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**REDD and Optimal Carbon Credits  
Trading**

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and  
Eiji Sawada**

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# REDD and Optimal Carbon Credits Trading

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**Abstract:** REDD is receiving considerable attention as an effective mechanism for offering incentives to developing countries to slow the rate of deforestation and forest degradation. We show that the trading ratio of credits that is consistent with the social optimum, is not one to one because of uncertainty, and can be more or less than unity, depending on the type of uncertainty. Moreover, we show that the trading ratio will always be less than unity. Furthermore, we present a condition where a country that had a higher level of forest management in the past is assigned a higher trading ratio. Finally, we demonstrate that if the level of forest management is controllable for each country, then REDD does not achieve the social optimum in general.

**Keywords:** REDD; Deforestation and forest degradation; Carbon credits trading; Trading ratio.

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## 1 Introduction

Reducing emissions from deforestation and forest degradation (REDD) is a mechanism designed to provide a financial incentive to developing countries to slow the rate of deforestation and forest degradation. In principle, the incentive is a framework in which developed countries provide economic assistance to developing countries to reduce their GHG emissions by slowing deforestation and forest degradation. In particular, it is likely that REDD will allow carbon credits to be issued to nations according to the reduction in carbon emissions that they achieve compared with a baseline level of carbon emissions from deforestation and forest degradation (Laurance, 2007).

REDD is an extension of reducing emissions from deforestation (RED), which was jointly proposed and advocated by Papua New Guinea and Costa Rica at the 11th Conference of the Parties (COP 11) to the United Nations Framework Convention on Climate Change (UNFCCC) in 2005. As its name suggests, RED focused only on carbon emissions from deforestation. By contrast, REDD recognizes the deterioration of forest quality as an important source of carbon emissions, in addition to deforestation; consequently, reductions in forest degradation also earn carbon credits under REDD. Through REDD, developing countries can benefit financially by trading the credits in the carbon market, thus endowing forest conservation with economic benefits. At the latest UNFCCC Conference of the Parties, COP 16, REDD was progressed substantially (Phelps et al., 2010).

However, REDD has some implementation problems. First, the method for setting baseline emission levels for each country is subject to criticism; the baseline level for each country is set equal to a multiyear average of historical carbon emissions from deforestation and forest degradation (Santilli, 2005; Miles and Kapos, 2008). This approach may result in unfair baselines being set, because countries that have restored forests in the past will have lower baselines and thus relatively limited issuance of carbon credits, whereas countries with little conservation in the past will be able to issue a large number of carbon credits (Ebeling and Yasue, 2008). Figure 1 shows that a larger amount of carbon abatement is estimated for country  $i$  as  $DC$  than country  $j$  as  $HG$  at time  $t_2$ , even though the two countries have an identical rate of carbon emissions from forests after  $t_1$ . This occurs because the baseline of the country  $j$  is more advantageous than that of the country  $i$ .

The second problem concerns the accuracy of estimations of the rate of deforestation and forest degradation. That is, there is considerable uncertainty about forest carbon estimates, because there exists no methodology for measuring the carbon stock directly (ITTO, 2011). In particular, the international tropical timber organization (ITTO) reports: “Data on the carbon stock in tropical forests is much more uncertain because only a few tropical countries have reliable forest inventory data.” (ITTO, 2011, p34). According to DeFries et al. (2005), even if accurate inventory data were available, accurate

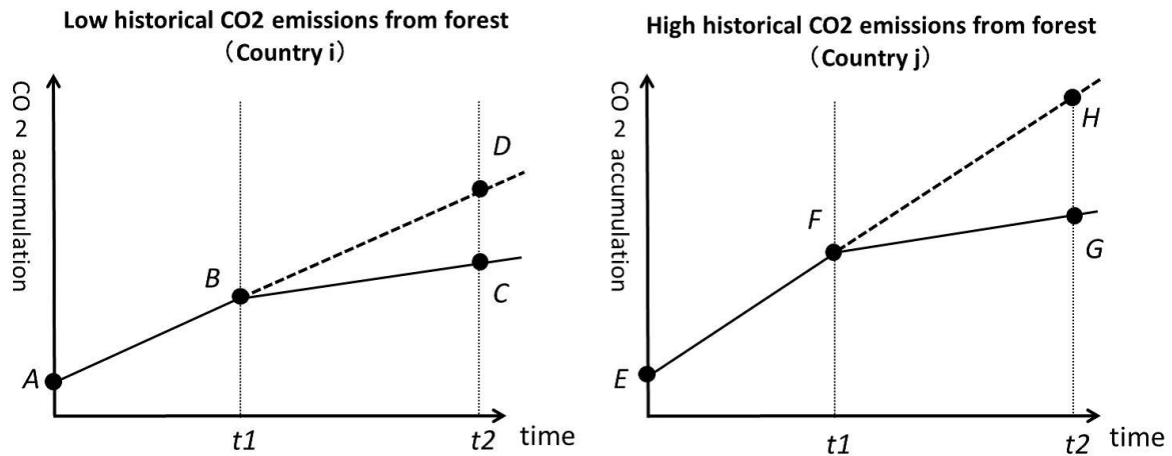


Figure 1: Unfair abatements/emissions because of baselines

estimation of carbon emissions from tropical rainforests would not be possible, because forest degradation is very difficult to quantify, although various methods are available for effectively monitoring and verifying tropical deforestation.

According to the Meridian Institute (2009), forest degradation is caused largely by illicit and environmentally unfriendly activities, such as illegal logging, nonmechanized traditional logging, unplanned conventional logging, excessive biomass extraction for fuel, shortening of the crop–fallow cycle, and forest fragmentation, which as a whole we refer to in this paper as “illegal logging”. In fact, the Brazilian government found in 1997 that 80% of logging in the Amazon was illegal (Laurance, 1998). In another example, ITTO (2011) shows that illegal logging and industrial mining in opened-up areas are the main causes of forest degradation in Gabon.

Sasaki and Putz (2009) examine illegal logging in Cambodia, where luxury-grade timber is exploited. Such selective logging of marketable tree species cannot be detected by satellites, although a new satellite technology is being developed (Asnar et al., 2005). This implies that accurately capturing the extent and amount of illegal logging is extremely difficult; hence, forest degradation, which is caused mainly by illegal logging, is especially hard to capture correctly. That is, estimates of the rate of forest degradation are highly inaccurate, not only because of the lack of an adequate database but also because of the

difficulty of monitoring the decline in forest quality. Therefore, the amount of carbon reductions achieved under REDD has to be estimated stochastically.

