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Tough Love and Intergenerational Altruism

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Tough Love and Intergenerational Altruism*

Vipul Bhatt† and Masao Ogaki‡

Abstract

This paper develops and studies a tough love model of intergenerational altruism. We model tough love by modifying the Barro-Becker standard altruism model in two ways. First, the child's discount factor is endogenously determined, so low consumption at a young age leads to a higher discount factor later in life. Second, the parent evaluates the child's lifetime utility with a constant high discount factor. The tough love model predicts that transfers from the parent will fall when the child's discount factor falls. This is in contrast with the standard altruism model, which predicts that transfers from parents are independent of exogenous changes in the child's discount factor.

1 Introduction

How different generations are connected to each other is an important economic issue with implications for individual economic behavior such as savings, investment in human capi-

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tal, and bequests, which in turn affect aggregate savings and growth. These interactions are also important from a policy perspective since they determine how families respond to public policies aimed at redistributing resources among family members. A commonly used paradigm to study such linkages is the standard altruism model proposed by Becker (1974) and Barro (1974) in which the current generation derives utility from its own consumption and the utility level attainable by its descendant(s). Using this framework, Barro found that there will be no net wealth effect of a change in government debt.

A striking prediction of the standard altruism model is that when the child becomes impatient, transfers from the parent to the child do not change when the child is borrowing constrained (as we show in section 3). This implication of the model is not consistent with recent empirical evidence on pecuniary and non-pecuniary parental punishments (see Bhatt (2011), Hao et al (2008), and Weinberg (2001) for empirical evidence). For example, imagine that a child befriends a group of impatient children and suddenly becomes impatient because of their influence. As a result, the child starts to spend more time playing with the new friends and less time studying. In the worst cases, the child starts to smoke, drink, or consume illegal drugs (see Ida and Goto (2009) for empirical evidence that shows the association of low discount factors and smoking). At least some parents are likely to respond by imposing pecuniary punishments, such as reducing allowances, or non-pecuniary punishments, such as grounding. Another feature of the standard altruism model is that it precludes parents from directly influencing their children's time preferences. However, there is empirical evidence that parents attempt to shape their children's economic behavior and attitudes, including time preferences, as reviewed below. In many recent theoretical contributions, preferences of children are not exogenous, but are shaped by the attitudes and actions of their parents and other role models. For example, in the literature on cultural transmission of preferences, Bisin and Verdier (2001) proposed a general model with endogenous cultural transmission mechanisms wherein parents take actions to affect children's traits, which as a special case can correspond to time preferences. In some other models, even though parents do not take actions with the deliberate intention of affecting their children's preferences, they end up doing so indirectly. For example, Fernandez et al (2004) used a dynamic model where mothers who work play an important role in the transmission of attitudes favoring the participation of women in the labor force to their sons. We will further discuss this issue by presenting empirical evidence for the parents' role in children's endogenous preference formation in the next section.

The main contribution of this paper is to propose a new theoretical model of parent–child interaction that incorporates a mechanism through which parents can affect their children's time preference formation. We develop a tough love model of intergenerational altruism, in which the parent is purely altruistic to the child, but exhibit tough love: he allows the child to suffer in the short run with the intent of helping her in the long run. The main prediction of our tough love altruism model is that transfers from the parent will fall when the child's discount factor falls exogenously. This is in sharp contrast to the prediction of the standard altruism model where the parent does not respond to such a change in the child's discount factor. An interpretation of this result is that parents with the tough love motive use pecuniary incentives to mold their children's time preferences. Since exogenous changes in the child's discount factor that make her impatient are likely to cause behavior that calls for parents' corrective actions, the prediction of the tough love altruism model is more consistent with the empirical evidence on parental punishments, as well as with the role of parents in shaping children's preferences as compared with that of the standard altruism model.

In the simple setting of a three-period economy with a single parent and single child with perfect information and borrowing constraints, we model parental tough love by combining two ideas that have been studied in the literature in various contexts. First, the child's discount factor is endogenously determined, so low consumption at a young age leads to a higher discount factor later in life. This is based on the endogenous discount factor models of Uzawa (1968), except that the change in the discount factor is immediate in Uzawa's formulation, whereas a spoiled child with high consumption progressively grows impatient in our formulation.[1] Second, the parent evaluates the child's lifetime utility with a constant discount factor that is higher than that of the child. Since the parent is the social planner in our simple model, this feature is related to recent models in which the discount factor of the social planner is higher than that of the agents.[2] In our model, these two features lead the parent to exhibit tough love behavior in which he takes into account the influence of income transfers to the child based on the latter's discount factor.

An argument for the plausibility of endogenous discounting can be found in Becker and Mulligan (1997). They model an individual whose discount factor depends on the remoteness or vividness of imagined future pleasures. Becker and Mulligan's model involves investment in human capital to increase the vividness of imagination. For the direction of the effect of wealth on the discount factor, this argument can be used to support both Fisher's conjecture that poor people are less patient (see Fisher 1930, p.72 for details) and Uzawa's (1968) hypothesis that poor people are more patient. Because richer people tend to invest more, their model typically implies that poorer people are less patient. On the other hand, if a child experiences low consumption, it should be easier for the child to imagine future misery more vividly. This argument implies that a child who experiences low consumption will tend to grow more patient. The child may experience low consumption either because the parent is poor or because the parent is concerned about spoiling the child. In our review of empirical evidence in the next section, we find mixed evidence for both directions, which seems to imply that both of these forces are working in practice. For the purpose of our paper, we abstract from the human capital aspect and adopt the formulation that a child

[1] Recent theoretical models that adopt the Uzawa-type formulation include Schmitt-Grohé and Uribe (2003) and Choi et al (2008).

[2] See Caplin and Leahy (2004), Farhi and Werning (2007), Phelan (2006), and Sleet and Yeltekin (2005, 2007).

who is spoiled by high consumption in childhood grows to be less patient.[3]

Turning to the plausibility of the parent using a higher discount factor than the child, an extreme case is a parent with a newborn baby. When the baby is born, it is very impatient and cries for food all the time but the parent does not give in to this persistent demand. This is likely because the parent evaluates the baby's utility over its lifetime with a higher discount factor as compared with the baby's very low discount factor. We think that it is likely that many parents continue to evaluate their children's lifetime utility when they are no longer babies. Mischel's (1961) results, which we mention in the next section, are consistent with our view. Parents may continue to do this until their children learn to be as patient as them.

As a model of parental punishments, our model is related to Weinberg's (2001) model.[4] He develops a static incentive model based on asymmetric information, whereas our model is dynamic without any uncertainty. The parent in Weinberg's model does not affect the child's preferences, whereas the parent in our model takes actions with explicit intent to affect the child's discount factor. In this paper, we are emphasizing the role of the parent in molding the time preference of the child. In this regard, our model is closely related to those of Akabayshi (2006) and Doepke and Zilibotti (2008). In these models also, the parent takes actions in order to influence the child's discount factor. In Akabayashi's model, the child has endogenous discounting, and the parent evaluates the child's lifetime utility with a fixed discount factor. Together with asymmetric information about the child's ability, Akabayashi's model can explain abusive repeated punishments by parents under certain parameter configurations. In Doepke and Zilibotti's model, the parent uses the child's discount factor to

[3] This paper focuses on the parent's role in preference formation. A related work is Mulligan (1998) on the altruistic preference formation of the parent toward the child.

[4] In a recent work, Slaví̌k and Wiseman (2009) have also proposed a model of tough love. These authors emphasize the moral hazard problem faced by parents in order to construct a model with a dynamic insurance strategy that involves providing greater inter vivos transfers to poor children and dividing bequests equally. Their model does not involve endogenous time discounting.

evaluate the child's lifetime utility. They use their model of occupational choice to account for a number of observations about the British Industrial Revolution. The main difference from our model is that these authors adopt a Becker-Mulligan formulation of endogenous discounting so that children become more patient when their human capital is higher. In contrast, we adopt an Uzawa-type formulation for our model.

The remainder of the paper is organized as follows. Section 2 reviews the empirical evidence related to the key assumptions and implications of the tough love altruism model. Section 3 explains the structure and main findings of our model with only a consumption good, and contrasts the implications of the model with those of the standard altruism model. Section 4 proposes two alternative models of altruism in order to show that both features discussed above (the endogenous discount factor of the child and the parent's evaluation of the child's lifetime utility with a high constant discount factor) are necessary in order for transfers to decrease when a child exogenously becomes impatient. Section 5 introduces leisure in the tough love altruism model with the objective of studying how parental transfers are affected by endogenous changes in the child's income caused by (exogenous) changes in her discount factor. Section 6 concludes.

