

Title	CCAPM with time-varying parameters : some evidence from Japan
Sub Title	
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Publisher	Keio Economic Society, Keio University
Publication year	2011
Jtitle	Keio Economic Society discussion paper series Vol.11, No.4 (2011. )
JaLC DOI	
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Notes	
Genre	Technical Report
URL	<a href="https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA10715850-00001104-0001">https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA10715850-00001104-0001</a>

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Keio Economics Society Discussion Paper Series  
KESDP 11-4  
CCAPM with Time-Varying Parameters:  
Some Evidence from Japan

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August 25, 2011

# CCAPM with Time-Varying Parameters: Some Evidence from Japan\*

Mikio Ito<sup>†</sup> and Akihiko Noda<sup>‡</sup>

## Abstract

We test the parameter constancy of the standard consumption based asset pricing model (CCAPM) by using the method of Hansen (1990) for the generalized empirical likelihood (GEL) estimates for the Japanese financial data; we estimate the time-varying parameters considering our non-linear state space model as a simultaneous equation system and using the method developed by Ito (2007) and the GEL estimators. The empirical results exhibit that both the parameters estimated of the CCAPM, the degree of risk aversion and the time discount rate, vary with time.

*JEL classification numbers:* C58; G12; E21

*Keywords:* Time-Varying Estimation; CCAPM; GEL; Kalman Smoothing

## 1 Introduction

Since the early 1970s literature including Cooley and Prescott (1973a,b, 1976), Rosenberg (1973), and Sarris (1973) was published, regression models with the time-varying parameter or “stochastic coefficients,” have attracted a lot of attention of researchers in empirical work, especially in forecasting applications. The coefficients evolving stochastically over time enable them to treat time series models with parameter instability.

Recently, more and more researchers in economics and finance have been interested in time-varying structure in the economy. Smets and Wouters (2003) consider a time-varying degree of risk aversion in their derivation of the foreign exchange rate equation in the dynamic stochastic general equilibrium (DSGE) model. Fernández-Villaverde and Rubio-Ramírez (2007) also investigate stability of structural parameters including preferences ones in DSGE context. In literature of consumption-based capital asset pricing model (CCAPM), Campbell and Cochrane (1999) suggest that asset markets confront fluctuating

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\*We would like to thank Colin McKenzie, Taisuke Otsu, Tatsuma Wada and Makoto Yano for their helpful comments and suggestions. All data and programs used for this paper are available on request.

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risk aversion or distorted beliefs and introduce habit formation to preferences of consumers in order to explain a wide variety of asset pricing phenomena.

Following their articles, a number of economists have made their effort to extend the habit formation model in a wide variety of approaches (see, for example, Brandt and Wang (2003) and Møller (2009)). Maki and Sonoda (2002) pay attention to the structural stability in the Japanese financial markets to address the equity premium and risk free rate puzzles. Hyde and Sherif (2005) test the structural stability of several types of CCAPMs using the European financial and economic data. Brunnermeier and Nagel (2008) attempt to test if risk aversion varies with time in response to a change in wealth using household-level panel data. In literature of the capital asset pricing model (CAPM), moreover, a number of articles introduce time-varying elements, for instance, the covariance matrix of the rate of returns, to the model in order to improve explanation of the model (see, for example, Bollerslev et al. (1988), Bodurtha Jr and Mark (1991) and Ng (1991)).

In this paper, we show a simple time-varying CCAPM to investigate the gradual change in financial markets. We first test the parameter constancy using the test of Hansen (1990) which enables us to treat the parameter variation as random; he considers the parameters varying as a martingale process covering a wide range of their variation, for example, a usual random walk. Secondly, considering the model as a state space model, we estimate the time varying parameters by using the generalized empirical likelihood (GEL) method. we construct moment restrictions for the time-varying CCAPM with stochastic parameters, subject to a random walk, and estimate them simultaneously; preceding studies on CCAPMs with time-varying parameters use a rolling method to estimate them and to time-varying parameter of degree of risk aversion in CCAPM and make statistical inference (see, for example, Kim (2009)). While the dynamics of the parameters is quite simple, it brings us plausible estimates of the time-varying parameters as Nyblom (1989) and Kitagawa (2010) suggest. Our empirical results suggest that the time-varying estimates wildly change over time in Japan.

Section 2 presents a review of a typical CCAPM and the GEL estimators. We include a brief procedure of testing parameter constancy developed by Hansen (1990). Section 3 describes the data used. Section 4 shows the empirical result that both the two parameters in CCAPM with the CRRA utility function vary with time. Section 5 is for conclusion.

## 2 Model and Empirical Method

In this section, we present a typical CCAPM and empirical methods to estimate the parameters. First, we give a brief review of a CCAPM with the CRRA utility function for the reader's convenience. Then, we present a moment restriction model, which enables us to cope with statistical procedures: estimating parameters and testing hypotheses concerning the overidentifying restrictions of the model. Next, we show methods to estimate the parameters in our CCAPM and to test the parameter constancy of the model based on the technique developed by Hansen (1990). Assuming both the two parameters in the model, the degree of risk aversion and the time discount rate, vary with time, we estimate the time-varying parameters considering our non-linear state space model as a simultaneous equation system and using the method developed by Ito (2007) and the GEL estimators.

## 2.1 CCAPM

We assume that the representative consumer at time 0 chooses his/her life-time consumption and holding of several assets to maximize his/her expected utility subject to the budget constraint. The consumer's optimization problem is

$$\text{Max } E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma} \right], \quad 0 < \beta < 1, \quad 0 < \gamma, \quad (1)$$

$$\text{s.t. } C_t + \sum_{i=1}^N p_{i,t} A_{i,t} = \sum_{i=1}^N [p_{i,t} + d_{i,t}] A_{i,t-1} + Y_t, \quad i = 1, 2, \dots, N, \quad (2)$$

where the subscript  $t$  indicates time,  $C_t$  is real per capita consumption,  $p_{i,t}$  is the price of the  $i$ th asset,  $d_{i,t}$  is the dividend of the  $i$ th asset,  $A_{i,t}$  is the amount of the per capita holdings of the  $i$ th asset,  $Y_t$  is real per capita labor income,  $N$  is the number of assets,  $\beta$  is the subjective time discount factor,  $\gamma$  is the relative risk aversion (RRA), and  $E_t[\cdot]$  is the expectation operator conditional on the information available. In equation (1), we assume that the utility function is in the constant relative risk aversion (CRRA) class.

Solving the above utility maximization problem, we derive the following Euler equation:

$$E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + R_{i,t+1}) - 1 \right] = 0, \quad i = 1, 2, \dots, N, \quad (3)$$

where  $R_{i,t+1}$  is the real return of the asset at time  $t + 1$ , which is defined as

$$R_{i,t+1} = \frac{p_{i,t+1} + d_{i,t+1}}{p_{i,t}} - 1, \quad i = 1, 2, \dots, N. \quad (4)$$

Note that the log-linearized form of (3) will be also used:

$$E_t \left[ \log \beta - \gamma \log \left( \frac{C_{t+1}}{C_t} \right) + \log(1 + R_{i,t+1}) \right] = 0, \quad i = 1, 2, \dots, N. \quad (5)$$

At this stage, we do not specify data generating process (DGP) of  $C_t$ 's and  $R_{i,t}$ 's. Later in this section, we will show how to estimate the parameters in this Euler equation using the conditional expectation operator and how to make statistical inferences on these parameters.

## 2.2 Moment Restriction Model

We present a framework, called the moment restriction model, allowing us to cope generally with economic models, such as CCAPM, in which the distribution of data is not specified. In particular, we transform the equation (3) or (5) into one without a conditional expectation operator to estimate its parameters.

