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Simulating fiscal sustainability in the US

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Abstract

We explain low interest rates of government bonds relative to the growth rate by providing a dynamic stochastic general equilibrium (DSGE) model of a production economy featuring intermediation costs and heterogeneous access to production among agents. We investigate fiscal sustainability by testing whether the expected debt-to-GDP ratio stabilizes or increases without bound. We update the future fiscal variables by specifying a fiscal rule that incorporates Obama's plan and the observed correlation between the primary surplus-to-GDP ratio and the GDP growth rate, and simulate the debt-to-GDP ratio. We report that the fiscal policy of the US is not sustainable in the sense that the debt-to-GDP ratio will increase without bound with a probability greater than 50 percent.

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1. Introduction

President Obama released the stimulus package of the huge fiscal expansion to stimulate recovery from the severe recession that was triggered by the Lehman shock. As shown in Table 1, according to that plan, the fiscal deficit relative to the GDP should be no less than 11.9 percent. Although he stressed that the budget deficit would be reduced to a half of its previous level in 2013, the possible huge amounts of outstanding government debt have raised great concerns about the sustainability of the US fiscal policy. The central question is whether the fiscal deficit of the US is sustainable.

Investigating fiscal sustainability, however, entails facing a puzzling fact. Interest rates on government bonds have remained quite low relative to the economic growth rate in the US. Figure 1 illustrates the time series of the Treasury bill rate, the interest rate of the long-term bond, and the growth rate for the period 1990–2007.[3] The averages of the long-term bond and the Treasury bill rate are 3.5 percent and 1.8 percent, respectively, while the average of the growth rate is 2.8 percent. With the annual discount factor that is extensively used in the business cycle literature (namely, $1/1.03$), the exogenous growth model predicts that the interest rate should be above 3 percent. The endogenous growth model predicts that, with log-utility, the interest rate should be higher than the growth rate by at least 3 percent.[4] Studying a "low" interest rate is crucial to investigating fiscal sustainability.[5]

We investigate the low interest rates of government bonds relative to the growth rate by providing a dynamic stochastic general equilibrium (DSGE) model incorporating the AK production technology. To motivate low interest rates, we introduce financial friction into the model by assuming intermediation costs for private lending and borrowing and the heterogeneity in the access to production among agents (e.g., Woodford, 1990 and Bohn, 1999). In addition, to capture the low interest rate on risky bonds, we consider the

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[3] All figures are measured in real terms.

[4] This value of the discount factor is equal to the estimate reported in the business cycle literature (see, e.g., Christiano and Eichenbaum, 1992 and Christiano, Eichenbaum and Evans, 2005).

[5] This tendency is not specific to the US. In fact, the average realized real rates of return on government bonds in major OECD countries over the past 30 years have been smaller than the real growth rate (e.g., Blanchard and Weil, 2001).

incomplete bond market where the government can issue only one-period bonds and private agents cannot insure away the income uncertainty.[6]

The introduction of the intermediation cost gives rise to a decline in the economic growth rate and, in addition, the decline in the interest rates. Whether the intermediation cost makes the fiscal policy more sustainable or not depends on the elasticity of substitution on consumption. If it is less than nearly 2, the high intermediation cost improves fiscal sustainability.

We evaluate sustainability by testing whether the debt-to-GDP ratio stabilizes or increases without bound. The approach of checking the intertemporal government budget constraint has a misleading implication for sustainability. In the presence of intermediation costs, any interest rate among various menus of government bonds and their combination is not appropriate for correct discounting. In addition, this criterion of fiscal sustainability is derived from the feasibility of tax revenues as well as the non-Ponzi-game conditions of private agents, and thus turns out to be more appropriate than the approach of checking only the intertemporal budget constraint of agents.

Fiscal sustainability depends on the fiscal rule. If we use a fiscal rule that incorporates Obama's plan and the observed correlation between the primary surplus-to-GDP ratio and the GDP growth rate, , the expected debt-to-GDP ratio would reach 1.62 in 100 years and afterwards would continue to diverge dramatically. The probability that the debt-to-GDP ratio will diverge is greater than 50 percent in 25 years and later, and we have to judge that the US fiscal policy is not sustainable.

The expected debt-to-GDP ratio depends on some key parameters including the financial intermediation cost, the elasticity of intertemporal substitution of consumption, and average real GDP growth rate. The calculated elasticity of intertemporal substitution of consumption is within the range over which the interest rate declines less than the growth rate in response to the reduction of the intermediation cost. In the US, the high intermediation cost contributes to fiscal sustainability. If the fiscal rule incorporates

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[6] Some studies cast doubts on the presence of complete bond markets. For example, Marcet and Scott (2009) find the persistency of the data for the US government debt, which is supportive of incomplete markets but is inconsistent with complete markets

Bohn's idea that a rational government should increase the primary surplus when the debt-to-GDP ratio is high, sustainability improves.

We do not rely on the risk-premium approach to explain low interest rates for two reasons (e.g., Mehra and Prescott, 1985, and Weil, 1989).[7] First, this approach explains only the low interest rate of the safe bond but does not explain low rates of the government bonds as a whole. Secondly, as the literature on the "risk-free rate puzzle" (e.g., Weil, 1989) points out, classes of simple utility functions do not succeed in explaining the low interest rate within admissible parameter values.

This paper contributes to the literature on methodology to test fiscal sustainability. One approach uses the intertemporal government budget constraint, including Hamilton and Flavin (1986) and Ahmed and Rogers (1995).[8] Bohn (1995) criticized this approach for the reason that safe government bonds do not reflect correct discounting. Another approach checks the behavior of the debt-to-GDP ratio, including Bohn (1998) and Ball et al. (1998). Bohn (1998) proposed a simple test to check whether the debt-to-GDP ratio displays a mean-reversion property.[9] Ball et al. (1998), in their famous paper entitled "Deficit Gamble," projected future growth rates and interest rates from past data and calculated the probability under which the debt-to-GDP ratio would enter a dangerous zone. Our approach is similar to the latter approach. In our model, when there is a significant intermediation cost, the government can run the Ponzi strategy, but even then, if the debt-to-GDP ratio is constant, the fiscal policy is sustainable.

Recently, some literature has studied fiscal sustainability by applying a dynamic stochastic general equilibrium (DSGE) model. Mendoza and Oviedo (2004, 2006)

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[7] Abel et al. (1989) and Bohn (1995) provide stochastic growth models in which the risk premium drives down the safe interest rate, often below the economic growth rate. Their argument, along with that of Zilcha (1992), demonstrates that the no-Ponzi condition for the intertemporal government budget constraint holds, even if the safe interest rate is below the growth rate.

[8] Other works include Wilcox (1989), Trehan and Walsh (1991), Hakkio and Rush (1991), Ahmed and Rogers (1995), Uctum and Wickens (2000), and Polito and Wickens (2005).

[9] Bohn (2005) applies his test to the historical data of the US and finds evidence supporting fiscal sustainability (see also Greiner and Kauermann, 2007). Mendoza and Ostry (2008) applies Bohn's test to industrial and emerging countries and finds evidence of fiscal solvency in both types of countries. Galí and Perotti (2003) and Wierts (2007), among others, apply Bohn's test to European countries.

develop small open economies to investigate how macroeconomic shocks affect government finances and estimate the amount of sustainable public debt in emerging market economies. Arellano (2008) also develops a small open-economy model to study sovereign default risk and its interaction with output and foreign debt. Sakuragawa and Hosono (2009) develop a closed economy model of an exchange economy to test fiscal sustainability of the Japanese economy.

This paper is also related to the theoretical literature that combines financial frictions with the heterogeneity in the access to production among agents to have implications for the lower interest rate than the growth rate. The literature includes Woodford (1990), Bohn (1999), Kiyotaki and Moore (2008), Hellwig and Lorenzoni (2009) and Kocherlakota (2009).

This paper is organized as follows. In Section 2, we outline the model. In Section 3, we develop the theoretical analysis. In Section 4, we describe the simulation procedure. In Section 5, we investigate the sustainability of the US public debt.