Sound forest management is required to combat illegal logging and the consequent forest degradation. Indeed, eradication of illegal logging is an important element of sustainable forest management (ITTO, 2011). Therefore, it is expected that improvements in forest management will result in improvements in the accuracy of estimating carbon reductions.

Uncertainty concerning carbon reduction must be significant, in particular when the reduction of carbon becomes tradable but the level of reduction is highly uncertain. Suppose that the actual reduction is lower than the expected amount but that credits are issued based on the expected amount. In this case, emissions generated based on the credits will exceed the actual reduction and thus the carbon credit trading system under REDD could in fact cause an increase in the rate of global warming. To mitigate this possible problem, it might be useful to distinguish between carbon reductions implemented in countries with and without significant uncertainty: the method adopted in this paper is to discount the tradable rates according to the level of uncertainty. That is, based on an expected carbon reduction, a country with less uncertainty concerning the size of the reduction is assigned more carbon credits and buyers of those credits can emit more carbon.

In the literature, various economic viewpoints on REDD have been put forward. For example, Phelps et al. (2010) consider the financial risk generated by uncertainty over future variables such as carbon demand under REDD. Fuss et al. (2011) focus on the impact of low-cost carbon reductions through REDD on investment in energy and on the development of clean technology. Bosetti et al. (2011) study the effects of RED on energy technology innovation. However, to our knowledge, no theoretical economic study has considered the problems arising from the uncertainty surrounding the reduction of carbon emissions from illegal logging or proposed a way to mitigate it.

The purpose of this paper is to develop a rule for carbon credit trading between developed countries (the North) and developing countries (the South) in an economy where carbon credits are issued under REDD with uncertainty surrounding illegal logging.

If a credit generated in a country with a lower level of forest management is discounted more, then such discounting might contribute to mitigating the effects of unfair baselines, because a country that has made greater forest conservation efforts in the past would have a higher level of forest management, and carbon credits issued by that country might be subject to slight discounting.

We analyze the trading ratio of carbon credits between the North and the South in terms of the social optimum in the North–South economy. We show that the trading ratio of credits is not one to one because of the uncertainty regarding the reduction in carbon emissions from deforestation. We also derive a trading ratio that is consistent with the social optimum, which can be more or less than unity, depending on the type of uncertainty. Moreover, by specifying some functions, we show that the trading ratio will always be less than unity. Furthermore, we provide the condition under which a country that commenced forest conservation at an earlier date is assigned a higher trading ratio in the REDD equilibrium. This condition is shown to be always satisfied when the social optimum results in a smaller reduction in deforestation for that country subject to the disadvantageous conditions associated with REDD. Finally, we demonstrate that REDD does not in general achieve the social optimum if each country controls its level of forest management. That is, although standard emissions trading achieves efficiency, REDD cannot be characterized as an instrument for efficiency.

Section 2 introduces the model and Section 3 presents a general analysis. In Section 4, we specify some functions and present more detailed results on the trading ratio. Section 5 discusses whether REDD is compatible with social optimality when the level of forest management is controllable for each country. Section 6 presents our conclusions.

## 2 The model

We consider an economy in which developed countries (the North) purchase carbon credits from developing countries (the South) issued under the REDD mechanism. For simplicity, the number of countries in the North is unity, and we assume the number in the South to be  $n$ . The North is obliged to reduce carbon emissions by  $\bar{x}$ , although it can purchase

carbon credits from the South generated through forest conservation under REDD.  $x$  is the level of carbon abatement of the North, so that  $\bar{x} - x$  represents the amount of carbon credits purchased from the South. The North's abatement cost function,  $C_n(x)$ , is assumed to be convex, with  $C_n(0) = 0$ ,  $C'_n > 0$  and  $C''_n > 0$ .  $C_n(\cdot)$  is the minimized cost given  $\bar{x}$ , so emissions trading without REDD (if such a scheme exists) is reflected in the function. Without REDD, the price of carbon credits,  $p$ , is determined at the level  $p = C'_n(\bar{x})$ .

Let us now incorporate REDD into our model. REDD allows carbon credits to be issued according to the size of the carbon reduction that each nation achieves by reducing deforestation and forest degradation, compared with the baseline level of carbon emissions, which is defined as the BAU level. That is, the South can obtain carbon credits if it reduces carbon emissions from the baseline level. The reduction in the area of deforestation in country  $i$  is represented by  $y_i$ . We refer to  $y_i$  as the level of conservation achieved by country  $i$ , which contributes to reductions in carbon emissions from forests. However, this contribution is offset partly by the expansion of illegal logging, as illegal logging will become easier as the forest area expands. We assume that illegal logging decreases the density of conserved forests, which reduces the amount of sequestered carbon. This is forest degradation in the context of REDD.

Thus, illegal logging should be viewed as reducing the contribution of the South's forest conservation to controlling climate change. Let  $\bar{M}^i$  be the level of carbon density of the forest without illegal logging per hectare conserved. We assume that illegal logging reduces the level of carbon density, which per hectare is expressed by the function  $M^i(y_i, e_i, \epsilon) \in (0, \bar{M}^i)$ , where  $e_i$  expresses the South's level of forest management and  $\epsilon \in (-\infty, \infty)$  is the stochastic uncertainty regarding how many people enter the forest and log timber illegally. We assume  $M^i_{y_i} > 0$  and  $M^i_{e_i} < 0$ . That is, illegal logging is increasing with the conserved area of forest and decreasing with the level of authority to manage the forest. Thus, the amount of sequestered carbon is represented by  $\bar{M}^i - M^i(y_i, e_i, \epsilon)$ , so that the total amount of carbon prevented from being released under REDD is expressed by:

$$G^i(y_i, e_i, \epsilon) \equiv y_i(\bar{M}^i - M^i(y_i, e_i, \epsilon)). \quad (1)$$



We call this “carbon abatement” under REDD. We suppose that, if the South conserves a larger area, total abatement will increase, i.e.:

$$G_{y_i}^i = \bar{M}^i - M^i(y_i, e_i, \epsilon) - y_i M_{y_i}^i(y_i, e_i, \epsilon) > 0. \quad (2)$$

That is, we do not consider the opposite case where carbon emissions from forest rise when the conserved area is increased.

