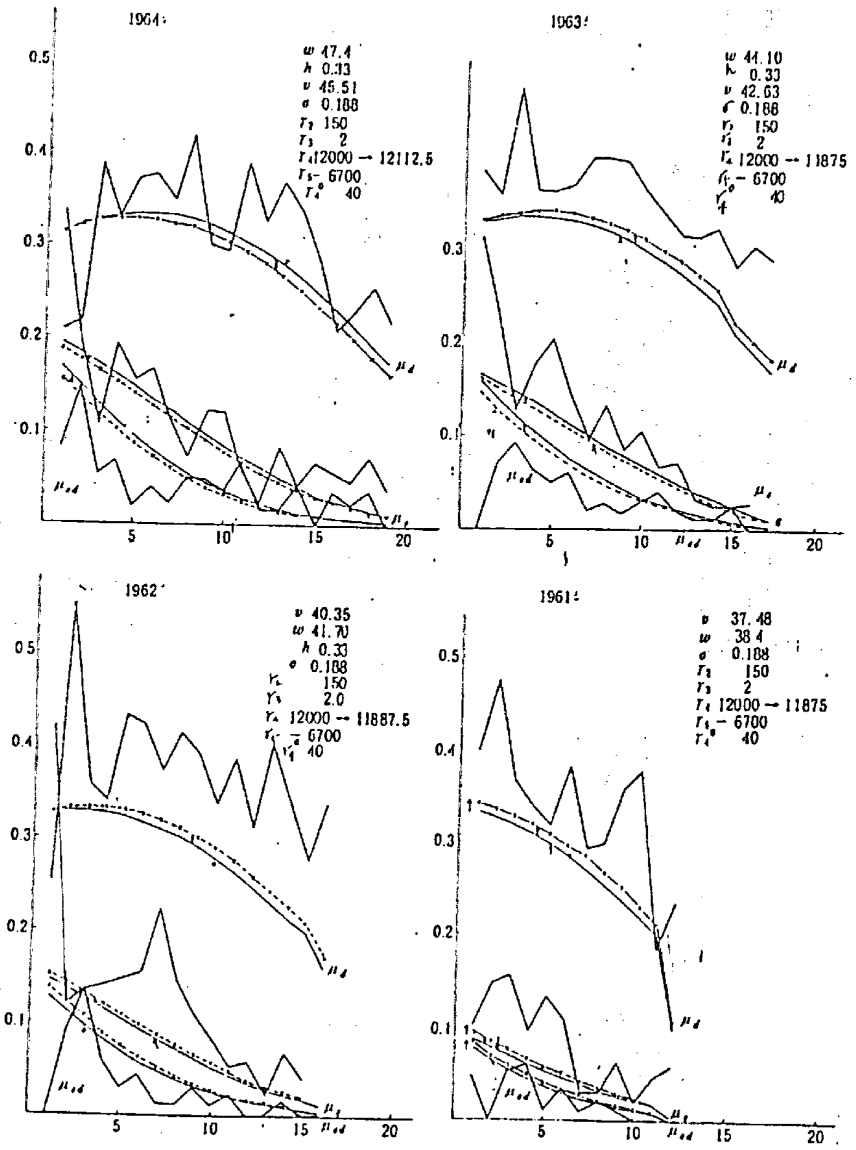

experiment number
2 (81—10—6—2)
1964
initial values
convergence values
v 45 → 45.48
σ 0.188 → 0.17675
r2 150 150
r3 2 2
r4 12000 12000
r5 − 6700 − 6700
r6° 40 → 25.3
w 47.4 47.4
1963
v 42 → 42.63
σ 0.188 0.17675
r2 150 150
r3 2 2
r4 12000 12000
r5 − 6700 − 6700
r6° 40 → 25.3
w 44.1 44.10
1962
v 45 → 45.48
σ 0.188 → 0.17675
r2 150 150
r3 2 2
r4 12000 12000
r5 − 6700 − 6700
r6° 40 → 25.3
w 47.4 47.4
1961
v 45 → 45.5
σ 0.188 → 0.17675
r2 150 150
r3 2 2
r4 12000 12000
r5 − 6700 − 6700
r6° 40 → 25.3
w 47.4 47.4
μd
μe
μed


experiment number
3 (81-10-19)
1964
1963
1962
1961
H = 0.33
W = 47.40
V = 45.51
W = 44.10
V = 42.63
W = 41.70
V = 40.35
W = 38.40
V = 37.48
V = 45.48
σ = 0.17675
(r_4 1) → μ1
(1 r_5 11) → (μ 1)

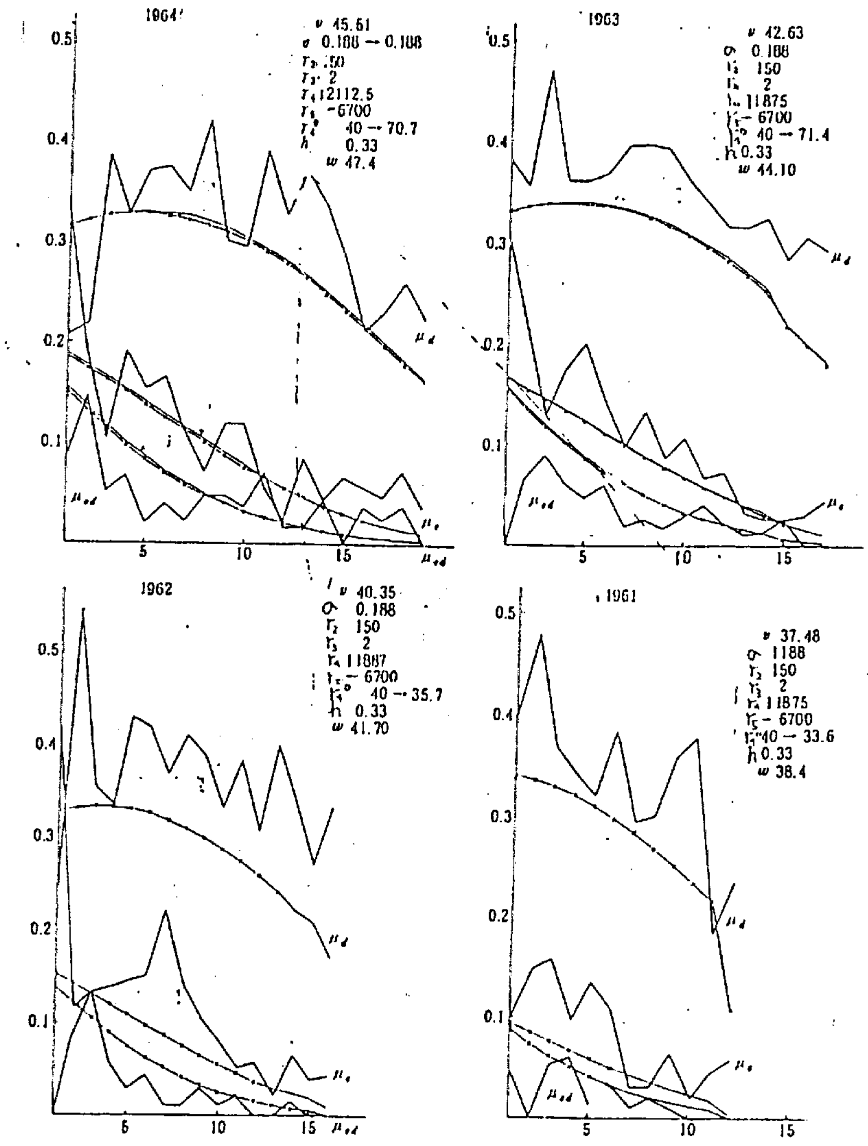

1964
H = 0.33
W = 47.40
V = 45.51
σ 0.188
r₂ 150.0
r₃ 2.0
r₄ 12000
r₅ − 6700 → − 6762.5
r₄° 40.0
1963
H = 0.33
σ 0.188 W = 44.10
V = 42.63
150.0
2.0
12000.0
− 6700.0 → − 6762.5
40.0
1962
H = 0.33
σ 0.188 W = 41.70
V = 40.35
150.0
2.0
12000.0
− 6700.0 → − 6762.5
40.0
1961
H = 0.33
σ 0.188 W = 38.40
V = 37.48
150.0
2.0
12000.0
− 6700.0 → − 6762.5
40.0


1964
H = 0.33
W = 47.40
V = 45.48
σ 0.17675
r₂ 150.0
r₃ 2.0
r₄ 12000 → 11775.0
r₅ − 6700
r₄° 25.3
1963
H = 0.33
σ 0.17675 W = 44.10
V = 42.63
150.0
2.0
12000.0 → 11775.0
6700.0
25.3
1962
H = 0.33
σ 0.17675, W = 41.70
150.0
2.0
12000.0 → 11775.0
6700.0
25.3
1961
H = 0.33
σ 0.17675 W = 38.40
V = 37.48
150.0
2.0
12000.0 → 11775.0
6700.0
25.3

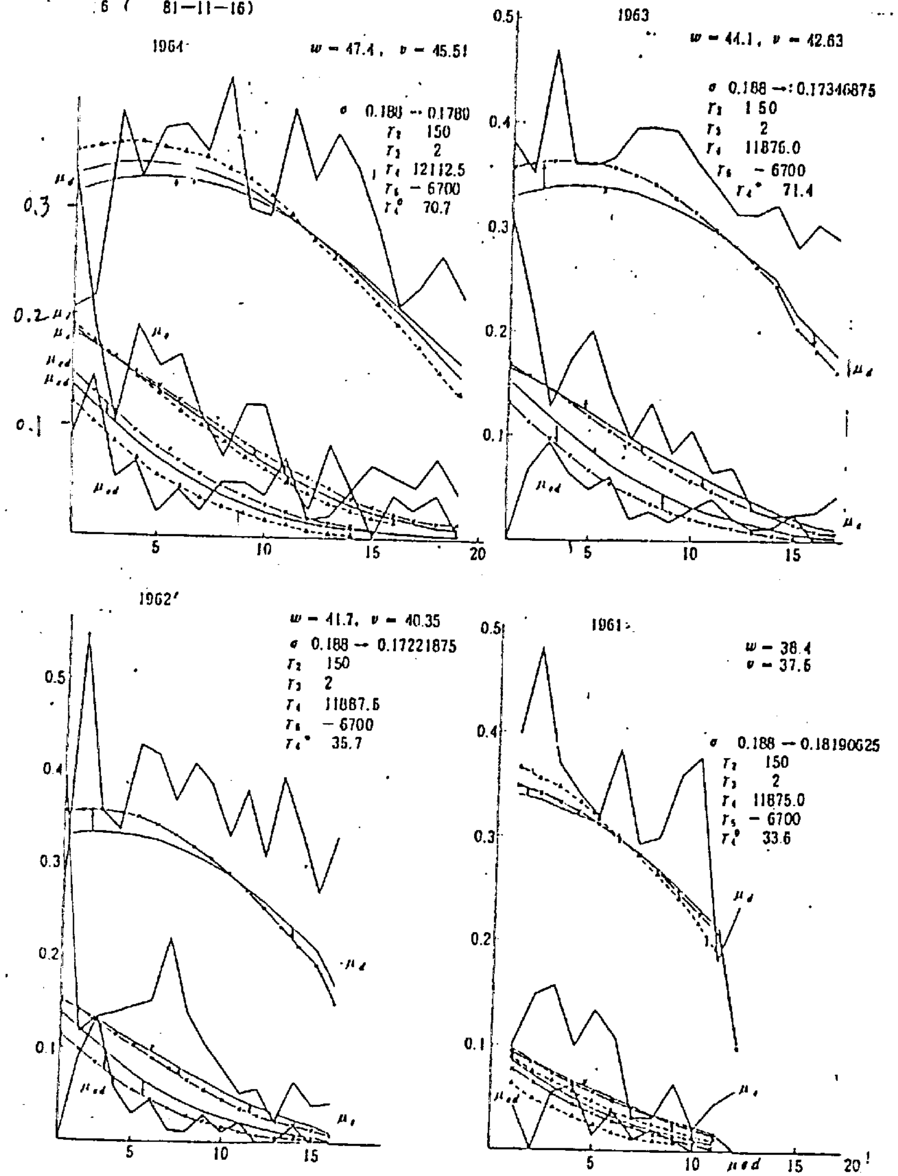
experiment 4 number ('81-11-07)