2. Model

Consider an economy made up of two types of agents that live infinitely, with the number of each normalized to be unity, and the third type of agents that live for two periods and act as intermediaries. We consider heterogeneous agents and financial friction in order to provide implications for low interest rates.[10]

Type E agents have access in all *even* periods to an AK production technology that transforms K_t units of the final good into random $(1+x_{t+1})K_t$ units after one period, while type O agents have access in all *odd* periods. Two reasons motivate the introduction of the AK model. First, fiscal sustainability is a long-run problem. Second, the AK model enables one to have the positive link between interest and growth rates that is observed in the time series. The rate of return on capital, x_{t+1}, is a random variable that follows a Markov process and takes values in a set, X_t. The history of the economy up to time t is

[10] Aiyagari and Gertler (1991) and Heaton and Lucas (1996) construct models with intermediation costs and heterogeneous agents.

denoted by $h_t = (x_t, x_{t-1}, \ldots)$, which takes values in a set H_t. Denote the probability of a variable, x_{t+1}, given a history h_t, by $\pi(x_{t+1}|h_t)$.

To simplify the notation, let there be one representative agent of each type so that the individual income $(1+x_{t+1})K_t$ denotes the aggregate income. There is no population growth. Both types have identical preferences over consumption and maximize $E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\alpha}}{1-\alpha}$, where α is the inverse of the elasticity of substitution of consumptions across periods, β (0< β <1) is the discount factor, and E_0 is the expectation operator. We impose the relevant condition on bounded utility by $E_t\{\beta(1+g_{t+1})^{1-\alpha}\}<1$, where g_t is the growth rate of the aggregate income argued below. The government spending, G_t, is a constant share of GDP to meet $G_t = zY_t$. The government finances its spending by imposing lump-sum taxes T_t and by issuing public debt.

At each period, finite N agents who act as intermediaries are born and live for two periods. They are endowed with a specific skill of intermediating finance between private agents, and maximize the second-period consumption less the amount of effort exerted by them.

We introduce financial friction by supposing that these agents have to bear a proportional intermediation cost, $\kappa > 0$, per unit of funds. The cost is measured in terms of the loss of effort. One may interpret κ as a cost of monitoring or identifying a borrower, or of verifying credit.

3. Theoretical Analysis

The intermediary issues securities that request the rate of repayment r_t^b to firms and guarantee the rate of return r_t to investors. In a world of competitive intermediation, intermediaries finally have to earn zero profit to satisfy

(1) $$1 + r_t = (1 + r_t^b)/(1+\kappa)$$

for any x_t. Note that both assets are risky in the sense that the rate of return depends on the productivity.

At the beginning of an even period t, type E agents face a shock x_t, receive capital income $(1+x_t)K_{t-1}$, repay $(1+r_t^b)B_{t-1}$, consume C_t, pay taxes T_t, and invest the remaining in the private security W_t and the public bond D_t. They maximize the following value function:

$$(2) \qquad V(B_{t-1}, K_{t-1}, x_t) = \max_{C_t, W_t, D_t} \frac{C_t^{1-\alpha}}{1-\alpha} + \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)\, \tilde{V}(W_t, D_t, x_{t+1})$$

subject to the budget constraint

$$(3) \qquad (1+x_t)K_{t-1} - (1+r_t^b)B_{t-1} = C_t + W_t + D_t + T_t .$$

On the other hand, at period t, type O receives interest incomes from the private security $(1+r_t)W_t$ and the public bond $(1+R_t)D_{t-1}$, and transfers Π_t from intermediaries. They consume $\tilde{C}_t$ and invest the remaining in capital to produce in the odd period. They maximize the following value function:

$$(4) \qquad \tilde{V}(W_{t-1}, D_{t-1}, x_t) = \max_{\tilde{c}_t, K_t, B_t} \frac{\tilde{C}_t^{1-\alpha}}{1-\alpha} + \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)\, V(B_t, K_t, x_{t+1})$$

subject to

$$(5) \qquad \Pi_t + (1+r_t)W_{t-1} + (1+R_t)D_{t-1} + B_t = \tilde{C}_t + K_t ,$$

where R_t is the interest rate on the government bond, and $\Pi_t (\equiv \kappa(1+R_t)W_{t-1})$ is the intermediary's profit that is transferred to them. The intermediary is compensated for the loss of effort by income, but the income accruing to the intermediary is transferred to households in a lump-sum fashion.

Assume that the equilibrium has an interior solution. Equilibrium conditions on K_{t+1}, W_{t+1}, B_{t+1}, and D_{t+1}, together with envelope conditions, lead to

$$(6) \qquad 1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)(1+x_{t+1}) \frac{\{\tilde{C}(h_t)\}^{\alpha}}{\{C(x_{t+1} | h_t)\}^{\alpha}}$$

$$(7) \qquad 1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1} | h_t)\{1 + r(x_{t+1} | h_t)\} \frac{\{C(h_t)\}^{\alpha}}{\{\tilde{C}(x_{t+1} | h_t)\}^{\alpha}}$$

(8) $$1=\beta \sum_{x_{t+1}\in X_{t+1}} \pi(x_{t+1}|h_t)\{1+r^b(x_{t+1}|h_t)\}\frac{\{\tilde{C}(h_t)\}^{\alpha}}{\{C(x_{t+1}|h_t)\}^{\alpha}},$$

and

(9) $$1=\beta \sum_{x_{t+1}\in X_{t+1}} \pi(x_{t+1}|h_t)\{1+R(x_{t+1}|h_t)\}\frac{\{C(h_t)\}^{\alpha}}{\{\tilde{C}(x_{t+1}|h_t)\}^{\alpha}}.$$

The market clearing in the good market is expressed as

(10) $$C_t+\tilde{C}_t+K_t+zY_t=(1+x_t)K_{t-1}.$$

The market clearing in the credit market is expressed as

(11) $$W_t=D_t.$$

Finally, the government's budget constraint is given by

(12) $$D_t=(1+R_t)D_{t-1}-T_t+G_t.$$

We say that the government's budget is *feasible* if the tax revenue does not exceed GDP at any time.

The competitive equilibrium is defined as a sequence of nine variables $\{\frac{C_{t+1}}{C_t},\frac{\tilde{C}_{t+1}}{\tilde{C}_{t+1}},\frac{K_{t+1}}{K_t},\frac{W_{t+1}}{W_t},\frac{B_{t+1}}{B_t},\frac{D_{t+1}}{D_t},r_t,r_t^b,R_t\}_{t=0}^{\infty}$, satisfying nine equations (1), (3),(6)–(12), the feasibility of the government budget, and relevant non-Ponzi-game (NPG) conditions, given the sequence of random variables $\{x_t\}_{t=0}^{\infty}$ and the sequence of the policy rule $\{T_t,D_t\}_{t=0}^{\infty}$, and given K_0 and D_0.

The fact that two-period-lived intermediaries have no intertemporal consideration simplifies the link among several interest rates. As for the link between x_t and r_t^b, competitive intermediation should lead to $x_t=r^b(x_t)$ for any x_t. Jointly with (1), the investors' rate of return r_t should also be dependent on x_t to satisfy $r_t=(1+r^b(x_t))/(1+\kappa)-1\equiv r(x_t)$. Equations (6) and (8) imply that the private security issued by the intermediary and the government bond are perfect substitutes for investors so that, without loss of generality, we may set $R(x_t)=r(x_t)$ for any x_t. We have

$R(x_t) < r^b(x_t)$; the government can borrow at a lower rate than private agents. The reason behind this finding is that loans to the government can be monitored with no cost, while loans to private agents need intermediation cost.

We use these features on several interest rates to argue on the non-Ponzi-game (NPG) conditions that formalize the limited willingness of agents to lend. The NPG condition for agents who have access to production at the current period and those who have no access are, respectively,

(NPG1) $$E_t \lim_{s\to\infty} \frac{K_{t+s} - B_{t+s}}{(1+r^b_{t+1})(1+r_{t+2})(1+r^b_{t+3})...} = 0,$$

and

(NPG2) $$E_t \lim_{s\to\infty} \frac{W_{t+s} + D_{t+s}}{(1+r_{t+1})(1+r^b_{t+2})(1+r_{t+3})...} = 0.$$

We assume that both conditions are satisfied. The agent alternates between a lender and a borrower every other period and discounts the future at rate $\sqrt{(1+r_t)(1+r^b_{t+1})} - 1$ (or $\sqrt{(1+r^b_t)(1+r_{t+1})} - 1$). On the other hand, the discounted value of debt that the government could earn by a Ponzi strategy is $E_t \lim_{s\to\infty} \frac{D_{t+s}}{(1+R_{t+1})(1+R_{t+2})...(1+R_{t+s})...}$. Importantly, because we have $r_t = R_t < r^b_t$, the latter value should not always converge to zero, even when both (NPG1) and (NPG2) are satisfied. As argued below, the difference in discounting has an important implication on conditions for fiscal sustainability.