In addition, we assume that the marginal abatement by conservation will not increase.

$$G_{y_i y_i}^i = -2M^i(y_i, e_i, \epsilon) - y_i M_{y_i y_i}^i(y_i, e_i, \epsilon) \leq 0 \quad (3)$$

Let  $\alpha_i$  represent the number of carbon credits to be issued to country  $i$  in the South against the amount of abatement implemented by the country through REDD.  $\alpha_i$  can be unity, but it is generally more or less than this value, as we explain later under optimality. Therefore, the expected total amount of credits that country  $i$  gains is equivalent to  $\alpha_i E[G^i(y_i, e_i, \epsilon)]$ . Hereafter, we call  $\alpha_i$  the “trading ratio”.

With respect to the cost of conservation, country  $i$ 's conservation cost function, which is the opportunity cost of conservation, is expressed by  $C_{s_i}(y_i)$  with the properties  $C'_{s_i} > 0$  and  $C''_{s_i} > 0$ .

Finally, we define the environmental damage function as  $D(Z)$  with  $D' > 0$  and  $D'' > 0$ , where  $Z$  is net total emissions, expressed by:

$$Z = \bar{z} - x - \sum_{k=1}^n G^k(y_k, e_k, \epsilon). \quad (4)$$

Here,  $\bar{z}$  denotes the total emissions before abatement. The model above is the one we will use in our analysis.

### 3 Optimal trading ratio of carbon credits

The trading ratio  $\alpha_i$  is very closely related to the efficiency of emissions trading in the situation where uncertainty exists with respect to the level of environmental damage. To shed light on this aspect, we consider a simple case where the South can control the extent of deforestation but the level of forest management is fixed for each country. Therefore,

forest degradation per hectare due to illegal logging is expressed by  $M^i(y_i, \bar{e}_i, \epsilon)$  given  $\bar{e}_i$ , so that the related abatement level is expressed by  $G^i(y_i, \bar{e}_i, \epsilon) \equiv y_i(\bar{M}^i - M^i(y_i, \bar{e}_i, \epsilon))$ .

The North is assumed to minimize the cost of complying with the emission constraints. That is, the objective function of the North is represented by:

$$\min_x C_n(x) + p(\bar{x} - x). \quad (5)$$

This leads to:

$$C'_n(x) = p. \quad (6)$$

However, the South minimizes the cost of reducing deforestation, taking into consideration the fact that the reduction generates carbon credits under REDD. That is, country  $i$  in the South has the following objective function:

$$\min_{y_i} C_{s_i}(y_i) - \alpha_i p E(G^i(y_i, \bar{e}_i, \epsilon)), i = 1, \dots, n. \quad (7)$$

This leads to:

$$\frac{C'_{s_i}(y_i)}{pE(G^i_{y_i}(y_i, \bar{e}_i, \epsilon))} = \alpha_i, i = 1, \dots, n. \quad (8)$$

On the other hand, the carbon credit market clearing condition is:

$$\bar{x} - x = \sum_{k=1}^n \alpha_k E(G^k(y_k, \bar{e}_k, \epsilon)). \quad (9)$$

Let  $y$  express  $(y_1, \dots, y_n)$ . The number of unknown variables  $(y, x, p)$  is  $n + 2$ , which coincides with that in equations (6), (8) and (9). Thus, the equilibrium solution can be obtained, which we denote by  $(y^r, x^r, p^r)$ .  $(y^r, x^r, p^r)$  is referred to as the REDD equilibrium.

The social optimum is defined as the level of forest conservation in the South and the level of emissions abatement in the North that minimize the total cost, including environmental damage, which is represented by solving:

$$\min_{x, y_i} C_n(x) + \sum_{k=1}^n C_{s_k}(y_k) + E(D(Z)), i = 1, \dots, n. \quad (10)$$

This leads to:

$$C'_n(x) = E(D') \quad (11)$$

$$C'_{s_i}(y_i) = E(D'G_{y_i}^i(y_i, \bar{e}_i, \epsilon)), i = 1, \dots, n. \quad (12)$$

The social optimum is expressed by  $(y^*, x^*)$ . Let us find the trading ratio  $\alpha^* \equiv (\alpha_1^*, \dots, \alpha_n^*)$  for which the REDD equilibrium coincides with the social optimum.

From the conditions (6), (8), (11) and (12), we derive the “optimal” trading ratio  $\alpha_i^*$ , under which the social optimum is achieved. That is, optimality requires:

$$\frac{C'_{s_i}(y_i^r)}{C'_n(x^r)} = \frac{C'_{s_i}(y_i^*)}{C'_n(x^*)} \quad (13)$$

so that it holds that:

$$\frac{\alpha_i^* p E(G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon))}{p} = \frac{E(D'G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon))}{E(D')}, i = 1, \dots, n. \quad (14)$$

Therefore, we obtain the following optimal trading ratio:

$$\begin{aligned} \alpha_i^* &= \frac{E(D'G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon))}{E(D')E(G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon))} \\ &= \frac{E(D')E(G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon)) + Cov(D', G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon))}{E(D')E(G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon))} \\ &= 1 + \frac{Cov(D', G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon))}{E(D')E(G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon))}, i = 1, \dots, n. \end{aligned} \quad (15)$$

Then, the sign of  $\alpha_i^*$  is positive under (2), because  $G_{y_i^*}^i$  is positive, so that  $D'G_{y_i^*}^i > 0$ . From (15), the sign of  $Cov(D', G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon))$  determines whether  $\alpha_i^*$  is greater, equal to or less than unity<sup>1</sup>. This is stated in the following proposition.

**Proposition 1.** *Under (2), the optimal trading ratio  $\alpha_i^*$  is always positive and expressed by (15). Moreover,  $\alpha_i^* \geq 1$  if  $Cov(D', G_{y_i^*}^i(y_i^*, \bar{e}_i, \epsilon)) \geq 0$ .*

Note that the optimal trading ratio  $\alpha_i^*$  depends on the reduction of deforestation not only in country  $i$ ,  $y_i^*$ , but also in other developing countries  $y_j^*$  ( $j \neq i$ ) through  $D'$ . We next consider how the optimal ratios differ between countries in the South.

