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MALTHUS ON SAY'S LAW

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Abstract: Say's law is one of the basic principles in the system of the English Classical School of economics in the 19th century. On the other hand, Malthus (as well as Sismondi in Switzerland) rejected the law, and has been regarded as a precursor of the Keynesian Revolution. However there still remain a few ambiguous points which require further clarification. For instance, we may ask: (1) what does Say's law mean? (There are several statements which are called by the same name.) (2) what kind of statement did Malthus intend to reject? This note is a trial to answer these questions.

Key words: Say's law, general glut, effective demand.

JEL Classification Number: B12, B31.

1. INTRODUCTION

There are a few notorious topics in the history of economics, which have aroused vehement lasting controversies without any satisfactory solutions. Say's law of market (or *loi des débouchés*) is the most tedious one among them.

According to Say's law, any general glut, that is overall excess supply (overproduction) in all markets can not occur although a partial excess supply in some particular market is possible. But there are several other statements called by the same name. For instance, "supply creates its own demand", "saving is always equal to investment", "labor market is always in full employment equilibrium", and so on. Many historians of economics often lump them all together indiscriminately.

What does Say's law mean? This is the first question to be examined and answered in this note.

The leading figures, including J.Mill, D.Ricardo and J.S.Mill, of the Classical School of economics basically accepted Say's law as a cornerstone of their system. On the other hand, T.R.Malthus and J.C.L.S. Sismondi, who are called contemporary "heretical economists", explicitly opposed to and rejected Say's law. Following the famous essay by Keynes¹, many economists have been regarding Malthus as a precursor of the

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¹ Keynes [3], *The collected Works* edition, Vol.X, Chap.12.

Keynesian Revolution. But the subtle difference between Malthus' view and Keynes' on this problem does not seem to have been scrutinized very well.

Thus we are led to our next indispensable task to make clear what is Say's law in the sense of Malthus. What specific statement did Malthus intend to refute ?

In sections 3–5, we try to clarify the meaning (or meanings) of Say's law after providing a formal presentation of the law. Some logical links connecting related statements are also cleared up as immediate corollaries.

Based upon such a formal discussion, we proceed, in section 6, to examining what Malthus really intended to claim when he opposed to Say's law. This is the principal concern of this note.

Finally, in section 7, we recapitulate T.Negishi's work² on Malthus' theory of optimal saving in order to supplement a little bit of mathematical rigor.

For the sake of readers' convenience, a brief history of Say's law in the classical era (from Say to Malthus) is described in section 2.

2. A SHORT HISTORY — FROM SAY TO MALTHUS

As I explained in the introduction, Say's law asserts that any general glut (overall excess supply) is not possible because the supply (production) of a commodity is nothing other than the demand for other commodities to be purchased by means of the produced commodity in question. In other word, if there exists any excess supply in a sector, there must exist excess demand in other sectors. That is all Say himself wrote concerning this problem in his *Traité d'économie politique* published in 1803. He spent only three and a half pages full of mysterious expressions to expose the famous assertion although it was enhanced in the second edition (1814). We now quote a few passages from *Traité*.

Chaque producteur produit d'une certaine marchandise une quantité fort supérieure à sa consommation.³

Ce que je viens de dire d'un seul homme industriel peut se dire de cent mille. Leur nation leur offrira d'autant plus de débouchés, qu'elle peut payer plus de choses, et elle peut payer plus de choses à proportion de ce qu'elle en produit davantage. L'argent ne remplit qu'un office passager dans ce double échange. Les échanges terminés, il se trouve qu'on a payé des produits avec des produits.⁴

It is J.Mill who gave a more definite expression to the law removing ambiguous fog rolling over Say's world. In advance of Mill's work, W. Spence, a physiocrat, published a tract, *Britain Independent of Commerce* in 1807, in which he opposed to

² Negishi [11] pp.145–152.

³ Say [15] (1803) tome I, p.152.

⁴ *ibid.*, p.154.

A. Smith's doctrine on economic effects of extravagance and parsimony. He pointed out the possibility that the demand might be too weak to absorb increased products of new capital, and arrived at the negative standpoint against Say's law. Mill published *Commerce Defended* in 1808 as a criticism against Spence.