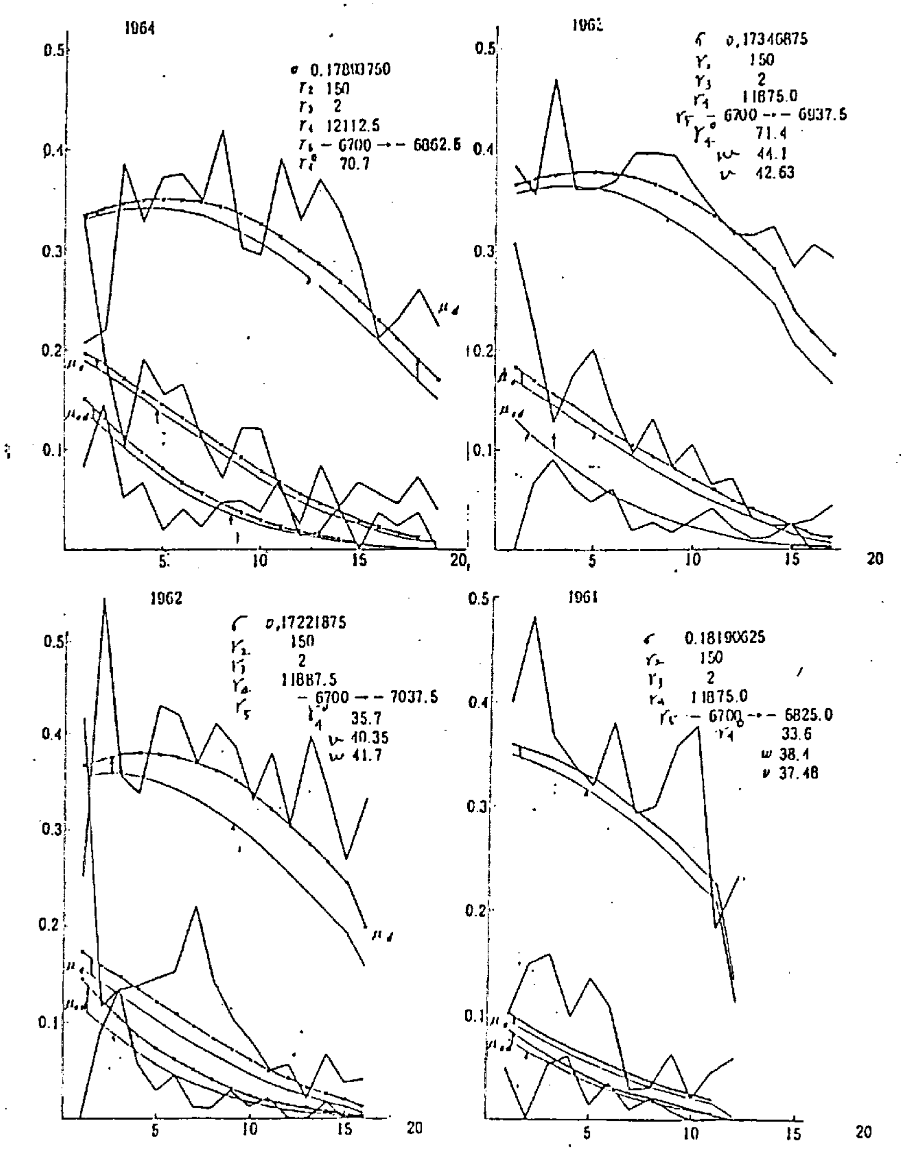
5 ('81-11-12)



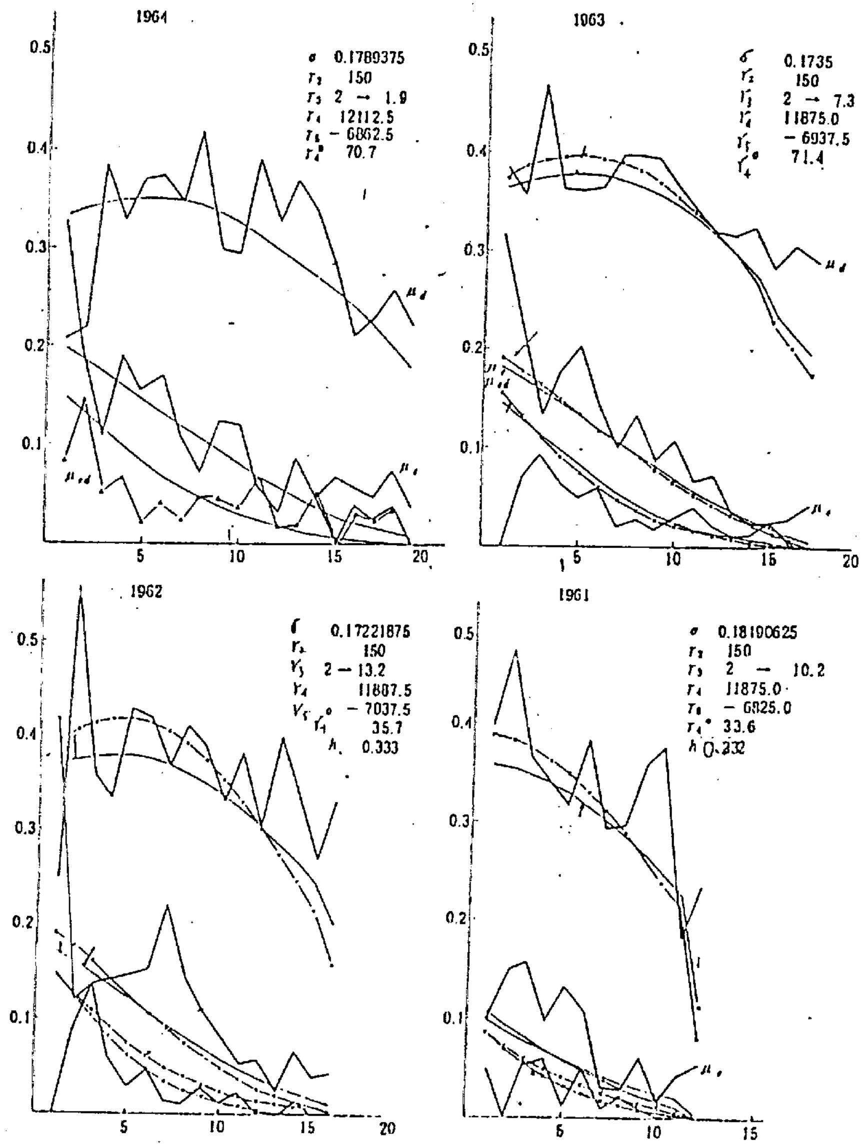
experiment number .6 (81—11—16)



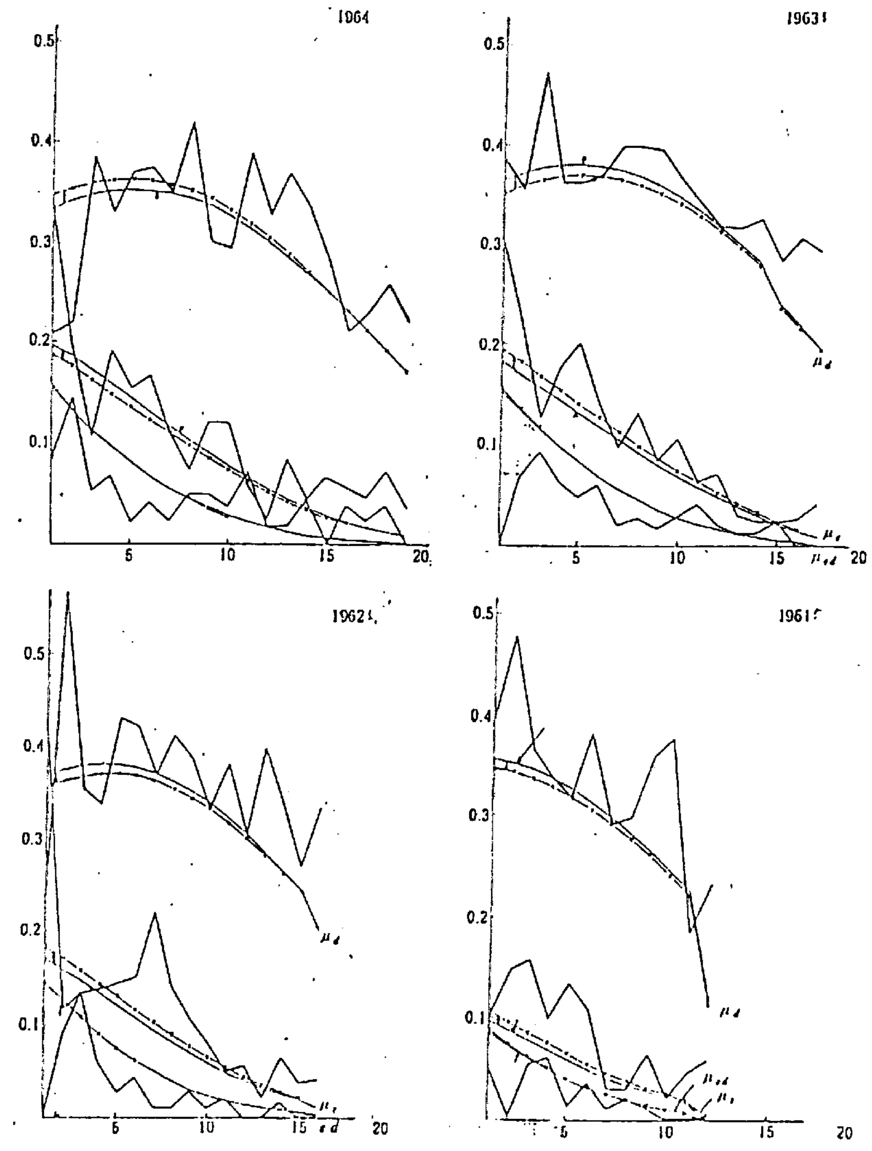
experiment number .7 (381—12—1)