<sup>1</sup>The literature on nonpoint source pollution discusses the nonunity trading ratio between point source and nonpoint source (see, for example, Horan (2001) and Horan and Shortle (2005)). In the literature, uncertainty regarding damage caused by natural phenomena such as the weather causes the ratio not to be equal to unity.

## 4 Comparison of optimal trading ratios

In this section, we specify our model to clarify further properties of the level of the optimal trading ratio and compare those ratios across different countries. First, let us assume that the type of forest is identical and the function regarding forest degradation is specified as:

$$G^i(y_i, e_i) = y_i(\bar{M} - m^i(y_i, e_i)f(\epsilon)), \epsilon \in (-\infty, \infty) \quad (16)$$

where  $m_{y_i}^i > 0$ ,  $m_{e_i}^i < 0$ ,  $m_{y_i e_i}^i < 0$ ,  $f'(\epsilon) > 0$  and  $\lim_{\epsilon \rightarrow -\infty} f(\epsilon) = 0$ .  $\bar{M}^i = \bar{M}$  implies that the carbon density of forests without illegal logging would be equivalent for all developing countries <sup>2</sup>.

Second, we assume that the damage function  $D(Z)$  is expressed as a quadratic function:

$$D(Z) = \frac{a}{2}Z^2. \quad (17)$$

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<sup>2</sup>Under (16), some of what is ambiguous under (1) is made clear. First, it holds that  $Cov(D', G_{y_i}^i(y_i^*, \bar{e}_i, \epsilon)) < 0$ . Hennessy and Feng (2008) provide a sufficient condition that determines the sign of the covariance using Chebyshev's inequality (they also show a necessary condition). By Chebyshev's inequality  $Cov(D', G_{y_i}^i(y_i^*, \bar{e}_i, \epsilon)) < 0$  if either  $D'$  or  $G_{y_i}^i(y_i^*, \bar{e}_i, \epsilon)$  is an increasing function of stochastic variable  $\epsilon$  and the other is a decreasing function of  $\epsilon$ . Under (1), we have:

$$\frac{\partial D'(Z)}{\partial \epsilon} = D'' \sum_{k=1}^n y_k M_{\epsilon}^k(y_k, e_k, \epsilon) \frac{\partial G_{y_i}^i(y_i, e_i, \epsilon)}{\partial \epsilon} = -M_{\epsilon}^i - y_i M_{y_i \epsilon}^i \quad \text{for } i = 1, \dots, n.$$

Even if we assume that  $M_{\epsilon}^i(y_i, e_i, \epsilon) > 0$  as under (16), which determines  $\frac{\partial D'(Z)}{\partial \epsilon} > 0$ ,  $\frac{\partial G_{y_i}^i(y_i, e_i, \epsilon)}{\partial \epsilon}$  is still ambiguous as long as the sign of  $M_{y_i \epsilon}^i$  is not identified. Under (16), however, it holds that:

$$\frac{\partial G_{y_i}^i(y_i, e_i, \epsilon)}{\partial \epsilon} = f'(\epsilon)(-m^i(y_i, e_i) - y_i m_{y_i}^i(y_i, e_i)) < 0,$$

which identifies  $Cov(D', G_{y_i}^i(y_i^*, \bar{e}_i, \epsilon)) < 0$ .

Second, the signs of  $\frac{\partial Var(G^i(y_i, e_i, \epsilon))}{\partial y_i}$  and  $\frac{\partial Var(G^i(y_i, e_i, \epsilon))}{\partial e_i}$  are derived easily under (16). In fact:

$$\begin{aligned} \frac{\partial Var(G^i(y_i, e_i, \epsilon))}{\partial y_i} &= 2y_i m^i(y_i, \bar{e}_i)(m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i))Var(f(\epsilon)) > 0 \\ \frac{\partial Var(G^i(y_i, e_i, \epsilon))}{\partial e_i} &= 2y_i^2 m^i(y_i, \bar{e}_i) m_{e_i}^i(y_i, \bar{e}_i)Var(f(\epsilon)) < 0. \end{aligned}$$

Then,  $Cov(D', G_{y_i}^i(y_i, \bar{e}_i, \epsilon))$  is calculated as <sup>3</sup>:

$$Cov(D', G_{y_i}^i(y_i, \bar{e}_i, \epsilon)) = -a (m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i)) \sum_{k=1}^n \left( \frac{Var(G^k(y_k, \bar{e}_k, \epsilon))}{y_k m_k(y_k, \bar{e}_k)} \right), \quad (18)$$

where  $Var(G^k(y_k, \bar{e}_k, \epsilon))$  is the variance in carbon reduction through forest conservation for country  $k$ . In view of proposition 1, a trading ratio of less than unity implies that the expected carbon reduction by the South will be “discounted” when that reduction is issued as carbon credits. It is also obvious that  $\alpha_i^*$  will be smaller if  $Var(G^k(y_k, \bar{e}_k, \epsilon))$  increases for some  $k$ . That is, if the uncertainty regarding the total reduction of carbon under REDD increases, then the discount must be strengthened. This is because such uncertainty can cause total emissions that exceed the constraint  $\bar{z}$ , so that the damage is more serious. A smaller trading ratio will play a role in alleviating the damage in such an extreme situation. These results are stated in the following proposition.

**Proposition 2.** *Assume (16) and (17). Then, the optimal trading ratio is less than unity, i.e.,  $\alpha_i^* < 1$  for  $i = 1, \dots, n$ . Moreover, the ratio declines if the level of uncertainty  $Var(G^k(y_k, \bar{e}_k, \epsilon))$  for some  $k$  increases.*

Next we analyze how the optimal trading ratios differ across developing countries. Under the specifications (16) and (17), (15) leads to:

$$\begin{aligned} \alpha_k^* &= 1 + \frac{-(m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k))(E(D'f(\epsilon)) - E(D')\mu)}{E(D')(\bar{M} - \mu(m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k)))} \\ &= 1 - \frac{E(D'f(\epsilon)) - E(D')\mu}{E(D')} \frac{m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k)}{\bar{M} - \mu(m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k))} \\ &= 1 - \frac{E(D'f(\epsilon)) - E(D')\mu}{E(D')} \frac{1}{\frac{\bar{M}}{m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k)} - \mu}, \end{aligned} \quad (19)$$

where  $\mu = \int_{-\infty}^{\infty} f(\epsilon) d\epsilon$ . From (19), the larger is  $\alpha_k^*$ , the smaller is  $m^k(y_k, \bar{e}_k) + y_k m_{y_k}^k(y_k, \bar{e}_k)$ . Thus, for arbitrary  $\alpha_i^*$  and  $\alpha_j^*$ ,  $\alpha_i^*$  is larger than  $\alpha_j^*$  if and only if:

$$m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i) < m^j(y_j, \bar{e}_j) + y_j m_{y_j}^j(y_j, \bar{e}_j), \quad (20)$$