Following Hansen (1982), we derive unconditional moment restrictions for the non-linear and log-linear versions of the CCAPM respectively with each  $N$  error vectors  $\mathbf{u}_{t+1}(\theta)$

depending on the underlying parameter vector  $\theta$  and the instruments,  $\mathbf{z}_t$  be a  $K$  vector of instruments known at time  $t$ . We define a  $m (= N \cdot K)$  moment indicator vector  $g_t(\theta)$  as

$$g_t(\theta) = \mathbf{u}_{t+1}(\theta) \otimes \mathbf{z}_t. \quad (6)$$

In case of the non-linear version, the error vector is defined as

$$\mathbf{u}_{t+1}(\theta) = \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \cdot (\mathbf{1} + \mathbf{R}_{t+1}) \right] - \mathbf{1},$$

where  $\mathbf{R}_{t+1} = (R_{1,t+1}, R_{2,t+1}, \dots, R_{N,t+1})'$ ,  $\mathbf{1} = (1, 1, \dots, 1)'$  and  $\theta = (\beta, \gamma)'$ . Similarly, in case of the log-linear version,

$$\mathbf{u}_{t+1}(\theta) = \left( \log \beta - \gamma \left( \frac{C_{t+1}}{C_t} \right) \right) \mathbf{1} + \boldsymbol{\rho}_{t+1},$$

where  $\boldsymbol{\rho}_{t+1} = (\log(1 + R_{1,t+1}), \log(1 + R_{2,t+1}), \dots, \log(1 + R_{N,t+1}))$ , and  $\theta = (\log \beta, \gamma)'$ .

Then we can represent our Euler equation as

$$E[g_t(\theta)] = \mathbf{0}, \quad (7)$$

where  $E[\cdot]$  is the unconditional expectation operator. We call this equation a moment restriction model.

Generally, let  $y_t$ 's, ( $t = 1, 2, \dots, T$ ), denote observations on a finite dimensional process, which is usually assumed to be stationary and strongly mixing (see Smith (2004)). The right-hand side in equation (6),  $g_t(\theta)$ , is called a moment indicator, which is not only a function with respect to the parameters concerned but also depends on the data,  $y_t$ , and potentially on the instruments,  $\mathbf{z}_t$ 's. When our attention is focused on the parameters to be estimated,  $y_t$  and  $\mathbf{z}_t$  are usually omitted.

In the next subsection, we present two methods to estimate the parameters in moment restriction models: the GEL.

## 2.3 GEL

This section gives the reader a brief review of the estimators of a moment restriction model, following Newey and Smith (2004). The model we presented above is one with  $m$  moment restrictions. Before we review estimators of the parameters in such a model, let us define some notation.  $\mathbf{x}_t$ , ( $t = 1, 2, \dots, T$ ) denotes *i.i.d.* observations of the data, and  $p$  (equal to 2 in our model) denotes the number of parameters to be estimated. We sometimes write  $g(\mathbf{x}, \theta)$  as an  $m$  vector of functions of the data observations and the parameters. We here assume that  $m \geq p$  and the model has a true parameter  $\theta_0$  satisfying the following condition:

$$E[g(\mathbf{x}, \theta_0)] = \mathbf{0},$$

where  $E$  is expectation taken with respect to the distribution of  $x_t$ 's.

We here summarize several notations:  $g_t(\theta) = g(x_t, \theta)$ ,  $\hat{g}(\theta) = T^{-1} \sum_{t=1}^T g_t(\theta)$  and  $\hat{\Omega}(\theta) = T^{-1} \sum_{t=1}^T g_t(\theta) g_t(\theta)'$ . The generalized method of moments (GMM) estimator of

Hansen (1982), which has been widely used, is  $\hat{\theta}_{GMM} = \arg \min_{\theta \in \Theta} \hat{g}(\theta)' \hat{\Omega}(\theta^*)^{-1} \hat{g}(\theta)$ . where  $\theta^*$  is a convergent estimate of  $\theta_0$ , and  $\hat{\Omega}(\theta^*)$  is an heteroskedasticity and auto-correlation consistent (HAC) matrix like the one proposed by Newey and West (1987), which depends on a kernel and its bandwidth that can be chosen using the procedure of Andrews (1991).

However, it is widely known that the GMM have poor small sample properties. Many econometricians have made an effort to improve these small sample properties and have suggested several alternative estimators. These include the EL estimator of Owen (1988), the CUE of Hansen et al. (1996), and the ET estimator of Kitamura and Stutzer (1997). These estimators belong to a class of GEL estimators as shown by Newey and Smith (2004). We adopt these estimator when we estimate our time varying CCAPM.

To explain GEL estimators, let  $\rho(v)$  be a concave function on a real open interval  $\mathcal{V}$  containing zero. The GEL estimator is defined

$$\hat{\theta}_{GEL} = \arg \min_{\theta \in \Theta} \sup_{\lambda \in \hat{\Lambda}_t(\theta)} \sum_{t=1}^T \rho(\lambda' g_t(\theta)), \quad (8)$$

where  $\hat{\Lambda}_t(\theta) = \{\lambda : \lambda' g_t(\theta) \in \mathcal{V}\}$ . Alternative estimators to GMM are characterized by specifying  $\rho(v)$ : in the case of the empirical likelihood (EL) estimator,  $\rho(v) = \ln(1 - v)$ , and in the case the exponential tilting (ET) estimator,  $\rho(v) = -e^v$ , as Kitamura and Stutzer (1997) showed. Furthermore, when  $\rho(v) = -(1 + v)^2/2$ , the GEL estimator is equivalent to the CUE estimator,

$$\hat{\theta}_{CUE} = \arg \min_{\theta \in \Theta} \hat{g}(\theta)' \hat{\Omega}(\theta)^- \hat{g}(\theta), \quad (9)$$

where  $A^-$  denotes any generalized inverse of a matrix  $A$  (see Theorem 2.1 in Newey and Smith (2004)). For convenience, we impose a normalization on  $\rho(v)$  as Newey and Smith (2004) suggest. Let  $\rho_j(v) = \partial^j \rho(v) / \partial v^j$  and  $\rho_j = \rho_j(0)$  for each  $j$ . We assume that  $\rho_1 = \rho_2 = -1$ .

Associated with each GEL estimator are implied probabilities for the data. Because these probabilities will be used for our empirical analysis, we give a brief review. Consider  $\rho(v)$ , an associated GEL estimator  $\hat{\theta}$ , and  $\hat{g}_t(\hat{\theta})$ . The implied probabilities are given by

$$\hat{\pi}_t = \frac{\rho_1(\hat{\lambda}' \hat{g}_t)}{\sum_{s=1}^T \rho_1(\hat{\lambda}' \hat{g}_s)}, \quad t = 1, 2, \dots, T, \quad (10)$$

where  $\hat{\lambda} = \arg \max_{\lambda} \sum_{t=1}^T \rho(\lambda' \hat{g}_t) / n$ . For any function  $f(x, \theta)$  and GEL estimator  $\theta$ , an efficient estimator  $\sum_{t=1}^T \hat{\pi}_t f(x_t, \hat{\theta})$  of  $E[f(x, \theta_0)]$  can be derived, as shown in Brown and Newey (1998).

We use a J-statistic to test the overidentifying restrictions; the J-statistic for GEL estimation is computed using the smoothed moment indicator  $g_T$  (see Section 4 in Smith (2004) for detail) and some kernel function  $k(\cdot)$ :

$$J = T \hat{g}_T(\hat{\theta})' \hat{\Omega}^{-1} \hat{g}_T(\hat{\theta}) / k_1^2,$$

where  $\hat{\Omega}$  is a consistent estimate of  $\Omega$  and  $k_1 = \int_{-\infty}^{\infty} k(a) da$ . Under the null hypothesis that equation (7) is true, the statistic is asymptotically distributed as  $\chi_{m-p}^2$ , where  $p$  is the number of parameters.