2 A Review of Empirical Evidence

In this section, we review the empirical evidence related to the key assumptions and implications of the tough love altruism model.

Endogenous discounting is an important assumption of our tough love model and we first review the existing empirical evidence for this assumption. In the literature, there are two competing hypotheses that allow for endogenous discount factor by linking patience to wealth. First is Fisher's hypothesis that the rich are more likely to be patient, and second is the Uzawa's hypothesis that implies the discount factor is decreasing in wealth.

Becker and Mulligan (1997) cite empirical evidence for endogenous discounting consistent with the Fisher hypothesis. Similarly, using the Panel Study of Income Dynamics (PSID), Lawrance (1991) employed the Euler equation approach to estimate the endogenous discount factor model and found evidence in favor of the discount factor increasing in wealth.

However, one has to be careful in evaluating the empirical evidence for endogenous discounting because of two problems. First, we have the endogeneity problem in that patient people with high discount factors tend to accumulate financial and human wealth. Thus, we may find that rich people have higher discount factors than poor people even when the discount factor of an individual is decreasing in wealth as in Uzawa's model.[5] The endogeneity problem mentioned above is addressed in Ikeda, Ohtake, and Tsutsui (2005). In their paper, they found that without accounting for the possible endogeneity between discount factors and wealth, the discount factor appears to be an increasing function of income/wealth. After taking into account the endogeneity problem, they find evidence in favor of the discount factor decreasing in wealth.[6] Another way to control for the endogeneity problem is to give different levels of consumption to the subjects before an experiment to see which subjects are more patient. Implementing this idea with human subjects is difficult, so rats were used instead. The results were in favor of the view that the discount factor is decreasing in wealth as reported in Kagel et al (1995, Ch. 7, Section 3).

The second problem in evaluating empirical evidence for endogenous discounting is that endogenous discounting and wealth-varying intertemporal elasticity of substitution (IES) can have similar implications in growing economies, and may be hard to distinguish from

[5]This issue is related to the literature on the importance of initial endowments on subsequent outcomes of a dynamic process (Heckman (1981, 1991)). As suggested by Heckman, it is important to distinguish between heterogeneity (how persistent is the effect of initial endowments on outcomes), and state dependence (whether subsequent experiences attenuate or accentuate the effect of initial endowments). It is possible that a raw correlation between wealth and consumption growth reflects a causal influence of wealth on consumption growth (state dependence), or the fact that individuals differ in time preferences and more patient people accumulate more wealth (heterogeneity).

[6]They control the endogeneity problem by analyzing how the discount factor changes with the size of a prize obtained in another experiment.

one another (Atkeson and Ogaki (1996)). Hence, although the Lawrance (1991) estimation method based on the instrumental variable approach could potentially resolve the endogeneity problem, she did not allow the IES to vary with wealth. Ogaki and Atkeson (1997) allow both the IES and the discount factor to vary with wealth for a panel data of households in Indian villages. They find evidence in favor of the view that the discount factor is constant and that the IES is increasing in wealth. It is possible that the discount factor is decreasing in wealth for richer households, but Lawrence found the opposite result by not allowing the IES to change. Ogawa (1993) argues that empirical results from Japanese aggregate data are consistent with a combination of Fisher's and Uzawa's hypotheses.

Overall, we think that the empirical evidence is consistent with the view that reality is best described by a combination of the two hypotheses. In our view, a child who experiences low consumption will grow to be more patient because he/she can more vividly imagine future misery. At the same time, a wealthier parent is more likely to invest in the child's human capital to help the child see the future more vividly. In this paper, we aim to develop a simple model that captures our intuition of tough love, which is that a parent allows suffering so that the child can learn to be more patient. Such a model will imply that transfers decrease when the child exogenously becomes impatient. For this purpose, we will assume that low childhood consumption leads to more patience (higher discount factor) in adulthood and will abstract from the human capital nature of endogenous discounting.

The tough love model presented in this paper hypothesizes a strong parental role in shaping child behavior and preferences. The main prediction of the model is that parents with tough love motives will provide lower childhood consumption to their children in order to influence their discount factor. Ideally, we would like to present evidence for such a parent-child interaction in data. However, to our knowledge, there is no existing study that seeks to answer this question directly. This is partially a consequence of lack of data (survey or experimental) on parental motives, childhood consumption, and discount factor of parents and their children. As result, we attempt to approach this issue indirectly by reviewing empirical evidence on three related questions.

Our first question is whether or not there is empirical evidence for parents' behavior influencing their children's discount factors as well as other economic preferences and attitudes. A necessary condition for parents' behavior to be able to affect children's time discounting factors is that genetic factors do not completely determine time discounting. Using a unique data set of twins in Japan, Hirata et al (2009) found empirical evidence in favor of this condition. Knowles and Postlewaite (2005) used data from the PSID to examine the relationship between parental attitudes toward planning for the future and their children's saving rates. They found that for the oldest children, the parents' attitudes explain one-third of the variance in savings rates that remains after controlling for income and demographics. Similarly, Webley and Nyhus (2006) used De Nederlandsche Bank household survey (DHS) data and found evidence to support the hypothesis that parental orientations, especially those related to intertemporal choice, affect the economic behavior of their children in both childhood and adulthood. In Webley and Nyhus' analysis, they observed high degrees of association between children's savings and parental savings, household income and economic socialization of parents. In the psychology literature, there is evidence in favor of the influence of parents in the development of children's willingness to delay rewards. Mischel (1961) studied children in the West Indian islands of Grenada and Trinidad. In both cultures, he found a significant relationship between absence of the father within a household and greater preference for immediate reward reflecting impatient child behavior. Such an association, among other things, suggests a strong role for the father in handing down values of thrift to his child.

Our second question is whether or not there is direct empirical evidence that some parents take actions with the intention of affecting their children's behavior. This issue of the relationship between various parenting styles, identified by varying degrees of control, has been addressed more directly in the psychology literature than in the economics literature.[7] For instance, Carlson and Grossbart (1988) used survey data on the mothers of schoolchildren (kindergarten through sixth grade) and divided them into groups based on the parenting style, ranging from neglecting to rigidly controlling. They found evidence suggesting that authoritative parents grant less consumption autonomy to their children, have greater communication with their children about consumption-related issues, set higher consumer socialization goals and exhibit greater monitoring of children's consumption vis--vis both permissive and authoritarian parents. Such a relationship holds in the data even after accounting for possible cultural differences. For example, Rose et al (2003) used data for India, Australia and Greece, and found evidence suggesting that authoritative parents more closely monitor their children's consumption compared with other parenting styles.

Our third question is whether or not there is empirical evidence related to the main implication of our model that a parent reduces his childs childhood consumption when the discount factor that the parent uses to evaluate the childs life time utility is higher than the childs discount factor. The data limitation is obviously a difficulty to find such evidence, but Kubota et al (2009) provided empirical evidence related to this. They used a unique U.S. and Japanese survey data that contained hypothetical survey questions concerning parents' tough love attitudes and their time discount factors for their own financial decisions. Their empirical results suggest that parents with lower time discount rates (higher discount factors) are more likely to have tough love attitudes to reduce consumption of a medicine when the medicine has a side effect to weaken the childs immune system after the child grows up.

[7] Baumrind (1966) identified three modes of parental control. The first mode is *permissive*, where parents act as a resource for their children and do not actively involve themselves in shaping the current and future behavior of the child. The second mode is *authoritarian*, where the parent uses a set standard of conduct that is theologically or religiously motivated and tries to shape and control the child's behavior with overt use of power. The third mode is *authoritative*, where the parent actively involves himself/herself in shaping the child's behavior and attitudes and uses reasoning and discipline to ensure a well-rounded long-term development of the child. The parent affirms the child's current behavior, separating right from wrong, and also sets standards for the child's future behavior.

3 A Consumption Good Economy

The main purpose of this section is to develop and analyze a model of altruism in which the parent's transfers decrease when the child exogenously becomes impatient. For this purpose, we modify the standard altruism model in two ways: the child's discount factor is endogenous in that higher consumption in childhood causes her discount factor to be lower, and the parent evaluates the child's lifetime utility with a high constant discount factor. The modified model is called the tough love altruism model. In order to gain a clear understanding of the properties of the model, we consider the simplest setting and compare the tough love model with the standard altruism model in this section.