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<sup>3</sup>See Appendix.

which leads to:

$$m^i(y_i, \bar{e}_i)(1 + \rho_{m_y}^i) < m^j(y_j, \bar{e}_j)(1 + \rho_{m_y}^j), \quad (21)$$

where  $\rho_{m_y}^k \equiv \frac{\partial m^k(y_k, \bar{e}_k)}{\partial y_k} \frac{y_k}{m^k(y_k, \bar{e}_k)} > 0$  is the elasticity of  $m^k(y_k, \bar{e}_k)$  with respect to  $y_k$ . This is stated in the following proposition.

**Proposition 3.** *Suppose (16) and (17). Then, the sign of the difference between the optimal trading ratios for countries  $i$  and  $j$  is expressed as follows:*

$$\alpha_i^* \gtrless \alpha_j^* \quad \text{if} \quad m^i(y_i, \bar{e}_i)(1 + \rho_{m_y}^i) \gtrless m^j(y_j, \bar{e}_j)(1 + \rho_{m_y}^j).$$

That is, the lower the carbon emissions per hectare due to illegal logging and the lower the elasticity of marginal carbon emissions for country  $i$  compared with those of country  $j$ , the higher the probability that  $\alpha_i^*$  exceeds  $\alpha_j^*$ .

It is possible that a country that has achieved a reduction in carbon emissions from deforestation and forest degradation before REDD is implemented cannot achieve a significant reduction in emissions because the country is assigned a disadvantageous baseline that ignores its past reduction efforts, meaning that it faces a higher opportunity cost of reducing emissions. This disadvantage reflects the unfairness of the baseline (Eberling and Yasue, 2008). If this country were to be assigned a higher trading ratio than other countries, the unfairness might be mitigated, because the country would have an advantageous trading ratio. In what follows, we examine whether this is true.

As an illustration, we assume that all the countries in the South are identical except for two variables related to past reductions in deforestation and forest degradation. One variable is the cost of reducing emissions: we assume that the larger the past reduction, the higher the opportunity cost of current reduction. The second variable is the level of forest management: we assume that a country's level of forest management will be more efficient the larger the area of forest that was conserved previously.

Stated formally, the function  $m^i$  is identical for all countries in the South, that is:

$$m^k(\cdot, \cdot) = m(\cdot, \cdot), \quad k = 1, \dots, n. \quad (22)$$

Moreover, let us assume that the cost function of reducing deforestation is identical for each country and is expressed as:

$$C_{s_k}(y) = A_k C(y), A_k > 0, C' > 0, C'' > 0. \quad (23)$$

$A_k$  is referred to as the “cost coefficient” for country  $k$ , which shows the cost differences between countries. A higher  $A_k$  indicates that a reduction in deforestation is costly.

Let the accumulated reduction in carbon emissions from deforestation and forest degradation in the past for country  $k$  be expressed by  $v_k$ . We say that country  $i$  is more *sustainable* than country  $j$  if  $v_i > v_j$ , because this may mean that country  $i$  saved a larger area of forest than country  $j$ . We assume that:

$$A_k = A(v_k), A' > 0 \quad (24)$$

and

$$\bar{e}_k = e(v_k), e' \geq 0. \quad (25)$$

Under our assumptions, if country  $i$  has made a greater effort to reduce carbon emissions from deforestation and forest degradation, compared with country  $j$ , which means  $v_i > v_k$ , it holds that:

$$\begin{aligned} \bar{e}_i &\geq \bar{e}_j \\ A_i &> A_j. \end{aligned} \quad (26)$$

In this case, will REDD result in  $\alpha_i^* > \alpha_j^*$ ? If so, we say that the optimal trading ratios in REDD *advantage* the more sustainable country.

To see this, let us focus on country  $j$  and differentiate totally the second condition of (12) with respect to  $A_j, y_j$  and  $\bar{e}_j$  at the social optimum. Note that we are interested in how  $y_i^*$  is different from  $y_j^*$  because of the difference between  $A_i$  and  $\bar{e}_i$ , which means that  $Z$  is fixed. With (18), this leads to:

$$dy_j = -\frac{(E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})de_j + C'dA_j}{A_j C'' + E(D'(Z))\mu + aV}, \quad (27)$$

where  $V = \sum_{k=1}^n \left( \frac{Var(G^k(y_k, \bar{e}_k, \epsilon))}{y_k m_k(y_k, \bar{e}_k)} \right)$ .

In view of (20) and (27),  $\alpha_i^* > \alpha_j^*$  holds if and only if  $dA_j$  and  $d\bar{e}_j$  satisfy:

$$\frac{dy_j}{de_j} < -\frac{m_{e_j} + y_j m_{y_j e_j}}{2m_{y_j} + y_j m_{y_j y_j}}, \quad (28)$$

where  $dy_j$  is given by (27). Rearranging (28) with respect to  $dA_j (> 0)$  and  $de_j (> 0)$  leads to:

$$dA_j > \left( \frac{A_j C'' + E(D'(Z))\mu + aV}{(2m_{y_j} + y_j m_{y_j y_j})C'} - \frac{(E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{C'} \right) de_j, \quad (29)$$

where  $m_e, m_{yy}, m_{ye}, C'$  and  $C''$  are all evaluated at  $(y_j^*, \bar{e}_j)$ . That is,  $\alpha_i^* > \alpha_j^*$  can hold under  $dA_j > 0$  and  $de_j > 0$ . This is claimed in the following proposition.

**Proposition 4.** *Assume (16), (17), (22), (23), (24) and (25). Moreover, assume that countries  $i$  and  $j$  satisfy (26) because  $v_i > v_j$ . Then, if it holds in the REDD equilibrium that:*

$$A_i - A_j > \left( \frac{(A_j C'' + E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{(2m_{y_j} + y_j m_{y_j y_j})C'} - \frac{(E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{C'} \right) (\bar{e}_i - \bar{e}_j),$$

then the optimal trading ratios advantage the more sustainable country, i.e., it holds that  $\alpha_i^* > \alpha_j^*$ .