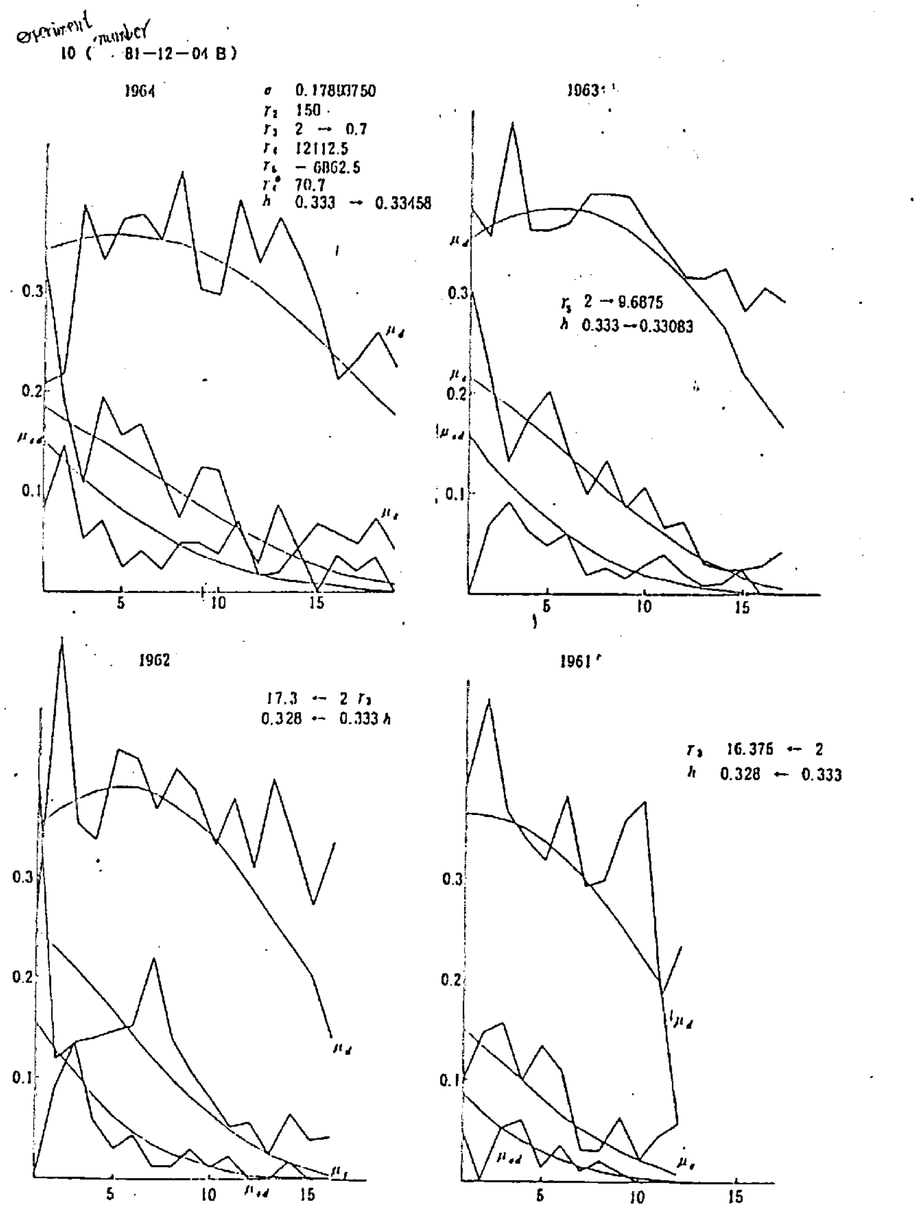


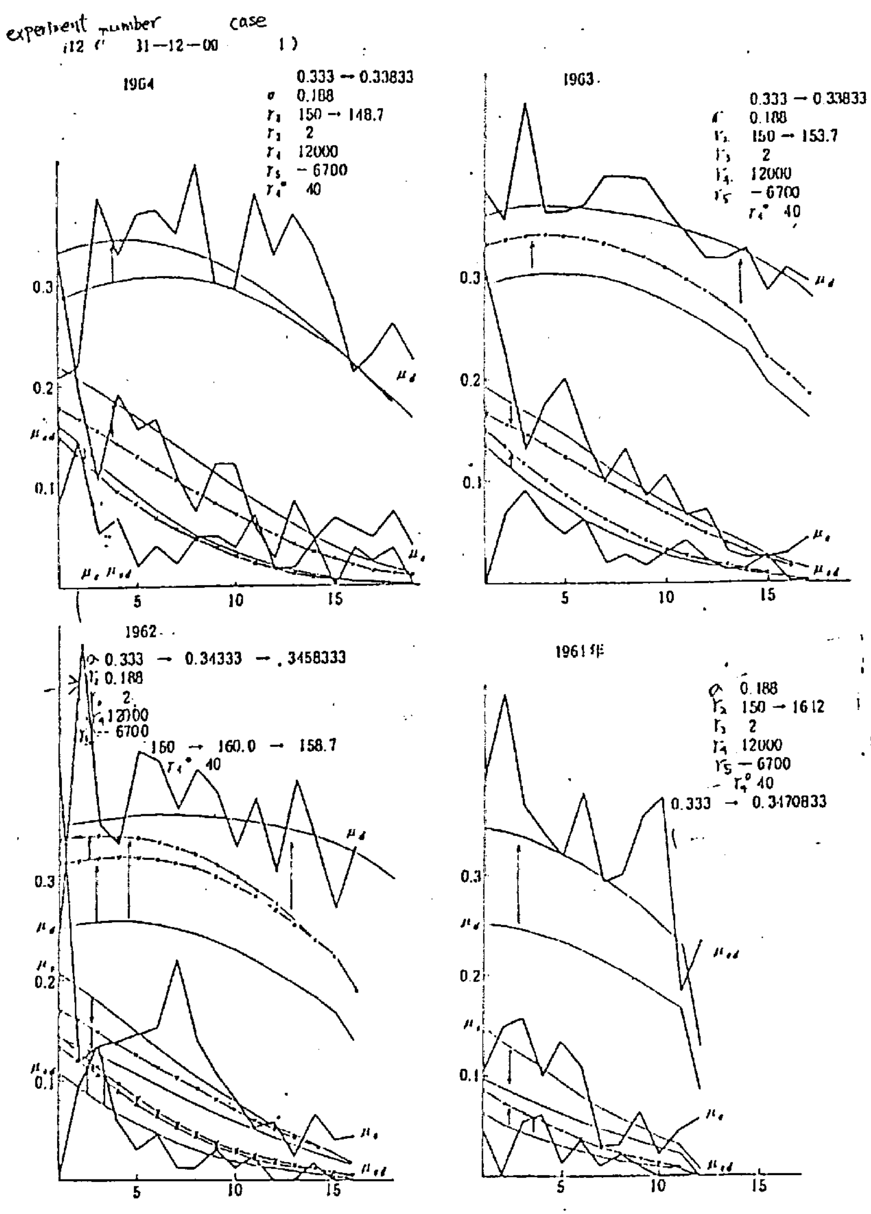
experiment number 8 (81—12—03)

