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ROYALTY AND LICENSE FEE UNDER VERTICAL DIFFERENTIATION IN OLIGOPOLY WITH OR WITHOUT ENTRY OF INNOVATOR: CREDIBILITY OF THREAT OF ENTRY BY TWO-STEP AUCTION

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Abstract: When an outside innovating firm has a technology to produce a higher quality good than the good produced at present, it can sell licenses of its technology to incumbent firms using a combination of royalty and fixed fee, or enter the market with or without license. We examine credibility of threat of entry of the innovating firm using a two-step auction in an oligopoly under vertical product differentiation. The credibility of threat of entry by two-step auction depends on the form of the cost function of the new technology, whether it is concave or convex.

Key words: Royalty and license fee, entry, oligopoly with vertical differentiation, two-step auction.

JEL Classification Number: D43, L13.

1. INTRODUCTION

In Proposition 4 of Kamien and Tauman (1986) it was argued that in an oligopoly when the number of firms is small (or very large), strategy to enter the market and at the same time license the cost-reducing technology to the incumbent firm (entry with license strategy) is more profitable than strategy to license its technology to the incumbent firm without entering the market (license without entry strategy) for the innovating firm. However, their result depends on their definition of license fee. They defined the license fee in the case of licenses without entry by the difference between the profit of

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an incumbent firm in that case and its profit before it buys a license without entry of the innovating firm. Although this is the standard definition of license fee, we may consider another definition. If an incumbent firm does not buy a license, the innovating firm may punish the incumbent firm by entering the market. The innovating firm can use such a threat if and only if it is a credible threat. In a duopoly case with one incumbent firm, when the innovating firm does not enter nor sell a license, its profit is zero; on the other hand, when it enters the market without license, its profit is positive. Therefore, threat of entry without license is credible under duopoly, and then even if the innovating firm does not enter the market, the incumbent firm must pay the difference between its profit when it uses the new technology and its profit when the innovating firm enters without license as a license fee. However, in an oligopoly with more than one incumbent firms, the credibility of threat of entry is a more subtle problem.

In this paper we extend this analysis to an oligopolistic situation with three firms, one outside innovating firm and two incumbent firms under vertical product differentiation. We examine the definitions of license fee for producing a higher quality good than the good produced at present considering the threat of entry by a two-step auction in the case of licenses without entry. Also we suppose that the innovating firm uses a combination of royalty per output and a fixed license fee.

A two-step auction, for example, in the case of a license to one incumbent firm without entry is as follows.

1. The first step.

   The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below. The innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.
2. The second step.

   The innovating firm sells a license to one firm at auction *with its entry*.

At the first step of the auction, each incumbent firm has a willingness to pay the following license fee;

> the difference between its profit when only this firm uses the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step each incumbent firm has an incentive to make a bid when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid. The decision of the innovator not to enter the market in the first step is commitment if the incumbent firms accept the offer. We need the minimum bidding price because if there is no minimum price, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding. Threat by such a two-step auction is credible if the total payoff of the innovating firm when it enters the market with a license to one firm is larger than its total payoff when it licenses to one firm without entering the market. A two-step auction in the case of licenses to two incumbent firms without entry is similar, and at the first step of the auction the incumbent firm has a willingness to pay the following license fee;

> the difference between its profit when both firms use the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step each incumbent firm has an incentive to make a bid even if the other firm makes a bid because if it does not make a bid, the auction proceeds to the next step.

In the next section we present a concise literature review. In Section 3 the model of this paper is described. We use a model of vertical differentiation according to Tanaka (2001). In Section 4 we consider various equilibria of the oligopoly. In Section 5 we present the license fees under entry with license strategy. In Section 6 we consider a two-step auction and present the definitions of license fees under license without entry strategy. In Sections 5 and 6 the following results about the optimal royalty rate for the innovator will be shown (see Proposition 1).

Entry with license to one firm case: If the goods are strategic complements, the optimal royalty rate is positive. If the goods are strategic substitutes, it may be positive or negative.

Entry with licenses to two firms case: If the goods are strategic complements, the optimal royalty rate is positive. If the goods are strategic substitutes, it may be positive or negative.

License to one firm without entry not using two-step auction case: If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it may be positive or negative.

License to one firm without entry using two-step auction case: If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it is positive.

Licenses to two firms without entry using or not using two-step auction case: The optimal royalty rate is positive.

In Section 6 we examine the credibility of threat of entry by two-step auction, and will show the following results (see Proposition 2).

1. If the cost function of the new technology is linear, the profit of the innovating firm when it enters the market with a license to one incumbent firm and its profit when it licenses to one incumbent firm without entering the market are equal. Therefore, entry with license to one firm case and license to one firm without entry case are equivalent. In this case we assume that threat of entry by two-step auction is credible.
2. If the cost function of the new technology is strictly convex, threat of entry by two-step auction is credible.
3. If the cost function of the new technology is strictly concave, threat of entry by

two-step auction is not credible.

In Section 7 we present an example with uniform distribution of consumers' taste parameter and a quadratic cost function. We will show that when the quality of the high-quality good is high, licenses to two firms without entry strategy is optimal; on the other hand, when the quality of the high-quality good is not high, entry with licenses to two firms strategy is optimal. In appendices we present analyses of demand and inverse demand functions. Analyses of optimal strategies in general distribution and cost functions case seems to be complicated. It is the theme of the future research.

Hattori and Tanaka (2018b) analyzed a two-step auction under vertical differentiation with only a fixed license fee, and showed that when the quality improvement (the difference between the quality of the high-quality good and the quality of the low-quality good) is small (or large), the two-step auction is (or is not) credible. This paper is an extention of Hattori and Tanaka (2018b) to a case of a combination of royalty and fixed license fee.