But wherein consist the collective means of payment of the whole nation ? Do they not consist in its annual produce, in the annual revenue of the general mass of its inhabitants ? But if a nation's power of purchasing is exactly measured by its annual produce, as it undoubtedly is; the more you increase the annual produce, the more by that very act you extend the national market, the power of purchasing and the actual purchases of the nation.⁵

Thus it appears that the demand of a nation is always equal to the produce of a nation. This indeed must be so; for what is the demand of a nation ? The demand of a nation is exactly its power of purchasing. But what is its power of purchasing ? The extent undoubtedly of its annual produce. The extent of its demand therefore and the extent of its supply are always exactly commensurate.⁶

Ricardo also supported Say-Mill's law in his *On the Principles of Political Economy, and Taxation* published in 1817.

Productions are always bought by productions, or by services; money is only the medium by which the exchange is effected. Too much of a particular commodity may be produced, of which there may be such a glut in the market, as not to repay the capital expended on it; but this cannot be the case with respect to all commodities.....⁷

As is well-known, Ricardo's viewpoint changed remarkably in the third edition (1821) of *Principles*.⁸

On the other hand, Malthus' attitude was completely opposite to Say-Mill-Ricardo. It can be verified in the correspondence between Ricardo and Mill, even before the publication of Ricardo's *Principles*, Malthus wrote to Ricardo on September 11, 1814:

In short I by no means think that the power to purchase necessarily involves a proportionate will to purchase; and I cannot agree with Mr. Mill in an ingenious position which he lays down in his answer to Mr. Spence, that in reference to a nation, supply can never exceed demand. A nation must certainly have the power of purchasing all that it produces, but I can easily

⁵ Mill [9] *Selected Economic Writings* edition, p.135.

⁶ *ibid.*, p.136.

⁷ Ricardo [13] pp.341-342.

⁸ *ibid.*, Chap.XXX1 (on machinery).

conceive it not to have the will and if we were to grow next year half as much corn again as usual, a great part of it would be wasted, and the same would be true if all commodities of all kinds were increased one half. It would be impossible that they should yield the expense of production. You have never I think taken sufficiently into consideration the wants and tastes of mankind. It is not merely the proportion of commodities to each other but their proportion to the wants and tastes of mankind that determines prices.⁹

Malthus' view never changed in his *Principles of Political Economy* published in 1820.

It has been thought by some very able writers, that although there may easily be a glut of particular commodities, there cannot possibly be a glut of commodities in general; because, according to their view of the subject, commodities being always exchanged for commodities, one half will furnish a market for the other half, and production being thus the sole source of demand, an excess in the supply of one article merely proves a deficiency in the supply of some other, and a general excess is impossible. M.Say, in his distinguished work on political economy, has indeed gone so far as to state that the consumption of a commodity by taking it out of the market diminishes demand, and the production of a commodity proportionably increases it.

This doctrine, however, to the extent in which it has been applied, appears to me to be utterly unfounded, and completely to contradict the great principles which regulate supply and demand.¹⁰

3. SOCIAL ACCOUNTING

No economist in the Classical School gave any definite formal expression to Say's law. Probably they, even Ricardo, did not like any rigid formalism as the vehicles of their reasoning. Nevertheless, I dare try to construct, in the following two sections, a specific framework as an auxiliary tool of talking about Say's law with less ambiguity.¹¹

Suppose that there are l kinds of ordinary goods and services other than money. We denote by $z(p) = (z_1(p), z_2(p), \dots, z_l(p))$ the excess demand function of goods and services, where $p = (p_1, p_2, \dots, p_l)$ is a vector of their prices in terms of money. The demand function of money is denoted by $m(p)$ and its quantity is fixed at \bar{M} . Then $\zeta(p) = (z(p), m(p) - \bar{M})$ is the excess demand function of goods-services together with money.

Making use of these notations, we can express Walras' law in the form

⁹ Ricardo [14] VI, pp.130–132.

¹⁰ Malthus [7] p.353.

¹¹ The formulation of Say's law shown below is due to Grandmont [2] pp.10–11 and Patinkin [12] pp.193–195, 355–359, 645–650.

$$(W) \quad \sum_{j=1}^l p_j z_j(p) + (m(p) - \bar{M}) = 0 \quad \text{for all } p.$$

Walras' law means that the sum of the values of excess demands (of all goods-services and money) evaluated by p is equal to zero for all p . On the other hand, Say's law can be formulated as

$$(S) \quad \sum_{j=1}^l p_j z_j(p) = 0 \quad \text{for all } p.$$

It says that the sum of the values of excess demands of all goods-services (excluding money) evaluated by p is zero for all p .