Let $C_t/\tilde{C}_t \equiv \theta_t$ denote the consumption ratio between two different types of agents. Limiting focus on an economy with θ_t being constant through time, we have the consumption growth rate as

(13) $$\frac{C(x_{t+1}|h_t)}{C(h_t)} = \frac{\tilde{C}(x_{t+1}|h_t)}{\tilde{C}(h_t)} \equiv 1 + g(x_{t+1}|h_t).$$

We use (13) to rewrite (7) as

(14) $$1 = \beta \sum_{X_t} \pi(x_{t+1}|h_t)\{1 + g(x_{t+1}|h_t)\}^{-\alpha} \theta^{\alpha} \{1 + r(x_{t+1})\},$$

which embodies the relationship between the growth and interest rates. On the other hand, (8), (9), (13), and (1) jointly imply $\theta=(1+\kappa)^{1/2\alpha}$. In the presence of an intermediation cost, agents consume more when they receive income and consume less when they do not. On the other hand, plugging (1) and $\theta=(1+\kappa)^{1/2\alpha}$ into (14), we have the following:

$$1=\beta\sum_{X_t}\pi(x_{t+1}|h_t)\{1+g(x_{t+1}|h_t)\}^{-\alpha}(1+x_{t+1})(1+\kappa)^{-1/2}. \tag{15}$$

Equations (14) and (15) fully determine the sequence of the growth and interest rates as the stochastic variable x_t evolves.

We explicitly solve (14) and (15). We denote the transition probability as $\pi(x_{t+1}=\lambda_j|x_t=\lambda_i)=\phi_{ij}$ in the set $X=\{x_1,...,x_n\}$. We denote $g(x_{t+1}=x_j|h_t)=g_j$, which implies that the growth rate of consumption depends only on the current rate of return on capital. This arises from the fact that the production technology is the "AK" type and the fact that the proportion of government expenditure in output is constant. Accordingly, we rewrite (15) as

$$1=\beta\sum_{j=1}^{n}\phi_{ij}(1+g_j)^{-\alpha}(1+x_j)(1+\kappa)^{-1/2} \text{ for } i=1,...,n. \tag{16}$$

We rewrite (14) similarly as

$$1=\beta\sum_{j=1}^{N}\phi_{ij}(1+g_j)^{-\alpha}\theta^{\alpha}(1+r_j) \text{ for } i=1,...,n. \tag{17}$$

Equations (16) and (17) constitute $2n$ equations for solving n growth rates and n interest rates. We show the case of $n=2$, with $X=\{x_1,x_2\}$. We solve four variables $\{g_1,g_2,r_1,r_2\}$ from the following four equations:

$$\begin{bmatrix}(1+g_1)^{-\alpha}(1+x_1)\\(1+g_2)^{-\alpha}(1+x_2)\end{bmatrix}=\beta^{-1}(1+\kappa)^{\frac{1}{2}}\begin{bmatrix}\phi_{11} & \phi_{12}\\ \phi_{21} & \phi_{22}\end{bmatrix}^{-1}\begin{bmatrix}1\\1\end{bmatrix}, \tag{18}$$

and

$$\begin{bmatrix}(1+r_1)(1+g_1)^{-\alpha}\\(1+r_2)(1+g_2)^{-\alpha}\end{bmatrix}=\beta^{-1}(1+\kappa)^{-\frac{1}{2}}\begin{bmatrix}\phi_{11} & \phi_{12}\\ \phi_{21} & \phi_{22}\end{bmatrix}^{-1}\begin{bmatrix}1\\1\end{bmatrix}. \tag{19}$$

Appendix A provides the procedure for solving the general case.

It is interesting to investigate how a change in κ influences g_{t+1} and r_{t+1}, and their relationship. Equation (18) implies that a one percent increase in κ leads to a decline in approximately $1/(2\alpha)$ percent of g_{t+1} for each state. On the other hand, (19), together with (18), implies that a one percent increase in κ leads to a decline of one percent of r_t for each state. If α is sufficiently small (less than 0.5), or equivalently, the elasticity of substitution of consumptions is sufficiently large, the growth rate declines more sharply than the interest rate as κ increases. Figure 2A illustrates the case for $\alpha = 0.4$.[11] By contrast, if α is sufficiently large, the growth rate declines less than the interest rate for higher intermediation costs. Figure 2B illustrates the case for $\alpha = 1$(log-utility). Whether higher intermediation costs make fiscal sustainability difficult depends on the elasticity of substitution of consumption.

Finally in this section, we check sustainability conditions for the fiscal policy. A fiscal policy rule is defined to be sustainable if the following two conditions are satisfied. First, the NPG condition of private agents is satisfied. When agents discount the future more than the government, the NPG condition of the government does not establish condition for sustainability. The government cannot run a Ponzi strategy so long as agents do not find it optimal to be on the lending side of the Ponzi scheme. The latter condition is expressed as $E_t \lim_{s\to\infty} \frac{D_{t+s}}{(1+r_{t+1})(1+r_{t+2}^b)(1+r_{t+3})...} = 0$, which is a weaker condition than the condition precluding the gain of the Ponzi scheme on the side of government $E_t \lim_{s\to\infty} \frac{D_{t+s}}{(1+R_{t+1})(1+R_{t+2})...(1+R_{t+s})...}$. Second, the government's budget is feasible, i.e., $T_t < Y_t$.

We consider this issue by studying a deterministic version of this economy because the deterministic model exploits general features that arise when there is an intermediation cost (see Appendix B for details). If the limit of the debt-to-GDP ratio takes some positive value, a balanced growth path (BGP) can reproduce the equilibrium. Along the BGP, K_{t+1}, W_{t+1}, B_{t+1}, and D_{t+1} grow at the same rate g, and then we use the deterministic version of (14), $\theta = (1+\kappa)^{1/2\alpha}$, and the bounded utility condition, $\beta(1+g)^{1-\alpha} < 1$, to derive the NPG condition as $D_t \lim_{s\to\infty}\{\beta(1+g)^{1-\alpha}\}^s = 0$. As Appendix B

[11] In Figure 2, we set x^a = 0.0515. See Table 2 for other parameter values.

proves in detail, the government budget is feasible so long as $\frac{D_t}{Y_t} < \frac{(1-z)(1+g)}{R-g}$ for $R > g$.[12] Suppose, for example, that $R-g = 0.01$, $z = 0.2$, and $g = 0.02$, then the inequality becomes $D_t/Y_t < 82.4$, the right-hand side of which is unrealistically high so that the latter condition is virtually not restrictive.

Behind the BGP, agents discount the future at the rate $\sqrt{(1+r_t)(1+r_{t+1}^b)}-1$, while the government discounts the future at a smaller rate $R_t (= r_t)$ than households. The limiting value of the debt $D_t \lim_{s\to\infty}\{\beta(1+g)^{1-\alpha}(1+\kappa)^{1/2}\}^s$ does not necessarily converge to zero, and goes to infinity for a higher κ, even if $\beta(1+g)^{1-\alpha} < 1$ is imposed. If the debt-to-GDP ratio converges to some value that is not too large, the fiscal policy is sustainable, despite the fact that the government can run the Ponzi strategy when there is a significant intermediation cost.

Conversely, if the debt-to-GDP ratio increases forever, the public debt crowds out private lending, and at some date, the credit market should disappear. Agents start financing investment only by their net worth, and the interest rate on government bonds should rise to exceed the growth rate of the government debt if agents are willing to hold the government bond. The government cannot rely on the Ponzi strategy and is forced to raise tax revenues to pay for the increasing debt, which is infeasible because the maximum growth rate of tax revenues is the GDP growth rate, which should be less than the growth rate of debt. If the debt-to-GDP ratio diverges, the fiscal policy is not sustainable. Summing up, checking whether the debt-to-GDP ratio converges or diverges is an appropriate criterion for checking fiscal sustainability.