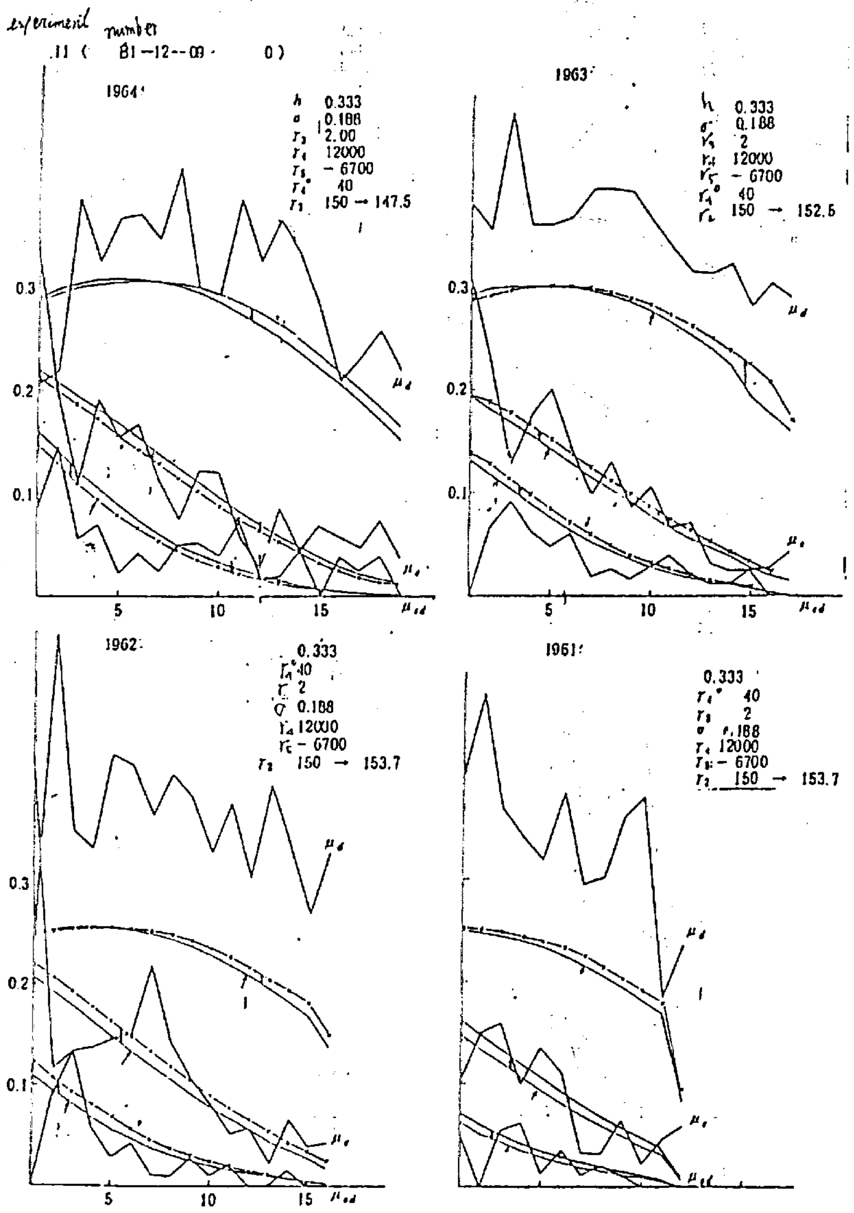


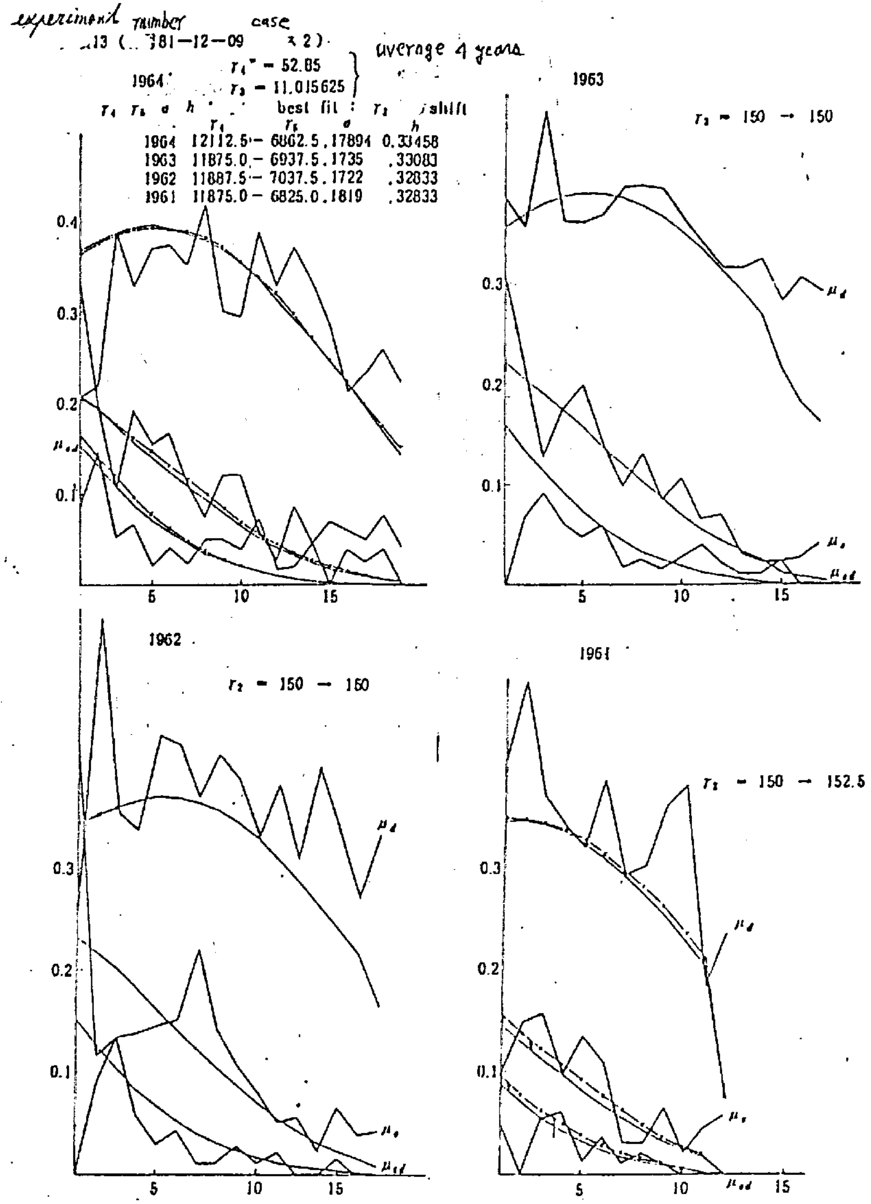
experiment number 19 (81—12—04 A)

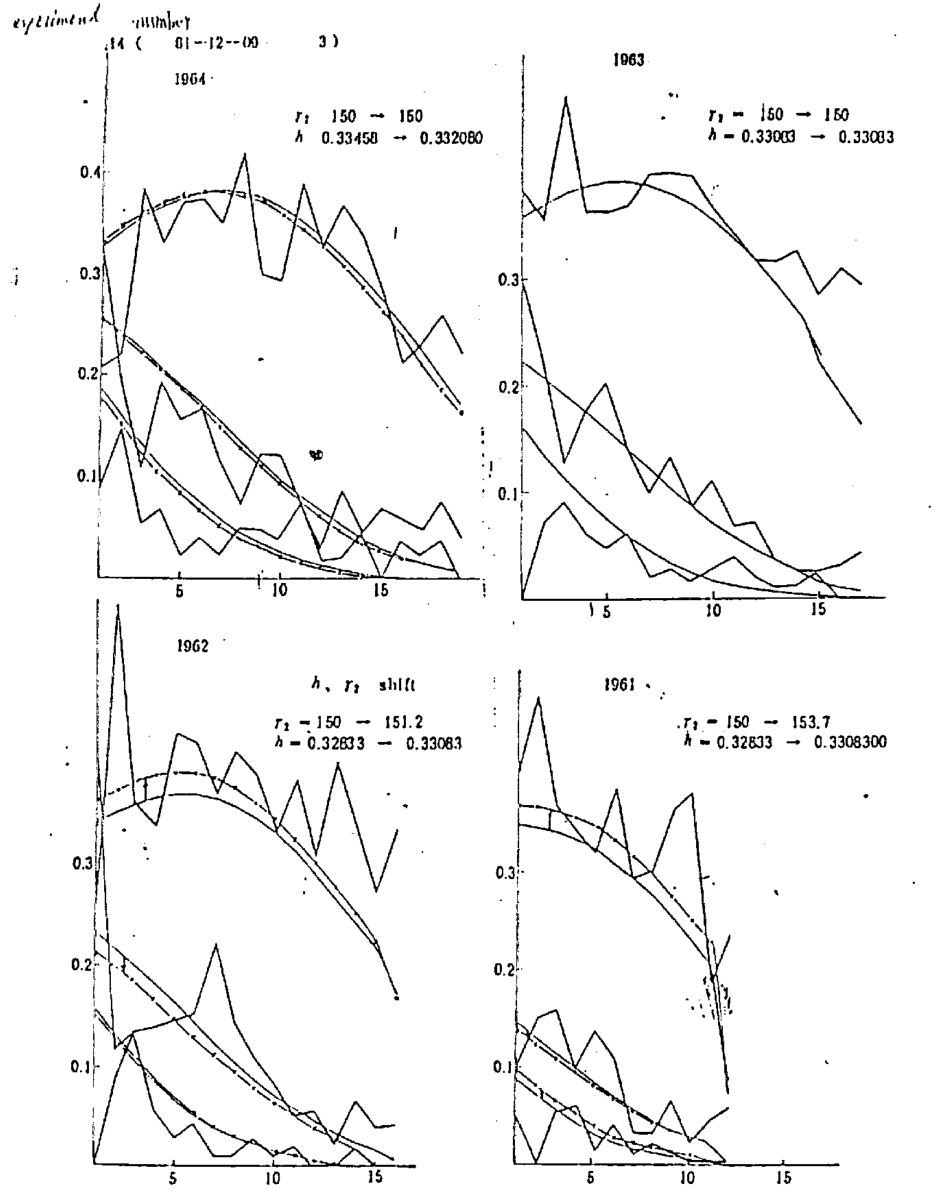


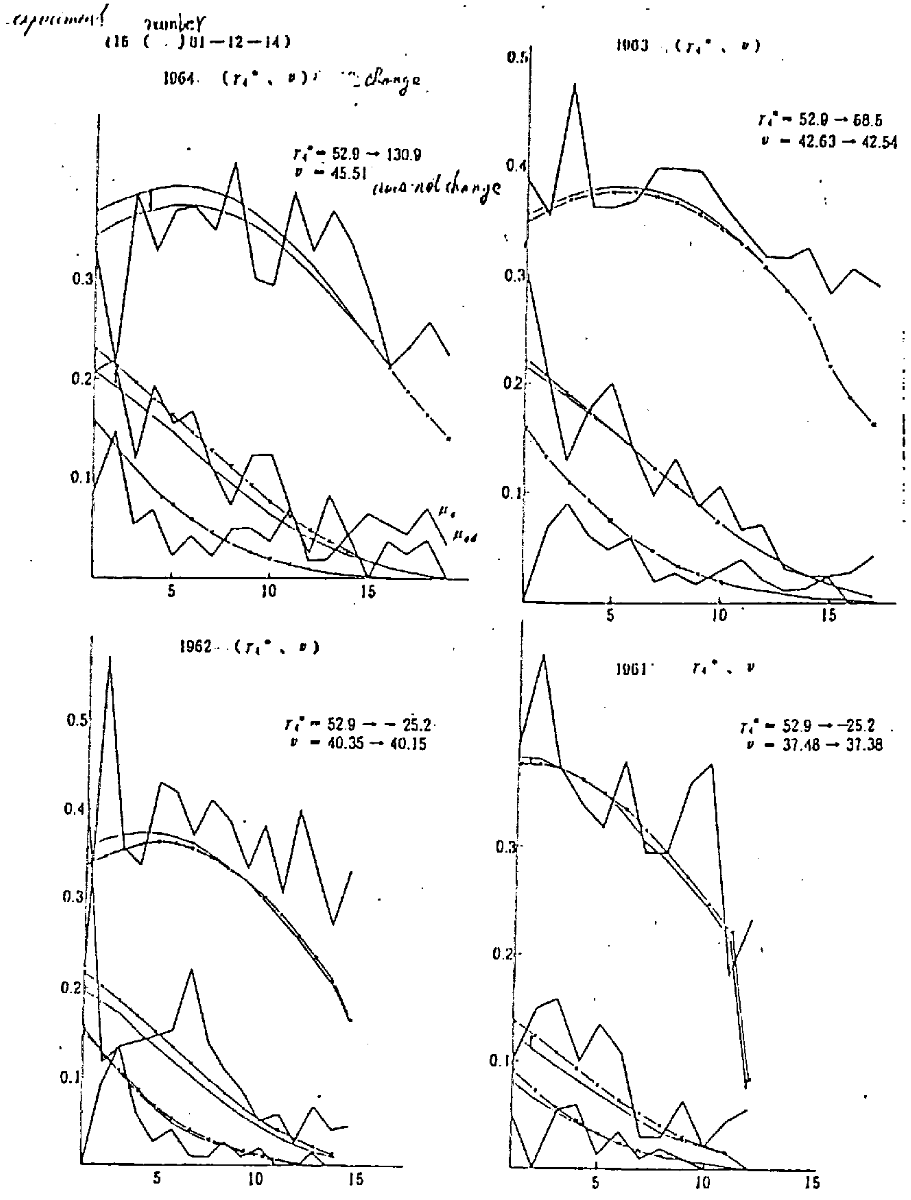

experiment number
10 (. 81—12—04 B)
1964
σ 0.17893750
r_2 150
r_3 2 → 0.7
r_4 12112.5
r_5 − 6862.5
r_4° 70.7
h 0.333 → 0.33458
1963
r_3 2 → 9.6875
h 0.333 → 0.33083
1962
17.3 ← 2 r_3
0.328 ← 0.333 h
1961
r_3 16.375 ← 2
h 0.328 ← 0.333
0.1
0.2
0.3
5
10
15
μ_d
μ_e
μ_{ed}


experiment number case
12 (31—12—00 1)
1964
0.333 → 0.33833
σ 0.188
r_2 150 → 148.7
r_3 2
r_4 12000
r_5 − 6700
r_4° 40
1963
0.333 → 0.33833
σ 0.188
r_2 150 → 153.7
r_3 2
r_4 12000
r_5 − 6700
r_4° 40
1962
σ 0.333 → 0.34333 → .3458333
r_2 0.188
r_3 2
r_4 12000
r_5 − 6700
150 → 160.0 → 158.7
r_4° 40
1961
σ 0.188
r_2 150 → 161.2
r_3 2
r_4 12000
r_5 − 6700
r_4° 40
0.333 → 0.3470833
0.1
0.2
0.3
5
10
15
μ_d
μ_e
μ_{ed}