If Walras' law (W) is presupposed, Say's law (S) is equivalent to

$$(S') \quad m(p) = \bar{M} \quad \text{for all } p;$$

that is the money market is always in equilibrium regardless of p .

We now classify goods-services (other than money) into the following four aggregate categories : primary factors (denoted by F), consumption goods (denoted by C), capital goods (denoted by I), and unproductive labor (denoted by L).

The economy under consideration is dichotomized into the production sector (P) and the household sector (H). The demand (purchase) and the supply (revenue) of each sector can be summarized as in Table 1. Each entry in the table signifies an *a priori* value in terms of money — in other words, scheduled value based on the decision making of each sector in advance of the actual trading. The symbol D_F stands for the value of demand for primary factors, S_C the value of supply of consumption goods, and so on. The demand for capital goods is divided into two parts, D_{I_1} and D_{I_2} ; the former is the replacement of the depreciation of capital, and the latter the net investment. The total supply S_I of capital goods is absorbed in either channel of D_{I_1} or D_{I_2} .

The meaning of the entry $\Pi - D_{I_2}$ will be explained later.

Table 1.

Household sector		Production sector	
D_C	S_F	D_F	S_C
D_L	S_L	D_{I_1}	S_I
		D_{I_2}	
		$\Pi - D_{I_2}$	

Table 2.

	demand	supply
primary factors (F)	P	H
consumption goods (C)	H	P
capital goods (I)	P	P
unproductive labor (L)	H	H

Each of four categories is demanded /supplied by either one of two sectors. Table 2 shows that, for instance, the primary factors (F) are demanded by the production sector (P) and supplied by the household sector (H), and so on. Goods-services of each category are supposed to be demanded and supplied by either one of (P) and (H).

4. SOME ARITHMETIC

From now on, we treat with F , C , I and L as if each of them were a single commodity.¹²

Based upon the notation in section 2, we can restate Walras' law (W) and Say's law (S) in alternative forms. The notation \equiv means that the both sides are identically equal.

$$(W) \quad (D_F - S_F) + (D_C - S_C) + (D_{I_1} + D_{I_2} - S_I) \\ + (D_L - S_L) + (D_M - \bar{M}) \equiv 0,$$

where D_M is the demand for money.

$$(S) \quad (D_F - S_F) + (D_C - S_C) + (D_{I_1} + D_{I_2} - S_I) \\ + (D_L - S_L) \equiv 0.$$

$S_C + S_I$ is the revenue of the production sector (P). D_F and D_{I_1} are the prime cost and the user cost, respectively, in Keynes' vocabulary.¹³ Consequently,

$$(4.1) \quad \Pi = (S_C + S_I) - (D_F + D_{I_1})$$

is the scheduled total profit of the sector (P). In view of (4.1), we may again rewrite Walras' law as

$$(4.2) \quad (\Pi - D_{I_2}) - (D_C - S_F) \equiv (D_M - \bar{M}) + (D_L - S_L).$$

The first term $(\Pi - D_{I_2})$ on the left-hand side of (4.2) is easily seen to be the net money flow of the sector (P) since

$$\Pi - D_{I_2} = (S_C + S_I) - (D_F + D_{I_1} + D_{I_2})$$

by (4.1), This is determined solely by the decision making of the sector (P). The second term $(D_C - S_F)$ is the net money inflow into the sector (P) from outside (the household

¹² The framework of this section is adapted from Lange [5] and Morishima [10] Chap.7.

¹³ cf. Keynes [4] pp.52-61.

sector (H)), which depends exclusively upon the decision making of the sector (H).

Thus, the equality $\Pi - D_{I_2} = D_C - S_F$ means that the scheduled profit Π and the net investment D_{I_2} are actually feasible for the sector (P) by means of the net money inflow. On the other hand, if the inequality $\Pi - D_{I_2} > D_C - S_F$ holds, for instance, either the scheduled profit Π or the scheduled net investment D_{I_2} is not feasible. (Either the profit must be decreased or the net investment must be increased.) We may follow a similar reasoning in the case of the converse inequality: $\Pi - D_{I_2} < D_C - S_F$.

According to (4.2), we obtain the equivalence relation as follows:

$$(4.3) \quad (\Pi - D_{I_2}) \begin{matrix} \geq \\ \leq \end{matrix} (D_C - S_F) \iff (D_M - \overline{M}) + (D_L - S_L) \begin{matrix} \geq \\ \leq \end{matrix} 0.$$

($A \Rightarrow B$ means that the statement A implies the statement B , $A \Leftrightarrow B$ means that A and B are equivalent.)