Our criterion of fiscal sustainability argues against the approach of evaluating sustainability in terms of the intertemporal government budget constraint in two respects. First, in the presence of intermediation costs, any interest rate among various menus of government bonds or their combination is not appropriate for correctly discounting the

[12] The government budget is feasible whenever $R \le g$,

value of future government debt.[13] Second, our criterion argues that even if the correct discounting is made, checking only the intertemporal budget constraint is not sufficient.

4. Calibration

In this section and the next, we simulate the model to investigate the fiscal sustainability of the US economy. Our methodology is to update the future fiscal variables by introducing a specified fiscal rule into the developed DSGE model and to simulate the debt-to-GDP ratio. The driving force of the growing economy is the rate of return on capital, which determines the growth rate and the interest rate. We specify the stochastic process for the rate of return on capital by discretizing a simple AR(1) process with nine states $(n = 9)$. The AR(1) process is described, with the serial correlation coefficient ρ and the average $\log(1+x^a)$, as

$$\log(1+x_{t+1}) = \rho\log(1+x_t) + (1-\rho)\log(1+x^a) + e_{t+1}, \tag{20}$$

where e_{t+1} for all t are random shocks that are independent and identically distributed as a normal distribution with standard deviation σ_e. Once three parameters, x^a, ρ , and σ_e, have been set, following the method developed by Tauchen and Hussey (1991), we construct the nine states $\{x_1,....,x_9\}$ and the transition probability ϕ_{ij} $(i, j = 1,...9)$.[14] We use (16) and (17) to solve for 18 variables (g_j, r_j) $(j = 1,...9)$, given the specified (x_j, ϕ_{ij}) (see Appendix A for details). We use (A1) in Appendix A and (20) finally to obtain the stochastic process for the GDP growth rate as a discretized version of an AR(1) process:

$$\log(1+g_{t+1}) = \rho\log(1+g_t) + (1-\rho)\log(1+g) + \varepsilon_{t+1}, \tag{21}$$

where $\varepsilon_{t+1}(\equiv e_{t+1}/\alpha)$ is a random shock with the standard deviation of $\sigma_\varepsilon(\equiv \sigma_e/\alpha)$ and g is the steady-state growth rate.

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[13] This finding is stronger than that of Bohn (1995), who develops a stochastic economy with complete markets to address a similar problem and demonstrates that the safe interest rate is not appropriate for correct discounting.

[14] Given that we set ρ at a relatively low value (0.535), Tauchen and Hussey's quadrature-based method delivers a good approximation (Flodén, 2008).

The debt-to-GDP ratio evolves from (12) as

$$d_{t+1} = \frac{(1+R_{t+1})}{(1+g_{t+1})} d_t - s_{t+1}, \quad (22)$$

where d_t and s_t are debt and primary surplus divided by GDP, respectively. These two equations, together with a fiscal policy rule that determines s_{t+1}, provide the full system. Before going on to the fiscal policy rule, we choose parameter values.

First, we choose the preference parameters, β and α. We set the annual discount factor β to 1/1.03=0.9709.[15] The inverse of the elasticity of intertemporal substitution, α, plays a central role in relating the interest rate to the growth rate, as captured by (19) (or (A2) in Appendix A). To set α, we note from (19) that

$$\log(1+R_j) = \alpha \log(1+g_j) + \gamma_j, \quad (23)$$

where γ_j is the logarithm of the *j*th element of the right-hand side in (19). We regressed the nominal yield to the Treasury bills on the nominal GDP growth rate using OLS. We limited the sample period to 1980–2007 to take into account the fact that financial liberalization was completed in 1980.[16] The estimation result is

$$R_t = \underset{(1.260)}{0.873} + \underset{(0.191)}{0.767} g_t, \quad Adj.R^2 = 0.368, \quad (24)$$

where the numbers in parentheses are standard errors. Following (23), we set α at 0.767, implying that the elasticity of intertemporal substitution is set at 1.304.[17] It is noteworthy that this figure is in the region of parameters in which a rise in the intermediation cost can improve fiscal sustainability.

Next, we specify the technology parameters, ρ and σ_e. Using the data on the GDP growth rate data over the period 1929–2007, we obtain the OLS estimation as

$$g_{t+1} = 0.038 + 0.535 g_t, \quad Adj.R^2 = 0.299. \quad (25)$$

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[15] This value of β is equal to the estimate reported in the business cycle literature, including Christiano and Eichenbaum (1992) and Christiano, Eichenbaum and Evans (2005).

[16] The Depository Institutions Deregulation and Monetary Control Act (DIDMCA) of 1980 was enacted in 1980.

[17] Sakuragawa and Hosono (2009) report the value of $\alpha = 0.688$ for the Japanese economy.

(0.009) (0.092)

We set ρ at 0.535 and σ_ε at 0.0407, where the latter is the root mean squared error of the regression. Given the chosen σ_ε and α, we set σ_e at 0.0312. We assume that the projected real GDP growth rate as of 2014 continues, on average, and set the average GDP growth rate, g, at 0.027.

Third, we set the financial intermediation cost, κ, at 0.038, the average of net interest margins between the bank loans and the bank deposits, calculated over the sample period for 1980–2007 (see Figure 3).[18] We use the chosen β, g, and κ to set x^a at 0.0709 to satisfy (16).

We complete the model by specifying the government's fiscal policy rule. Up to 2014, we used projections released by the Office of Management and Budget (2008), the so-called "Obama's plan." According to this plan, the primary deficit is projected to be as large as 11.9 percent of GDP at its maximum and to continue up to 2014. We use Obama's projections as initial values of primary balances as a proportion of GDP and real GDP growth rate for the period of 2006–2014.

We obtain the rule for after 2014. The fiscal rule is interpreted as a consequence of the conflict of interests among many pressure groups and so changes little unless the political situation surrounding them changes greatly. Based on this idea, we specify the fiscal rule by the regression. We regress the primary balance as a proportion of GDP on the real GDP growth rate and the one-period lagged primary balance. The GDP growth rate is expected to capture the business cycle effects. When the economic boom comes, an increase in tax revenues improves the fiscal stance. The lagged variable captures the persistency of the government expenditure and tax revenues.[19] To be specific, we describe a fiscal policy rule as

(26) $$s_t = \gamma_0 + \gamma_1 g_t + \gamma_2 s_{t-1}.$$

Using the data over the period 1970–2007, we obtain the regression result as

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[18] These data are derived from the FDIC Quarterly Banking Profile (Federal Deposit Insurance Corporation).

[19] Galí and Perotti (2003) estimate the fiscal policy rules for European countries including the lagged deficit in the explanatory variables. See also Wierts (2007).

(27) $$s_t = -0.015 + 0.500 g_t + 0.847 s_{t-1}, \quad Adj.R^2 = 0.672.$$
$$(0.004) \quad (0.093) \quad (0.100)$$

Based on (27), we set γ_1=0.500, and γ_2=0.847. We use γ_0 as a free parameter and set γ_0=–0.0135 so as to make the average primary surplus zero, given an average projected GDP growth rate after 2014 of 0.027. We set the initial values of the primary surplus-GDP ratio based on Obama's plan as of 2006 (see Table 1).

We simulate the model recursively by generating $(g_{t+1}, R_{t+1}, s_{t+1})$ for the stochastic process of x_{t+1}, and obtaining d_{t+1}, given the starting value of d_0.

Table 2 summarizes the parameters that we use for the baseline calibration. The simulation procedure is described in Appendix C.

5. Simulation Results

A. Baseline forecasts

Incorporating Obama's projection of the primary surplus and the GDP growth rate into the model, we find that the projected debt-to-GDP ratio as of 2014, the end year of Obama's plan, is 0.953. Given this value of the debt-to-GDP ratio as the initial condition, we simulate the debt-to-GDP ratio under the fiscal policy rule (27). We also report the probability of the debt-to-GDP ratio exceeding its initial value of 0.953.