Imagine a three-period model economy with two agents, the parent and the child. For simplicity, we consider the case of a single parent and a single child. The three periods considered are childhood, work and retirement.[8] The model has seven features. First, the timing of the model is assumed to be such that the life of the parent and the child overlaps in the first two periods of the child's life. Hence, the parent has the child in the second period of his own life, which in turn corresponds to the first period of the child's life. Second, the parent not only cares about his own consumption, but is also altruistic toward the child. He assigns a weight of η to his own utility, where $0 < \eta < 1$. Third, the parent receives an exogenous income, denoted by y_p, in period 2 of his life. For simplicity, we assume that there is no bequest motive and also that the parent receives no income in the last period of his life but simply consume savings from the previous period. Fourth, the parent maximizes utility over the last two periods of life by choosing consumption and transfers to his child, denoted by C_2^p and T, respectively, in period 2 of life and consuming savings in the last

[8]For expositional ease, we begin by making the simplifying assumption that these three periods are of equal duration. Note that results presented in this section as well as in section 4 are robust to varying durations for the three periods. Further, in section 5 we relax this assumption and study the model with varying durations for childhood, work, and retirement.

period of life.[9] Fifth, the child is assumed to be a nonaltruist and derives utility only from her own consumption stream $\{C_t\}_{t=1}^{3}$.[10] We assume that the child's income in periods 1 and 2, denoted by y_1 and y_2, respectively, is given exogenously and she receives no income in the last period of life. Sixth, the child is assumed to be borrowing constrained in period 1. Lastly, there is no uncertainty in the economy.

3.1 Standard Altruism Model

We start our analysis with the standard altruism model. In this model, both the parent and the child use the same constant discount factor when evaluating the child's future utility. The parent's problem is:

$$\max_{C_2^p, T} \Bigg\{ \eta \Big[v(C_2^p) + \widetilde{\beta} v(R(y_p - C_2^p - T)) \Big] + \widetilde{\beta}(1-\eta) \Big[u(C_1^*) + \beta_2 u(C_2^*) \tag{1}$$
$$+ \beta_2 \beta_3 u(R^2(y_1 + T + \frac{y_2}{R} - C_1^* - \frac{C_2^*}{R})) \Big] \Bigg\},$$

subject to:

$$C_1 = y_1 + T, \tag{2}$$

[9] Given the timing of our model, this implies that transfers, T, are made only in period 1. Further, we assume that transfers are made from the parent to the child and there are no reverse transfers.

[10] In this simple consumption good economy, we view consumption as a composite good that may include leisure activities such as TV time, video game time etc. In section 5, we extend this basic setup and introduce leisure as a second good.

and:

$$\{C_1^*, C_2^*\} \equiv \arg\max_{C_1,C_2} \Big[u(C_1) + \beta_2 u(C_2) + \beta_2\beta_3 u(R^2(y_1 + T + \frac{y_2}{R} - C_1 - \frac{C_2}{R}))\Big], \quad (3)$$

where $v(.)$ and $u(.)$ are standard concave period utility functions of the parent and the child, respectively. $\widetilde{\beta}$ is the parent's own discount factor whereas β_t is the period t discount factor used to evaluate the child's future utility. R is the gross nominal interest rate.

We can simplify the parent's problem by making two modifications. First, we are interested in the case where the borrowing constraint is binding for the child and assume that the parameters are such that the constraint is binding. We substitute out the borrowing constraint faced by the child in period 1 in the parent's problem described above. Second, we can reduce the dimensionality of the maximization problem by solving for optimal C_2^p as a function of transfers T and other model parameters and then substituting it out from the parent's maximization problem. We denote the resulting indirect utility function of the parent by $V(R(y_p - T), \widetilde{\beta})$. After incorporating these modifications, we can rewrite the parent's optimization problem as:

$$\max_T \left\{\eta\, V(R(y_p - T), \widetilde{\beta}) + \widetilde{\beta}(1-\eta)\Big[u(y_1 + T) + \beta_2 u(C_2^*) + \beta_2\beta_3 u(R(y_2 - C_2^*))\Big]\right\}, \quad (4)$$

subject to:

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_3 u(R(y_2 - C_2))\Big]. \quad (5)$$

Let us focus on the child's optimization program. From the first-order condition for the child's problem described in equation (5), we obtain:

$$u_{C_2}(C_2) - \beta_3 R u_{C_2}(R(y_2 - C_2)) = 0, \tag{6}$$

where:

$$u_x(x) \equiv \frac{\partial u(x)}{\partial x}.$$

Assuming that the utility function satisfies conditions for the Implicit Function Theorem,[11] we can solve equation (6) for C_2 as a function of the model parameters and the state variables:

$$C_2^* = C_2(y_2, \beta_3, R). \tag{7}$$

The optimal period 2 consumption for the child is independent of the period 1 transfers of the parent and hence can be dropped from the parent's optimization program. Hence, we can rewrite the parent's problem described by equations (4) and (5) as:

$$\max_T \left[\eta V(R(y_p - T), \widetilde{\beta}) + \widetilde{\beta}(1-\eta)u(y_1 + T)\right]. \tag{8}$$

From the first-order condition for the above problem and the implicit function theorem, we obtain:

$$T^* = T(y_p, y_1, \widetilde{\beta}, R, \eta). \tag{9}$$

We now consider a comparative statics exercise for the standard altruism model wherein we decrease the child's discount factor β_3 and observe how this rise in the child's impatience is accommodated by the parent in terms of a change in period 1 transfers. From equation (9),

[11] u(.) is continuously differentiable with a nonzero *Jacobian*.

optimal period 1 transfers by the parent in the standard altruism model are in fact independent of the child's discount factor. Hence, such an exogenous change in the child's discount factor will have no effect on the period 1 transfers made by the parent.[12] As discussed earlier, this implication of the model does not seem to be consistent with data where we find that both pecuniary and non-pecuniary punishments are used by parents to influence their children's behavior and outcomes.

3.2 Tough Love Altruism Model

We propose a tough love altruism model that provides for a channel through which parents can influence their child's economic behavior.[13] We introduce the tough love motive of the parent via asymmetric time preferences between generations and endogenous discounting. In this model, the parent uses a constant and high discount factor, denoted by $\beta_{t,p}$, to evaluate the child's lifetime utility. The child herself uses a discount factor that is endogenously determined as a decreasing function of period 1 consumption:

$$\beta_{t,k}(C_1) \quad ; \quad \frac{\partial \beta_{t,k}}{\partial C_1} < 0.$$

With the borrowing constraint faced by the child in period 1, her period t discount factor is given by $\beta_{t,k}(y_1 + T)$.

[12] Note that changes in the parent's own discount factor will affect transfers. However, here we are imagining a sudden change in social norms that affects only the child's discount factor with no effect on the parent's discount factor.

[13] The discussion presented here postulates a model of parental tough love in the context of a single generation. An interesting extension is to model tough love in a dynastic framework exemplified in the context of the standard altruism model by Barro (1974). In such a model, the child will be a repeater in the dynamic process and will pass on the discount factor she inherited from her tough love parent to her own offspring. We are investigating the implications of such a framework for tough love altruism in a separate paper that is a work in progress (Bhatt et al (2011)).

In this model, the parent solves the following optimization problem:

$$\max_{T} \left\{ \eta \, V(R(y_p - T), \widetilde{\beta}) + \widetilde{\beta}(1-\eta)\Big[u(y_1 + T) + \beta_{2,p} u(C_2^*) + \beta_{2,p}\beta_{3,p} u(R(y_2 - C_2^*)) \Big] \right\}, \tag{10}$$

subject to:

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}(y_1 + T) u(R(y_2 - C_2)) \Big]. \tag{11}$$

From the first-order condition for the child's problem described in equation (11) and the implicit function theorem, in principle we can solve for the optimal C_2 as a function of the model parameters and the state variables.[14]

$$C_2^* = C_2(y_2, \beta_{3,k}(y_1 + T), R) \tag{12}$$

3.2.1 Relationship between transfers and the child's discount factor

One of the distinguishing predictions of our tough love altruism model concerns the relationship between the optimal parental transfers and the child's discount factor. In what follows, we provide an analytical result that formalizes this relationship in our model. For this purpose, we assume the following specification for the child's discount factor:

[14] It can be easily shown that

$$\frac{\partial C_2^*}{\partial \beta_0} < 0 \quad \frac{\partial C_2^*}{\partial T} > 0$$

.

$$\beta_{t,k}(y_1+T)=\beta_0+\psi(y_1+T) \quad ; \quad \psi'(y_1+T)<0.$$

In this specification, β_0 is introduced for the purpose of performing comparative statics for exogenous changes in the child's discount factor, which do not change the sensitivity of the discount factor to changes in period 1 consumption. We are interested in establishing a relationship between optimal parental transfers, T^* and β_0 for the parent with the tough love motive. Our intuition is that the parent with a tough love motive will reduce transfers in response to an exogenous decrease in the child's discount factor, i.e. $\dfrac{\partial T^*}{\partial \beta_0}>0$. Using the optimization conditions for the parent and child problems, we next derive the expression for $sign\left(\dfrac{\partial T^*}{\partial \beta_0}\right)$.