## 2.4 Extension of Hansen's Parameter Constancy test for GEL

In this paper, we test whether the parameters estimated by GEL are constant or not, using the Hansen's parameter constancy test. (See the Appendix for the main idea and the procedure.) His test is based on the first order condition (FOC) of underlying estimators: OLS, GLS, IV, ML and GMM. While Hansen does not consider the GEL as the underlying estimator, his test can be extended to the GEL.

In the following part, we present the specification of the first order conditions to derive the GEL estimators (CUE, EL, and ET) for CCAPMs considering a moment restriction model  $E[g(\mathbf{x}_t, \theta)] = \mathbf{0}$ . Suppose that the moment indicator  $g(\cdot, \cdot)$ ,  $p$  vector valued function, is twice continuously differentiable and that  $\{g(\mathbf{x}_t, \theta)\}$  is *i.i.d.* According to Hansen (1990), the expressions are used for the GMM.

$$m_t(\theta, \tau) = g_t(\theta),$$

$$Q_T(\theta, \tau) = \left( \sum_{t=1}^T \frac{\partial}{\partial \theta} g_t(\theta) \right)' \Omega_T^{-1},$$

and

$$m_{Tt}(\theta, \tau) = \left( \sum_{t=1}^T \frac{\partial}{\partial \theta} g_t(\theta) \right)' \Omega_T^{-1} g_t(\theta),$$

where  $\Omega_T$  is some natural estimate of  $E[g(\mathbf{x}_t)g(\mathbf{x}_t)']$ . The expressions used for the GEL are derived from the first order condition for the GEL estimator as Newey and Smith (2004) show:

$$T \left[ \sum_{t=1}^T \hat{\pi}_t G(\mathbf{x}_t, \hat{\theta}) \right]' \left[ \sum_{t=1}^T \hat{k}_t g_t(\hat{\theta}) g_t(\hat{\theta})' \right]^{-1} \hat{g}(\hat{\theta}) = T Q_T(\theta, \tau) \frac{1}{T} \sum_{t=1}^T g_t(\theta) = 0,$$

where  $G(\mathbf{x}_t, \hat{\theta})$  is the gradient of  $g$  with respect to  $\theta$  which is valued at  $\hat{\theta}$ ,

$$\begin{aligned} m_{Tt}(\theta, \tau) &= Q_T(\theta, \tau) \hat{g}_t(\theta) \\ &= T \left( \sum_{t=1}^T \hat{\pi}_t G_t(\hat{\theta}) \right)' \left( \sum_{t=1}^T \hat{k}_t g_t(\hat{\theta}) g_t(\hat{\theta})' \right)^{-1} \hat{g}_t(\theta), \end{aligned}$$

$\pi_t$ , called the implied probability, is defined as

$$\hat{\pi}_t = \frac{\rho_1(\hat{\lambda}' g_t(\hat{\theta}))}{\sum_{s=1}^T \rho_1(\hat{\lambda}' g_s(\hat{\theta}))},$$

$\hat{\lambda}$  is the auxiliary vector for GEL setting, and

$$\hat{k}_t = \frac{k(\hat{v}_t)}{\sum_{s=1}^T k(\hat{v}_s)}, \quad \text{where } \hat{v}_t = \hat{\lambda}' \hat{g}_t, \quad k(v) = \frac{\rho(v) + 1}{v} \quad (v \neq 0, \quad k(0) = -1),$$

and  $\rho(\cdot)$  is a conjugate function of a member of the Cressie-Read estimator family of discrepancies (see Newey and Smith (2004, pp.223–224) for detail). Note that the weights in the above equation are  $\hat{k}_t = \hat{\pi}_t$  for EL,  $k_t = 1/T$  for CUE.



## 2.5 Estimation of Time-Varying Parameters in CCAPM

The test of parameter constancy of the standard CCAPM in the last subsection concerns that the parameters vary over time. In this subsection, considering the alternative hypothesis of the test in the previous subsection, which implies the parameter variation as random, we show a simple time-varying CCAPM (TV-CCAPM) to investigate the possibility of gradual change of economic structure.

When we estimate the TV-CCAPM, there are two difficulties: (i) the GEL estimators are usually unstable in numerical calculation and (ii) as many parameters as double number of sample periods are to be estimated. Specifically, it is hard to choose appropriate starting values in numerical optimization. Our method of estimation of the parameters in the TV-CCAPM has two stages: (i) we preliminarily estimate them using the Kalman smoothing for a state space representation of the log-linearized TV-CCAPM and thus (ii) we compute the TV-CCAPM with the starting values obtained at the first stage.

Following Nyblom (1989), who introduces a martingale formulation allowing for substantial flexibility, we simply assume that both the parameters in our model, the degree of risk aversion and the time discount rate, are subject to the following restrictions:

$$\beta_t = E_t[\beta_{t+1}], \quad (t = 1, \dots, T-1),$$

$$\gamma_t = E_t[\gamma_{t+1}], \quad (t = 1, \dots, T-1).$$

This setting supposes that the value of a parameter in the specific period is equal to conditional expectation of the parameter in the previous period. We pose this assumption for a first approximation of dynamics of the underlying parameters to capture gradual changes in financial markets; one could say that this assumption has no theoretical background.

When we estimate the parameters, we adopt more specific form of the above restrictions:

$$\beta_{t+1} = \beta_t + \varepsilon_t^\beta, \quad (t = 1, \dots, T-1), \quad \varepsilon_t^\beta \sim i.i.d.(0, \sigma_\beta^2), \quad (11)$$

$$\gamma_{t+1} = \gamma_t + \varepsilon_t^\gamma, \quad (t = 1, \dots, T-1), \quad \varepsilon_t^\gamma \sim i.i.d.(0, \sigma_\gamma^2). \quad (12)$$

For convenience of calculation, we use log-linearized equations in order to estimate the time-varying parameters:

$$\log(1 + R_{1,t}) = -\log \beta + \gamma \log \left( \frac{C_t}{C_{t-1}} \right) + u_{1t}, \quad (t = 1, \dots, T-1),$$

$$\log(1 + R_{2,t}) = -\log \beta + \gamma \log \left( \frac{C_t}{C_{t-1}} \right) + u_{2t}, \quad (t = 1, \dots, T-1).$$

Assuming that both the two parameters,  $\gamma$  and  $\beta$ , vary with time, we first adopt the following equations.

$$\log(1 + R_{1,t}) = -\log \beta_t + \gamma_t \log \left( \frac{C_t}{C_{t-1}} \right) + u_{1t}, \quad (t = 1, \dots, T-1), \quad (13)$$

$$\log(1 + R_{2,t}) = -\log \beta_t + \gamma_t \log \left( \frac{C_t}{C_{t-1}} \right) + u_{2t}, \quad (t = 1, \dots, T-1). \quad (14)$$

We here consider the system of the above equations, (13) and (14) of CCAPM together with the ones of the parameter dynamics, equations (11) and (12) as an ordinary state space model: the equations (13), (14) are for the observation equations and (12) and (11) for the state equations. Note that both the two parameters are state variables to be estimated.

In order to estimate the state variables in a state space model, one usually use Kalman's techniques, the Kalman filter and smoothing. Instead, we use other method developed by one of the authors of this paper in order to estimate state variables in a linear state space model. Ito (2007) constructs a simple linear regression model with some priors, which enables us to estimate the state variables by OLS, GLS, 2SLS or SUR, while Durbin and Koopman (2001) consider a unified linear structural equation for the underlying state space model, which is hard to estimate.