We now look at Walras' law (W) from another angle. D_{I_1} being the user cost in the sense of Keynes, $S_C + S_I - D_{I_1}$ corresponds to the (net) income, which we denote by Y . Then Walras' law (W) can be rewritten as

$$(4.4) \quad \begin{aligned} Y - D_C - D_{I_2} \\ \equiv (D_F - S_F) + (D_L - S_L) + (D_M - \overline{M}). \end{aligned}$$

Replacing Y by $Y' = S_C + S_I$ (gross income), we obtain another expression of (4.4):

$$(4.5) \quad \begin{aligned} Y' - D_C - (D_{I_1} + D_{I_2}) \\ \equiv (D_F - S_F) + (D_L - S_L) + (D_M - \overline{M}). \end{aligned}$$

$D_{I_1} + D_{I_2}$ is nothing other than the gross investment. Since $Y - D_C$ in (4.4) is the net saving and $Y' - D_C$ in (4.5) is, so to speak, the gross saving. (4.4) and (4.5) tell us that

$$(4.6) \quad \begin{aligned} &\text{net saving} - \text{net investment} \\ &= \text{gross saving} - \text{gross investment} \\ &= (D_F - S_F) + (D_L - S_L) + (D_M - \overline{M}). \end{aligned}$$

5. VARIANTS OF SAY'S LAW

We have already confirmed that

$$(5.1) \quad (S) \iff D_M - \overline{M} \equiv 0$$

under the presupposition of Walras' law (W).

We now suppose Say's law (S) in addition to Walras' law (W), and compare the following three statements.

$$(L) \quad D_L - S_L \equiv 0.$$

The unproductive labor market is always in equilibrium. (Or there is no unproductive labor market.)

$$(I) \quad \Pi - D_{I_2} \equiv D_C - S_F.$$

The scheduled profit and net investment are always realizable.

$$(II) \quad Y - D_C - D_{I_2} = Y' - D_C - (D_{I_1} + D_{I_2}) \equiv D_F - S_F.$$

The difference between saving and investment (net or gross) is always equal to the value of excess demand for the primary factor (which may be identified with productive labor).

Under the assumptions (W) and (S), the above three statements are mutually equivalent.

It is not necessary to give any "proof" of this fact in detail. The equivalence (L) \Leftrightarrow (I) immediately follows from (4.2), and (L) \Leftrightarrow (II) from (4.6).

If we presuppose (W) and (L) instead, then (S), (I) and (II) are equivalent.

Finally assume also (L) in addition to (W) and (S). Under these strong assumptions. We can easily show the equivalence of the following two statements.

$$(III) \quad Y - D_C - D_{I_2} = Y' - D_C - (D_{I_1} + D_{I_2}) \equiv 0.$$

The saving and the investments (net or gross) are always equal.

$$(IV) \quad D_F - S_F \equiv 0.$$

The market for primary factors (productive labor) is always in full employment.

We started by (S) as a basic statement of Say's law. However other impressive statements (I)–(IV) are also referred to by the same name of Say's law. Some logical relations examined above are summarized in Table 3.

Table 3.

Assumptions	Results
(W)	(S) $\Leftrightarrow D_M \equiv \bar{M}$
(W),(S)	(L) \Leftrightarrow (I) \Leftrightarrow (II)
(W),(L)	(S) \Leftrightarrow (I) \Leftrightarrow (II)
(W),(S),(L)	(III) \Leftrightarrow (IV)

6. MALTHUS' VIEW

Chapter VII of Malthus' *Principles* is devoted to the study of a "practical question" — "what are the most immediate and effective stimulants to the continued creation and progress of wealth".¹⁴ Malthus counted a couple of principal factors which stimulate the "progress of wealth". One factor is the increase of population and the accumulation of capital is the other.

Any increase of population (that of employment) generates a larger demand for commodities, which invokes an incentive to increase production. But the sufficient growth of subsistence (circulating capital) may not accompany the increase of population. In this case, the increased production can not find an ample demand which absorbs the supply. Thus the increase of population (employment) can not by itself, sustain any continuing growth of wealth.