Proposition 1 *A necessary and sufficient condition for* $\dfrac{\partial T^*}{\partial \beta_0}>0$ *is*

$$\left[1+R(\beta_p-\beta_{3,k})\frac{u''(R(y_2-C_2^*))}{u'(R(y_2-C_2^*))}\frac{\partial C_2^*}{\partial \beta_0}-(\beta_p-\beta_{3,k})\frac{\partial^2 C_2^*}{\partial \beta_0 \partial T}\right]>0. \tag{13}$$

Proof: See Appendix A for a proof. ∎

This is the main result of this paper. In order to facilitate intuitive interpretation of the condition in this proposition, we give the next proposition. It shows that the condition specified in equation (13) is related to the convexity of the marginal utility and the child's impatience (both the absolute level and the relative level compared with the parent's discount factor used to evaluate the child's lifetime utility).

Proposition 2 *The following three conditions are jointly sufficient for the condition in Proposition 1 to hold*

i) $u'''(.)\geq 0$,

ii) $\beta_p \geq \beta_{3,k}$, *and*

iii) $\beta_{3,k} RG \leq 1$,

where

$$G = \left(\frac{u'''(C_3^*)}{u''(C_3^*)} \right) \frac{\partial C_2^*}{\partial \beta_0}.$$

Proof: See Appendix B for a proof. ∎

The first condition in Proposition 2 implies convexity of the marginal utility function. This condition is satisfied by many functional forms that are used for the utility function in the consumption literature: it is satisfied with the strict inequality for the power utility function, and with equality for the quadratic utility function. The second condition implies that the child is relatively inpatient compared with the parent's norm in the sense that the child's discount factor is less than or equal to the discount factor used by the parent to evaluate the child's lifetime utility.[15] This condition is consistent with the first assumption of our tough love altruism model that the parent uses a high constant discount factor to evaluate the child's lifetime utility. The third condition is trivially satisfied if $u'''(C_3^*) = 0$. If $u'''(C_3^*) > 0$, the condition requires a certain level of impatience of the child, where the level is not directly affected by the parent's norm. For example, if we assume a power utility function, then the third condition can be expressed as:

$$\beta_{3,k} R \leq \left(\frac{\sigma}{R} \right)^{\frac{1}{\sigma}} \tag{14}$$

as shown in Appendix C. If $\sigma = R$, then this condition reduces to $\beta_{3,k} R \leq 1$. If $\sigma = 1$, then this condition reduces to $\beta_{3,k} \leq 1/R^2$, which implies $\beta_{3,k} \leq 0.69$ if $R = 1.2$. Assuming

[15] Note that since $\frac{\partial T^*}{\partial \beta_0}$ is strictly positive for $\beta_p = \beta_{3,k}$, we have a positive relationship between T^* and β_0 even when $\beta_p < \beta_{3,k}$, as long as the difference is small in magnitude.

$R = 1.2$, the condition implies that β_3 is less than equal to 0.96, 1, and 1.07 when σ is equal to 1.5, 1.61, and 2, respectively.

Thus, the condition (14) is satisfied by all β_3 less than one if $\sigma \geq 1.61$ when $R = 1.2$. However, the condition may seem stringent for smaller values of σ. Here, it should be noted that the upper bound in the condition (14) is not meant to be sharp because the condition is sufficient but is not necessary. Transfers can have a positive relationship with β_0 with much larger values of $\beta_{3,k}$. To provide greater insight on this issue we numerically solve the optimization problem of the parent with tough love motive. The objective of this exercise is to numerically find the magnitude of the greatest upper bound at which a decrease in β_0 leads to higher parental transfers. For this purpose, we impose the following parameterization:[16]

$$u(C) = v(C) = \frac{C^{1-\sigma}}{1-\sigma}. \tag{15}$$

The discount factor is given by:

$$\beta(y_1 + T) = \beta_0 + \frac{1}{1 + a(y_1 + T)} \quad where \quad a > 0 \; and \; \beta_0 \leq 0. \tag{16}$$

We solve the problem described in equations (10) and (11) numerically as a nonlinear root finding problem using the above parametric specification and a given set of parameter values.[17]

[16] Our simulation results are robust to alternative parametric specifications of the utility function and also to a wide range of model parameter values.

[17] We have chosen our parameter values to be consistent with consensus estimates reported in the literature. When such estimates are not available, we have used the optimality conditions of our model and used micro data to approximate the parameter values. For σ, we are using a value of 1.5, which implies an elasticity of intertemporal substitution of around 0.67. In the literature, many studies have used micro data and have estimated this parameter to be between 0.4 and 0.7 (see Hall (2009), Ogaki and Reinhart (1998)). For deriving a value for η, given our parametric specification and under the assumption that β_p and β_k are approximately close to each other, from the parent's first-order condition we obtain:

$$\frac{C_p^{-\sigma}}{C_1^{-\sigma}} = \frac{1-\eta}{\eta}.$$

We consider comparative statics for exogenous changes in the discount factor of the child as captured by a change in the parameter β_0. We first solve the model for the parametric specification given in (15) and (16) with a given set of model parameter values. This gives us the benchmark optimal transfers T^*. We then decrease β_0 and trace out the parental response in terms of transfers. The results of this exercise are summarized in Table 1.

Table 1. Effect of an exogenous decrease in the child's discount factor

Global Parameters

$\eta = 0.66$; $R = 1.2$;
$\widetilde{\beta} = \beta_p = 0.99$; $y_1 = 1$; $y_2 = 10$; $y_p = 10$; $a = 0.02$

	(1) $\beta_0 = 3.0$	(2) $\beta_0 = 2.5$	(3) $\beta_0 = 2.0$	(4) $\beta_0 = 1.5$	(5) $\beta_0 = 1.0$	(6) $\beta_0 = 0.5$	(7) $\beta_0 = 0.0$
Panel 1: $\sigma = 1.5$							
T^*	1.7345	1.7347	1.7348	1.7347	1.7344	1.7330	1.7282
$\beta_{3,k}$	3.9481	3.4481	2.9481	2.4481	1.9481	1.4482	0.9483
Panel 2: $\sigma = 0.7$							
T^*	0.7312	0.7329	0.7347	0.7365	0.7372	0.7341	0.7156
$\beta_{3,k}$	3.9665	3.4665	2.9665	2.4664	1.9664	1.4665	0.9668

As seen in the first panel of Table 1, when we decrease β_0 from 3.0 to 0.0, parental transfers first increase and then start to decrease with the peak around $\beta_0 = 2.0$, which corresponds to $\beta_{3,k} = 2.9481$. Hence, for the benchmark case, the greatest upper bound far exceeds $\beta_{3,k} = 1$. In order to check the robustness of this result, the second panel of Table 1 reports results for the case with $\sigma = 0.7$ with the other parameters unchanged from the benchmark case. Again we find that as decrease β_0 from 3.0 to 0.0, parental transfers first increase and then start to decrease with the peak around $\beta_0 = 1.0$, which corresponds

We used $\sigma = 1.5$ and data from the Consumer Expenditure Survey (CEX) on per capita annual consumption expenditure for individuals aged 25 or below to approximate C_1. We used per capita annual consumption expenditure for individuals aged 65 or above to approximate C_p. Then, the above optimality condition $\eta = 0.66$. Finally, for parameter a, we assumed a value of 0.02, although we also tried alternative values of 0.01 and 0.04 and found that the results are robust to these alternative values for parameter a.

to $\beta_{3,k} = 1.9664$. These numerical results indicate that the condition in Proposition 1 is satisfied by a wide range of reasonable parameter values even for low values of σ.

Proposition 1 formalizes the main prediction of our tough love altruism model that an exogenous decrease in the child's discount factor will lead to a fall in parental transfers under some regularity conditions. This prediction is in sharp contrast to that in the standard altruism model where parental transfers were independent of the child's discount factor.[18]

3.2.2 Comparative statics for changes in the child's income and family income

We constructed the tough love model, so that transfers decrease when the child's discount factor exogenously falls. We now present comparative statics results for changes in the child's current and permanent income as well as for changes in family permanent income. The objective of this exercise is to challenge our tough love model by first illustrating its predictions with respect to aforesaid income changes and then comparing these predictions with the existing empirical evidence.