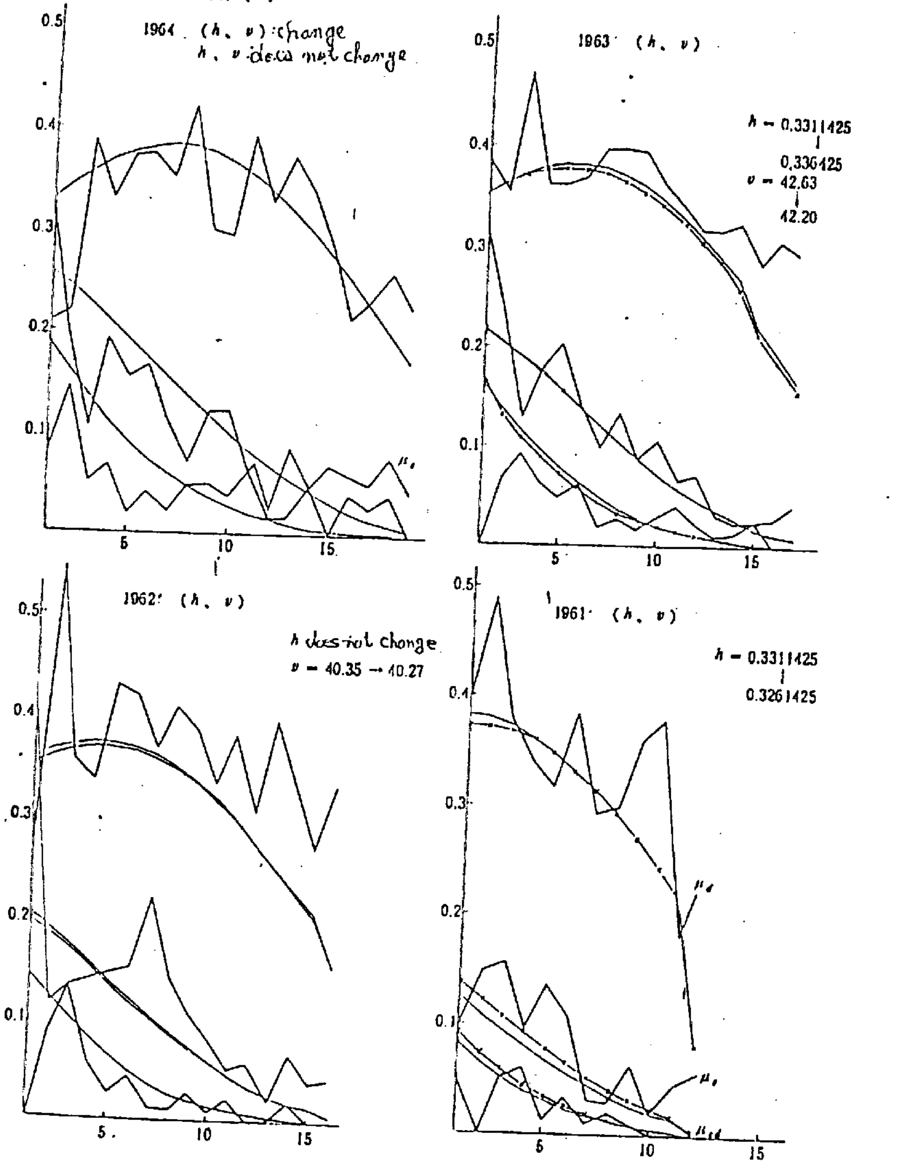

experiment number
.11 (81-12-09 0)
1964
h 0.333
σ 0.188
r_3 2.00
r_4 12000
r_5 − 6700
r_4^* 40
r_2 150 → 147.5
1963
h 0.333
σ 0.188
r_3 2
r_4 12000
r_5 − 6700
r_4^* 40
r_2 150 → 152.5
1962
0.333
r_4^* 40
r_3 2
σ 0.188
r_4 12000
r_5 − 6700
r_2 150 → 153.7
1961
0.333
r_4^* 40
r_3 2
σ 0.188
r_4 12000
r_5 − 6700
r_2 150 → 153.7
μ_d
μ_e
μ_{ed}


experiment number case
13 (81-12-09 × 2)
r_4^* = 52.85
r_3 = 11.015625
average 4 years
1964
r_4 r_5 σ h best fit : r_2 shift
1964 12112.5 − 6862.5 .17894 0.33458
1963 11875.0 − 6937.5 .1735 .33083
1962 11887.5 − 7037.5 .1722 .32833
1961 11875.0 − 6825.0 .1819 .32833
1963
r_2 = 150 → 150
1962
r_2 = 150 → 150
1961
r_2 = 150 → 152.5
μ_d
μ_e
μ_{ed}

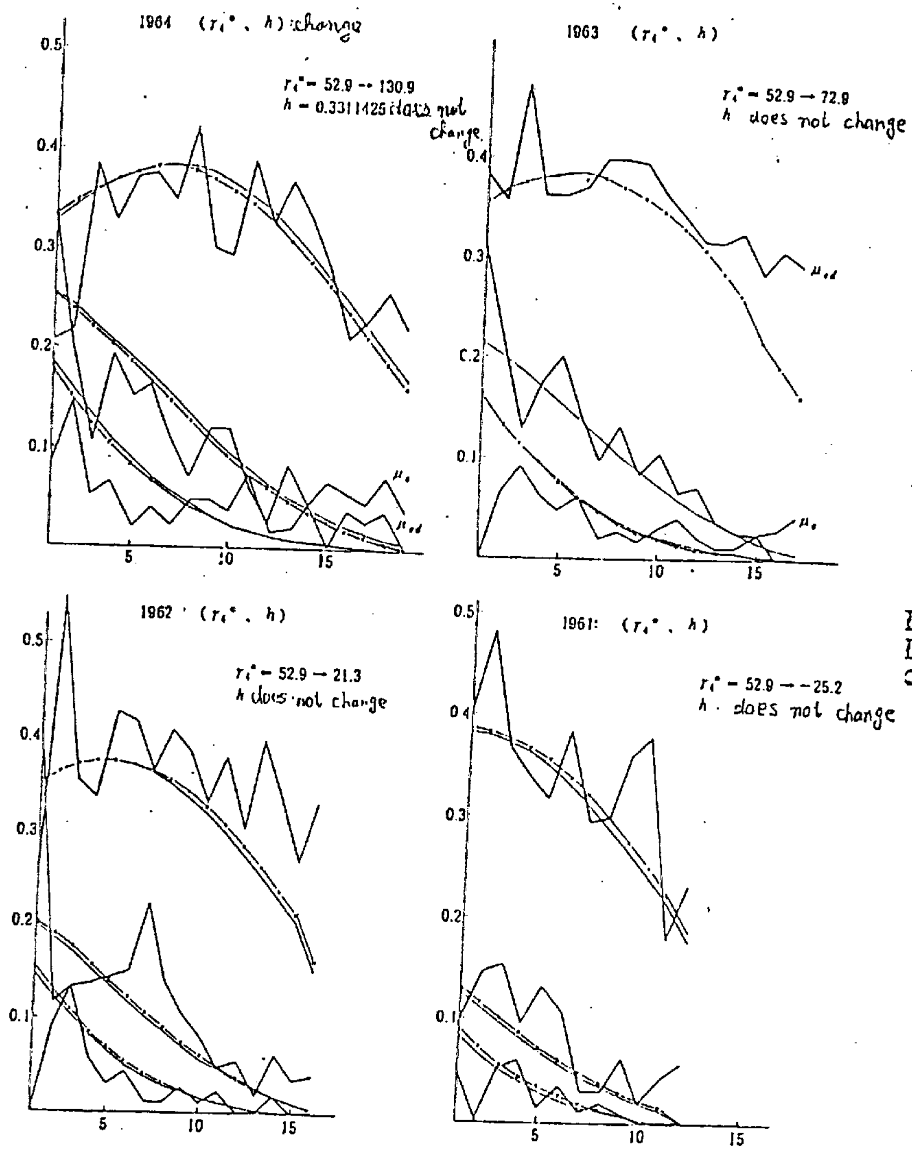

experiment number
14 (81-12-09 3)
1964
r_2 150 → 150
h 0.33458 → 0.332080
1963
r_2 = 150 → 150
h = 0.33083 → 0.33083
1962
h, r_2 shift
r_2 = 150 → 151.2
h = 0.32833 → 0.33083
1961
r_2 = 150 → 153.7
h = 0.32833 → 0.3308300