It will be said perhaps that the increase of population will lower wages, and, by thus diminishing the costs of production, will increase the profits of the capitalists and the encouragement to produce. Some temporary effect of this kind may no doubt take place, but it is evidently very strictly limited. The fall of wages cannot go on beyond certain point without not only stopping the progress of the population but making it even retrograde; and before this point is reached, it will probably happen that the increase of produce occasioned by the labour of the additional number of persons will have so lowered its value, as more than to counterbalance the fall of wages, and thus to diminish instead of increase the profits of the capitalists and the power and will to employ more labour.¹⁵

On the other hand, assume that capitalists save more (by restricting consumption of luxuries) and a part of unproductive labor is converted to productive one. For the sake of simplicity, total population is assumed to be constant. While production = supply is enhanced very much, the total labor is constant, by assumption, and capitalists' propensity to consume declines. The demand is gradually weakened relative to increased production. Thus accumulation of capital without sufficient growth of demand can not be maintained.

In the case supposed there would evidently be an unusual quantity of commodities of all kinds in the market, owing to the unproductive labourers of the country having been converted, by the accumulation of capital, into productive labourers, while the number of labourers altogether being the same, and the power and will to purchase for consumption among landlords and capitalists being by supposition diminished, commodities would necessarily fall in value, compared with labour, so as to lower profits almost to nothing,

¹⁴ Malthus [7] p.347.

¹⁵ *ibid.*, p.349.

and to check for a time further production. But this is precisely what is meant by the term glut, which, in this case, is evidently general not partial.¹⁶

Under these circumstances, I would ask, how it is possible to suppose that the increased quantity of commodities, obtained by the increased number of productive labourers, should find purchasers, without such a fall of price as would probably sink their value below the costs of production, or, at least, very greatly diminish both the power and the will to save.¹⁷

..... because it is precisely the high profits of stock occasioned by the demand for commodities, and the consequent demand for the means of producing them, which at once give the power and the will to accumulate.¹⁸

Thus neither too much population (consumption) relative to productivity nor too much accumulation (productivity) relative to demand does not lead to “the continued creation and progress of wealth”. The solution should be found between the extremes. Malthus’ idea seems to have come from his knowledge of calculus he learned during his days in Cambridge as a math-student. (cf. section 7.)

If consumption exceed production, the capital of the country must be diminished, and its wealth must be gradually destroyed from its want of power to produce; if production be in a great excess above consumption, the motive to accumulate and produce must cease from the want of will to consume. The two extremes are obvious; and it follows that there must be some intermediate point, though the resources of political economy may not be able to ascertain it, where, taking into consideration both the power to produce and the will to consume, the encouragement to the increase of wealth is the greatest.¹⁹

In the last section, we examined some implications of Say’s law and logical links among related statements. Now what can we say about Malthus’ attitude toward these statements ?

The passages quoted above show the definite negative attitude of Malthus against the basic statement (S), which says that the sum of the values of excess demands of all the commodities (other than money) is always equal to zero.

He did not mention the statement $D_M - \overline{M} \equiv 0$. Since (S) and $D_M - \overline{M} \equiv 0$ are equivalent under the assumption (W), he should have rejected $D_M - \overline{M} \equiv 0$. This overlook seems to be due to the lack of sufficient formal analysis concerning Say’s

¹⁶ *ibid.*, p.354.

¹⁷ *ibid.*, p.353. Similar arguments appear in his letter to Ricardo dated July 7, 1821. Ricardo [14] IX, pp.9–11.

¹⁸ *ibid.*, pp.374–375.

¹⁹ *ibid.*, p.9.

law. However it deserves a special notice that Malthus' reasonings were performed independently of the role of money.²⁰

The statement (I) was not referred to, probably by the same reason.

It may be surprising, especially for the historians wearing Keynesian glasses, that Malthus did admit the statement (III), which asserts saving and investment are always equal. ((II) was not mentioned.)

..... such increase of power and will to save must almost always be accompanied by a proportionate increase of capital.²¹

However Malthus never agreed with the statement (IV), which asserts the always full employment in labor market. We exemplify it by quoting a few sentences from his letter to Ricardo, dated July 16, 1821.

..... particularly as this stagnation must inevitably throw the rising generation out of employment. We know from repeated experience that the money price of labour never falls till many workmen have been for some time out of work.²²

Malthus admitted (III), but, on the other hand, he negated (IV). This is not a contradiction because he did not approve (S).

Finally, there remains the statement (I) that the scheduled profit and investment are always realizable. It is this statement (I), which Malthus tried to refute vehemently as is illuminated by the above quotations.