2. CONCISE LITERATURE REVIEW

Many references analyzed the relation between the technology licensor and licensee. Royalties per output, fixed license fees, combinations of them, and auctions, are discussed by Katz and Shapiro (1985) and Kamien and Tauman (2002). Sen and Tauman (2007) compared the license system when the licensor is an outsider and that when it is an incumbent firm, using combination of royalties and fixed fees. The production capacity, however, is externally given, and they did not consider a choice of entry. Thus, we need more discussion for the optimal strategies of outside innovators with an option to enter the market. Duchene, Sen and Serfes (2015) considered new entrants with old technology, and showed that a low license fee can be used to deter entry of potential entrants. But the firm with new technology is assumed to be an incumbent, and its choice of entry is not analyzed.

Hoppe, Jehiel and Moldovanu (2006) analyzed the relation between market structure and licensing by auction. They considered licensing strategies of outsider licensor who decides whether or not to license its innovation to potential entrants and incumbents, and showed that a licensor can increase the auction fees by manipulating the number of licenses to potential entrants, and licensing to more firms makes the market more competitive. However, they did not consider a choice by the innovator to enter or not.

Hattori and Tanaka (2015) and (2016a) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Rebolledo and Sandonís (2012) presented an analysis of the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016) analyzed problems about product innovation, that is, introduction of a higher quality good in a duopoly with vertical product differentiation.

Some other related studies are Kabiraj (2004), Wang and Yang (2004), Chen (2017), Creane, Chiu and Konishi (2013), La Manna (1993), Watanabe and Muto (2008), Pal (2010), Rebolledo and Sandonís (2012). See Hattori and Tanaka (2018a) for the contents of these references.

3. THE MODEL

The model of this paper is according to Tanaka (2001) (also see Mussa and Rosen (1978) and Bonanno and Haworth (1998)). There are three firms, Firms A, B and C. Firm A can produce the high-quality good whose quality is q_H, and Firms B and C produce the low-quality good whose quality is q_L, where $q_H > q_L > 0$. The values of q_H and q_L are fixed. Both of the high-quality and the low-quality goods are produced at the same cost.

At present only Firms B and C produce the low-quality good. Firm A is an outside innovator, and it may sell licenses to use its technology for producing the high-quality good to no or one or two incumbent firms (Firms B and C), and it can enter the market with the high-quality good. Call Firm A the innovating firm and Firms B and C the incumbent firms.

Firm A has five options.

1. To enter the market, and license its technology to no incumbent firm.
2. To enter the market, and license its technology to one incumbent firm.
3. To enter the market, and license its technology to two incumbent firms.
4. To license its technology to one incumbent firm, but not enter the market.
5. To license its technology to two incumbent firms, but not enter the market.

The cost function of the firms is $c(\cdot)$, which is twice continuously differentiable. There is no fixed cost; thus $c(0) = 0$.

In the market there is a continuum of consumers with the same income. They have different values of the taste parameter ξ. A consumer buys at most one unit of the good. If a consumer with parameter ξ buys one unit of a good of quality k at price p, his utility is equal to the income plus $-p + \xi k$. If a consumer does not buy the good, his utility is equal to the income. The parameter ξ is distributed according to a distribution function $\rho = F(\xi)$ in the interval $0 < \xi \leq 1$. It is twice continuously differentiable. By ρ we denote the probability that the taste parameter is smaller than or equal to ξ. The size of consumers is one. The inverse function of $F(\xi)$ is denoted by $G(\rho)$. Note that $G(1) = 1$.

Let p_L and x_L be the price and supply of the good of quality q_L; p_H and x_H be the price and supply of the good of quality q_H; and let x_A, x_B and x_C be the outputs of Firms A, B and C.

We consider the sub-game perfect equilibrium of a game with the following structure.

1. In the first stage Firm A chooses one of above five options. In the cases of one or two licenses without entry, it determines whether it uses a two-step auction or not.
2. According to a choice in the first stage, Firm A determines the number of licenses it sells, and it enters the market or not. The licenses are sold through one-step or two-step auctions.
3. In the third stage the firms determine their outputs.

4. GENERAL ANALYSIS

4.1. *Entry without license case*

Suppose that Firm A (the innovating firm) enters the market without license to Firm B nor C. Then, Firm A supplies the high-quality good and Firms B and C supply the low-quality good. Let ξ_L be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\xi_L = \frac{p_L}{q_L}.$$

Let ξ_H be the value of ξ for which the corresponding consumer is indifferent between buying the low-quality good and buying the high-quality good. Then

$$\xi_H = \frac{p_H - p_L}{q_H - q_L}.$$

Let $x_H = x_A$ and $x_L = x_B + x_C$. The inverse demand function is described as follows.

1. When $x_H > 0$ and $x_L > 0$, we have $p_H = (q_H - q_L)G(1 - x_H) + q_L G(1 - x_H - x_L)$ and $p_L = q_L G(1 - x_H - x_L)$.
2. When $x_H > 0$ and $x_L = 0$, we have $p_H = q_H G(1 - x_H)$ and $p_L = q_L G(1 - x_H)$.
3. When $x_H = 0$ and $x_L > 0$, we have $p_H = q_H - q_L + q_L G(1 - x_L)$ and $p_L = q_L G(1 - x_L)$.
4. When $x_H = 0$ and $x_L = 0$, we have $p_H = q_H$ and $p_L = q_L$.

Since $G(1) = 1$, this is a continuously differentiable function with the domain $0 \leq x_H \leq 1$ and $0 \leq x_H \leq 1$. About details for derivation of the inverse demand function please see Appendix A.3.