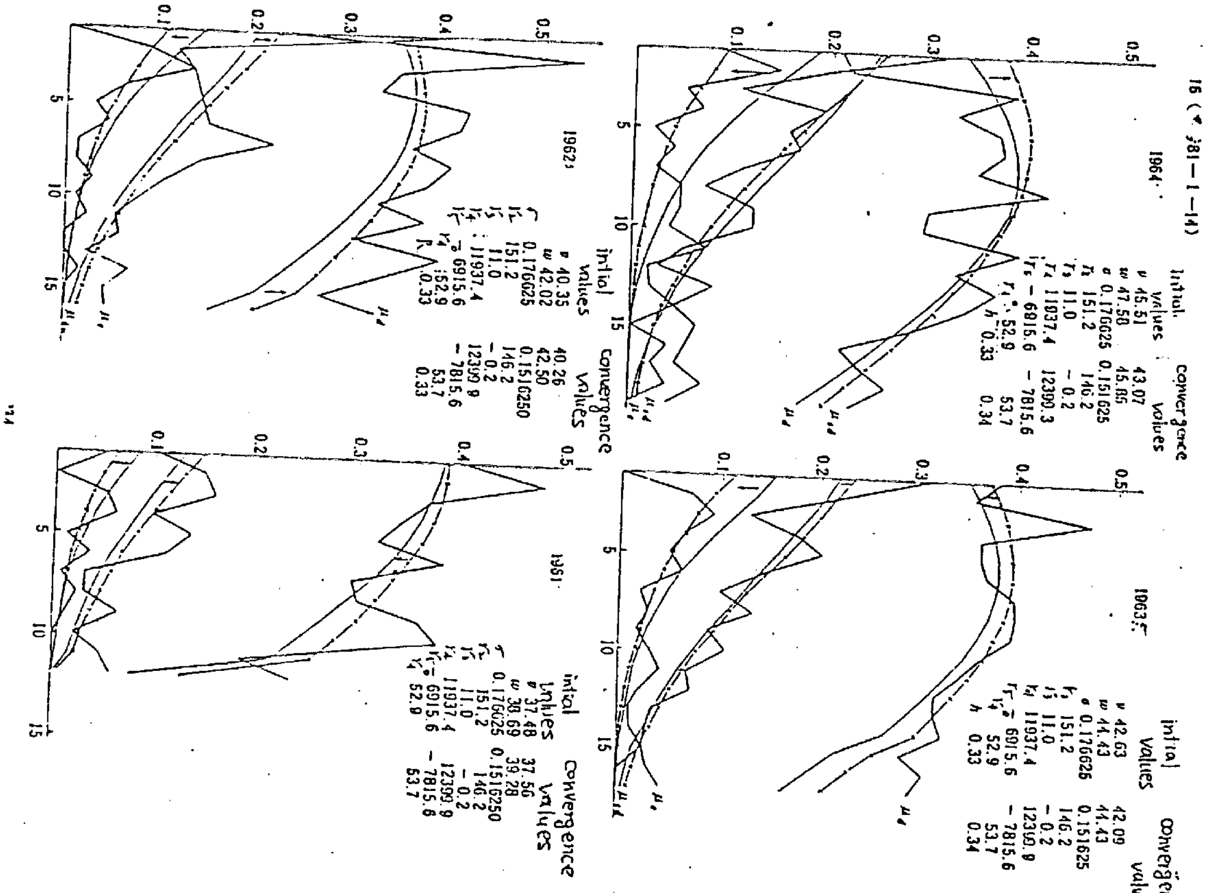

experiment number
(15 ( )81-12-14)
1964 (r_4^*, v) change
r_4^* = 52.9 → 130.9
v = 45.51 does not change
1963 (r_4^*, v)
r_4^* = 52.9 → 68.6
v = 42.63 → 42.54
1962 (r_4^*, v)
r_4^* = 52.9 → −25.2
v = 40.35 → 40.15
1961 r_4^*, v
r_4^* = 52.9 → −25.2
v = 37.48 → 37.38

experiment number 16 (81—12—14) (a)



experiment number 16 (81—12—14) (b)




16 (ゞ 381—1—14)
1964
initial values / convergence values
ν 45.51 43.07
w 47.50 45.85
σ 0.176625 0.151625
r₁ 151.2 146.2
r₂ 11.0 −0.2
r₄ 11937.4 12399.3
r₅ −6915.6 −7815.6
r₄ 52.9 53.7
h 0.33 0.34
1963
initial values / convergence values
ν 42.63 42.09
w 44.43 44.43
σ 0.176626 0.151625
r₁ 151.2 146.2
r₂ 11.0 −0.2
r₄ 11937.4 12399.9
r₅ 6915.6 −7815.6
r₄ 52.9 53.7
h 0.33 0.34
1962
initial values / convergence values
ν 40.35 40.26
w 42.02 42.50
σ 0.176625 0.1516250
r₁ 151.2 146.2
r₂ 11.0 −0.2
r₄ 11937.4 12399.9
r₅ 6915.6 −7815.6
r₄ 52.9 53.7
h 0.33 0.33
1961
initial values / convergence values
ν 37.48 37.56
w 38.69 39.28
σ 0.176625 0.1516250
r₁ 151.2 146.2
r₂ 11.0 −0.2
r₄ 11937.4 12399.9
r₅ 6915.6 −7815.6
r₄ 52.9 53.7

Numerical values for parameters which are held constant for the estimation of γ_4 are shown in table (in experiment 1). Estimates obtained for γ_4 are indicated with an arrow. The estimates for the four years strikingly resemble each other, but on the other hand, the fitting of theoretical value to the observed value of μ^e, μ^d and μ^{ed} is not good. Hence, there remain considerable discrepancies between observed and estimated values as shown in Fig( ) and it is therefore necessary to change the numerical values attached to the preference parameters, γ_2, γ_3, γ_5, σ and γ_4^0 among those parameters we first reestimate σ and γ_4^0.

Experiment 5.

the initial values for w , v and h

	1964	1963	1962	1961
w	47.40	44.10	41.70	38.40
v	45.51	42.63	40.35	37.48

$h = 1/3$

	σ	γ_2	γ_3	γ_4	γ_5	γ_4^0	Φ
1964	0.188 → 0.188	150	2	12112.5	− 6700	40 → 70.7	431.05
1963	0.188 → 0.188	150	2	11875.5	− 6700	40 → 71.4	↓
1962	0.188 → 0.188	150	2	11887.5	− 6700	40 → 35.7	
1961	0.188 → 0.188	150	2	11875	− 6700	40 → 33.6	429.66

In this experiment σ and γ_4^0 are allowed to vary simultaneously. For the value of γ_4, we employ the better one obtained in experiment 4.

Estimates for σ and γ_4^0 are shown, together with values for the parameters which are held constant, in table ( ). σ and γ_4^0 are allowed to vary among the years 1961 through 1964. Results show that estimates for σ are almost same as the initial values, but that estimates for γ_4^0 in 1961 and 1962 and in 1963 and 1964 respectively resemble each other. In terms of fitting, it is not clear there has been any significant improvement.

Experiment 6.

	64	63	62	61	
r_5	− 6700	− 6700	− 6700	− 6700	
r_4	12112.5	11875.0	11887.5	11875.0	Φ
r_4^0	70.7	71.4	35.7	33.6	429.66
r_3	2.0	2.0	2.0	2.0	↓
w	47.4	44.1	41.7	38.4	389.80
v	45.51	42.63	40.35	37.48	
σ	0.188 ↓ 0.1789375	0.188 ↓ 0.17346875	0.188 ↓ 0.17221875	0.188 ↓ 0.18190625	

One point to be noticed among the results of experiment 5 is the apparent constancy of the estimates of σ over time, that is, they are the same for three decimals. However, the estimates(or convergence values)seem to be affected by the numerical values for the other parameters which are held constant for estimation. In order to check this point, we tentatively replace the initial valucs for γ_4^0 used in experiment 5 by the ones estimated in experiment (5).

We allow σ only to vary and let the other parameters be given as shown in the table. Estimated values for σ are stable over time, that is, the estimates closely resemble each other and differences between initial and convergence values are small for each year. This is a favorable result that may ensure the stability of the preference parameters over time.

However, on the other hand, the fitting of theoretical values to observed values is not so easily improved, as shown in Fig (   ). In particular, there was systematic underestimation for the three years, 1961 through 1963.

Experiment 7.

	19 64	63	62	61
r_2	150	(左と同じ)		
r_3	2			
r_4	11875.0	←		11875.0
r_5	− 6700 ↓ − 6862.5	− 6700 ↓ − 6937.5	− 6700 ↓ − 7037.5	− 6700 ↓ − 6825.0
r_4^0	70.7	71.4	35.7	33.6
σ	0.17893750	0.17346875	0.17221875	0.18190625
v	45.51	42.63	40.35	37.48
w	47.40	44.10	41.70	38.40

In the prvious experiments, 1 through 6, we did not allow γ_5 to vary. Here, γ_5 is allowed to vary in order to improve the fitting for 1961 through 1963. In this experiment, we do not use the a priori information that γ_5 should be of the same value during the four years, 1961 through 1964, because we want to test the stability of estimated values for γ_5 among the years.

	1961	62	63	64
γ_5	−6700 ↓ −6825.0	−6700 −7037.5	−6700 −6937.5	−6700 −6862.5

γ_2	γ_3	γ_4	γ_4^0	σ
150	2	1187.50	(33.6, 35.7, 71.4, 70.7)	(0.1819, 0.1722, 0.1735, 0.17[illegible])

v
(37.48, 40.35, 42.63, 45.51)

w
(38.40, 41.70, 44.10, 47.40)

1961 estimates for μ^d and $\hat{\mu}^d$, approach the observed values, while $\hat{\mu}^{ed}$'s exceed the those values. $\hat{\mu}^d$ approaches the observed values but not sufficiently.

1962 $\hat{\mu}^e$ and $\hat{\mu}^d$ sufficiently approach the observed values, but μ^{ed}'s come off the observables.

1963 the same tendency as 1962.

1964 the same as 1962.

It can be seen that the estimates for γ_5 for each year are stable during the estimating period. This is a favorable result for the postalate that preference parameters are stable inter years.

Experiment 8.

	64	63	62	61	
σ	0.1789375	0.1735	0.17221875	0.18190625	
γ_2	150	150	150	150	Φ
γ_3	$2 \rightarrow 1.9$	$2 \rightarrow 7.3$	$2 \rightarrow 13.2$	$2 \rightarrow 10.2$	313.033
γ_4	12112.5	11875.0	11887.5	11875.0	↓
γ_5	−6862.5	−6937.5	−7037.5	−6825.0	284.946
γ_4^0	70.7	71.4	35.7	33.6	

We have not allowed γ_3 to vary in the previous experiment. In the same manner as for γ_5 in experiment 7, we allow γ_3 to vary in this experiment. γ_5 is fixed at the values obtained in experiment 7, and the values for other parameters, except for γ_3, are given as in experiment 7.

	1961	62	63	64
γ_3	10.2	13.2	7.3	1.9

As shown in Fig ( ), the effect of changing the value of γ_3 is remarkable. Considerable improvement in fitting is observed, for the first time, for years 1961 through 1963.

Experiment 9.

	64	63	62	61	
σ	0.1789375	0.1735	0.17221875	0.18190625	
r_2	150	150	150	150	
r_3	2	2	2	2	Φ
r_4	12112.5	11875.0	11887.5	11875.0	313.0334
r_5	−6862.5	−6937.5	−7037.5	−6825.0	↓
r_4''	70.7	71.4	35.7	33.6	310.8691
h	0.333 ↓ 0.33458	0.333 ↓ 0.332	0.333 ↓ 0.332	0.333 ↓ 0.332	

In this experiment, parameter $\bar{h}$ only is allowed to vary. Other parameters are fixed at the values for experiment 7.