Table 3 reports the simulation results. The benchmark case shows that the average growth rate is 2.7 percent and the average interest rate is 3.17 percent, with the gap being 0.47 percentage points. The expected debt-to-GDP ratio reaches 1.62 in 100 years and continues to diverge dramatically afterwards. The probability that the debt-to-GDP ratio will exceed its initial value is greater than 50 percent in 25 years and later.

The path of the expected debt-to-GDP ratio and the probability that the debt-to-GDP ratio will exceed its initial value are highly sensitive to the intermediation cost. If we set $\kappa = 0$, the average interest rate and the GDP growth increase to 7.09 percent and 5.23 percent, respectively, with the gap widening to 1.86 percentage points. Consequently, the expected debt-to-GDP ratio increases more rapidly than in the benchmark case and reaches 5.74 in 100 years. The smaller intermediation costs make the fiscal sustainability more difficult. This is because the estimated elasticity of intertemporal substitution of consumption is well below two.

So far, we have assumed that the long-run average real GDP growth rate is 2.7 percent. If we assume instead that it is 2 percent, the expected debt-to-GDP ratio grows more rapidly than in the baseline case and reaches 1.92 in 100 years.[20] As the average real GDP growth rate declines, the real interest rate also declines, but the gap widens, resulting in a higher expected debt-to-GDP ratio.

The sensitivity of the interest rate to the growth rate also depends on the intertemporal rate of substitution. If we return the average real GDP growth rate to 2.7 percent, as in the benchmark case, but set the inverse of the elasticity of intertemporal substitution at one (the log-utility case), the average real interest rate becomes 3.82 percent, making the sustainability conditions more difficult to meet: the expected debt-to-GDP ratio reaches 2.72 in 100 years.

B. Alternative fiscal policy rules

Fiscal sustainability depends on the fiscal rule. A simple way to restore sustainability would be to raise the average primary surplus by raising the value of γ_0 in the rule (26). We found that the primary surplus that is 0.56 percent of GDP is enough to stabilize the expected debt-to-GDP ratio at its initial value and, thus, to make debt sustainable.

A more flexible and maybe more interesting way to restore sustainability is to change the fiscal policy rule. As Bohn (1998) states, a rational government should increase the primary surplus when the debt-to-GDP ratio is high. We incorporate his idea into the fiscal rule by assuming that the primary surplus-to-GDP ratio depends on the lagged value debt-to-GDP ratio as well. To be specific, we have

(28) $$s_t = \gamma_0 + \gamma_1 g_t + \gamma_2 s_{t-1} + \gamma_3 d_{t-1}.$$

[20] We adjust the constant term of the fiscal policy rule (25) so that the average primary surplus is zero under the real growth rate of 2 percent.

We use the same parameters for $\gamma_1 (= 0.500)$ and $\gamma_2 (= 0.847)$, as in the baseline case, and set γ_3 arbitrarily at several values.[21]

Table 4A shows the expected debt-to-GDP ratios for each value of γ_3. We find that a sufficiently positive response to the debt-to-GDP ratio ($\gamma_3 \geq 0.002$) makes government debt sustainable. Table 4B reports the expected primary surplus-to-GDP ratio, showing that it is initially higher but eventually lower as γ_3 increases.

6. Conclusion

We have accounted for low interest rates on government bonds relative to GDP growth rates under a plausible degree of risk aversion by presenting a DSGE model featuring intermediation costs and heterogeneous access to production among agents. Feeding a fiscal policy rule into the model, we have presented a new test of fiscal sustainability. Specifically, we have investigated whether, under a specific fiscal policy rule, the expected debt-to-GDP ratio stabilizes or increases without bound. This criterion of fiscal sustainability, derived from the NPG conditions of private agents as well as the feasibility of tax revenues, turns out to be appropriate in the economy with financial intermediation costs. Applying our approach to the US fiscal policy, we find that the US government debt is not sustainable as long as we specify the fiscal policy rule as the one incorporating Obama's plan and the observed correlation between the primary surplus-to-GDP ratio and the GDP growth rate. The expected debt-to-GDP ratio depends on some key parameters including financial intermediation costs, intertemporal rate of substitution of consumption, and average real GDP growth rate, though our main result seems to be robust under a plausible range of parameters as long as we assume the above fiscal policy rule. If, instead, we incorporate the fiscal rule of Bohn (1998), positing that a rational government should increase the primary surplus when the debt-to-GDP ratio is high, we find that a sufficiently positive response of primary surplus to debt can restore fiscal sustainability.

Our results suggest that some form of fiscal adjustment will be required in the future. Given demographic aging and rising costs of health care (Congressional Budget Office,

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[21] We estimated (28) but found that the coefficient on d_{t-1} was positive but not significant.

2009), the magnitude of fiscal adjustment may be large, though precise forms of adjustment are hard to foresee.[22]

One caveat is necessary in utilizing our model as a test of fiscal sustainability. Our model does not incorporate the default cost into the model so that the calculated interest rate can be regarded as a lower bound for possible scenarios. Our test implies that the convergence of the debt-to-GDP ratio is necessary but not sufficient for sustainability. For example, when bad states arise many times, the high default cost raises the interest rate for long periods, and the debt-to-GDP ratio may increase without bound, even in the case when it converges in the absence of the default cost. One direction is to make our criterion a necessary and sufficient condition for sustainability by extending the model to consider the default cost explicitly.[23]

Appendix A: A General Solution for Growth and Interest Rates

We derive the equilibrium growth and interest rates. We first solve for $\{g_1, g_2,,, g_n\}$. Letting G_α denote the $n\times 1$ vector with its j th element of $(1+g_j)^{-\alpha}$, Φ denote the $n\times n$ matrix with the (i, j) element of $\phi_{i,j}$, $\tilde{X}$ denote the $n\times n$ diagonal matrix with the (j, j) element of $1+x_j$, and I denote the $n\times 1$ unit vector, we can rewrite (16) as $I = \beta(1+\kappa)^{-\frac{1}{2}}\Phi\tilde{X}G_\alpha$, which leads to

(A1) $$G_\alpha = \beta^{-1}(1+\kappa)^{\frac{1}{2}}\tilde{X}^{-1}\Phi^{-1}I \cdot$$

Next we solve for $\{r_1, r_2,,, r_n\}$. Letting P denote the $n\times 1$ vector with its jth element of $(1+r_j)$ and $\tilde{G}_\alpha$ denote the $n\times n$ diagonal matrix with the (j, j) element of $(1+g_j)^{-\alpha}$, we can rewrite (17) as $I = \beta\theta^\alpha\Phi\tilde{G}_\alpha P$, which leads to

(A2) $$P = \beta^{-1}\theta^{-\alpha}\tilde{G}_\alpha^{-1}\Phi^{-1}I = \beta^{-1}(1+\kappa)^{-\frac{1}{2}}\tilde{G}_\alpha^{-1}\Phi^{-1}I,$$

[22] Davig et al. (2009) examine the long-run macroeconomic consequences of great uncertainty about the future fiscal and monetary policies, focusing on the growing US "unfunded liabilities," such as Federal Social Security, Medicare, and Medicated spending.

[23] Arellano (2008) assumes that in a small open economy, sovereign default entails temporary exclusion from international financial markets and direct output costs.

where the second equality comes from (14). Letting X denote the $n\times 1$ vector with its j th element of $1+x_j$, we can rewrite (16) as $I=\beta(1+\kappa)^{-\frac{1}{2}}\Phi\tilde{G}_\alpha X$. Substituting this into (A2), we obtain

(A3) $$P=(1+\kappa)^{-1}X .$$

Finally, we compute the steady-state values of g_t and r_t. Let the vector λ denote the stationary distribution of Φ. Then the steady state values of g_t and r_t are obtained by $g=\sum_{j=1}^{n} g_j\lambda_j$ and $r=\sum_{j=1}^{n} r_j\lambda_j$, respectively, where λ_j is the j th element of λ.

Appendix B: Fiscal Sustainability in a Deterministic Economy

We investigate conditions for fiscal sustainability by using a deterministic economy with the rate of return on capital being constant over time. A fiscal policy rule is defined to be sustainable first if the NPG condition of private agents is satisfied, and second if the government's budget is feasible. We begin by studying properties of the balanced growth path (BGP) and then go to sustainability conditions.