We represent our state space model to be estimated as a combined system of linear equations in a matrix form as follows:

$$\begin{bmatrix} \log(1 + R_{1,1}) \\ \log(1 + R_{2,1}) \\ \log(1 + R_{1,2}) \\ \log(1 + R_{2,2}) \\ \vdots \\ \log(1 + R_{1,T}) \\ \log(1 + R_{2,T}) \\ \log \beta_0 \\ 0 \\ \vdots \\ 0 \\ \gamma_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \begin{array}{ccc|ccc} -1 & 0 & O & \log \frac{C_1}{C_0} & 0 & O \\ -1 & 0 & & \log \frac{C_1}{C_0} & 0 & \\ 0 & -1 & & 0 & \log \frac{C_2}{C_1} & \\ 0 & & & 0 & \log \frac{C_2}{C_1} & \ddots \\ & & \ddots & & & \ddots \\ O & & -1 & & & \log \frac{C_T}{C_{T-1}} \\ & & -1 & & & \log \frac{C_T}{C_{T-1}} \\ \hline 1 & & O & & & \\ \vdots & & & & & \\ -1 & & & & O & \\ \vdots & & & & & \\ O & & -1 & 1 & & \\ \hline & & & & 1 & \bar{O} \\ & & & & & \\ & & O & & -1 & \ddots \\ & & & & \ddots & \ddots \\ & & & & & -1 & 1 \end{array} \end{bmatrix} \begin{bmatrix} \log \beta_1 \\ \log \beta_2 \\ \vdots \\ \log \beta_T \\ \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_T \end{bmatrix} + \begin{bmatrix} \epsilon_{1,1} \\ \epsilon_{1,2} \\ \epsilon_{2,1} \\ \epsilon_{2,2} \\ \vdots \\ \epsilon_{T,1} \\ \epsilon_{T,2} \\ \eta_1 \\ \eta_2 \\ \vdots \\ \eta_T \\ \xi_1 \\ \xi_2 \\ \vdots \\ \xi_T \end{bmatrix}, \quad (15)$$

where  $\beta_0$  and  $\gamma_0$  are priors, for which we use the each estimates of the CCAPM **with constant parameters** for the whole sample. We can estimate both the two parameters in TV-CCAPM with CRRA utility function by 2SLS. Let  $\tilde{\beta} = (\tilde{\beta}_1 \tilde{\beta}_2 \cdots \tilde{\beta}_T)$  denote the time-varying estimates of  $\beta$  and  $\tilde{\gamma} = (\tilde{\gamma}_1 \tilde{\gamma}_2 \cdots \tilde{\gamma}_T)$  the ones of  $\gamma$ <sup>1</sup>.

At the second stage, we estimate the following moment restrictions model for our TV-CCAPM:

$$E_t[g_t(x_t, \theta)] = 0, \text{ for } t = 1, \dots, T, \quad (16)$$

where

$$\begin{aligned} g_t(x_t, \theta) &= (g_t^1(x_t, \theta), g_t^2(x_t, \theta), g_t^3(x_t, \theta)), \\ g_t^1(x_t, \theta) &= \left[ \beta_t \left( \frac{C_{t+1}}{C_t} \right)^{\gamma_t} (1 + R_{1,t+1}) - 1 \right] \cdot \mathbf{z}_t, \\ g_t^2(x_t, \theta) &= \left[ \beta_t \left( \frac{C_{t+1}}{C_t} \right)^{\gamma_t} (1 + R_{2,t+1}) - 1 \right] \cdot \mathbf{z}_t, \end{aligned}$$

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<sup>1</sup>We should select hyper parameters associated with the weights which are used in estimation; here we avoid discussions on their selection of a non-linear state space model.

$$g_t^3(x_t, \theta) = \theta_t - \theta_{t-1},$$

$\mathbf{z}_t$  represents the vector of instrumental variables, and  $\theta = (\beta, \gamma) \in \mathbf{R}^{2T}$ . When we estimate  $\theta$  by using the GEL methods, we use  $\hat{\theta} = (\hat{\beta}, \hat{\gamma})$  that we obtain at the first stage as the starting value of numerical computation.

### 3 Data

In this paper, quarterly data from 1980Q3 to 2009Q4 are used. The per capita consumption are computed as “Nondurable goods plus service consumption (benchmark year 2000)” divided by the estimates of the total population reported in the *Annual Report on National Accounts* in Japan. The per capita consumption data are seasonally adjusted using the X-12 ARIMA procedure. The returns on short-term instruments are employed as the return on the risk-free asset and these are obtained from Nikko Financial Intelligence. The Fama-French’s market portfolio returns are treated as the returns on the risky asset and these are obtained from Nikkei Portfolio Master.<sup>2</sup> To deflate all series, the “Nondurable plus service consumption” deflator published in the *Annual Report on National Accounts* is used.<sup>3</sup> Figure 1 presents time series plots for each variables.

(Figure 1 around here)

Lagged values of the real consumption growth rate, the real return on the risk-free asset, and the real return on the market portfolio are used as instruments.

For the GEL estimator, all variables that appear in the moment conditions should be stationary. To check whether the variables satisfy the stationarity condition, We use the ADF-GLS test of Elliott et al. (1996). Table 1 provides some descriptive statistics and the results of the ADF-GLS tests. For all the variables, the ADF-GLS test rejects the null hypothesis that the variable contains a unit root at conventional significance levels.<sup>4</sup>

(Table 1 around here)

### 4 Empirical Results

First, we present preliminary estimations of two basic parameters in CCAPMs, non-linear and log-linear models, using the GEL estimators. Second, we examine whether the parameters are constant or not using the Hansen’s (1990) parameter constancy test. Third, we estimate the time-varying parameters in the CCAPM framework.

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<sup>2</sup>Fama-French’s market factors in Japan are calculated by following Kubota and Takehara (2007).

<sup>3</sup>The “Nondurable plus service consumption” deflator is a weighted inflation rate using “Nondurable goods” and “Service” deflators that are also published in the *Annual Report on National Accounts*.

<sup>4</sup>We confirm that there are no size distortions that Elliott et al. (1996) and Ng and Perron (2001) pointed out in making the ADF-GLS test for small samples (see a column  $\hat{\phi}$  of the table 1 for details). Therefore, we use the Modified Bayesian Information Criterion (MBIC), not the Modified Akaike Information Criteria (MAIC) to select an optimal lag order for the ADF-GLS tests.

## 4.1 Preliminary Estimates

Table 2 shows the preliminary results for non-linear and log-linear CCAPMs with the GEL estimators (CUE, EL, and ET) using the whole sample.

(Table 2 around here)

In GEL estimations, we choose the truncated kernel proposed by Kitamura and Stutzer (1997) and Smith (1997) to smooth the moment conditions (that is, equation (8) in our case) because Anatolyev (2005) demonstrates that, in the presence of correlation in the moment function, the smoothed GEL estimator is efficient.<sup>5</sup> In addition, we employ an appropriate HAC covariance matrix of Andrews (1991) to reduce estimation biases. The estimates of  $\beta$  and  $\gamma$  are statistically significant at conventional levels. The estimates of  $\beta$  range from 0.9981 to 0.9987 (0.9981 to 0.9986); the estimates of  $\gamma$  range from 0.8026 to 0.8969 (0.7944 to 0.8644) in the case of a non-linear (log-linear) CCAPM. The  $p$ -values for the Hansen's J-test are large enough that we cannot reject the null that the moment conditions hold. Therefore, we obtain almost the same empirical results regardless of the formulation: non-linear or log-linear.

## 4.2 Parameter Constancy Tests

Now, we investigate whether there exhibits a parameter constancy in the non-linear and log-linear CCAPMs using Hansen's (1990) tests under random parameters hypothesis. Few studies examining the possibility of gradual changes in the parameters in CCAPMs estimate the time-varying parameters without using the method of rolling.