The profits of Firms A, B and C are written as

$$\pi_A = [(q_H - q_L)G(1 - x_A) + q_L G(1 - x_A - x_B - x_C)]x_A - c(x_A),$$
$$\pi_B = q_L G(1 - x_A - x_B - x_C)x_B - c(x_B),$$
$$\pi_C = q_L G(1 - x_A - x_B - x_C)x_B - c(x_C).$$

The first order conditions for profit maximization of Firms A, B and C are

$$\frac{\partial \pi_A}{\partial x_A} = (q_H - q_L)G(1 - x_A) + q_L G(1 - x_A - x_B - x_C) - [(q_H - q_L)G'(1 - x_A) + q_L G'(1 - x_A - x_B - x_C)]x_A - c'(x_A) = 0,$$
$$\frac{\partial \pi_B}{\partial x_B} = q_L G(1 - x_A - x_B - x_C) - q_L G'(1 - x_A - x_B - x_C)x_B - c'(x_B) = 0,$$
$$\frac{\partial \pi_C}{\partial x_C} = q_L G(1 - x_A - x_B - x_C) - q_L G'(1 - x_A - x_B - x_C)x_C - c'(x_C) = 0.$$

The second order conditions are

$$\frac{\partial^2 \pi_A}{\partial x_A^2} = -2[(q_H - q_L)G'(1 - x_A) + q_L G'(1 - x_A - x_B - x_C)] + [(q_H - q_L)G''(1 - x_A) + q_L G''(1 - x_A - x_B - x_C)]x_A - c''(x_A) < 0,$$

$$\frac{\partial^2 \pi_B}{\partial x_B^2} = -q_L[2G'(1 - x_A - x_B - x_C) - G''(1 - x_A - x_B - x_C)x_B] - c''(x_B) < 0,$$

$$\frac{\partial^2 \pi_C}{\partial x_C^2} = -q_L[2G'(1 - x_A - x_B - x_C) - G''(1 - x_A - x_B - x_C)x_C] - c''(x_C) < 0.$$

Hereafter we assume that the second order conditions in each case are satisfied.

Denote the equilibrium profits of Firms A, B and C in this case by π_A^{e0}, π_B^{e0} and π_C^{e0}. Note that $\pi_B^{e0} = \pi_C^{e0}$.

4.2. Entry with license to one firm case

Suppose that Firm A enters the market and licenses its technology for producing the high-quality good to one of the incumbent firms. We assume that it is Firm C. Then, Firms A and C produce the high-quality good, and Firm B produces the low-quality good. Let $x_H = x_A + x_C$ and $x_L = x_B$. The inverse demand function is the same as that in the previous case.

Denote the royalty per output and the fixed license fee by r and L. The profits of Firms A, B and C are

$$\pi_A = [(q_H - q_L)G(1 - x_A - x_C) + q_L G(1 - x_A - x_B - x_C)]x_A - c(x_A),$$

$$\pi_B = q_L G(1 - x_A - x_B - x_C)x_B - c(x_B),$$

$$\pi_C = [(q_H - q_L)G(1 - x_A - x_C) + q_L G(1 - x_A - x_B - x_C)]x_C - c(x_C) - rx_C - L.$$

The first order conditions are

$$(q_H - q_L)G(1 - x_A - x_C) + q_L G(1 - x_A - x_B - x_C) \tag{1a}$$
$$- [(q_H - q_L)G'(1 - x_A - x_C) + q_L G'(1 - x_A - x_B - x_C)]x_A - c'(x_A) = 0,$$

$$q_L G(1 - x_A - x_B - x_C) - q_L G'(1 - x_A - x_B - x_C)x_B - c'(x_B) = 0, \tag{1b}$$

$$(q_H - q_L)G(1 - x_A - x_C) + q_L G(1 - x_A - x_B - x_C) \tag{1c}$$
$$- [(q_H - q_L)G'(1 - x_A - x_C) + q_L G'(1 - x_A - x_B - x_C)]x_C - c'(x_C) - r = 0.$$

Denote the equilibrium profits of Firms A, B and C by π_A^{e1}, π_B^{e1} and π_C^{e1}. Differentiating (1a), (1b) and (1c) with respect to r, we obtain $\frac{dx_A}{dr}$, $\frac{dx_B}{dr}$ and $\frac{dx_C}{dr}$. About details of them see Appendix B. We have $\frac{dx_C}{dr} < 0$. If the goods are strategic substitutes, $\frac{dx_A}{dr}$ and $\frac{dx_B}{dr}$ are positive. If the goods are strategic complements, $\frac{dx_A}{dr}$ and $\frac{dx_B}{dr}$ are negative.

4.3. Entry with licenses to two firms case

Suppose that Firm A enters the market and licenses its technology for producing the high-quality good to both incumbent firms. Then, all firms produce the high-quality good. Let ξ_0 be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then

$$\xi_0 = \frac{p_H}{q_H}.$$

Let $x_H = x_A + x_B + x_C$. The inverse demand function is described as follows.

1. When $x_H > 0$, we have $p_H = q_H G(1 - x_H)$.

2. When $x_H = 0$, we have $p_H = q_H$.

Since $G(1) = 1$, this is a continuously differentiable function with the domain $0 \leq x_H \leq 1$. About details for derivation of the inverse demand function please see Appendix A.5.