From this proposition, we immediately obtain the following property.

**Corollary 1.** *Under the assumptions of proposition 4 with  $\bar{e}_i = \bar{e}_j$ , then  $A_i > A_j$  always leads to  $\alpha_i^* > \alpha_j^*$ .*

From (27), the social optimum requires  $y_i^*$  to be equal to or smaller than  $y_j^*$  if and only if the numerator of the RHS of (27) is positive, i.e.,  $dA_j$  and  $de_j$  satisfy:

$$dA_j \geq -\frac{(E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{C'(y_j)} de_j. \quad (30)$$

If (30) is true, it is easy to see that the sufficient condition of proposition 4 is satisfied, because:

$$\frac{(A_j C'' + E(D'(Z))\mu + aV)(m_{e_j} + y_j m_{y_j e_j})}{(2m_{y_j} + y_j m_{y_j y_j})C'} < 0. \quad (31)$$

Thus, we also have the following result.



**Proposition 5.** *Under the assumptions of proposition 4, if it holds that  $y_i^* \leq y_j^*$  in the social optimum, which is equivalent to (30) being satisfied, then the optimal trading ratios always advantage the more sustainable country, i.e., it holds that  $\alpha_i^* > \alpha_j^*$ .*

As a consequence of determining the optimal trading ratio under REDD, this proposition says that if a country's reduction of deforestation is smaller because that country began forest conservation at an earlier date, the country must be assigned a higher trading ratio. This might mitigate the disadvantageous conditions for the country under REDD.

By (1), emissions abatement is determined by the product of conservation area and carbon density. Thus, a smaller conservation area does not necessarily imply a smaller carbon reduction. Does proposition 5 also hold with emissions abatement? We examine whether more sustainable countries can have advantageous trading ratios if their emissions abatement is smaller because of their earlier conservation efforts.

First, we review our definition of emissions abatement. From (16) and (22), the level of emissions abatement is expressed as:  $G^k(y_k, e_k) = y_k(\bar{M} - m(y_k, e_k)f(\epsilon))$ . By totally differentiating  $G^k(y_k, \bar{e}_k)$  with respect to  $y_j$  and  $\bar{e}_j$  at the social optimum, we obtain the following equation:

$$dG^j = dy_j(\bar{M} - m(y_j, e_j)f(\epsilon) - y_j m_{y_j} f(\epsilon)) - y_j m_{\bar{e}_j} f(\epsilon) d\bar{e}_j. \quad (32)$$

Assume that  $dG^j = G^i - G^j \leq 0$  holds, that is, the emissions abatement of more sustainable countries is not greater than that of less sustainable countries. Then,  $dG^j \leq 0$  can be rearranged by (32) as follows:

$$\frac{dy_j}{d\bar{e}_j} \leq \frac{y_j m_{\bar{e}_j} f(\epsilon) d\bar{e}_j}{\bar{M} - f(\epsilon)(m(y_j, \bar{e}_j) + y_j m_{y_j})}. \quad (33)$$

Because a necessary and sufficient condition of  $\alpha_i^* > \alpha_j^*$  is (28), if the RHS of (33) is greater than the RHS of (28), then  $G^i(y_i^*, \bar{e}_i) \leq G^j(y_j^*, \bar{e}_j)$  always means  $\alpha_i^* > \alpha_j^*$ . By

calculating the difference between the RHS of (33) and the RHS of (28):

$$\begin{aligned}
& \frac{y_j m_{\bar{e}_j} f(\epsilon) d\bar{e}_j}{\bar{M} - f(\epsilon)(m(y_j, \bar{e}_j) + y_j m_{y_j})} + \frac{m_{e_j} + y_j m_{y_j e_j}}{2m_{y_j} + y_j m_{y_j y_j}} \\
&= \frac{y_j m_{\bar{e}_j} f(\epsilon) d\bar{e}_j (2m_{y_j} + y_j m_{y_j y_j}) + (m_{e_j} + y_j m_{y_j e_j})(\bar{M} - f(\epsilon)(m(y_j, \bar{e}_j) + y_j m_{y_j}))}{(\bar{M} - f(\epsilon)(m(y_j, \bar{e}_j) + y_j m_{y_j}))(2m_{y_j} + y_j m_{y_j y_j})} \\
&= \frac{G_{y_j e_j}^j G_{y_j}^j - G_{e_j}^j G_{y_j y_j}^j}{G_{y_j}^j G_{y_j y_j}^j} < 0. \tag{34}
\end{aligned}$$

The sign of (34) is negative because by (2) and (3),  $G_{y_j}^j > 0$ ,  $G_{y_j y_j}^j \leq 0$ . Furthermore, by  $m_{e_j} < 0$ ,  $G_{e_j}^j > 0$ , and by  $m_{e_j} < 0$  and  $m_{y_j e_j} < 0$ ,  $G_{y_j e_j}^j > 0$ . The above analysis can be summarized as the following result.

**Proposition 6.** *Under the assumptions of proposition 4, if it holds that  $G_i^* \leq G_j^*$  in the social optimum, which is equivalent to (30) being satisfied, then the optimal trading ratios always advantage the more sustainable country, i.e., it holds that  $\alpha_i^* > \alpha_j^*$ .*