(Table 3 around here)

Table 3 presents the results of parameter constancy tests as shown in subsection 5. The first and second row of the table 3 show the individual parameter constancy test results for  $\beta$  and  $\gamma$ , respectively. We can not reject the null of constant parameters against the parameter variation as random walk and conclude that each parameters are constant individually. However, the third row of the table 3 shows the results of joint parameter constancy test,  $\beta$  and  $\gamma$  are both time-varying, we reject the null of constant parameters against the parameter variation as random walk at the 1% statistical significant level. Therefore, we estimate the time-varying parameters of CCAPMs to investigate whether gradual changes occur in the Japanese financial market.

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<sup>5</sup>We employ the smoothed GEL estimator, but the optimal kernel weights do not exceed one. This suggests that the kernel smoothing has no effect.

### 4.3 Time-Varying Estimates

Figures 2 to 5 provide the time-varying GEL estimates of  $\beta$  and  $\gamma$ , and present the filtered estimates of Hodrick and Prescott (1997).

(Figures 2 to 5 around here)

We confirm that both the parameters change over time regardless of the formulation: non-linear or log-linear. In particular, we find that: (i) the filtered estimates of  $\beta$  decline (increase) as the economy slowed (improved), and (ii) the filtered estimates of  $\gamma$  decline over time. The first result suggests consumers' reluctant (willing) attitudes about his/her investment in the recession (economic recovery).<sup>6</sup> The second result may reflect that Japanese consumers have become more and more active in the financial markets because of financial deregulations since 1985, although it is hard to interpret the trending  $\gamma$  in the whole sample.

## 5 Conclusion

By adopting the GEL estimators, one could accurately estimate the parameters of CCAPMs using the data of the Japanese financial markets as Noda and Sugiyama (2010) and Ito and Noda (2012) successfully show. However, since we have experienced several financial crises such as the recent one after Lehman's fall, we should expect that the structure in financial markets usually confront sudden or gradual changes; we should test the constancy of parameters of models to explain our financial system. Thus, we investigate possibilities of gradual change in the Japanese financial markets.

Our empirical results exhibit gradual changes in the Japanese financial markets by making the test of the null of constant parameters against the parameter variation as a random walk process and by estimating time-varying parameters of the CCAPM with the CRRA utility function; we set up a CCAPM associated with time-varying parameters as a non-linear state space model with observation equations (moment restrictions) and with state equations representing random walk of the parameters in the CCAPM: the time discount rate and the degree of risk aversion. We successfully estimate the time-varying parameters by the technique developed by one of the authors of this paper based on the Kalman smoothing. Furthermore, setting the non-linear Euler equations of CCAPM and the martingale formulation of random parameters as moment restrictions, we estimate the time-varying parameters by using GEL. Our estimates show that both the two parameters, especially the degree of risk aversion, vary over time.

We finally conclude that the economic agents in the Japanese financial markets have been changing their preferences in subjective discount rate as well as risk aversion. This finding possibly reflects a link between fluctuations in real economy and economic agents' preferences. We can not rely on the simple model of asset pricing with a few factor such as the CCAPM.

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<sup>6</sup>The filtered estimates of  $\beta$  exceed one in some periods, but Kocherlakota (1990) shows that there is equilibrium where the subjective discount rate is over one.

## Appendix: Hansen's Parameter Constancy Test

This appendix presents a parameter constancy test which treats the parameter variation as a random walk. We consider the situation in which the consumer's preferences vary gradually over time. Specifically, following Hansen (1990), we consider a parameter vector  $\theta_t$  and some increasing sequence of  $\sigma$ -field  $\{\mathcal{F}_t\}$  to which  $\theta_t \in \mathbf{R}^k$  is adapted. Set

$$\Delta_t = \theta_t - \theta_{t-1},$$

and assume

$$E[\Delta_t | \mathcal{F}_{t-1}] = \mathbf{0}, \quad E[\Delta_t \Delta_t'] = \delta^2 G_{t-1},$$

for some known matrix  $G_{t-1}$ . This martingale type of parameter variation, which Nyblom (1989) introduced, has substantial flexibility;  $\{\theta_t\}$  obeying a random walk process as an example covers very wide range of variations (see Kitagawa (2010,ch.11), for detail).

Our null and alternative hypotheses are

$$H_0 : \delta^2 = 0, \quad H_1 : \delta^2 > 0.$$

Hansen (1990) proposes two statistics as a good approximation to the LM test of  $H_0$  against  $H_1$ , inspired by Nyblom (1989), who considers maximum likelihood problems. By focusing the first order conditions, Hansen reconstructs Nyblom's idea to treat general nonlinear models; he represents the first order conditions such as the normal equations of OLS and GLS estimations as follows:

$$\mathbf{0} = FOC(\hat{\theta}, \hat{\tau}),$$

where the right hand side represents  $k$  equations with  $k$  unknowns. Quite often, the FOC can be written in the following form:

$$FOC(\theta, \tau) = Q_T(\theta, \tau) \sum_{t=1}^T m_t(\theta, \tau),$$

where  $Q_T(\cdot, \cdot)$  is a sequence of  $k \times q$  ( $q \geq k$ ) depending of the data and  $\tau$  is a random vector of some preliminary estimator used for estimating  $\theta$  such as the implied probabilities and the auxiliary variables in GEL estimation. Note that the first order conditions are representable in the following way:

$$\mathbf{0} = \sum_{t=1}^T m_{Tt}(\hat{\theta}, \hat{\tau}), \quad \text{where } m_{Tt}(\theta, \tau) = Q_T(\theta, \tau) m_t(\theta, \tau).$$

In order to apply Hansen's test to our inference, we should represent the first order conditions of the estimators in this paper in Hansen's way; we use the following expressions:  $\hat{m}_t = m_t(\hat{\theta}, \hat{\tau})$ ,  $\hat{m}_{Tt} = m_{Tt}(\hat{\theta}, \hat{\tau})$ , and  $\hat{Q}_T = Q_T(\hat{\theta}, \hat{\tau})$ .

We define the partial sums of the FOCs:

$$S(\pi) = \sum_{t=1}^{[T\pi]} m_t(\theta_0, \tau_0).$$

where  $\pi$  is a real number in the unit interval and  $[T\pi]$  is the integer part of  $T\pi$ . The estimated correspondence is

$$\hat{S}(\pi) = \sum_{t=1}^{[T\pi]} \hat{m}_t.$$

Note that both the two expressions can be represented as follows:

$$S_T(\pi) = \sum_{t=1}^{[T\pi]} m_{Tt}(\theta_0, \tau_0) = Q_T(\theta_0, \tau_0)S(\pi),$$

and

$$\hat{S}_T(\pi) = \sum_{t=1}^{[T\pi]} \hat{m}_{Tt} = \hat{Q}_T \hat{S}(\pi).$$

The variance of  $S_T(\pi)$  and the estimated correspondence are

$$V(\pi) = E[S_T(\pi)S_T(\pi)'],$$

and

$$\hat{V}(\pi) = \sum_{t=1}^{[T\pi]} \hat{m}_{Tt} \hat{m}'_{Tt} = \hat{Q}_T \sum_{t=1}^{[T\pi]} \hat{m}_t \hat{m}'_t \hat{Q}'_T,$$

respectively.