1961	62	63	64
0.332*	0.332	0.332	0.335

(* initial values are all 0.333)

The estimates of $\bar{h}$ for four years are stable over time. However, no remarkable improvement can be seen in the fitting of the curves.

Experiment 10.

	64	63	62	61	
r_3	$2 \rightarrow 0.6875$	$2 \rightarrow 9.6875$	$2 \rightarrow 17.3125$	$2 \rightarrow 16.375$	Φ
h	$0.333 \rightarrow 0.33458$	$0.333 \rightarrow 0.33083$	$0.333 \rightarrow 0.32833$	$0.333 \rightarrow 0.32833$	313.0334
r_4	12112.5	11875.0	11887.5	11875.0	↓
r_5	− 6862.5	− 6937.5	− 7037.5	− 6825.0	267.657
r_4^0	70.7	71.4	35.7	33.6	
σ	0.17894	0.1735	0.1722	0.1819	

Taking into account the results obtained in experiment 8 and 9, we allow γ_3 and $\bar{h}$ to vary simultaneously.

Changes in the estimated values for γ_3 from the initial values especially in 1963 and 64, and the estimated values fit the observed values somewhat better.

	1964	63	62	61
γ_3	0.6875	9.6876	17.3125	16.375
$\bar{h}$	0.3346	0.3308	0.3283	0.3283

Experiment 11.

We have not yet allowed γ_2 to vary. In this experiment, γ_2 only is varied, other parameters being fixed at the initial values given in experiment 1. Estimated values for γ_2 in years 1961 through 1964 are as follows:

1961	62	63	64
153.7	153.7	152.7	147.5

Overtime, the estimates are fairly stable. The results of fitting are fairly good for the 1964 data but underestimates were obtained for the other years.

Experiment 12.

	1964	1963	1962	1961	Φ
γ_2	150 → 148.7	150 → 153.7	150 → 158.7	150 → 161.2	543.298 ↓
h	0.333 → 0.33833	0.333 → 0.33833	0.333 → 0.3458333	0.333 → 0.3470833	407.7284

We allow γ_2 and $\bar{h}$ to vary simultaneously in this experiment. The values for γ_4^0, γ_3, $\bar{h}$, γ_4, γ_5 and σ are fixed at those values used in experiment 11. The results are listed in the table below.

	1961	62	63	64
γ_2	161.2	158.7	153.7	148.7
h	0.3471	0.3458	0.3383	0.3383

Little difference is found between the values for γ_2 estimated in this experiment and those in experiment 11. The values for γ_2's and h's are respectively fairly similar among the years.
The μ^d's for the years 1961 and 1963 are still underestimated and also are underestimated for the upper principal earners' income classes in the year 1962. The fitting of the μ_e's are somewhat improved for 1964.

Experiment 13.

As is shown in experiment 5, discrete changes in the estimates for γ_4^0 are observed as time passed; that is, those estimates for 1963 and 1964 are larger than those for 1961 and 1962. In successive experiments, we fixed γ_4^0's at those values obtained in experiments. Also we have found that estimated values for other preference parameters, γ_2, γ_4 and so on, are fairly stable over time, with the exception of γ_3. Therefore, it is reasonable to hypothesize that all the preference parameters,

$\gamma_2 (\equiv -1)$, γ_3, γ_4, γ_5, γ_4^0 and σ, are respectively constant over the years 1961 to 1964. That is, significant differences in the estimates for γ_4^0 and σ, respectively, might stem from the inadequacy of the values for other preference parameters used in experiment 5 and 8. (The significant differences in estimates among years appeared in experiments 5 and 8).

Hence, calculating mean values of the estimates for γ_4^0 and σ, respectively for the time period 1961 through 1964, we obtain $\gamma_4^0 = 52.85$ and $\gamma_3 = 11.0156$.

Making use of those values for γ_3 and γ_4 and those values for other preference parameters given in experiment 9, we allow γ_2 only to vary. By doing so we obtain estimates for γ_2 as follows.

1961	62	63	64
152.5	150.0	150.0	151.2

It should be noted that differences in yearly estimates of γ_2 are less than those obtained in experiment 12 where we did not use mean values for γ_4^0 and γ_3. Also the magnitude of the objective function Φ in this experiment is smaller than that obtained in experiment 12.

Thus, it may be concluded that the results of this experiment tend to verify the above hypothesis.

Experiment 14.

	1964	1963	1962	1961	
γ_2	150.0	150.0	151.2	153.7	151.225
h	0.33208	0.33083	0.33083	0.33083	0.3311425
γ_4	12112.5	11875.0	11887.5	11875.0	11937.375
γ_5	−6862.5	−6937.5	−7037.5	−6825.0	−6915.625
σ	0.17894	0.1735	0.1722	0.1819	0.176625
γ_4^0	52.85	52.85	52.85	52.85	(52.85)
γ_3	11.015625	11.015625	11.015625	11.015625	(11.015625)
v	45.51	42.63	40.35	37.48	
w	47.4	44.1	41.7	38.4	

The stability of the estimates for γ_2 over time is by this experiment which allows γ_2 and $\bar{h}$ to vary simultaneously. The results are shown in the following table.

	1961	62	63	64
γ_2	153.7	151.2	150.0	150.0
h	0.3308	0.3308	0.3308	0.3321

Although slight successive reduction on the estimates for γ_2 are observed during the four years, the estimates are fairly stable and those for $\bar{h}$ are extremely stable. The magnitude of the objective function in this experiment is also reduced in comparison to that value calculated with the initial values of preference parameters.

Experiment 15.

Taking into account the results of experiments 13 and 14, that estimated values for the parameters are stable over time, the values for preference parameters, γ_4, γ_5, σ, γ_3 and γ_2, are respectively fixed at mean values of estimates for the years, 1961 through 1964 in this experiment.

We allow two of the three parameters h, ν and γ_4^0, to vary simultaneously as is shown in the following table, (a) through (c).

a) h and ν are varied.

	1964	63	62	61
h	0.3311	0.3364	0.3311	0.3261
ν	45.51	42.20	40.27	37.97

There parameters are not preference parameters which are assumed to be constant, but rather are the assigned hours of work and the earning rate for selfemployed work which may vary over time.

b) γ_4^0 and h are varied simultaneously. The estimates for γ_4^0 have been observed to change over time since experiment 5. In this experiment, we examine if estimates for γ_4^0 vary when using the set of parameters fixed at mean values. The results are:

	1964	63	62	61
γ_4^0	130.9	72.9	-21.3	-25.2
h	0.3311	0.3311	0.3311	0.3311

The variation in the estimates for γ_4^0 reappears. However, the magnitude of the objective function is less favorable in comparison to the value obtained in experiment 14 where γ_4^0 was held constant over time. Hence, it can be seen that allowing estimates for γ_4^0 to vary has no merit in improving the fitting. That is, can obtain a better set of preference parameters by holding γ_4^0 constant and choosing better values of the other parameters.

(c) γ_4^0 and ν are allowed to vary.

The estimation results are as follows.

	1961	62	63	64	Φ
γ_4^0	-25.2	-25.2	68.5	130.9	327.078
ν	37.38	40.15	42.54	45.51	306.992

In this experiment variation in the estimates of γ_4^0 reappears as in experiment (b). In this case, although the maginitude of Φ is a little smaller than that in (b), it is larger than Φ in experiment 13 or 14 where γ_4^0 is held constant. Hence, it can be seen in this case also, that allowing γ_4^0 to vary over time has no merit in improving the fitting of estimated to observed valves.

Experiment 16.

	1964	1963	1962	1961
v	45.51 ↓ 43.07	42.63 ↓ 42.09	40.35 ↓ 40.26	37.48 ↓ 37.56
w	38.69 ↓ 39.28	42.02 ↓ 42.50	44.43 ↓ 44.43	38.69 ↓ 39.28
σ	0.176625 ↓ 0.151625	0.176625 ↓ 0.151625	0.176625 ↓ 0.151625	0.176625 ↓ 0.151625
γ_2	151.2 ↓ 146.2	151.2 ↓ 146.2	151.2 ↓ 146.2	151.2 ↓ 146.2
γ_3	11.0 ↓ −0.2	11.0 ↓ −0.2	11.0 ↓ −0.2	11.0 ↓ −0.2
γ_4	11937.4 ↓ 12399.3	11937.4 ↓ 12399.9	11937.4 ↓ 12399.9	11937.4 ↓ 12399.9
γ_5	−6915.6 ↓ −7815.6	−6915.6 ↓ −7815.6	−6915.6 ↓ −7815.6	−6915.6 ↓ −7815.6
γ_4^0	52.9 ↓ 53.7	52.9 ↓ 53.7	52.9 ↓ 53.7	52.9 ↓ 53.7
$\bar{h}$	0.33 ↓ 0.34	0.33 ↓ 0.33	0.33 ↓ 0.34	0.33 ↓ 0.33

Φ

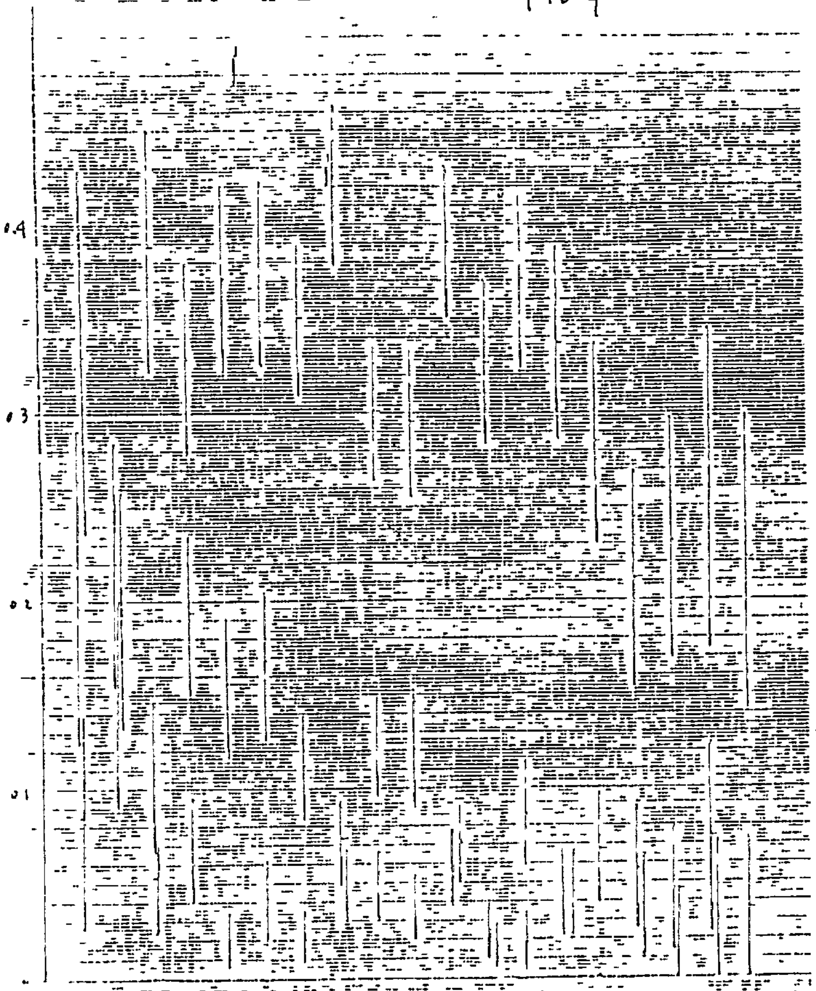
323.03041 ↓ 211.33099

Taking into account the results obtained by the previous experiments, it may be argued that there is no strong evidence contradicting the assumption of the constancy of preference parameters over time. Therefore, if the parameters are, at least locally, identifiable we will obtain more favorable estimation results by making use of the priori information that preference parameters are constant over the years.