Effect of changes in the child's current income

One of the most important implications of the standard altruism model is the redistributive neutrality property (also called the transfer derivative restriction). The standard altruism model implies that an exogenous dollar decrease in the child's income coupled with a dollar increase in the parent's income will lead to a dollar increase in transfers from the parent to the child. Empirical evidence on the redistributive neutrality property is mostly negative and although many studies have found an inverse relationship between transfers and the

[18]We have also studied the version of our model where we add the bequest motive for the parent. We find that the main result of the paper remains qualitatively unchanged. Consistent with our intuition of parental tough love, the parent with a tough love motive respond to a fall in his child's discount factor by reducing transfers and increasing bequests. In contrast, the parent in the standard altruism model increase transfers and decrease bequests in response to a fall in the child's discount factor.

recipient's current income, the magnitude is much smaller than one-for-one. For instance, Altonji et al (1997) used Panel Survey of Income Dynamics (PSID) data and found that transfers only increase by 13 cents even when the recipient child is borrowing constrained.

Our tough love altruism model also implies redistributive neutrality. Because the parent optimizes the child's consumption level in the first period, if an exogenous factor changes the distribution of income for the parent and the child, the parent neutralizes the change by changing transfers.[19] However, this redistributive neutrality only holds for exogenous current income changes. In section 5, we address this issue by allowing for leisure as a second good in the utility function. Consistent with empirical evidence, we find that our tough love altruism model predicts a less than one-for-one inverse relationship between parental transfers and the child's current income.

Effect of changes in the child's permanent income

In the literature on parent child interactions, an important issue relates to the compensatory nature of parental transfers, wherein one can argue that the lower is the ability of the child, the greater will be the resource transfer from parents. To the extent that the child's permanent income is a reasonable proxy for her ability, one way to address this issue is by predicting how a parent with tough love adjusts transfers in response to an exogenous change in the child's permanent income.

For conducting this experiment, we consider comparative statics for exogenous changes in the child's period 2 income (y_2). For this purpose, we first solve the model for the parametric specification given in (15) and (16) and a given set of model parameter values. This gives us the benchmark optimal transfers T^*. We then decrease y_2 exogenously, while increasing the

[19] For brevity, we have not provided proofs for the redistributive neutrality property for all the models presented in this paper. These analytical results are available from the authors upon request.

parent's income by the same amount. This adjustment is necessary to keep family permanent income constant. The results for a given set of model parameter values are summarized in Table 2. As seen in Table 2, controlling for family permanent income, a decrease in a child's permanent income leads to an increase in parental transfers. Hence, our tough love altruism model predicts that the parent transfers more if the child is less able, measured by a decrease in the child's permanent income.

Table 2. Effect of a decrease in the child's permanent income on transfers

Global Parameters

$\eta = 0.66$; $\sigma = 1.5$; $R = 1.2$;
$\widetilde{\beta} = \beta_p = 0.99$; $\beta_0 = 0$; $y_1 = 1$; $a = 0.02$

	(1) $y_2 = 10; y_p = 10$	(2) $y_2 = 8; y_p = 11.67$
T^*	1.7282	2.1419
Child's Permanent Income	11.0615	9.8085

The empirical evidence on the relationship between the child's permanent income and transfers is mixed but more in favor of a negative relationship. For instance, Altonji et al (1997) used PSID and found a negative relationship between transfers and the recipient's permanent income.[20] Hence, the prediction of our model is consistent with the empirical evidence and to the extent that permanent income reflects the child's ability, our model predicts greater transfers to the less able child.

[20] Using data from the Health and Retirement Survey (HRS), McGarry (1999) found a positive relationship between the recipient's permanent income and the amount of transfers. However, this study uses education of the respondent as a proxy for the permanent income, which may be missing important aspects of permanent income. Further, when estimating the relationship between the recipient's permanent income and amount of transfers, McGarry (1999) did not adjust the parent's permanent income in order to keep family permanent income constant. We believe that the Altonji et al (1997) result is more robust as they used better measures of permanent incomes of the parent and the child and also controlled for family permanent income when evaluating the relationship between recipients' permanent income and their transfers.

3.2.3 Relationship between family permanent income and the child's discount factor

In this subsection, using simulations, we illustrate the prediction of our tough love model for the effect of family income on the child's discount factor. We use comparative statics for an exogenous change in the family's permanent income. For this purpose, we first solve the model for the parametric specification given in (15) and (16) and a given set of model parameter values. This gives us the benchmark optimal transfers T^* and the child's discount factor $\beta(C_1^*)$. We then increase y_p implying a higher level for the family's permanent income. The results for this exercise are summarized for a given set of model parameter values in Table 3. As seen in Table 3, an increase in family permanent income leads to an increase in parental transfers and a lower realized discount factor for the child. Hence, our tough love altruism model predicts a negative relationship between wealth and the discount factor.

Table 3. Effect of an increase in family permanent income on the child's discount factor

Global Parameters $\eta = 0.66$; $\sigma = 1.5$; $R = 1.2$; $\widetilde{\beta} = \beta_p = 0.99$; $\beta_0 = 0$; $y_1 = 1$;$y_2 = 10$; $a = 0.02$		
	(1) $y_p = 10$	(2) $y_p = 11.67$
T^*	1.7282	2.1420
Child's Discount Factor ($\beta(C_1^*)$)	0.9483	0.9409

Based on our earlier discussion (Section 2), there are two competing hypotheses regarding the relationship between wealth and discount factors: Fisher's hypothesis of a positive relationship and Uzawa's negative relationship hypothesis. Becker and Mulligan (1997) provided a theoretical framework for endogenous discounting, where an individual's discount factor is affected by investment in future-specific human capital intended to improve the vividness of his/her imagination. We also presented a brief discussion of the existing empirical evidence and it seems mixed at best. We believe that the mixed nature of empirical findings is consistent with the view that reality is probably best described by a combination of the two hypotheses. In our view, a child who experiences low consumption will grow to be more patient because he/she can more vividly imagine future misery. At the same time, a wealthier parent is more likely to invest in the child's human capital to help the child see the future more vividly. In this paper, we aim to develop a simple model that captures our intuition of tough love, which is that a parent allows suffering so that the child can learn to be more patient. Consistent with this tough love intuition, in our model low childhood consumption leads to more patience (higher discount factor) in adulthood. Hence, the prediction of a negative relationship between family permanent income and the child's discount factor illustrated here is a manifestation of our tough love intuition. We believe that the addition of the human capital nature of endogenous discounting propounded by Becker and Mulligan (1997) may help our model to best capture the reality. However, for simplicity, we abstract from the human capital approach in our paper and leave such an extension for future work.

4 How Important is Tough Love?

The main result of our tough love altruism model is that the parent will decrease transfers in response to an exogenous decrease in the child's discount factor. Our model modifies the standard altruism model in two ways. Do we need both of these modifications in order to obtain this result? In order to answer this question, we analyze two alternative models of altruism. First, we modify the standard altruism model by assuming that the parent evaluates the child's lifetime utility with a higher constant discount factor than that of the child. However, we do not introduce endogenous discounting in this model. This model is called the paternalistic altruism model. Second, we modify the standard altruism model by introducing endogenous discounting on the part of the child. However, we assume that the parent will use the child's endogenous discounting to evaluate the child's lifetime utility.

4.1 Paternalistic Altruism Model

In this model, both the parent and the child use constant discount factors to evaluate future utility. However, unlike the standard altruism model, here the discount factor used by the parent is higher than the child's discount factor, i.e. $\beta_{t,p} > \beta_{t,k}$, where $\beta_{t,p}$ is the discount factor used by the parent to evaluate the child's future utility and $\beta_{t,k}$ is the discount factor used by the child in period t. The parent's problem is given by:

$$\max_{T} \left\{ \eta\, V(R(y_p - T), \widetilde{\beta}) + \widetilde{\beta}(1-\eta)\Big[u(y_1 + T) + \beta_{2,p}u(C_2^*) + \beta_{2,p}\beta_{3,p}u(R(y_2 - C_2^*))\Big] \right\}, \tag{17}$$

subject to:

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}u(R(y_2 - C_2))\Big]. \tag{18}$$

As before, we solve the child's optimization problem first, which gives us the optimal period 2 consumption of the child:

$$C_2^* = C_2(y_2, \beta_{3,k}, R). \tag{19}$$

The optimal period 2 consumption of the child is independent of the period 1 transfers of the parent, so it can be dropped from the parent's optimization program. We rewrite the parent's problem described by equations (17) and (18) as:

$$\max_{T} \left[\eta V(R(y_p - T), \widetilde{\beta}) + \widetilde{\beta}(1-\eta)u(y_1 + T)\right]. \tag{20}$$

From the first-order condition for the above problem, in principle we can solve for the optimal period 1 transfers as:

$$T^* = T(y_p, y_1, \widetilde{\beta}, R, \eta). \tag{21}$$

We now consider an exogenous decrease in the child's discount factor, $\beta_{3,k}$. From equation (21), optimal period 1 transfers by the parent are independent of the discount factor of the child. Therefore, like the standard altruism model, in this model there is no effect of a decrease in the discount factor on the period 1 transfers. Thus, we cannot replicate the tough love altruism prediction of lower parental transfers in response to a decrease in the child's discount factor by only introducing paternalistic altruism. Next, we show that only adding endogenous altruism to the standard altruism model is also not sufficient to generate the positive relationship between parental transfers and the child's discount factor implied by our tough love altruism model.