The profits of the firms are

$$\pi_A = q_H G(1 - x_A - x_B - x_C)x_A - c(x_A),$$
$$\pi_B = q_H G(1 - x_A - x_B - x_C)x_B - c(x_B) - rx_B - L,$$
$$\pi_C = q_H G(1 - x_A - x_B - x_C)x_C - c(x_C) - rx_C - L.$$

The first order conditions are

$$q_H G(1 - x_A - x_B - x_C) - q_H G'(1 - x_A - x_B - x_C)x_A - c'(x_A) = 0, \quad (2a)$$
$$q_H G(1 - x_A - x_B - x_C) - q_H G'(1 - x_A - x_B - x_C)x_B - r - c'(x_B) = 0, \quad (2b)$$
$$q_H G(1 - x_A - x_B - x_C) - q_H G'(1 - x_A - x_B - x_C)x_C - r - c'(x_C) = 0. \quad (2c)$$

Denote the equilibrium profits of Firms A, B and C by π_A^{e2}, π_B^{e2} and π_C^{e2}. Differentiating (2a), (2b) and (2c) with respect to r, we obtain $\frac{dx_A}{dr}$, $\frac{dx_B}{dr}$ and $\frac{dx_C}{dr}$. About details of them see Appendix C. We have $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$. If the goods are strategic substitutes, $\frac{dx_A}{dr}$ is positive. If the goods are strategic complements, $\frac{dx_A}{dr}$ is negative.

4.4. *License to one firm without entry case*

Suppose that Firm A sells a license of its technology to one of the incumbent firms and does not enter the market. We assume that it is Firm C. Firm B still produces the low-quality good. Let $x_H = x_C$ and $x_L = x_B$. The inverse demand function is the same as that in the entry without license case.

The profits of Firms B and C are

$$\pi_B = q_L G(1 - x_B - x_C)x_B - c(x_B),$$
$$\pi_C = [(q_H - q_L)G(1 - x_C) + q_L G(1 - x_B - x_C)]x_C - c(x_C) - rx_C - L.$$

The first order conditions are

$$q_L G(1 - x_B - x_C) - q_L G'(1 - x_B - x_C)x_B - c'(x_B) = 0, \quad (3a)$$

$$(q_H - q_L)G(1 - x_C) + q_L G(1 - x_B - x_C) - [(q_H - q_L)G'(1 - x_C) + q_L G'(1 - x_B - x_C)]x_C - r - c'(x_C) = 0. \quad (3b)$$

Denote the equilibrium profits of Firms B and C by π_B^{l1} and π_C^{l1}. Differentiating (3a) and (3b) with respect to r, we obtain

$$\frac{dx_B}{dr} = -\frac{-q_L G'(1 - x_B - x_C) + q_L G''(1 - x_B - x_C)x_B}{\Gamma},$$

and

$$\frac{dx_C}{dr} = \frac{-2q_L G'(1 - x_B - x_C) + q_L G''(1 - x_B - x_C)x_B - c''(x_B)}{\Gamma} < 0,$$

where

$$\begin{aligned}\Gamma =&[-2q_L G'(1-x_B-x_C)+q_L G''(1-x_B-x_C)x_B-c''(x_B)]\theta_C\\&-[-q_L G'(1-x_B-x_C)+q_L G''(1-x_B-x_C)x_B]\\&\quad\times[-q_L G'(1-x_B-x_C)+q_L G''(1-x_B-x_C)x_C],\end{aligned}$$

$$\begin{aligned}\theta_C =&-2[(q_H-q_L)G'(1-x_C)+q_L G'(1-x_B-x_C)]+[(q_H-q_L)G''(1-x_C)\\&+q_L G''(1-x_B-x_C)]x_B-c''(x_C).\end{aligned}$$

From the stability conditions of oligopoly (Seade (1980) and Dixit (1986)), $\Gamma > 0$. If the goods are strategic substitutes, $\frac{dx_B}{dr} > 0$, and if they are strategic complements, $\frac{dx_B}{dr} < 0$.

4.5. *Licenses to two firms without entry case*

Suppose that Firm A sells licenses of its technology to two incumbent firms and does not enter the market. Then, Firms B and C produce the high-quality good. Let $x_H = x_B + x_C$. The inverse demand function is the same as that in the entry with licenses to two firms case.

The profits of the firms are

$$\pi_B = q_H G(1-x_B-x_C)x_B - c(x_B) - rx_B - L,$$
$$\pi_C = q_H G(1-x_B-x_C)x_C - c(x_C) - rx_C - L.$$

The first order conditions are

$$q_H G(1-x_B-x_C) - q_H G'(1-x_B-x_C)x_B - r - c'(x_B) = 0, \tag{4a}$$
$$q_H G(1-x_B-x_C) - q_H G'(1-x_B-x_C)x_C - r - c'(x_C) = 0. \tag{4b}$$

Denote the equilibrium profits of Firms B and C by π_B^{l2} and π_C^{l2}. In this case we have $x_B = x_C$. Differentiating (4a) and (4b) with respect to r, we obtain

$$\frac{dx_B}{dr} = \frac{dx_C}{dr} = \frac{-q_H G'(1-x_B-x_C) - c''(x_B)}{\Gamma'},$$

where

$$\begin{aligned}\Gamma' =&[-2q_H G'(1-x_B-x_C)+q_H G''(1-x_B-x_C)x_B-c''(x_B)]\times\\&[-2q_H G'(1-x_B-x_C)+q_H G''(1-x_B-x_C)x_C-c''(x_C)] > 0.\end{aligned}$$

From the stability conditions, we have

$$\begin{aligned}&|-2q_H G'(1-x_B-x_C)+q_H G''(1-x_B-x_C)x_B-c''(x_B)|\\>&|-q_H G'(1-x_B-x_C)+q_H G''(1-x_B-x_C)x_B|,\end{aligned}$$

and

$$\begin{aligned}&|-2q_H G'(1-x_B-x_C)+q_H G''(1-x_B-x_C)x_C-c''(x_C)|\\>&|-q_H G'(1-x_B-x_C)+q_H G''(1-x_B-x_C)x_C|.\end{aligned}$$

Thus, $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$.