We first investigate the equilibrium of the BGP along which K_t, W_t, B_t, and D_t grow at the same rate g .Because the private credit market is active, four deterministic versions of first-order conditions (6)–(9) and the intermediary's zero profit condition lead to

(B1) $$r^b=x,$$

(B2) $$R=r=\frac{1+x}{1+\kappa}-1,$$

(B3) $$g=\beta^{\frac{1}{\alpha}}(1+x)^{\frac{1}{\alpha}}(1+\kappa)^{-\frac{1}{2\alpha}}-1,$$

and $\theta=\{(1+x)/(1+r)\}^{\frac{1}{2\alpha}}=(1+\kappa)^{\frac{1}{2\alpha}}$. From (B1)–(B3), and $\beta(1+g)^{1-\alpha}<1$, we have

(B4) $$1+g<\sqrt{(1+r)(1+r^b)}<1+r^b .$$

We use (B1)–(B3) to derive the NPG conditionas

(B5) $$\lim_{s\to\infty}\frac{D_{t+s}}{\{(1+r)(1+r_b)\}^{s/2}}=D_t\lim_{s\to\infty}\{\beta(1+g)^{1-\alpha}\}^s=0 .$$

We next check the feasibility of the government budget that requires $T_t < Y_t$ at any time. Because the government debt D_t grows at rate g along the BGP, we rewrite the budget constraint (12) as

(B6) $$\frac{T_t}{Y_t} = z + \frac{R-g}{1+g}\frac{D_t}{Y_t}.$$

If $R \le g$, $T_t < Y_t$ is always satisfied because $z < 1$. If $R > g$, $T_t < Y_t$ is satisfied if $\frac{D_t}{Y_t} < \frac{(1-z)(1+g)}{R-g}$. We summarize this argument as follows.

Result 1

Suppose that the economy exhibits the BGP with an active private credit market. Then a fiscal policy is sustainable if the debt-to-GDP ratio is not too large and satisfies $\frac{D_t}{Y_t} < \frac{(1-z)(1+g)}{R-g}$, where R and g satisfy (B2) and (B3).

We next go to the case when the private credit market is inactive; i.e., $W_t = B_t = 0$. Agents who have just had access to the production purchase only government bonds D_t^E, facing the budget constraint $(1+x)K_{t-1} + (1+R_t)D_{t-1}^O = C_t + D_t^E + T_t$. Likewise, those who are going to have access to the production purchase government bonds D_t^O and invest in capital K_t, facing $(1+R_t)D_{t-1}^E = \tilde{C}_t + K_t + D_t^O$, where $D_t \equiv D_t^E + D_t^O$. The first-order conditions on D_t^E, D_t^O, and K_t are $\beta(1+R)\tilde{C}_{t+1}^{-\alpha} = C_t^{-\alpha}$, $\beta(1+R)C_{t+1}^{-\alpha} = \tilde{C}_t^{-\alpha}$, and $\beta(1+x)C_{t+1}^{-\alpha} = \tilde{C}_t^{-\alpha}$, respectively. Along the BGP, we obtain

(B7) $$R = x,$$

(B8) $$g = \beta^{\frac{1}{\alpha}}(1+x)^{\frac{1}{\alpha}} - 1,$$

and $\theta = 1$. From (B7)–(B8), and $\beta(1+g)^{1-\alpha} < 1$, we have

(B9) $$g < R.$$

Now agents discount the future at rate R, and the NPG condition is expressed as

(B10) $$\lim_{s\to\infty}\frac{D_{t+s}}{(1+R)^s}=D_t\lim_{s\to\infty}\{\beta(1+g)^{1-\alpha}\}^s=0\,.$$

From (B6), likewise, we can confirm the feasibility. We summarize in the following result.

Result 2

Suppose that the economy exhibits the BGP with inactive private credit market. Then a fiscal policy is sustainable if the debt-to-GDP ratio is not too large and satisfies $\frac{D_t}{Y_t}<\frac{(1-z)(1+g)}{R-g}$, where R and g satisfy (B6) and (B7).[24]

Given the above preparation, we derive conditions for fiscal sustainability.

Proposition 1

A fiscal policy is sustainable if the debt-to-GDP ratio converges to a value less than $\frac{(1-z)(1+g)}{R-g}$.

Proof: If $0<\lim_{t\to\infty}\frac{D_t}{Y_t}<\infty$, from Results 1 and 2, the BGP, with or without the credit market, can reproduce the equilibrium, and thus the NPG conditions of agents are satisfied. In addition, likewise from Results 1 and 2, the equilibrium satisfies the feasibility of the government budget if $\frac{D_t}{Y_t}<\frac{(1-z)(1+g)}{R-g}$. If $\lim_{t\to\infty}\frac{D_t}{Y_t}=0$, the result is trivial. Q.E.D.

Proposition 2

A fiscal policy is not sustainable if the debt-to-GDP ratio diverges to infinity.

[24] Parameter restrictions to guarantee the bounded utility are $\beta^{\frac{1}{\alpha}}(1+x)^{\frac{(1-\alpha)}{\alpha}}(1+\kappa)^{\frac{(\alpha-1)}{2\alpha}}<1$ when the private credit market is active and $\beta^{\frac{1}{\alpha}}(1+x)^{\frac{(1-\alpha)}{\alpha}}<1$ when it is not active.

Proof: If $\lim_{t\to\infty}\frac{D_t}{Y_t}=\infty$, the credit market disappears at some date τ. At any date $t(\geq\tau)$, because the credit market is inactive, as has been studied above, agents discount the future at rate R, and the NPG conditionrequires

(B11) $$\frac{D_\tau\prod_{s=1}^{\infty}(1+g_{\tau+s}^D)}{\prod_{s=1}^{\infty}(1+R_{\tau+s})}=0\,.$$

On the other hand, the government's budget constraint (12) is now rewritten as

(B12) $$\frac{T_t}{Y_t}=z+\frac{R_t-g_t^D}{1+g_t^D}\frac{D_t}{Y_t}\,.$$

The feasibility implies $T_t/Y_t<1$ as $t\to\infty$, which should require $R_t<g_t^D$ as $t\to\infty$ when $\lim_{t\to\infty}\frac{D_t}{Y_t}=\infty$, a contradiction to (B11). Therefore, if $\lim_{t\to\infty}\frac{D_t}{Y_t}=\infty$, it is impossible that the NPG condition and the feasibility hold together. Q.E.D.

Remark

The NPG condition is not sufficient for fiscal sustainability because this condition does not exclude the case for $g_t<g_t^D<R_t$. When $g_t^D<R_t$, the NPG condition is satisfied but when $g_t<g_t^D$, debt-to-GDP ratio grows unboundedly.

Appendix C: Simulation Procedure

We simulate the model by dividing the forecasting periods into two: Obama's forecast period (2006–2014) and the postforecast period (2014–2100).

A. Obama's forecast period (2006–2014)

Step 1. We construct R_{t+1} by assuming perfect foresight. Specifically, we substitute the forecast values of g_{t+1} into the deterministic version of (14),

$$\log(1+R_{t+1})=\alpha\log(1+g_{t+1})-\frac{1}{2}\log(1+\kappa)-\log\beta\,.$$

Step 2. We construct d_{t+1} by substituting R_{t+1} and the forecast values of g_{t+1} and s_{t+1} into (22) with a starting value of d_0 as of year 2006.

B. Postforecast period (2014–2100)

Step 1. We generate a series of x_{t+1} for 987 periods starting from the initial value drawn from the stationary distribution of x_t.

Step 2. Given a series of x_{t+1}, we obtain g_{t+1} and R_{t+1} from (A2) and (A5).

Step 3. Given a series of g_{t+1}, we construct s_{t+1} recursively from the fiscal policy rule (26) with a starting value of s_0 as of year 2014.

Step 4. We construct d_{t+1} by substituting R_{t+1}, g_{t+1}, and s_{t+1} into (22) with a starting value of d_0 as of year 2014.