4.2 Endogenous Altruism Model

In this model, as was assumed in the tough love altruism model, the discount factor used by the child is endogenously determined as a decreasing function of period 1 consumption:

$$\beta_{t,k}(c_1) \quad ; \quad \frac{\partial \beta_{t,k}}{\partial C_1} < 0.$$

With the borrowing constraint faced by the child in period 1, the discount factor is given by $\beta_{t,k}(y_1 + T)$. However, unlike the tough love altruism model, the parent also uses the above discount factor for evaluating the child's future utility. So the key difference is the assumption:

$$\beta_{t,p}(x) = \beta_{t,k}(x) = \beta_t(x).$$

The parent's problem in this model is given by:

$$\max_T \Bigg\{ \eta\, V(R(y_p - T), \widetilde{\beta}) + \widetilde{\beta}(1-\eta)\Big[u(y_1 + T) + \beta_2(y_1 + T)u(C_2^*) \\ + \beta_2(y_1 + T)\beta_3(y_1 + T)u(R(y_2 - C_2^*))\Big]\Bigg\}, \tag{22}$$

subject to:

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_3(y_1 + T)u(R(y_2 - C_2))\Big]. \tag{23}$$

As before, we first solve the child's optimization problem, which gives us the optimal period 2 consumption:

$$C_2^* = C_2(y_2, \beta_3(y_1 + T), R). \tag{24}$$

We solve the problem described in equations (22) and (23) numerically as a nonlinear root finding problem. The solution method, parameterization, and the parametric values used are identical to those we used for the comparative statics exercises for the tough love altruism model in subsection 3.2.

We consider an exogenous decrease in the discount factor of the child achieved by decreasing the preference parameter β_0, and then trace out the effect of this change on the period 1 transfers, T. The results for the assumed set of model parameter values are summarized in Table 4. We find that as β_0 is reduced monotonically, parents in the endogenous altruism model will reduce transfers.

Table 4. Endogenous Altruism Model

Global Parameters

$\eta = 0.66$; $\sigma = 1.5$; $R = 1.2$;

$y_1 = 1$;$y_2 = 10$; $y_p = 10$; $a = 0.02$;$\tilde{\beta} = 0.99$

Optimum	$\beta_0 = 0$	$\beta_0 = -0.4$	$\beta_0 = -0.6$	$\beta_0 = -0.8$
T^*	2.0206	1.9415	1.8995	1.8526
C_1^*	3.0206	2.9415	2.8995	2.8526
C_2^*	5.2495	6.1446	6.8350	7.9305
C_3^*	5.7006	4.6265	3.7981	2.4834
$\beta(C_1^*)$	0.9430	0.5444	0.3452	0.1460

The results of this exercise seem to suggest that endogenous discounting is enough to obtain the main result of our tough love altruism model. With the given set of parameter values, this model also predicts a positive relationship between parental transfers and the child's discount factor. However, unlike the results of the tough love model, the result reported in Table 4 is very sensitive to the assumption made on σ. Table 5 below presents simulation results with $\sigma < 1$. Now we find that as β_0 falls, transfers increase monotonically. Hence, with the endogenous altruism model, the direction of the relationship between parental transfers and the child's discount factor rests critically on the parametric assumptions. This is in sharp contrast to our tough love altruism model where this relationship is positive regardless of the parametric specification.

Table 5. Endogenous Altruism Model

Global Parameters				
$\eta = 0.66$; $\sigma = 0.7$; $R = 1.2$;				
$y_1 = 2$;$y_2 = 10$; $y_p = 10$; $a = 0.02$;$\tilde{\beta} = 0.99$				
Optimum	$\beta_0 = 0$	$\beta_0 = -0.4$	$\beta_0 = -0.6$	$\beta_0 = -0.8$
T^*	0.0428	0.1955	0.2751	0.3483
C_1^*	1.0428	1.1955	1.2751	1.3483
C_2^*	4.8784	6.7003	7.8960	9.1850
C_3^*	6.1459	3.9596	2.5247	0.9780
$\beta(C_1^*)$	0.9796	0.5766	0.3751	0.1737

Thus to conclude, the results of this section show that in order to obtain the prediction that the parent's transfer decreases in response to an exogenous decrease in the child's discount factor, we need to introduce both endogenous discounting and paternalistic evaluation by the parent of the child's lifetime utility.

5 Tough Love Altruism Model with Leisure

We constructed the tough love model so that transfers decrease when the child's discount factor exogenously falls. We now turn to another type of comparative statics for the purpose of challenging our tough love model. Here, we examine the model's properties related to the relationship between the child's income and parental transfers. Until now, we have considered an economy where agents derive utility only from consumption. To examine the role of the child's income in a more realistic way, we now generalize our setup by allowing for leisure as a choice variable for the child. This is motivated by empirical evidence against the standard altruism model's redistributive neutrality property. As we have discussed earlier, the empirical evidence for redistributive neutrality is largely unfavorable. Although our tough love model also implies redistributive neutrality, such neutrality only holds for exogenous current income changes. We study below how endogenous changes in income caused by an exogenous change in the child's discount factor are related to transfers.

For this experiment, we extend our model in an important dimension. Until now, for notational simplicity, we assumed that the three periods of the child's life are of equal duration. In reality, we can expect them to vary. Allowing the duration to vary, we denote that of the childhood period by τ_1, that of the work period by τ_2, and that of the retirement period by τ_3.[21] We imagine the childhood period of the model to correspond with the period around high school and the early years of college in which children may engage in part-time work (e.g. 16–20 years of age) and set the duration to be 5 years.[22] The benchmark duration of the work period of the model is set to be 40 years, and corresponds to ages between 21 and 60 years. The benchmark duration for the retirement period is set to be 20 years, and corresponds to ages between 61 and 80 years.

We continue to assume perfect information. In our setup, this implies that the parent can fully observe the child's effort level. The remaining model assumptions are retained with transfers being made only in period 1 and with the child being borrowing constrained in period 1. The following notation is used. L_1 and L_2 denote the amount of leisure consumed by the child in period 1 and period 2, respectively. w_1 and w_2 denote the wage income of the child in the two periods. For simplicity, we assume that the child earns no wage income in period 3 and simply consumes her past savings. The parent's problem is:

[21] For simplicity, we abstract from the child's early life in which he does not face the work–leisure choice.

[22] Cunha, Heckman, Lochner and Masterov (2006) present a survey of empirical evidence that later interventions in adolescent years can affect noncognitive skills such as patience, self-control, temperament, time preferences, etc., while these interventions cannot affect cognitive skills.

$$\max_{T} \left\{ \eta\, V(R(y_p - T), \widetilde{\beta}, \tau_1, \tau_2) + \widetilde{\beta}(1-\eta)\Big[\tau_1\ u(w_1(1-L_1^*) + T, L_1^*) + \beta_{2,p}\ \tau_2\ u(C_2^*, L_2^*) \right.$$
$$\left. + \beta_{2,p}\beta_{3,p}\ \tau_3\ u(R(w_2(1-L_2^*) - C_2^*))\Big] \right\}, \tag{25}$$

subject to:

$$\{C_2^*, L_1^*, L_2^*\} \equiv \arg\max_{C_2, L_1, L_2} \Big[\tau_1\ u(w_1(1-L_1) + T, L_1) + \beta_{2,k}(w_1(1-L_1) + T)\ \tau_2\ u(C_2, L_2)$$
$$+ \beta_{2,k}(w_1(1-L_1) + T)\beta_{3,k}(w_1(1-L_1) + T)\ \tau_3 u(R(w_2(1-L_2) - C_2))\Big]. \tag{26}$$

We solve the above problem numerically as a nonlinear root finding problem and for that purpose we impose the following parametric specification:

$$u(C, L) = Log(C) + d\frac{L^{1-\gamma}}{1-\gamma}\ \ (Child's\ Utility\ Function), \tag{27}$$

$$v(C) = Log(C)\ \ (Parent's\ Utility\ Function). \tag{28}$$

The child's discount function is given by:

$$\beta(w_1(1-L_1) + T) = \beta_0 + \frac{1}{1 + a(w_1(1-L_1) + T)} \tag{29}$$
$$where \quad a > 0\ and\ \beta_0 < 0.$$

There are two important parameters of the period utility function to which we need to assign values for solving the optimization problem numerically. The first parameter, γ, is the reciprocal of the IES for labor/leisure. For men, most estimates of the intertemporal labor-supply elasticity are between 0 and 0.5. (MaCurdy (1981), Altonji (1986), Blundell and MaCurdy (1999), and French (2004)). John Ham (2003) used an implicit contract model and found this elasticity to be 0.9. To be consistent with the literature, we report results for two values of γ. The first is $\gamma = 1.11$, which is consistent with the elasticity of 0.9. The second is $\gamma = 2$, which is consistent with the elasticity of 0.5.