5. ROYALTY AND LICENSE FEES IN THE CASES OF LICENSES WITH ENTRY

In the cases of licenses with entry the fixed license fee is equal to the usual willingness to pay for the incumbent firms. We follow the arguments by Kamien and Tauman (1986) and Sen and Tauman (2007) about license fees by auction.

5.1. License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the technology for producing the high-quality good with entry of Firm A and its profit when only the rival firm buys the license with entry of Firm A.

This is because each incumbent firm knows that there will be one licensee regardless of whether or not it buys a license. The incumbent firms B and C have the same willingness to pay, so even when one of them does not make a bid, the rival firm gets the license. The fixed license fee is

$$L^{e1} = (\pi_C^{e1} + L^{e1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{e1} = \pi_B^{e1}$. The total payoff of Firm A in this case is written as

$$\begin{aligned}\varphi^{e1} =& \pi_A^{e1} + rx_C + L^{e1}\\ =& [(q_H - q_L)G(1 - x_A - x_C) + q_L G(1 - x_A - x_B - x_C)]x_A - c(x_A)\\ &+ [(q_H - q_L)G(1 - x_A - x_C) + q_L G(1 - x_A - x_B - x_C)]x_C\\ &- c(x_C) - (q_L G(1 - x_A - x_B - x_C)x_B - c(x_B)).\end{aligned}$$

Using the first order conditions, the condition for maximization of φ with respect to r is written as follows.

$$\begin{aligned}\frac{d\varphi^{e1}}{dr} &= r\frac{dx_C}{dr} - (q_H - q_L)G'(1 - x_A - x_C)\left(x_C\frac{dx_A}{dr} + x_A\frac{dx_C}{dr}\right)\\ &- q_L G'(1 - x_A - x_B - x_C)\left[(x_C - x_B)\frac{dx_A}{dr} + (x_A - x_B)\frac{dx_C}{dr} + (x_A + x_C)\frac{dx_B}{dr}\right]\\ &= 0.\end{aligned}$$

Then, we get the optimal royalty rate for Firm A as follows.

$$\begin{aligned}\tilde{r}^{e1} =& \frac{(q_H - q_L)G'(1 - x_A - x_C)}{\frac{dx_C}{dr}}\left(x_C\frac{dx_A}{dr} + x_A\frac{dx_C}{dr}\right)\\ &+ \frac{q_L G'(1 - x_A - x_B - x_C)}{\frac{dx_C}{dr}}\left[(x_C - x_B)\frac{dx_A}{dr} + (x_A - x_B)\frac{dx_C}{dr} + (x_A + x_C)\frac{dx_B}{dr}\right].\end{aligned}$$

We can show the following lemma.

LEMMA 1. *If the goods are strategic complements, the optimal royalty rate is positive. If the goods are strategic substitutes, it may be positive or negative.*

Proof. See Appendix D. □

5.2. Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the technology for producing the high-quality good with entry of Firm A and its profit when only the rival firm buys the license with entry of Firm A.

This is because each incumbent firm knows that there will be one licensee when it does not buy a license. In this case there is a minimum bidding price which is equal to the willingness to pay for the incumbents because without the minimum bidding price no firm makes a positive bid. The fixed license fee is

$$L^{e2} = (\pi_C^{e2} + L^{e2}) - \pi_B^{e1}.$$

This means $\pi_C^{e2} = \pi_B^{e1}$. The total payoff of Firm A is

$$\begin{aligned}\varphi^{e2} =& \pi_A^{e2} + r(x_B + x_C) + 2L^{e2} = q_H G(1 - x_A - x_B - x_C)(x_A + x_B + x_C) \\ & - c(x_A) - c(x_B) - c(x_C) - 2\pi_B^{e1}.\end{aligned}$$

Note that π_B^{e1} is constant and irrelevant to determination of the royalty rate in this case because it is determined in the case of entry with a license to one firm. Using the first order conditions, the condition for maximization of φ^{e2} with respect to r is written as follows.

$$\begin{aligned}\frac{d\varphi^{e2}}{dr} =& r\left(\frac{dx_B}{dr} + \frac{dx_C}{dr}\right) - q_H G'(1 - x_A - x_B - x_C)(x_B + x_C)\frac{dx_A}{dr} \\ & - q_H G'(1 - x_A - x_B - x_C)(x_A + x_C)\frac{dx_B}{dr} \\ & - q_H G'(1 - x_A - x_B - x_C)(x_A + x_B)\frac{dx_C}{dr} = 0.\end{aligned}$$

The optimal royalty rate is

$$\tilde{r}^{e2} = \frac{q_H G'(1 - x_A - x_B - x_C)}{\frac{dx_B}{dr} + \frac{dx_C}{dr}}\left[(x_B + x_C)\frac{dx_A}{dr} + (x_A + x_C)\frac{dx_B}{dr} + (x_A + x_B)\frac{dx_C}{dr}\right].$$

If the goods are strategic complements, $\tilde{r}^{e2} > 0$. If the goods are strategic substitutes, $\tilde{r}^{e2}$ may be positive or negative.

6. ROYALTY AND LICENSE FEES IN THE CASE OF LICENSES WITHOUT ENTRY: TWO-STEP AUCTION

6.1. One-step auction

If the licenses are auctioned off to the incumbent firms by one-step auction, the fixed license fee is determined by the usual willingness to pay for the incumbent firms described in Kamien and Tauman (1986) and Sen and Tauman (2007).