Some historians interpreted Malthus' theory as discussing the adjustment process of production caused by a gap between saving and investment or the underemployment equilibrium. These interpretations are not correct. The central problem Malthus focused upon is the defect of the intensity of demand relative to the effective demand, which makes scheduled profit and investment unsustainable.²³

7. A THEOREM ON OPTIMAL SAVING

As we have seen in section 6, Malthus' critical examination of Say's law is motivated by his aim to identify "the most immediate and effective stimulants to the continued creation and progress of wealth". He arrives at the answer that what causes "the power and will to accumulate" is the high level of profit realized by ample demand. If the saving is scarce, the capital formation is forced to be poor. Conversely, too much saving may

²⁰ Stigler also pointed out that "his theory is entirely non-monetary in nature". cf. Stigler [16] pp.595–596, [17] p.319.

²¹ Malthus [7] p.373.

²² Ricardo [14] IX, p.20.

²³ As is well-known, Say's law has a close connection with the quantity theory of money. However this problem is not studied here. See Grandmont [2] as well as Wicksell [18].

lead to weakened effective demand in comparison with the fortified production power. One of Malthus' principal objects in his *Principles* is to find "some intermediate point" between the extremes which assures "the encouragement to the increase of wealth" to be the greatest.

Negishi [11] rigorously formulated this problem and gave a characterization of solutions. This final section is just a mathematical complement to Negishi's work. At the same time, I try to illuminate the relevance of the theory to understand Malthus' peculiar view on Say's law.²⁴

Following Negishi, we suppose that a product (say, corn) is produced by productive labor L through a smooth production function $f(L)$, which satisfies $f'(L) > 0$ and $f''(L) < 0$. A part of $f(L)$ is absorbed in unproductive consumption and the remaining part is used for advanced circulating capital to employ productive labor, the former is denoted by C and the latter by $f(L) - C$, respectively. w stands for wage rate and r for profit rate.

Then we must have

$$(7.1) \quad wL = f(L) - C.$$

The right-hand side of (7.1) is interpreted as the saving. r is defined by

$$r = \frac{f(L) - wL}{wL}, \text{ i.e., } w(1+r)L = f(L),$$

which gives

$$(7.2) \quad w(1+r) = f'(L).$$

Again following Negishi, we interpret wL as "the encouragement to the increase of wealth". Hence Malthus' problem is summarized as;

$$(7.3) \quad \text{Maximize } W \equiv wL = f(L) - C.$$

If we define a function $\Phi(r, w, C)$ by

$$(7.4) \quad \Phi(r, w, C) = \begin{pmatrix} wL - f(L) + C \\ w(1+r) - f'(L) \end{pmatrix},$$

then the equations (7.1) and (7.2) can be rewritten as

$$(7.5) \quad \Phi(r, w, C) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Denoting by $D_{(r,w)}\Phi$ (resp. $D_C \Phi$) the derivative of Φ with respect to (r, w) , (resp. with respect to C), we obtain

$$(7.6) \quad D_{(r,w)}\Phi = \begin{pmatrix} 0 & L \\ w & 1+r \end{pmatrix}, \quad D_C \Phi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

²⁴ The relation between Malthus' theory of optimal saving and Say's law has often been overlooked. Such a misunderstanding seems to have occurred because the modern theory of optimal saving since F.Ramsay does not have any relation with Say's law.

1° The case of $L = \bar{L}$ (full employment).

In this case, $D_{(r,w)}\Phi$ is evaluated as

$$(7.7) \quad \det D_{(r,w)}\Phi |_{L=\bar{L}} = -w\bar{L} \neq 0.$$

Hence, by the implicit function theorem, the equation (7.5) can be solved locally with respect to (r, w) in the form

$$(7.8) \quad \begin{pmatrix} r \\ w \end{pmatrix} = \varphi(C).$$

Since the inverse matrix $(D_{(r,w)}\Phi)^{-1}$ of $D_{(r,w)}\Phi$ is

$$(D_{(r,w)}\Phi)^{-1} = -\frac{1}{w\bar{L}} \begin{pmatrix} 1+r & -\bar{L} \\ -w & 0 \end{pmatrix}$$

by (7.7), it follows that

$$(7.9) \quad \varphi'(C) = \begin{pmatrix} dr/dC \\ dw/dC \end{pmatrix} = -(D_{(r,w)}\Phi)^{-1} D_C \Phi = \begin{pmatrix} (1+r)/w\bar{L} \\ -1/\bar{L} \end{pmatrix}.$$

It implies that

$$(7.10) \quad \frac{dr}{dC} = \frac{1+r}{w\bar{L}} > 0,$$

$$(7.11) \quad \frac{dw}{dC} = -\frac{1}{\bar{L}} < 0.$$

The curve AB in Fig. 1 shows the relation between C and r . It is an increasing curve because of (7.10).