Step 5. We repeat Steps 1–4 N times to obtain the distribution of d_{t+1}. Indexing each series by i, the expected value of d_t and the probabilities that d_t exceeds its critical values $\bar{d}$ are computed as $E[d_t] = \frac{1}{N}\sum_{i=1}^{N} d_{i,t}$ and

$\Pr ob\left[d_t \geq \bar{d}\right] = \frac{1}{N}\sum_{i=1}^{N} I\left(d_{i,t} \geq \bar{d}\right)$, where $\bar{d} = 0.953$, $N = 10000$, and

$$I\left(d_{i,t} \geq \bar{d}\right) = \begin{cases} 1 & if \quad d_{i,t} \geq \bar{d} \\ 0 & otherwise \end{cases}.$$

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Table 1: Obama's Plan (2009)

	Real GDP growth rate	Primary balance/GDP	Gross federal debt/GDP	Long-term interest rate (real)[*14]	Short term interest rate (real)[*25]	Changes in GDP deflator (chain type)
2006	2.6%	–0.2%	0.656	2.1%	2.0%	2.7%
2007	2.3%	0.6%	0.656	2.0%	1.8%	2.6%
2008	–0.2%	–1.4%	0.702	1.3%	–1.1%	2.5%
2009	0.6%	–11.9%	0.904	2.5%	–1.0%	1.7%
2010	5.0%	–7.6%	0.981	3.1%	0.5%	1.5%
2011	5.0%	–4.4%	1.011	3.4%	2.0%	1.5%
2012	3.4%	–1.3%	1.006	3.5%	2.3%	1.6%
2013	2.7%	–0.6%	0.997	3.4%	2.2%	1.7%
2014	2.7%	–0.4%	0.998	3.3%	2.1%	1.8%

*1 Ten-year constant maturities. Adjusted by changes in the GDP deflator.

*2 Yield on actively traded three-month treasury bills adjusted to constant maturities. Adjusted by changes in the GDP deflator.

Table 2: Parameters (Baseline Calibration)

Preference		
β	1/1.03	Discount factor
α	0.767	Inverse of elasticity of intertemporal substitution
Technology		
x^a	0.0709	Average return to capital
ρ	0.535	Serial correlation of return to capital
σ	0.0312	Standard deviation of error term in return to capital
Financial Intermediation		
κ	0.038	Financial intermediation cost
Fiscal Policy Rule		
	–0.0135	Constant
	0.847	Coefficient on previous-year primary surplus/GDP
	0.5	Coefficient on GDP growth rate

Table 3: Baseline Simulation Results: Expected Debt-to-GDP Ratios

After	25 years	50 years	100 years	500 years	1000 years
Benchmark	1.05	1.22	1.62	12.88	165.45
	(54.2%)	(53.7%)	(54.7%)	(56.0%)	(56.1%)
No intermediation cost	1.45	2.30	5.74	7948.30	67351402.39
	(70.5%)	(70.0%)	(72.8%)	(76.0%)	(76.0%)
Low GDP growth rate	1.08	1.32	1.92	27.90	785.23
	(55.2%)	(55.7%)	(57.2%)	(59.0%)	(58.9%)
Log utility	1.20	1.58	2.72	212.67	49730.10
	(63.0%)	(63.6%)	(66.4%)	(71.9%)	(72.1%)

Numbers in parentheses are the probabilities that the debt-to-GDP ratio will exceed its initial value of 0.953.

*1 Benchmark (average GDP growth rate: 2.7%, financial intermediation cost: 3.8%).

*2 No intermediation cost (average GDP growth rate: 5.23%, financial intermediation cost: 0%).

*3 Low growth (average GDP growth rate: 2%, financial intermediation cost: 3.8%).

*4 Log-utility ($\alpha=1$) (average GDP growth rate: 2.7%, financial intermediation cost: 3.8%).

Table 4: Simulation Results under Bohn's Fiscal Policy Rule

A. Expected Debt-to-GDP Ratios

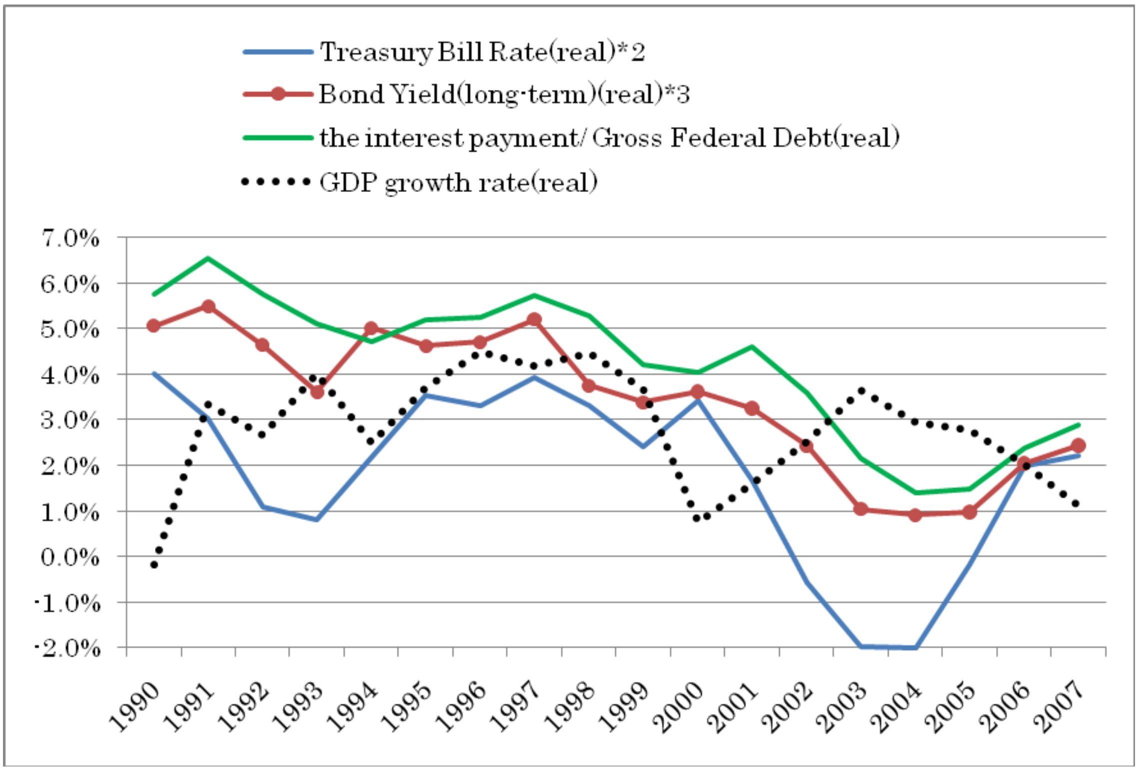
After	25 years	50 years	100 years	500 years	1000 years
$\gamma_3 = 0.001$	1.06	1.20	1.44	2.86	3.77
	(54.7%)	(54.4%)	(54.4%)	(58.5%)	(61.9%)
$\gamma_3 = 0.002$	1.05	1.18	1.32	1.62	1.60
	(53.9%)	(53.7%)	(53.7%)	(57.3%)	(56.8%)
$\gamma_3 = 0.005$	1.04	1.10	1.17	1.16	1.15
	(52.8%)	(52.7%)	(54.9%)	(54.1%)	(53.9%)

Numbers in parentheses are the probabilities that the debt-to-GDP ratio will exceed its initial value of 0.953.

B. Expected Primary Surplus-to-GDP Ratios (%)

After	25 years	50 years	100 years	500 years	1000 years
$\gamma_3 = 0.001$	−0.10%	0.29%	0.44%	1.31%	1.94%
$\gamma_3 = 0.002$	0.11%	0.24%	0.62%	0.84%	1.00%
$\gamma_3 = 0.005$	0.08%	0.55%	0.44%	0.71%	0.69%

Figure 1: US Data (1990–2007)[*1]



*1 Treasury Bill Rate (real) = Treasury Bill Rate (nominal) – changes in the GDP deflator

Bond Yield (long-term) (real)

= Bond Yield (long-Term) (nominal) – changes in the GDP deflator

Interest payment/ Gross Federal Debt (real)

= Interest payment/ Gross Federal Debt – changes in the GDP deflator.

*2 Ten-year constant maturities.

*3 Yield on actively traded three-month treasury bills adjusted to constant maturities.