6.1.1. License to one firm

The willingness to pay for each incumbent firm is equal to

> the difference between its profit when only this firm uses the technology for producing the high-quality good without entry of Firm A and its profit when only the rival firm buys the license without entry of Firm A.

Then, the fixed license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{l1}.$$

This equation means $\pi_C^{l1} = \pi_B^{l1}$. Denote L in this case by $\tilde{L}^{l1}$, and denote the total payoff of Firm A in this case by $\tilde{\varphi}^{l1}$ to distinguish it from the total payoff in the two-step auction case. It is

$$\tilde{\varphi}^{l1} = rx_C + \tilde{L}^{l1} = [(q_H - q_L)G(1 - x_C) + q_L G(1 - x_B - x_C)]x_C - c(x_C) - (q_L G(1 - x_B - x_C)x_B - c(x_B)).$$

Using the first order conditions, the condition for maximization of $\tilde{\varphi}^{l1}$ with respect to r is written as

$$\frac{d\tilde{\varphi}^{l1}}{dr} = (r + q_L G'(1 - x_B - x_C)x_B)\frac{dx_C}{dr} - q_L G'(1 - x_B - x_C)x_C \frac{dx_B}{dr} = 0.$$

Then, we obtain the optimal royalty rate for Firm A as follows.

$$r^{l1} = -\frac{q_L G'(1 - x_B - x_C)}{\frac{dx_C}{dr}} \left(x_B \frac{dx_C}{dr} - x_C \frac{dx_B}{dr} \right).$$

Denote it by $\tilde{r}^{l1}$. If the goods are strategic substitutes, $\tilde{r}^{l1} < 0$. If the goods are strategic complements, $\tilde{r}^{l1}$ may be positive or negative.

6.1.2. Licenses to two firms

The willingness to pay for each incumbent firm in this case is equal to

> the difference between its profit when two firms use the technology for producing the high-quality good without entry of Firm A and its profit when only the rival firm buys the license without entry of Firm A.

There is a minimum bidding price which is equal to the willingness to pay for the incumbents. The fixed license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{l1}.$$

This means $\pi_C^{l2} = \pi_B^{l1}$. Denote L in this case by $\tilde{L}^{l2}$, and denote the total payoff of Firm A by $\tilde{\varphi}^{l2}$. It is

$$\tilde{\varphi}^{l2} = r(x_B + x_C) + 2\tilde{L}^{l2} = q_H G(1 - x_B - x_C)(x_B + x_C) - c(x_B) - c(x_C) - 2\pi_B^{l1}.$$

Note that π_B^{l1} is constant and irrelevant to determination of the royalty rate because it is determined in the case of a license to one firm without entry. The condition for maximization of $\tilde{\varphi}^{l2}$ with respect to r is

$$\frac{d\tilde{\varphi}^{l2}}{dr} = r\left(\frac{dx_B}{dr} + \frac{dx_C}{dr}\right) - q_H G'(1 - x_B - x_C)x_B \frac{dx_C}{dr} - q_H G'(1 - x_B - x_C)x_C \frac{dx_B}{dr} = 0.$$

The optimal royalty rate is

$$r^{l2} = \frac{q_H G'(1 - x_B - x_C)}{\frac{dx_B}{dr} + \frac{dx_C}{dr}} \left(x_B \frac{dx_C}{dr} + x_C \frac{dx_B}{dr} \right).$$

Denote it by $\tilde{r}^{l2}$. This is positive.

6.2. Two-step auction

We consider a two-step auction for each case.

6.2.1. License to one firm

In this case the two-step auction is carried out as follows.

1. The first step.

   Firm A sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and Firm A imposes a predetermined (positive or negative) royalty per output on the licensee. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.
2. The second step.

   Firm A sells a license to one firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a willingness to pay the following license fee;

> the difference between its profit when only this firm uses the technology for producing the high-quality good *without entry* of Firm A and its profit when only the rival firm buys the license *with entry* of Firm A.

Then, the fixed license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{l1} = \pi_B^{e1}$. Denote L in this case by $\hat{L}^{l1}$.

In the first step each incumbent firm has an incentive to make a bid with the license fee $\hat{L}^{l1}$ when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid. We need the effective minimum bidding price $\hat{L}^{l1}$ because the profit of a non-licensee is π_B^{l1} which is larger than π_B^{e1}. If the minimum price does not function effectively, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

Denote the total payoff of Firm A in this case by $\hat{\varphi}^{l1}$. Then,

$$\hat{\varphi}^{l1} = rx_C + \hat{L}^{l1} = [(q_H - q_L)G(1 - x_C) + q_L G(1 - x_B - x_C)]x_C - c(x_C) - \pi_B^{e1}.$$

Note that π_B^{e1} is a constant number which is determined in the entry with a license to one firm case. The condition for maximization of φ with respect to r is

$$\frac{d\hat{\varphi}^{l1}}{dr} = r\frac{dx_C}{dr} - q_L G'(1 - x_B - x_C)x_C\frac{dx_B}{dr} = 0.$$

Then, we obtain the optimal royalty rate for Firm A as follows.

$$r^{l1} = \frac{q_L G'(1 - x_B - x_C)x_C}{\frac{dx_C}{dr}}\frac{dx_B}{dr}.$$

Denote it by $\hat{r}^{l1}$. If the goods are strategic substitutes, $r^{l1} < 0$, and if they are strategic complements, $r^{l1} > 0$.

6.2.2. Licenses to two firms

We consider the following two-step auction

1. The first step.

   Firm A sells licenses to two firms at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and Firm A imposes a predetermined (positive or negative) royalty per output on the licensee. If both firms make bids, they get licenses. If at least one of the firms does not make a bid, then the auction proceeds to the next step.
2. The second step.