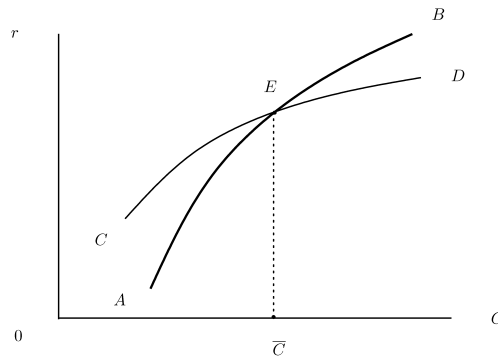


Fig 1.

2° $L \leq \bar{L}$ (underemployment).

We now go over to the general case which admits underemployment. We assume (Malthus would also agree) that the scheduled employment N is expressed as an increasing function of r ;

$$(7.12) \quad N = N(r) \quad , \quad N'(r) > 0.$$

Substituting (7.12) into (7.1) and (7.2), we obtain

$$(7.1') \quad wN(r) = f(N(r)) - C,$$

$$(7.2') \quad w(1+r) = f'(N(r)),$$

which can be expressed as

$$(7.13) \quad \Psi(r, w, C) = \begin{pmatrix} wN(r) - f(N(r)) + C \\ w(1+r) - f'(N(r)) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

corresponding to (7.5). Since

$$(7.14) \quad D_{(r,w)}\Psi = \begin{pmatrix} wN'(r) - f'(N(r))N'(r) & N(r) \\ w - f''(N(r))N'(r) & 1+r \end{pmatrix},$$

and $w < f'$ by (7.2), it can be verified that

$$(7.15) \quad \Delta \equiv \det D_{(r,w)}\Psi < 0.$$

Then the equation (7.13) is (locally) solvable with respect to (r, w) . The inverse matrix $(D_{(r,w)}\Psi)^{-1}$ is evaluated as

$$(7.16) \quad (D_{(r,w)}\Psi)^{-1} = \frac{1}{\Delta} \begin{pmatrix} 1+r & -N \\ -w + f''N' & wN' - f'N' \end{pmatrix}.$$

Consequently, we obtain

$$(7.17) \quad \begin{pmatrix} dr/dC \\ dw/dC \end{pmatrix} = -\frac{1}{\Delta} \begin{pmatrix} 1+r & -N \\ -w + f''N' & wN' - f'N' \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ = \frac{1}{A} \begin{pmatrix} 1+r \\ -w + f''N' \end{pmatrix},$$

where

$$(7.18) \quad -A = (1+r)N'(w - f') - N(w - f''N') = \Delta.$$

The increasing curve CD in Fig. 1 expresses the relation between C and r determined by (7.17). (It is not certain whether AB is steeper than CD or not.)

In the case $N(r) < \bar{L}$, the part AE of the curve AB is not realized. Since $N(r) > \bar{L}$ is impossible, the part ED must be excluded. Thus the viable relation between C and r is described by the bold curve CEB .

We now return back to Malthus' problem (7.3). As I have already shown, the relation between (r, w) and C which satisfy the equation (7.13) can be (locally) expressed as $(r, w) = (r(C), w(C))$. Hence (7.3) can be rewritten as

$$(7.3') \quad W = w(C)N(r(C)) = f(N(r(C))) - C.$$

Differentiating W with respect to C , we obtain, by (7.17), that

$$(7.19) \quad \frac{dW}{dC} = f'(N)N'(r)\frac{dr}{dC} - 1 = f'N'\frac{(1+r)}{A} - 1 = 0.$$

The equation (7.19) gives the solution $r = r^*$. We can find the optimal value C^* of C correspondingly.

Look at Fig. 2. The relation between C and W under full employment is shown by the curve $A'B'$. This is exactly the graph of $W = f(\bar{L}) - C$. In the general case, r is determined by C , and W is determined by r , in turn. Hence the relation between C and W is described by a curve $C'D'$. \bar{C} is the value of C corresponding to $L = \bar{L}$. Unemployment appears in the case $C^* < \bar{C}$.