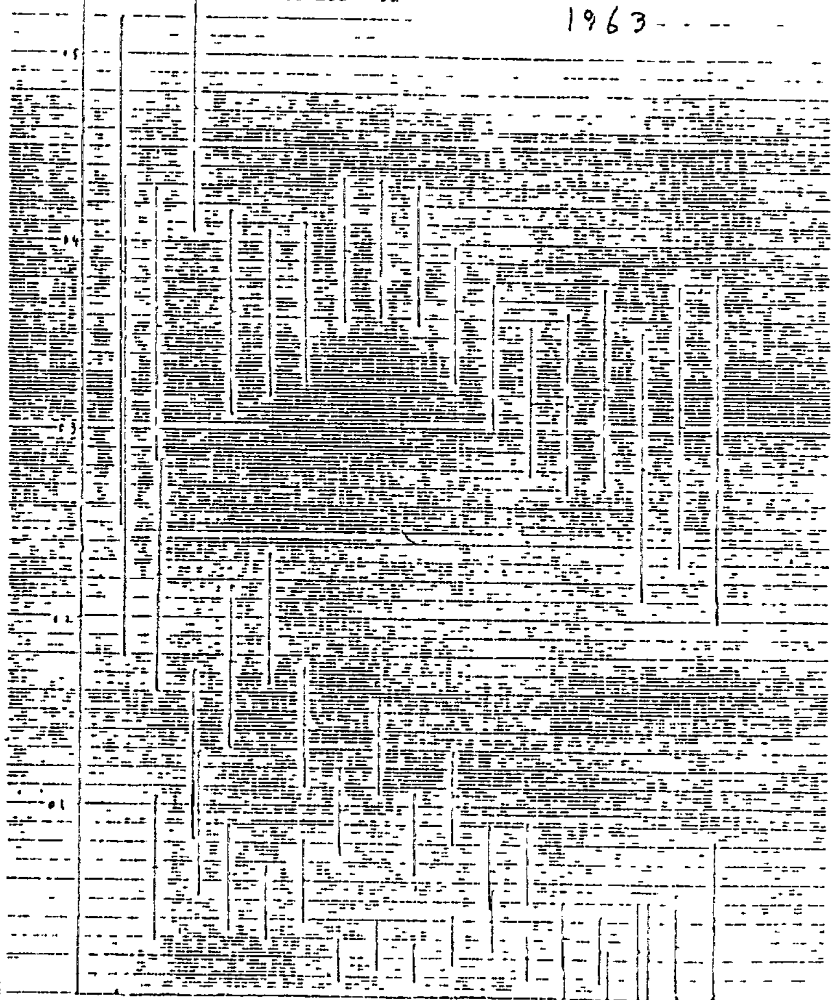
Hence, in this experiment we use as initial values for the parameters the average values for four years with respect to preference parameters, γ_2, γ_3, γ_4, γ_5, γ_4^0 and σ which are listed in the table in experiment 15. Other parameters, ω, ν and $\bar{h}$ are of course allowed to vary over time. Initial values for these are also listed in the table. Making use of these values as initial values for the parameters, we can estimate all the parameters by allowing all of them to vary simultaneously. All the estimates for the preference parameters are restricted to be constant over time. The results are shown in the above table ( ).

The steepest ascent method was employed for estimation. The speed of convergence in the process of obtaining estimates was faster than that in experiment 15. It can be seen that we attained the best fitting results amongst all the estimates obtained in section VI. That is, the problem of systematic underestimation for μ^d was resolved except for the lower income classes in 1964, and fittings for μ^e and μ^{ed} were improved.

Tab. VI-7-1 1964



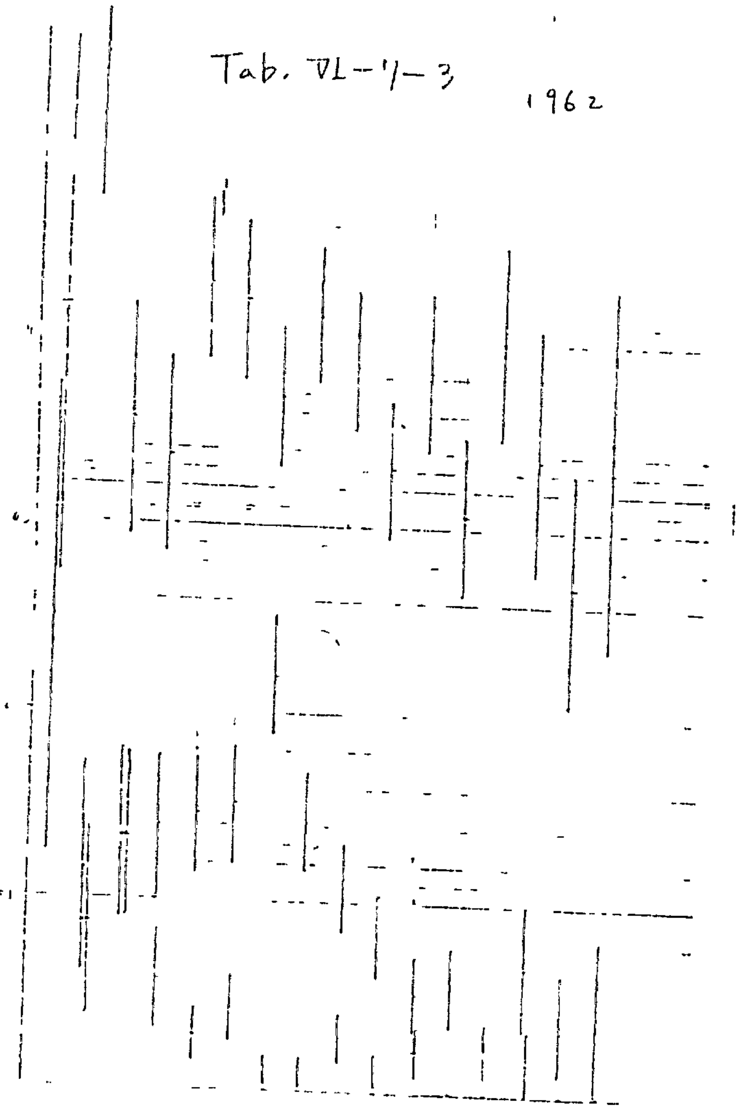
Tab. VI-7-2 1963



The perpendicular segments indicate standard deviation of observed participation ratios μ^{E}, μ^{R} and μ^{EL} respectively.

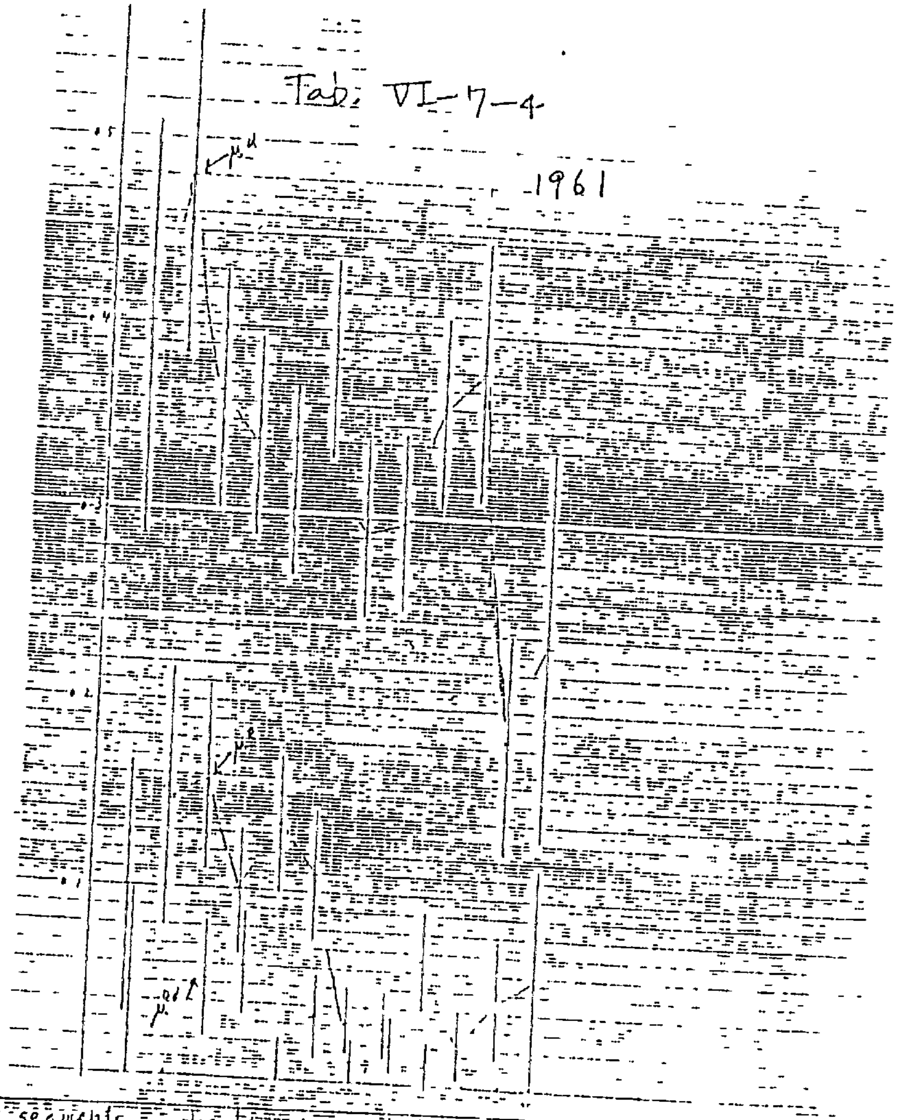
Tab. VI-7-3

1962



Tab. VI-7-4

1961



The perpendicular segments indicate standard deviation of observed participation ratios μ^e, μ^d and μ^{ed} respectively.