   Firm A sells a license to *one* firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a willingness to pay the following license fee;

> the difference between its profit when two firms use the technology for producing the high-quality good *without entry* of Firm A and its profit when only the rival firm buys the license *with entry* of Firm A.

The minimum bidding price should be equal to this willingness to pay. Then, the fixed license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{e1}.$$

This means $\pi_C^{l2} = \pi_B^{e1}$. Denote L in this case by $\hat{L}^{l2}$.

In the first step each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceeds to the next step.

Denote the total payoff of Firm A in this case by $\hat{\varphi}^{l2}$. It is

$$\hat{\varphi}^{l2} = r(x_B + x_C) + 2\hat{L}^{l2} = q_H G(1 - x_B - x_C)(x_B + x_C) - c(x_B) - c(x_C) - 2\pi_B^{e1}.$$

Note that π_B^{e1} is constant and irrelevant to determination of the royalty rate in this case. The condition for maximization of $\hat{\varphi}^{l2}$ with respect to r is

$$\frac{d\hat{\varphi}^{l2}}{dr} = r\left(\frac{dx_B}{dr} + \frac{dx_C}{dr}\right) - q_H G'(1 - x_B - x_C)\left(x_B\frac{dx_C}{dr} + x_C\frac{dx_B}{dr}\right) = 0.$$

$$\tilde{r} = \frac{q_H G'(1 - x_B - x_C)}{\frac{dx_B}{dr} + \frac{dx_C}{dr}} \left(x_B \frac{dx_C}{dr} + x_C \frac{dx_B}{dr} \right).$$

Denote it by $\hat{r}^{l2}$. It is positive, We see $\hat{r}^{l2} = \tilde{r}^{l2}$, but the total payoff of Firm A with two-step auction and that without two-step auction are different.

Summarizing the results about the optimal royalty rates for Firm A.

PROPOSITION 1. **Entry with license to one firm case:** *If the goods are strategic complements, the optimal royalty rate is positive. If the goods are strategic substitutes, it may be positive or negative.*

Entry with licenses to two firms case: *If the goods are strategic complements, the optimal royalty rate is positive. If the goods are strategic substitutes, it may be positive or negative.*

License to one firm without entry not using two-step auction case: *If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it may be positive or negative.*

License to one firm without entry using two-step auction case: *If the goods are strategic substitutes, the optimal royalty rate is negative. If the goods are strategic complements, it is positive.*

Licenses to two firms without entry using or not using two-step auction case: *The optimal royalty rate is positive.*

6.3. *Credibility of threat of entry*

In this subsection we will prove our main results. Firm A uses a two-step auction if and only if the threat by the existence of the second step of the auction is credible, and it is credible if and only if the total payoff of Firm A when it enters the market with a license to one firm is larger than (or equal to) its payoff when it does not enter and sells a license to one firm not using a two-step auction. Therefore, if

$$\pi_A^{e1} + \tilde{r}^{e1} x_C + L^{e1} \geq \tilde{r}^{l1} x_C + \tilde{L}^{l1},$$

threat of entry by two-step auction is credible. On the other hand, if

$$\tilde{r}^{l1} x_C + \tilde{L}^{l1} > \pi_A^{e1} + \tilde{r}^{e1} x_C + L^{e1},$$

the two-step auction is not credible.

We show the following proposition. Note that we assume $c(0) = 0$, that is, the fixed cost is zero.

PROPOSITION 2. 1. *If the marginal cost is constant, that is, the cost function is linear, entry with license to one firm case and license to one firm without entry case are equivalent, and so threat of entry by two-step auction is credible.*

2. *If the cost function of the firms is strictly convex, threat of entry by two-step auction is credible.*
3. *If the cost function of the firms is strictly concave, threat of entry by two-step auction is not credible.*

Proof. 1. First consider the case of entry with a license to one firm. Note that Firm

A can control the output of each firm by the royalty rate. Let $\bar{q} = x_A + x_C$. Denote the constant marginal cost by c, and denote the total payoff of Firm A by φ^{e1}. It is written as

$$\begin{aligned}\varphi^{e1} =&\pi_A^{e1} + rx_C + L^{e1} = [(q_H - q_L)G(1 - x_A - x_C) + q_L G(1 - x_A - x_B - x_C)]x_A \\ &- cx_A + [(q_H - q_L)G(1 - x_A - x_C) + q_L G(1 - x_A - x_B - x_C)]x_C - cx_C \\ &- (q_L G(1 - x_A - x_B - x_C)x_B - cx_B) \\ =&[(q_H - q_L)G(1 - \bar{q}) + q_L G(1 - \bar{q} - x_B)]\bar{q} - c\bar{q} - (q_L G(1 - \bar{q} - x_B)x_B - cx_B).\end{aligned}$$

If the marginal cost is constant, $c'' = 0$. Thus, $\frac{d\bar{q}}{dr} = \frac{dx_A}{dr} + \frac{dx_C}{dr}$ and $\frac{dx_B}{dr}$ in Section 4.2 are written as (see also Appendix B)

$$\frac{d\bar{q}}{dr} = \frac{[-(q_H - q_L)G'(1 - \bar{q}) + q_L G'(1 - \bar{q} - x_B)]\theta_B}{\Delta'},$$

$$\frac{dx_B}{dr} = -\frac{[-(q_H - q_L)G'(1 - \bar{q}) + q_L G'(1 - \bar{q} - x_B)]\sigma_B}{\Delta'},$$

where

$$\theta_B = -q_L[2G'(1 - \bar{q} - x_B) - G''(1 - \bar{q} - x_B)x_B] - c''(x_B),$$