When Hansen (1990) considers the parameter variation as random, he defines a LM type of statistic

$$LM(\pi) = \hat{S}_T(\pi)' G_T(\pi) \hat{S}_T(\pi). \quad (\text{A.1})$$

His major contribution is to propose two candidates of estimates of  $G_T$  as we show below. He rewrites the equation (A.1):

$$L = \frac{1}{T} \sum_{t=1}^{T-1} \left[ \left( \sum_{s=1} \hat{m}'_{Ts} \right) G_{Tt} \left( \sum_{s=1} \hat{m}_{Ts} \right) \right] = \int_0^1 \hat{S}_T(\pi)' G_T(\pi) \hat{S}_T(\pi) d\pi.$$

Hansen carefully chooses  $G_{Tt}$  so as to have a convenient limiting distribution and have power against alternative of interest, the martingale formulation. His choice of  $G_{Tt}$  is a matrix which is constant across time, say,  $G$ . The statistic is simply represented as:

$$L_C = \text{tr} \left\{ \frac{1}{T} \left[ \sum_{t=1}^{T-1} \hat{S}_T \left( \frac{t}{T} \right) \hat{S}_T \left( \frac{1}{T} \right)' \right] G \right\} = \text{tr} \left\{ \int_0^1 \hat{S}_T(\pi) \hat{S}_T(\pi)' d\pi G \right\},$$

where he chooses  $G$  as  $\hat{V}(1)^{-1}$ . Hansen shows that the statistic has the asymptotic distribution of  $L_C$  invariant to nuisance parameters as follows:

$$L_C = \text{tr} \left\{ \int_0^1 \hat{S}_T(\pi) \hat{S}_T(\pi)' d\pi \hat{V}(1)^{-1} \right\},$$

under the assumption of such constancy.

Hansen shows the limiting distributions of both the two statistics,  $L_C$  and  $LM(\cdot)$  using the multivariate standard Brownian motion  $B(\cdot)$  and the Brownian bridge defined as

$$BB_r(\pi) = B(\pi) - \pi B(1),$$

$$L_C \implies \int_0^1 BB_r(\pi)' BB_r(\pi) d\pi,$$

$$LM(\pi) \implies Q_k(\pi) = \frac{BB_r(\pi)' BB_r(\pi)}{\pi(1 - \pi)},$$

where " $\implies$ " denotes weak convergence of probability measures. He also presents the critical values of  $L_C$  (see Hansen (1990) Table 1). Using Hansen's method, we can test the parameter constancy in CCAPM setting the alternative hypothesis that the parameters vary as a martingale process. In Appendix, we summarize specifications of  $m_t$ ,  $m_{Tt}$ , and  $Q_t$  to build the statistics above.

The procedure of the parameter constancy test of Hansen (1990) is summarized as follows.

1. Estimate the model parameters under the assumption with no parameter variation.
2. Build  $\hat{m}_t$ ,  $\hat{m}_{Tt}$  and  $\hat{Q}_t$ .
3. Calculate  $\hat{S}$  and  $\hat{V}$ .
4. Compute the values of the test statistic,  $L_C$ .

The reader should note that it is unnecessary to estimate any time-varying estimates of underlying models.

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Figure 1: Time Series Plots

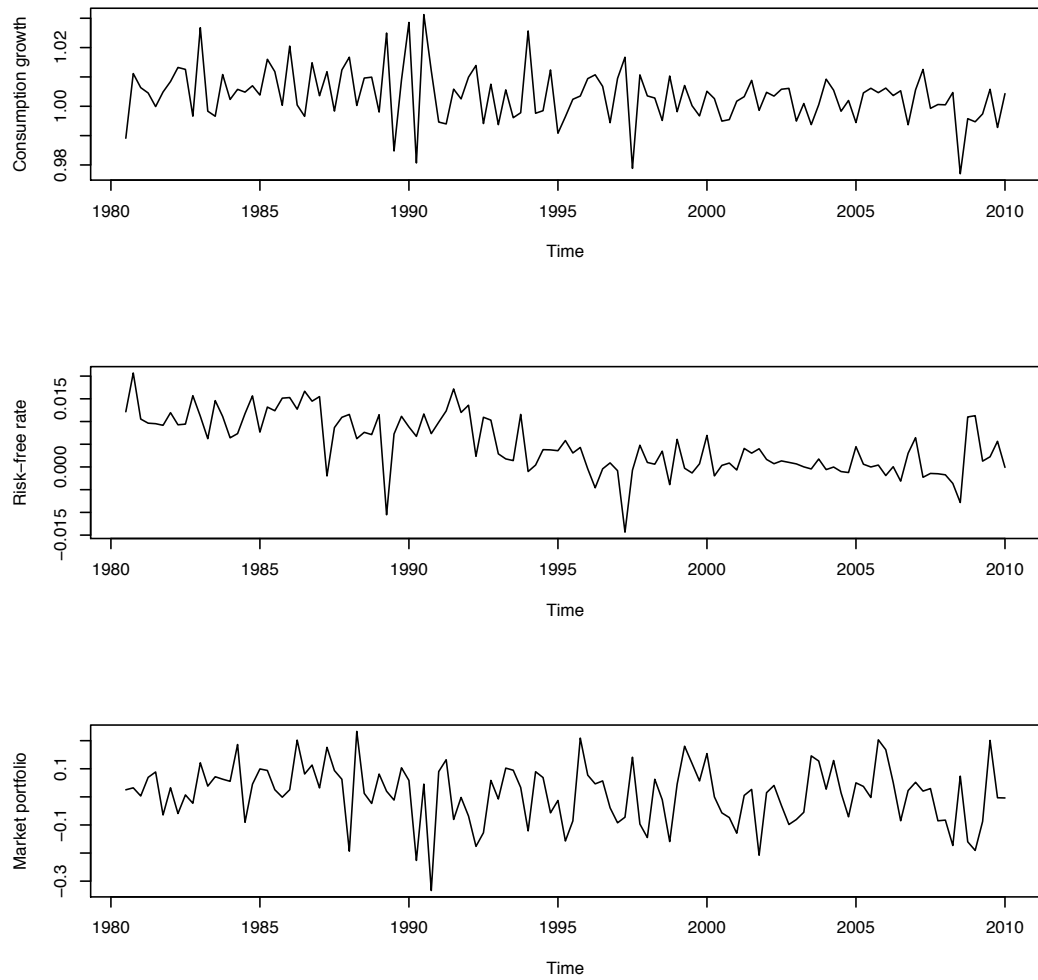


Table 1: Descriptive Statistics and Unit Root Tests

	Mean	SD	Min	Max	ADF-GLS	Lag	$\hat{\phi}$	$\mathcal{N}$
$CG_t$	1.0037	0.0091	0.9770	1.0312	-2.7188	2	0.1359	118
$R_t^f$	0.0050	0.0063	-0.0143	0.0207	-8.4604	0	0.2403	118
$R_t^m$	0.0121	0.1041	-0.3335	0.2331	-10.0600	0	0.0724	118

“ $CG_t$ ” denotes the gross real per capita consumption growth, “ $R_t^f$ ” denotes the real return on risk-free asset, “ $R_t^m$ ” denotes the real return on market portfolio, “ADF-GLS” denotes the ADF-GLS test statistics, “Lag” denotes the lag order selected by the MBIC, “ $\hat{\phi}$ ” denotes the coefficients vector in the GLS detrended series (see equation (6) in Ng and Perron (2001)), and “ $\mathcal{N}$ ” denotes the number of observations. In computing the ADF-GLS test, a model with a time trend and a constant is assumed. The critical values at the 10% significance level for the ADF-GLS test is “-2.62”. The null hypothesis that each variable has a unit root is clearly rejected at the 10% significance level. R version 2.13.1 was used to compute the statistics.