Figure 2A (Case for $\alpha < 0.5$)

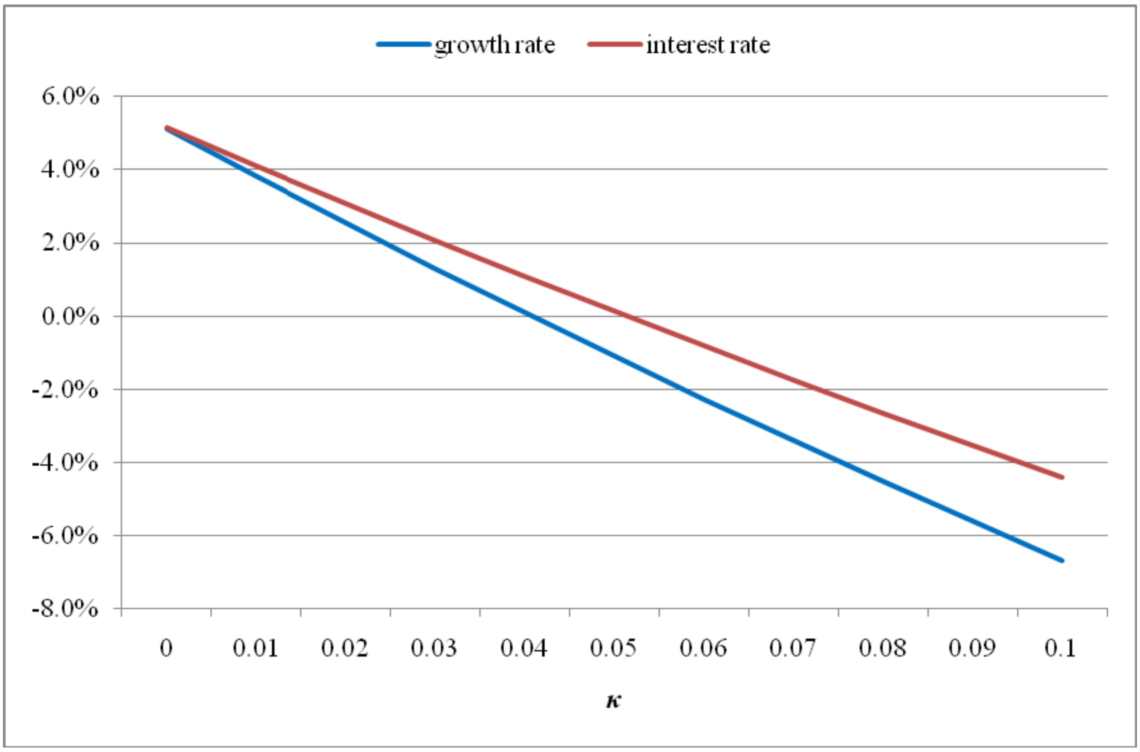


Figure 2B (Case for $\alpha > 0.5$)

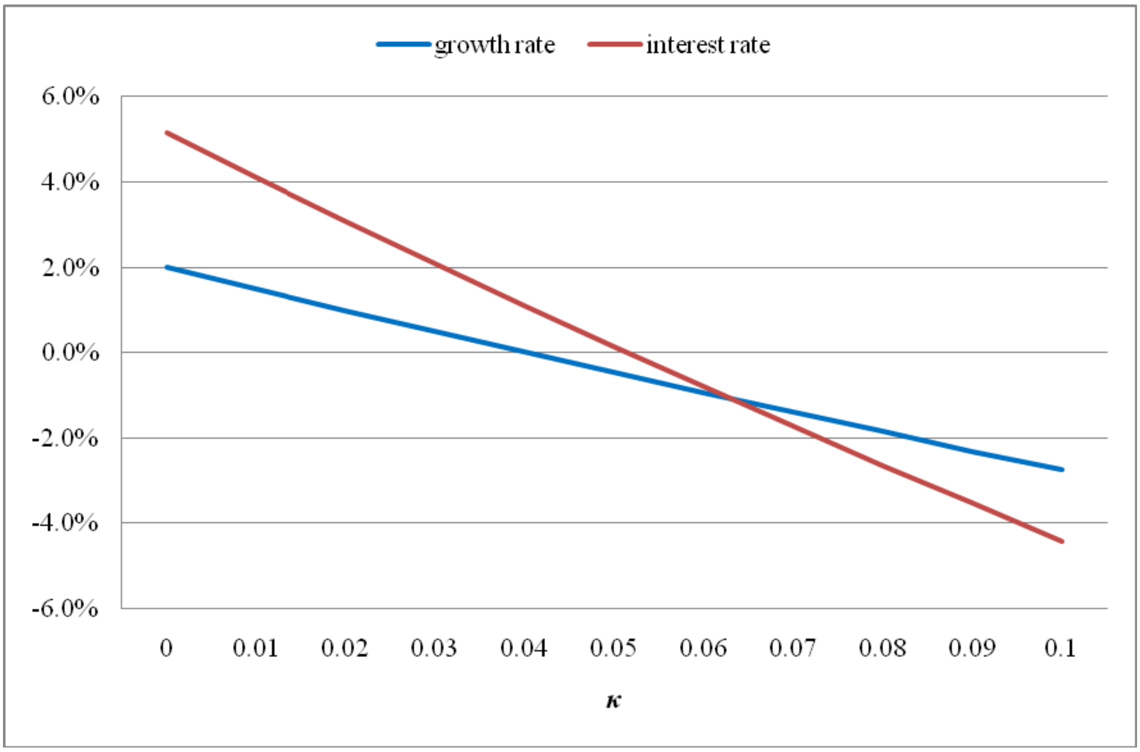
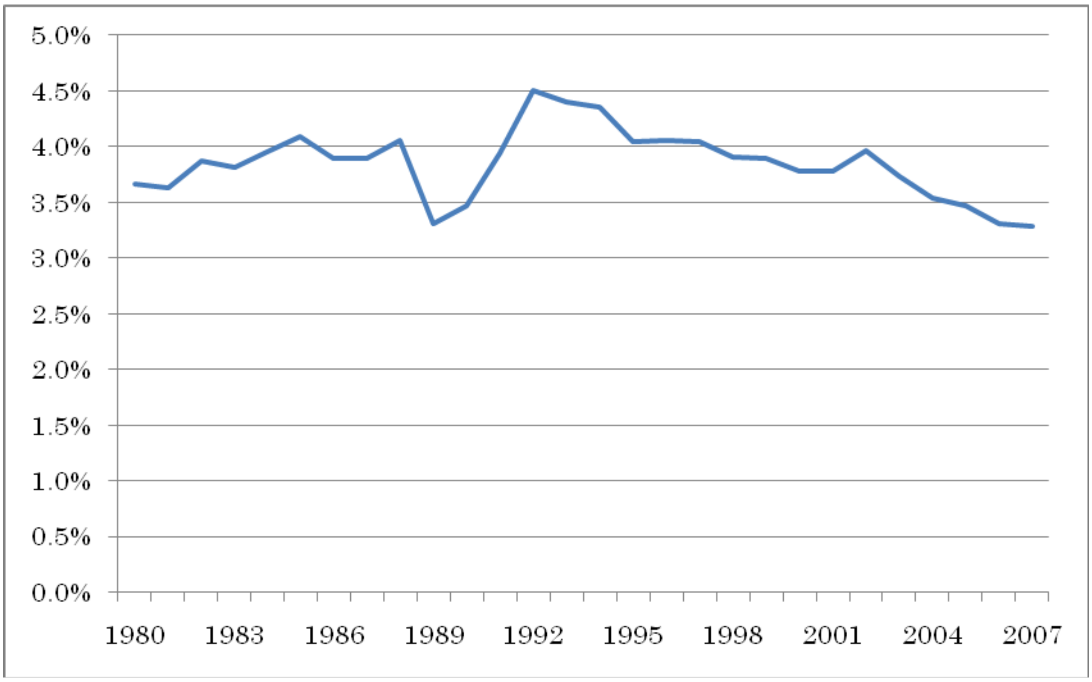


Figure 3: Net Interest Margin[*1]



*1 In FDIC Quarterly Banking Profile, net interest margin is defined as follows.

$$\text{net interest margin} = \frac{\text{interest income - interest expense}}{\text{average earming assets}}$$

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