The second parameter, d, captures the weight of leisure in the child's period utility function. In the real business cycle literature, the weight on leisure in the utility function is usually calibrated so that in the steady state the representative household spends about one-third of its total time working. In our model, given our parametric specification for the period utility, we obtain the following optimality condition that determines period t consumption-leisure choice:

$$d = \frac{W_t}{C_t} * L_t^{-\gamma}.$$

We set $L_t = 2/3$ and used the average value for labor income to consumption expenditure (for the period 1980–2009) from National Income and Product Account (NIPA) to approximate $\frac{W_t}{C_t}$. Then, using $\gamma = 1.11$, we obtain $d = 1.33$ and using $\gamma = 2$, we obtain $d = 1.9125$. We report simulation results for each combination of γ and d separately.

Table 6 summarizes the results of the simulations for a decrease in the parameter β_0. We observe that as β_0 falls from 0 to - 0.01, the parent with a tough love motive lowers transfers to the child. At the same time, there is also a fall in the child's income in the first period corresponding to the fall in β_0.

Thus, in our tough love altruism model, the parent's transfers and the child's income fall at the same time even though the child is borrowing constrained. Whether or not

Table 6. Tough Love Altruism Model with Leisure

Global Parameters				
$\eta = 0.66$; $r = 1.02$; $a = 0.02$; $\widetilde{\beta} = \beta_p = 0.99$; $w_1 = 1$; $w_2 = 10$; $y_p = 10$; $\tau_1 = 5$; $\tau_2 = 40$; $\tau_3 = 20$				
	(1)		(2)	
	$\gamma = 1.11$; $d = 1.33$		$\gamma = 2$; $d = 1.9125$	
	$\beta_0 = 0$	$\beta_0 = -0.01$	$\beta_0 = 0$	$\beta_0 = -0.01$
T^*	0.1869	0.1076	0.5198	0.4498
Child's First Period Income	3.1522	2.8155	1.0392	0.8275

this feature of our model can explain the finding of Altonji et al is an empirical problem that requires careful study of the PSID data. This depends, among other things, on how income changes are divided into endogenous and exogenous changes. However, the model does imply that the parent's transfers and the recipient's income can move in the same direction even when the recipient is borrowing constrained. This can potentially reconcile the apparent inconsistency between empirical results against the redistributive neutrality property and Laitner and Thomas' (1996) result in favor of parents' altruism for children. They used Teachers Insurance and Annuity Association-College Retirement Equities Fund (TIAA-CREF) data and focused on bequests as the channel for parental altruism. They found that for the subsample of respondents characterized by willingness to leave a bequest, the projected amount of the bequest is largest for households with lowest assessments of their children's likely earnings in the future.

6 Conclusion

In the simple setting of a three-period economy with a single parent and single child, perfect information, and borrowing constraints, we develop a model of intergenerational altruism wherein the tough love motive for parents is a driving force behind the parent's behavior.

In our tough love altruism model, the child's discount factor is endogenously determined, and the parent evaluates the child's lifetime utility with a constant discount factor that is higher than that of the child. With our modeling, we try to capture our intuition of tough love: in order to teach a child to be patient, the parent is willing to let the child suffer in the short run. In order to capture this intuition in a simple model, we abstract from the human capital nature of endogenous discounting.

The main prediction of our tough love model, for which we provide an analytical proof, is that an exogenous decrease in the child's discount factor lowers parental transfers. This prediction of our model is in contrast with that of the standard altruism model, in which the parent does not change transfers when the child becomes impatient. Since exogenous changes in the child's discount factor that make her impatient are likely to cause behavior that calls for the parent's corrective actions, the tough love altruism model is more consistent with empirical evidence on parental punishments as well as the role of parents in shaping children's preferences as compared with the standard altruism model.

Another contribution of our paper relates to the empirical evidence against the standard altruism model's redistributive neutrality property (also called the transfer derivative restriction). Our tough love altruism model also implies redistributive neutrality. However, this redistributive neutrality only holds for exogenous income changes, while in the data, we can have both endogenous as well as exogenous changes in income. In the version of the tough love altruism model with endogenous leisure choices for the child, we investigate how endogenous changes in income caused by an exogenous change in the child's discount factor are related to transfers. We find that an exogenous change in the discount factor to make the child more impatient can cause both lower income and lower transfers from the parent even when the child is borrowing constrained. This prediction of our model may be able to explain the empirical findings by Altonji et al (1997) depending on how income changes are divided into endogenous and exogenous changes among other factors.

An important stylized fact for the U.S. economy is that the distribution of wealth is very concentrated and skewed to the right. Castaneda, Diaz-Gimenez, and Rios-Rull (2003) emphasized that standard explanations based on household decision-making models with homogeneous preferences fail to account for this observed heterogeneity in wealth distribution. There is some evidence that heterogeneity in discount factors may be important in understanding differences in savings rates and hence in wealth accumulation. For example, Krusell and Smith (1998) found that incorporation of discount-rate heterogeneity markedly decreases the gap between model predictions and the observed wealth distribution. An interesting feature of our model is that it suggests a pecuniary channel through which parents with tough love motives can instill the virtue of patience in their children. Since not all parents will exhibit such tough love tendencies or the same degree of tough love, our model offers one rationale for individuals discounting the future at different rates depending on their parental tough love. To the extent that there is a link between heterogeneity in discount factors and heterogeneity in savings rates, this feature of our model has implications for the observed heterogeneity in wealth in the U.S.

In the future, it will be interesting to analyze the characteristics of parents who exhibit tough love in their children's upbringing. Bhatt and Ogaki (2009) suggest that the worldview of the parent may be an important factor. For example, how the parent views suffering may be important. If the parent views suffering as meaningless, it is harder for him to let the child suffer. If the parent views suffering as meaningful (e.g., educational), then it is easier for him to let the child suffer. A new direction for research in this field will be to utilize existing or new survey data on parents' view on suffering to infer their capacity to exhibit tough love as explained in Kubota et al (2011). It will also be of interest to measure both the child's and the parent's discount factors, and use them to directly test the implications of our tough love model. Research efforts to conduct experiments for parent-child pairs for this purpose have already started.

Another interesting future research question is to explore channels that may compete with parental tough love in influencing their children's discount factor. For instance, parents may make efforts to select the peers and/or friends of their children. This can be achieved, presumably at some cost, for example, the cost of selecting a good neighborhood or a good school. It would be interesting to conceive of an alternative model where parents can incur such selection costs to ensure transmission of desired discount factors to their children. We can then analyze how large these costs have to be in order to substitute for tough love parenting methods suggested by our model.[23]

Finally, in this paper, we have abstracted from the Becker–Mulligan type of human capital investment, which increases the discount factor for the child. In the future, it will be interesting to incorporate such an aspect into our tough love altruism model and investigate the impact of such modification on the predictions of our model regarding the relationship between parental transfers and the child's discount factor on one hand, and between parental transfers and their children's income (ability) on the other.

[23] We thank an anonymous referee for bringing this point to our notice as a potential future work.