Table 2: Preliminary Results

	Non-Linear			Log-Linear		
	CUE	EL	ET	CUE	EL	ET
$\hat{\beta}$	0.9985 [0.0008]	0.9987 [0.0009]	0.9981 [0.0008]	0.9985 [0.2247]	0.9986 [0.2273]	0.9981 [0.2206]
$\hat{\gamma}$	0.8502 [0.2306]	0.8969 [0.2403]	0.8026 [0.2204]	0.8351 [0.2233]	0.8644 [0.2258]	0.7944 [0.2191]
$p_J$	0.6227	0.6184	0.6142	0.6955	0.6941	0.6874
DF	8			8		

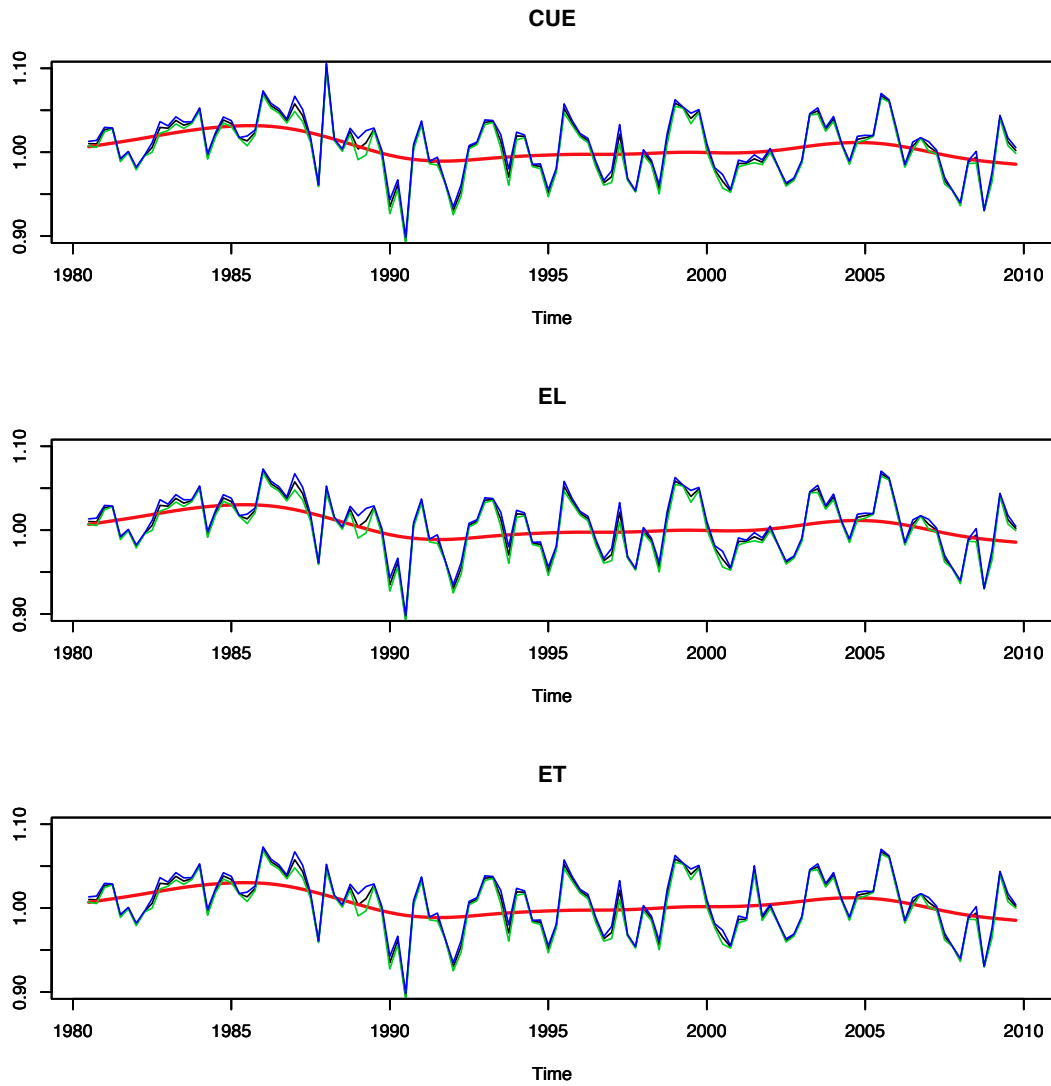
“ $\hat{\beta}$ ” denotes the estimate of the subjective discount rate (Log-Linear estimates of  $\beta$  is transformed exponentially), “ $\hat{\gamma}$ ” denotes the estimate of the degree of the relative risk aversion, “ $p_J$ ” denotes the  $p$ -value for Hansen’s J-test, and “DF” denotes the degrees of freedom for the Hansen’s J-test. Andrews (1991) adjusted standard error for each estimates are reported in brackets. R version 2.13.1 was used to compute the estimates, the starting values of the parameters are set equal to  $\beta = 1$  and  $\gamma = 1$ .

Table 3: Parameter Constancy Tests

	Non-Linear			Log-Linear			CV	DF
	CUE	EL	ET	CUE	EL	ET		
$\beta$	-	-	-	0.10	0.11	0.15	0.75	1
$\gamma$	-	-	-	0.10	0.11	0.15		
$\beta, \gamma$	2.36	2.22	2.36	1.17	1.10	1.36	1.07	2

“CV” denotes asymptotic critical values at 1% significant levels for each statistics (see Hansen (1990) Tables 1), and “DF” denotes the degree of freedom for the Hansen’s (1990) hazard statistics. R version 2.13.1 was used to compute the statistics.

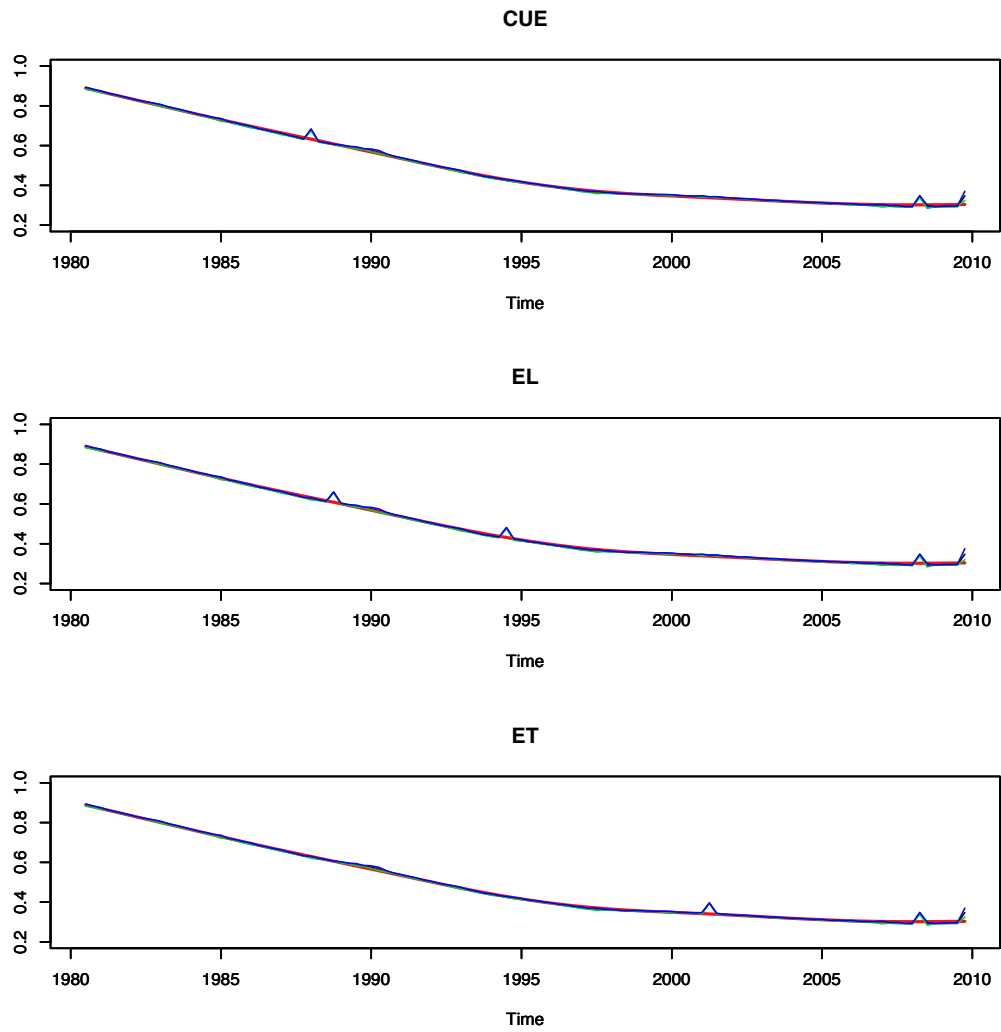
Figure 2: Non-Linear TV-Estimates of  $\beta$



Note: The blue and green lines are 95 % asymptotic confidence intervals.