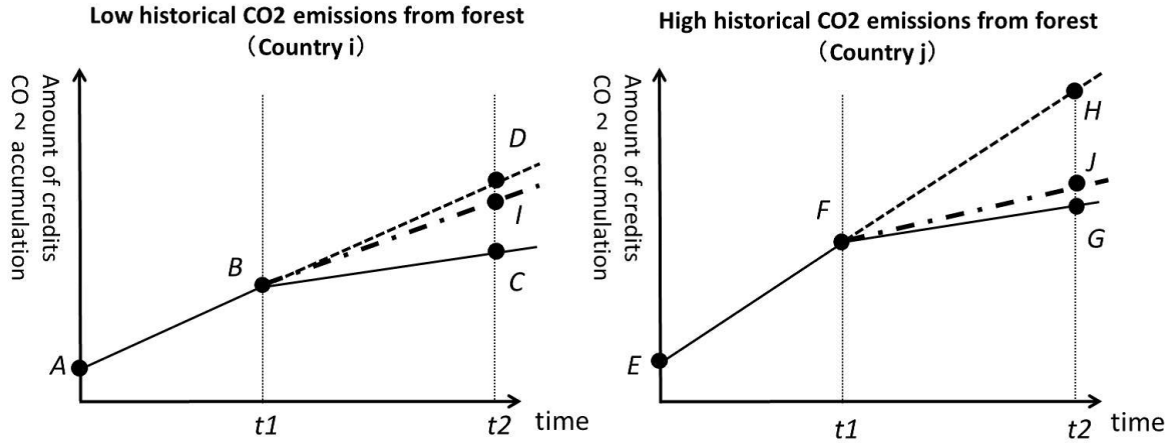


Figure 2: Mitigation of unfairness using the optimal trading ratio

Figure 2 illustrates the case in which the unfairness is mitigated by carbon trading under the optimal trading ratio. The amount of credits issued to country  $i$  equals  $\alpha_i^*$  times the carbon abatement DC, i.e., IC. Similarly, the amount of credits issued to country  $j$  equals JG. Although the level of carbon emissions in country  $i$  is lower than that in

country  $j$ , the difference is reduced because the optimal trading ratio for country  $i$  is always larger than that of country  $j$ . Figure 2 depicts a special case where the amount of credits issued to country  $i$  exceeds that issued to country  $j$ .

## 5 Control of the level of forest management and implementation of the social optimum under REDD

So far, we have assumed that the countries in the South control deforestation, and the level of forest management is assumed to be given. Now we relax this constraint and assume that the level of forest management is controllable, as is the reduction of deforestation. However, this presents a problem concerning the possibility of the social optimum under REDD.

Let the developing countries control the level of forest management  $e_i$  given the management cost  $w_i$ . In this case, the objective of country  $i$  in the South is expressed by:

$$\min_{y_i, e_i} C_{s_i}(y_i) + w_i e_i - \alpha_i p E(G^i(y_i, e_i, \epsilon)), i = 1, \dots, n. \quad (35)$$

This leads to:

$$\frac{C'_{s_i}(y_i)}{p E(G^i_{y_i}(y_i, e_i, \epsilon))} = \alpha_i, i = 1, \dots, n. \quad (36)$$

$$\frac{w_i}{p E(G^i_{e_i}(y_i, e_i, \epsilon))} = \alpha_i, i = 1, \dots, n. \quad (37)$$

The objective of the North remains unchanged, leading to (6). The carbon market equilibrium in this case is represented by  $(y^{rr}, e^{rr}, x^{rr}, p^{rr})$ .

The social optimization problem is expressed as:

$$\min_{x, y_i, e_i} C_n(x) + \sum_k^n (C_{s_k}(y_k) + w_i e_i) + E(D(Z)), i = 1, \dots, n. \quad (38)$$

This leads to:

$$C'_n(x) = E(D') \quad (39)$$

$$C'_{s_i}(y_i) = E(D' G^i_{y_i}(y_i, e_i, \epsilon)), i = 1, \dots, n. \quad (40)$$

$$w_i = E(D' G^i_{e_i}(y_i, e_i, \epsilon)), i = 1, \dots, n. \quad (41)$$

The social optimum is expressed as  $(y^{**}, e^{**}, x^{**})$ .

It is obvious that  $(y^{rr}, e^{rr}, x^{rr}, p^{rr}) = (y^{**}, e^{**}, x^{**})$  if and only if the following holds:

$$E(D'G_{e_i}^i(y_i^{rr}, e_i^{rr}, \epsilon)) E(G_{y_i}^i(y_i^{rr}, e_i^{rr}, \epsilon)) = E(G_{e_i}^i(y_i^{rr}, e_i^{rr}, \epsilon)) E(D'G_{y_i}^i(y_i^{rr}, e_i^{rr}, \epsilon)). \quad (42)$$

However, we obtain:

$$\begin{aligned} E(D'G_{e_i}^i)E(G_{y_i}^i) &= E(D'G_{e_i}^i G_{y_i}^i) - Cov(D'G_{e_i}^i, G_{y_i}^i) \\ E(D'G_{y_i}^i)E(G_{e_i}^i) &= E(D'G_{y_i}^i G_{e_i}^i) - Cov(D'G_{y_i}^i, G_{e_i}^i). \end{aligned} \quad (43)$$

Therefore, (42) can be achieved under REDD if and only if:

$$Cov(D'G_{e_i}^i, G_{y_i}^i) = Cov(D'G_{y_i}^i, G_{e_i}^i). \quad (44)$$

This result is presented in the following proposition.

**Proposition 7.** *Suppose that each country in the South can control the level of forest management under (2). Then REDD does not achieve the social optimum unless it holds that  $Cov(D'G_{e_i}^i, G_{y_i}^i) = Cov(D'G_{y_i}^i, G_{e_i}^i)$ .*

(44) does not in general hold. Under REDD, each country in the South might control both its level of forest management and deforestation, so we may conclude that it is hard to characterize REDD as a measure compatible with the social optimum.

## 6 Concluding remarks

In this paper, we examined the optimal rule of carbon credit trading between the North and the South under REDD, where the South can reduce its carbon emissions by decreasing deforestation and increasing the level of forest management. In addition, we took into consideration the fact that the area of conserved forests is uncertain because of illegal logging activities.

By including this aspect of reality, given the level of forest management, we showed that the optimal trading ratio is not in general unity, depending on the stochastic relationships between marginal damage, the marginal abatement of carbon through the reduction of deforestation, and marginal illegal logging of conserved forests.

Under some specifications, we also demonstrated that the optimal trading ratio is always less than unity and that the ratio decreases depending on the level of uncertainty. Moreover, we examined whether a country that had a relatively high level of forest management in the past and thus higher opportunity costs of conservation can be given a higher trading ratio, that is, whether the optimal trading ratios advantage more sustainable countries. We derived a condition showing that this is true. In particular, we demonstrated that if a country achieves a smaller reduction of deforestation, it must always be assigned a higher trading ratio. In this sense, this paper demonstrated that determining the trading ratio in the optimal way may mitigate the problem of unfairness arising under REDD, as discussed in the literature.