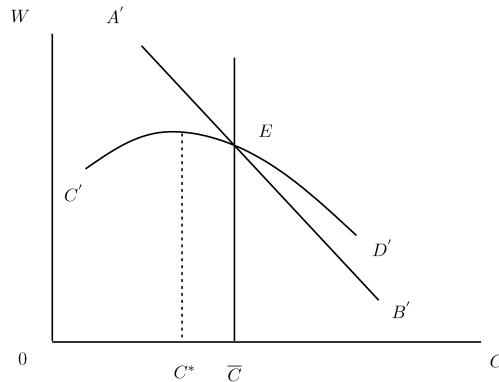


Fig 2.

Negishi's model gives a simple framework for a formal analysis of Malthus' intermediate value problem, which incorporates a part of Malthus' view on Say's law (in particular, on the role of the rate of profit). We should keep in mind carefully its implication that unemployment may not be resolved even if the optimal situation (in this context) is attained.

Negishi summarizes his evaluation of Malthus' view as follow :

Malthus is concerned, not with Keynesian effective demand, but with Smithian effectual demand, which assures the individual capitalists motives to produce.²⁵

My interpretation of Malthus, exposed in section 6, is compatible with Negishi's one. However the meaning of Malthusian effective demand is quite subtle and requires further investigation. I do not go into details here.²⁶

Postscript. Helpful comments by the anonymous referee is gratefully acknowledged. I added a few paragraphs at the end of this note in reply to his constructive criticism.

REFERENCES

- [1] Eltis, W.A., "Malthus's Theory of Effective Demand and Growth", *Oxford Economic Papers*, **32**(1980), 19–56.
- [2] Grandmont, J-M., *Money and Value*, (Cambridge Univ. Press, Cambridge) 1983.
- [3] Keynes, J.M., *Essays in Biography*, (Macmillan, London) 1933. Also in A.Robinson and D. Moggridge eds., *The Collected Writings of John Maynard Keynes*, Vol.X, (Macmillan, London/Basingstoke) 1972.
- [4] ———, *General Theory of Employment, Interest and Money*, (Macmillan, London) 1936.
- [5] Lange, O., "Say's Law: A Restatement and Criticism", in O. Lange, F. McIntyre and T.O. Yntema eds., *Studies in Mathematical Economics and Econometrics*, (Univ. Chicago Press, Chicago) 1942, 49–68.
- [6] Malthus, T.R., *An Inquiry into the Nature and Progress of Rent*, (John Murray, London) 1815. The second edition is included in Malthus [8].
- [7] ———, *Principles of Political Economy*, (John Murray, London) 1820.
- [8] ———, *The Pamphlets of Thomas Robert Malthus*, (Kelley, New York) 1970.
- [9] Mill, J., *Commerce Defended*, (C.and R. Baldwin, London) 1808. The second edition (1808) is included in *James Mill Selected Economic Writings*, D. Winch ed., (Oliver & Boyd, Edinburgh and London) 1966. [10] Morishima, M., *Ricardo's Economics*, (Cambridge Univ. Press, New York) 1989.
- [11] Negishi, T., *History of Economic Theory*, (Elsevier, Amsterdam) 1989.
- [12] Patinkin, D., *Money, Interest and Prices*, 2nd ed., (Harper & Row, New York / Evanston / London) 1965.
- [13] Ricardo, D., *On the Principles of Political Economy, and Taxation*, 3rd ed., (John Murray, London) 1821.
- [14] ———, *The Works and Correspondence of David Ricardo*, P. Sraffa ed., 11 Vol's, (Cambridge Univ. Press, London / New York) 1951–1973.
- [15] Say, J-B., *Traité d'économie politique*, 2Vol's, (Deterville, Paris) 1803. second édition, 2Vol's, (Antoine-Augustin, Renouard, Paris) 1814.
- [16] Stigler, G. J., "Sraffa's Ricardo", *American Economic Review*, **43** (1953), 586–599. Also in Stigler [17], 302–325.
- [17] ———, *Essays in the History of Economics*, (Chicago Univ. Press, Chicago) 1965.
- [18] Wicksell, K., *Lectures on Political Economy*, Vol.II, ed. with an introduction by L.Robbins, (Routledge & Kegan Paul, London) 1935.

Added in proof H.D.Kurz, "Malthus and the Classics (Not Walras and the Marginalists) as the Major Inspiring Source in the History of Economic Thought", in E.Bellino and S.Neruzzi eds., *Pasinetti and the Classical Keynesians: Nine Methodological Issues*, (Cambridge University Press, Cambridge), 2021.

²⁵ Negishi [11] p.152.

²⁶ Some of Malthus' key concepts were already established in Malthus [6]. See also Eltis [1].