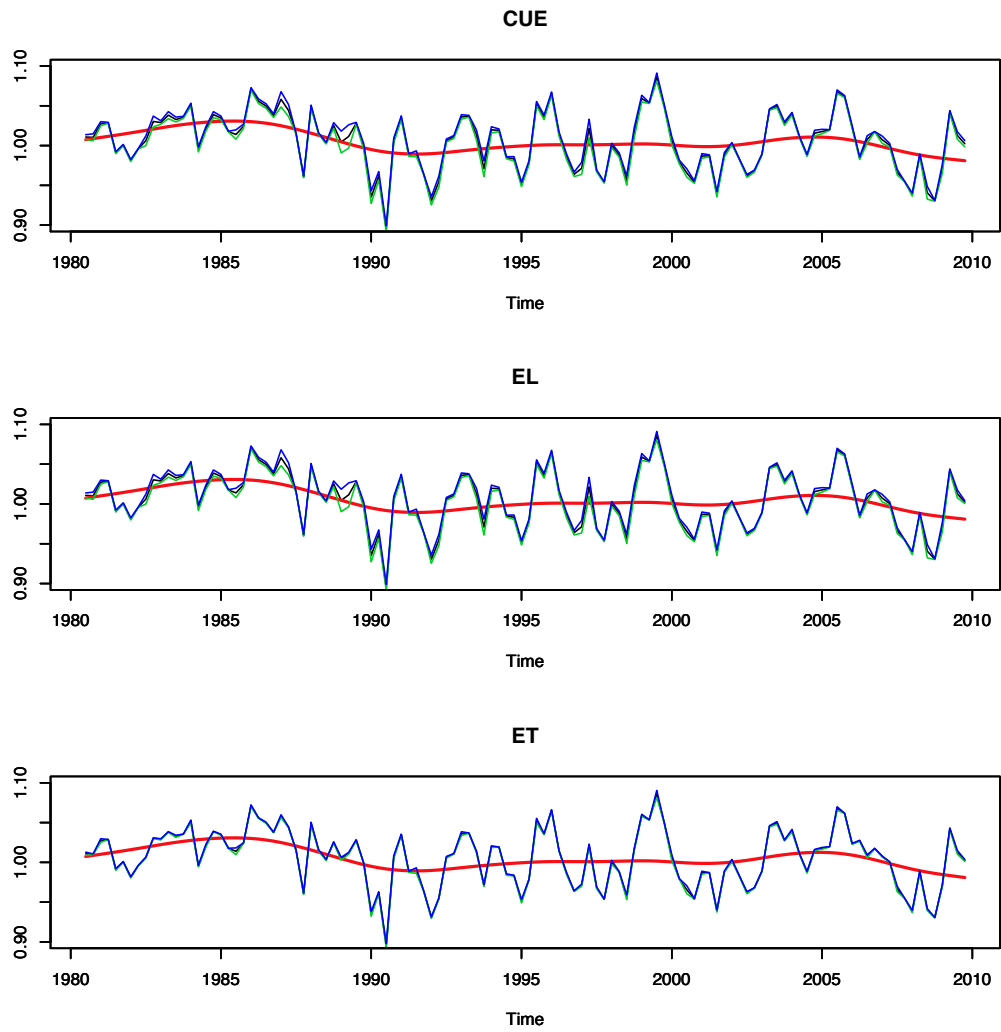


Figure 3: Non-Linear TV-Estimates of  $\gamma$



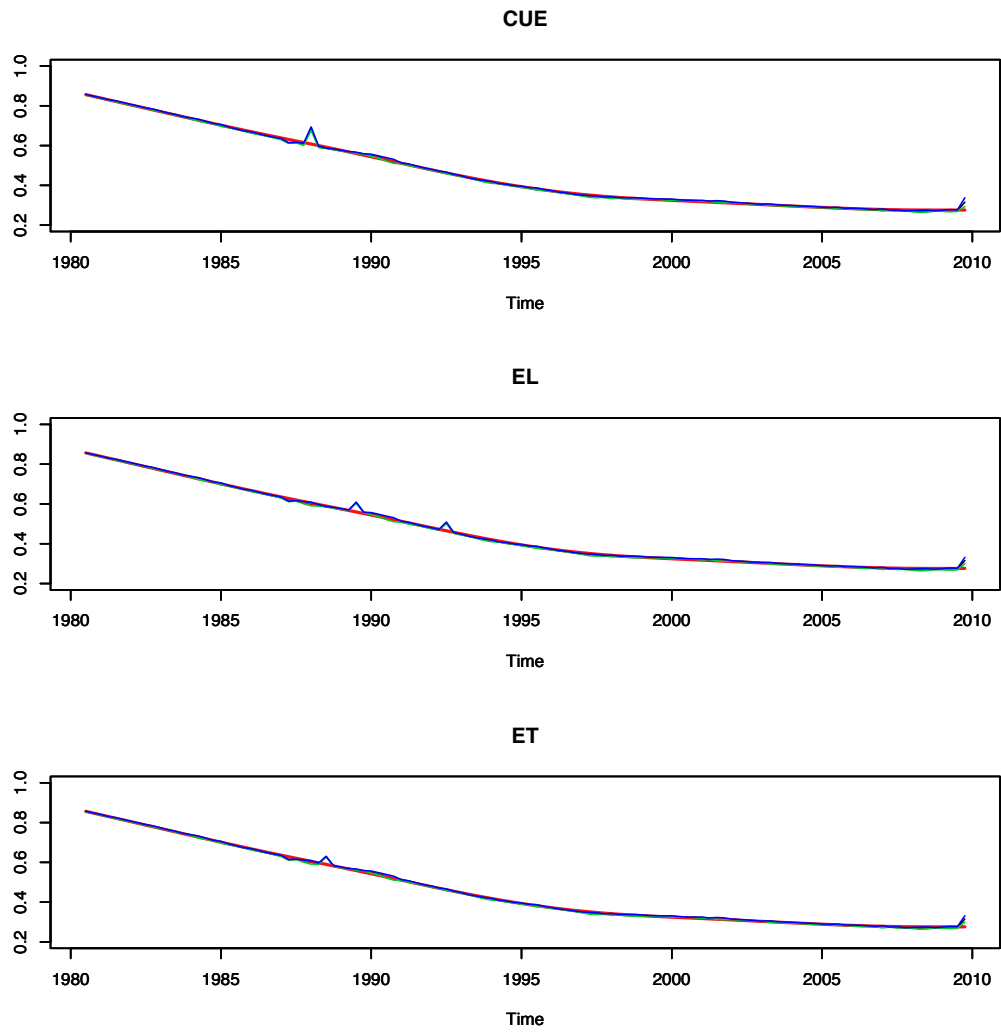
Note: The blue and green lines are 95 % asymptotic confidence intervals.

Figure 4: Log-Linear TV-Estimates of  $\beta$



Note: The blue and green lines are 95 % asymptotic confidence intervals.

Figure 5: Log-Linear TV-Estimates of  $\gamma$



Note: The blue and green lines are 95 % asymptotic confidence intervals.

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