However, we also demonstrated that if the level of forest management is controllable for the South, then REDD does not in general lead to the social optimum. That is, REDD cannot be characterized as a method of achieving efficiency, as can be said of standard emissions trading. This drawback requires some form of remedy. For example, suppose that REDD targets only the areas that achieve a certain level of forest management and that carbon credits are issued based on the determined level of forest management. Under this scheme, each participating country might set a level of forest management higher than that required unless setting a higher level generates some other benefits, so that each country's level of forest management must be equivalent, i.e.,  $e_i = e_j = \bar{e}$  where  $\bar{e}$  is the determined level. Then, each participating country controls the level of deforestation and our analysis will apply.

Needless to say, this scheme also has some problems. First, the resulting equilibrium will be the second best, not the first best one, so that the problem of REDD in terms of achieving the social optimum remains unsolved. Another problem concerns the incentive to raise the level of forest management; the scheme does not provide an incentive to set the level beyond the required minimum for participating countries. In addition, it will allow nonparticipating countries to increase their levels of deforestation and forest degradation. Therefore, it is important to study comprehensively ways to solve the social optimization problem of REDD posed in this paper.

Finally, this paper has two limitations. First, our study assumed that the carbon

credit price is given and not affected by the introduction of REDD into the global carbon market. However, as Bossetti et al. (2011) stress, the amount of credits issued under REDD will not be negligible and therefore will have an influence on the carbon market. This aspect is important; Bossetti et al. link the effects of REDD to innovation in the energy sector, via a reduction in the price of carbon credits. Second, REDD will be implemented over the long term, so the forest conservation decisions are made over time. Therefore, a dynamic model may provide some key insights into the nature and function of REDD that are not examined in the static model in this paper. These interesting analyses are left for future studies.

## 7 Appendix

We show that the expressions for (18) are derived under (16) and (17). The variance in carbon reduction is expressed as:

$$\begin{aligned}
& \text{Var}(G^i(y_i, \bar{e}_i, \epsilon)) \\
&= E \left( (G^i(y_i, \bar{e}_i, \epsilon))^2 \right) - (E(G^i(y_i, \bar{e}_i, \epsilon)))^2 \\
&= E \left( (y_i(\bar{M}_i - m^i(y_i, \bar{e}_i)f(\epsilon)))^2 \right) - (E(y_i(\bar{M}_i - m^i(y_i, \bar{e}_i)f(\epsilon))))^2 \\
&= y_i^2 (\bar{M}_i^2 - 2\bar{M}_i m^i(y_i, \bar{e}_i)\mu + (m^i(y_i, \bar{e}_i))^2 E((f(\epsilon))^2)) - y_i^2 (\bar{M}_i^2 - 2\bar{M}_i m^i(y_i, \bar{e}_i)\mu + (m^i(y_i, \bar{e}_i))^2 \mu^2) \\
&= (y_i m^i(y_i, \bar{e}_i))^2 (E((f(\epsilon))^2) - m^i(y_i, \bar{e}_i)^2 \mu^2) \\
&= (y_i m^i(y_i, \bar{e}_i))^2 \text{Var}(f(\epsilon)). \tag{45}
\end{aligned}$$

We demonstrate that (18) is derived as follows:

$$\begin{aligned}
& \text{Cov}(D', G_{y_i}^i(y_i, \bar{e}_i, \epsilon)) \\
&= E(D' G_{y_i}^i(y_i, \bar{e}_i, \epsilon)) - E(D') E(G_{y_i}^i(y_i, \bar{e}_i, \epsilon)) \\
&= E(D'(\bar{M} - f(\epsilon)(m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i))) - E(D') E(\bar{M} - f(\epsilon)(m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i))) \\
&= -E(D' f(\epsilon)(m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i))) + E(D') E(f(\epsilon)(m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i))) \\
&= -E(D' f(\epsilon))(m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i)) + E(D') \mu(m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i)) \\
&= -(m^i(y_i, \bar{e}_i) + y_i m_{y_i}^i(y_i, \bar{e}_i))(E(D' f(\epsilon)) - E(D') \mu). \tag{46}
\end{aligned}$$

From the specified form of  $D(\cdot)$ ,  $E(D'f(\epsilon)) - E(D')\mu$  becomes:

$$\begin{aligned}
& E(D'f(\epsilon)) - E(D')\mu \\
&= E(aZf(\epsilon)) - E(aZ)\mu \\
&= aE\left(\bar{z} - x - \sum_{k=1}^n G^k(y_k, \bar{e}_k, \epsilon)f(\epsilon)\right) - aE\left(\bar{z} - x - \sum_{k=1}^n G^k(y_k, \bar{e}_k, \epsilon)\right)\mu \\
&= -aE\left(\sum_{k=1}^n G^k(y_k, \bar{e}_k, \epsilon)f(\epsilon)\right) + aE\left(\sum_{k=1}^n G^k(y_k, \bar{e}_k, \epsilon)\right)\mu \\
&= -aE\left(\sum_{i=1}^n (y_i(\bar{M}_k - m^k(y_k, \bar{e}_k)f(\epsilon)))f(\epsilon)\right) + aE\left(\sum_{k=1}^n (y_k(\bar{M}_k - m^k(y_k, \bar{e}_k)f(\epsilon)))\right)\mu \\
&= aE\left(\sum_{k=1}^n y_k m^k(y_k, \bar{e}_k)f(\epsilon)f(\epsilon)\right) - aE\left(\sum_{k=1}^n y_k m^k(y_k, \bar{e}_k)f(\epsilon)\right)\mu \\
&= a\sum_{k=1}^n y_k m^k(y_k, \bar{e}_k)E((f(\epsilon))^2) - a\sum_{k=1}^n y_k m^k(y_k, \bar{e}_k)f(\epsilon)\mu^2 \\
&= a\sum_{k=1}^n (y_k m^k(y_k, \bar{e}_k)(E((f(\epsilon))^2) - \mu^2)) \\
&= a\sum_{k=1}^n (y_k m^k(y_k, \bar{e}_k)Var(f(\epsilon))). \tag{47}
\end{aligned}$$

Rearranging (47) using (45), we obtain:

$$a\sum_{k=1}^n (y_k m^k(y_k, \bar{e}_k)Var(f(\epsilon))) = a\sum_{k=1}^n \left(\frac{Var(G^k(y_k, \bar{e}_k, \epsilon))}{y_k m^k(y_k, \bar{e}_k)}\right). \tag{48}$$



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