Appendix A : A proof for Proposition 1

In this appendix we provide an analytical proof of our main result specified in equation (13):

$$\frac{\partial T^*}{\partial \beta_0} > 0 \quad iff \quad \left[1 + R(\beta_p - \beta_{3,k})\frac{u''(R(y_2 - C_2^*))}{u'(R(y_2 - C_2^*))}\frac{\partial C_2^*}{\partial \beta_0} - (\beta_p - \beta_{3,k})\frac{\partial^2 C_2^*}{\partial \beta_0 \partial T}\right] > 0.$$

We start the derivation of the above result by noting that the parent accounts for C_2^* when maximizing utility by choosing transfers, T. From the first-order condition for the parent's problem described in equations (10) and (11), we obtain:

$$V'(R(y_p - T^*), \widetilde{\beta}) = \left(\frac{1-\eta}{\eta}\right)\frac{\widetilde{\beta}}{R}\left(u'(y_1 + T^*) + \beta_p u'(C_2^*)\frac{\partial C_2^*}{\partial T} - \beta_p^2 R u'(R(y_2 - C_2^*))\frac{\partial C_2^*}{\partial T}\right),$$

where T^* denotes optimal parental transfers, $V'(.) = \dfrac{\partial V(.)}{\partial T}$, and $u'(.) = \dfrac{\partial u(.)}{\partial T}$.

Now consider an exogenous change in the child's discount factor (β_k) captured by a change in β_0. From the parent's first-order condition described above we obtain:

$$\frac{\partial T^*}{\partial \beta_0} = A * \left(\frac{\partial}{\partial \beta_0}\left[\beta_p u'(C_2^*)\frac{\partial C_2^*}{\partial T} - \beta_p^2 R u'(R(y_2 - C_2^*))\frac{\partial C_2^*}{\partial T}\right]\right),$$

where:

$$A = - \frac{\left(\frac{1-\eta}{\eta}\right)\widetilde{\beta}}{V''(R(y_p - T^*), \widetilde{\beta}) + \left(\frac{1-\eta}{\eta}\right)\widetilde{\beta}u''(y_1 + T^*)}.$$

Given concavity of V(.) and u(.), we know that $A > 0$. Hence:

$$sign\left(\frac{\partial T^*}{\partial \beta_0}\right) = sign\left(\frac{\partial}{\partial \beta_0}\left[\beta_p u'(C_2^*)\frac{\partial C_2^*}{\partial T} - \beta_p^2 Ru'(R(y_2 - C_2^*))\frac{\partial C_2^*}{\partial T}\right]\right).$$

Then $\dfrac{\partial T^*}{\partial \beta_0} > 0$ if and only if:

$$\frac{\partial}{\partial \beta_0}\left[\beta_p u'(C_2^*)\frac{\partial C_2^*}{\partial T} - \beta_p^2 Ru'(R(y_2 - C_2^*))\frac{\partial C_2^*}{\partial T}\right] > 0.$$

Now using the first-order condition for the child's problem, we can rewrite the above condition as follows.

$$\frac{\partial}{\partial \beta_0}\left[-(\beta_p - \beta_{3,k})Ru'(R(y_2 - C_2^*))\frac{\partial C_2^*}{\partial T}\right] > 0$$

It is straightforward to show that the LHS of the above expression is given by:

$$Ru'(R(y_2 - C_2^*))\frac{\partial C_2^*}{\partial T}\left(1 + R(\beta_p - \beta_{3,k})\frac{u''(R(y_2 - C_2^*))}{u'(R(y_2 - C_2^*))}\frac{\partial C_2^*}{\partial \beta_0} - (\beta_p - \beta_{3,k})\frac{\partial^2 C_2^*}{\partial \beta_0 \partial T}\right).$$

Since $\dfrac{\partial C_2^*}{\partial T} > 0$ and $u'(.) > 0$, the above expression is strictly positive if and only if:

$$\left[1 + R(\beta_p - \beta_{3,k})\frac{u''(R(y_2 - C_2^*))}{u'(R(y_2 - C_2^*))}\frac{\partial C_2^*}{\partial \beta_0} - (\beta_p - \beta_{3,k})\frac{\partial^2 C_2^*}{\partial \beta_0 \partial T}\right] > 0$$

This establishes our claim in equation (14). Now, given positive marginal utility, concavity of $u(.)$, and $\dfrac{\partial C_2^*}{\partial \beta_0} < 0$, the sufficient conditions for the above expression to be strictly positive are:

i) $\beta_p \geq \beta_{3,k}$ and

ii) $\dfrac{\partial^2 C_2^*}{\partial \beta_0 \partial T} \leq 0.$

Appendix B: A proof for Proposition 2

In this appendix we show that $\frac{\partial^2 C_2^*}{\partial \beta_0 \partial T} \leq 0$ will depend on the convexity of the marginal utility (as captured by the positive third derivative of the utility function) and the impatience level of the child.

We start our derivation with the partial derivative of the optimal second period consumption with respect to parental transfers:

$$\frac{\partial C_2^*}{\partial T} = \frac{\beta'(y_1 + T)Ru'(R(y_2 - C_2^*))}{[u''(C_2^*) + \beta_{3,k}R^2u''(R(y_2 - C_2^*))]}, \tag{30}$$

where

$$\beta_{3,k} = \beta_0 + \psi(y_1 + T).$$

Differentiating with respect to β_0

$$\frac{\partial^2 C_2^*}{\partial \beta_0 \partial T} = \frac{N}{D}, \tag{31}$$

where

$$D = [u''(C_2^*) + \beta_{3,k}R^2u''(R(y_2 - C_2^*))]^2 > 0.$$

and

$$N = [u''(C_2^*) + \beta_{3,k}R^2u''(R(y_2 - C_2^*))][-\psi'(y_1 + T)R^2u''(R(y_2 - C_2^*))\frac{\partial C_2^*}{\partial \beta_0}]$$
$$-\psi'(y_1 + T)Ru'(R(y_2 - C_2^*))[u'''(C_2^*)\frac{\partial C_2^*}{\partial \beta_0} + R^2u''() - \beta_{3,k}R^3u'''(R(y_2 - C_2^*))\frac{\partial C_2^*}{\partial \beta_0}].$$

Since, D is always positive, the sign of $\frac{\partial^2 C_2^*}{\partial \beta_0 \partial T}$ depends on the sign of N. Now, the sign of

N will be the same as the sign of:

$$sign([u'''(C_2^*)\frac{\partial C_2^*}{\partial \beta_0} + R^2 u''(R(y_2 - C_2^*)) - \beta_{3,k} R^3 u'''(R(y_2 - C_2^*))\frac{\partial C_2^*}{\partial \beta_0}]).$$

If the above expression is negative then $N < 0$.

Hence, the condition for $N < 0$ is:

$$[u'''(C_2^*)\frac{\partial C_2^*}{\partial \beta_0} + R^2 u''(C_3^*)\{1 - (\beta_{3,k} R \frac{u'''(C_3^*)}{u''(C_3^*)}\frac{\partial C_2^*}{\partial \beta_0}\}] < 0.$$

The above condition holds if,

i) $u'''(.) \geq 0$ and

ii) $\beta_{3,k}\ R\ G \leq 1$,

where

$$G = \left(\frac{u'''(C_3^*)}{u''(C_3^*)}\right)\frac{\partial C_2^*}{\partial \beta_0}.$$

Appendix C.

In this appendix we use the power utility function to interpret the following condition:

$$\beta_{3,k}\ R\ G \leq 1,$$

where

$$G = \left(\frac{u'''(C_3^*)}{u''(C_3^*)}\right)\frac{\partial C_2^*}{\partial \beta_0}.$$

We assume that the period utility function is given by:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

Using the above specification of the utility function, from the child's optimization problem, we get:

$$C_2^* = \frac{Ry_2}{R + (\beta_{3,k}R)^{\frac{1}{\sigma}}}$$

$$C_3^* = R(y_2 - C_2^*) = (\beta_{3,k}R)^{\frac{1}{\sigma}} C_2^*$$

$$\frac{\partial C_2^*}{\partial \beta_0} = -\frac{R^2 y_2 (\beta_{3,k}R)^{\frac{1-\sigma}{\sigma}}}{\sigma[R + (\beta_{3,k}R)^{\frac{1}{\sigma}}]^2}$$

$$\frac{u'''(C_3^*)}{u''(C_3^*)} = -(\sigma+1)(\beta_{3,k}R)^{-\frac{1}{\sigma}}\left(\frac{R+(\beta_{3,k}R)^{\frac{1}{\sigma}}}{Ry_2}\right)$$

Hence, for the power utility case we get:

$$G = \left(\frac{\sigma+1}{\sigma}\right)\frac{1}{\beta_{3,k}[R+(\beta_{3,k}R)^{\frac{1}{\sigma}}]}$$

Using the above expression for G we can rewrite the inequality of interest as:

$$\beta_{3,k}R\left(\frac{\sigma+1}{\sigma}\right)\frac{1}{\beta_{3,k}[R+(\beta_{3,k}R)^{\frac{1}{\sigma}}]} \leq 1$$

Rearranging, we get the following condition:

$$\beta_{3,k}R \leq \left(\frac{\sigma}{R}\right)^{\frac{1}{\sigma}}$$

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