$$\sigma_B = -q_L G'(1 - \bar{q} - x_B) + q_L G''(1 - \bar{q} - x_B)]x_B.$$

The condition for maximization of φ^{e1} with respect to r is

$$\lambda_1 \frac{d\bar{q}}{dr} - \lambda_2 \frac{dx_B}{dr} = 0, \tag{5}$$

where

$$\begin{aligned}\lambda_1 =&(q_H - q_L)G(1 - \bar{q}) + q_L G(1 - \bar{q} - x_B) \\ &- [(q_H - q_L)G'(1 - \bar{q}) + q_L G'(1 - \bar{q} - x_B)]\bar{q} - c + q_L G'(1 - \bar{q} - x_B)x_B, \\ \lambda_2 =& q_L G(1 - \bar{q} - x_B) - q_L G'(1 - \bar{q} - x_B)x_B - c + q_L G'(1 - \bar{q} - x_B)\bar{q}.\end{aligned}$$

From (1a) and (1c) we have

$$\begin{aligned}&(q_H - q_L)G(1 - \bar{q}) + q_L G(1 - \bar{q} - x_B) - [(q_H - q_L)G'(1 - \bar{q}) \\ &+ q_L G'(1 - \bar{q} - x_B)]\bar{q} - c = r - [(q_H - q_L)G(1 - \bar{q}) + q_L G(1 - \bar{q} - x_B)] + c.\end{aligned}$$

From this and (1b), (5) is rewritten as

$$\begin{aligned}&\{r - [(q_H - q_L)G(1 - \bar{q}) + q_L G(1 - \bar{q} - x_B)] + c + q_L G'(1 - \bar{q} - x_B)x_B\} \frac{d\bar{q}}{dr} \\ &- q_L G'(1 - \bar{q} - x_B)\bar{q}\frac{dx_B}{dr} = 0.\end{aligned}$$

Then, the optimal royalty rate is written as

$$\begin{aligned}\tilde{r}^{e1} =&(q_H - q_L)G(1 - \bar{q}) + q_L G(1 - \bar{q} - x_B) - c - q_L G'(1 - \bar{q} - x_B)x_B \\ &- q_L G'(1 - \bar{q} - x_B)\bar{q}\frac{\sigma_B}{\theta_B}.\end{aligned}$$

The first order condition for Firm C, (1c), with $r = \tilde{r}^{e1}$ is rewritten as

$$(q_H - q_L)G(1 - \bar{q}) + q_L G(1 - \bar{q} - x_B)$$

$$
\begin{aligned}
&-[(q_H-q_L)G'(1-\bar{q})+q_LG'(1-\bar{q}-x_B)]x_C-c\\
&-(q_H-q_L)G(1-\bar{q})-q_LG(1-\bar{q}-x_B)+c+q_LG'(1-\bar{q}-x_B)x_B\\
&+q_LG'(1-\bar{q}-x_B)\bar{q}\frac{\sigma_B}{\theta_B}\\
=&-[(q_H-q_L)G'(1-\bar{q})+q_LG'(1-\bar{q}-x_B)]x_C+q_LG'(1-\bar{q}-x_B)x_B\\
&+q_LG'(1-\bar{q}-x_B)\bar{q}\frac{\sigma_B}{\theta_B}=0.
\end{aligned}
$$

With $x_A+x_C=\bar{q}$, this and the first order condition for Firm A, (1a),

$$
\begin{aligned}
&(q_H-q_L)G(1-\bar{q})+q_LG(1-\bar{q}-x_B)-[(q_H-q_L)G'(1-\bar{q})\\
&+q_LG'(1-\bar{q}-x_B)]x_A-c=0
\end{aligned}
$$

imply

$$
\begin{aligned}
&(q_H-q_L)G(1-\bar{q})+q_LG(1-\bar{q}-x_B)-[(q_H-q_L)G'(1-\bar{q})\\
&+q_LG'(1-\bar{q}-x_B)]\bar{q}-c+q_LG'(1-\bar{q}-x_B)x_B\\
&+q_LG'(1-\bar{q}-x_B)\bar{q}\frac{\sigma_B}{\theta_B}=0.
\end{aligned}
\tag{6}
$$

Next consider the case of license to one firm without entry not using a two-step auction. Let $\bar{q}=x_C$. Denote the total payoff of Firm A in this case by $\tilde{\varphi}^{l1}$. It is written as

$$\tilde{\varphi}^{l1}=[(q_H-q_L)G(1-\bar{q})+q_LG(1-\bar{q}-x_B)]\bar{q}-c\bar{q}-(q_LG(1-\bar{q}-x_B)x_B-cx_B).$$

This is the same as φ^{e1}. If $c''=0$, $\frac{d\bar{q}}{dr}=\frac{dx_C}{dr}$ and $\frac{dx_B}{dr}$ in Section 4.4 are written as

$$\frac{d\bar{q}}{dr}=\frac{\theta_B}{\Gamma},\quad \frac{dx_B}{dr}=-\frac{\sigma_B}{\Gamma},$$

θ_B and σ_B in this case are the same as those in the previous case. The condition for maximization of $\tilde{\varphi}^{l1}$ with respect to r is

$$
\begin{aligned}
&\{[(q_H-q_L)G(1-\bar{q})+q_LG(1-\bar{q}-x_B)]-[(q_H-q_L)G'(1-\bar{q})\\
&+q_LG'(1-\bar{q}-x_B)]\bar{q}-c+q_LG'(1-\bar{q}-x_B)x_B\}\frac{d\bar{q}}{dr}\\
&-[q_LG(1-\bar{q}-x_B)-q_LG'(1-\bar{q}-x_B)x_B-c+q_LG'(1-\bar{q}-x_B)\bar{q}]\frac{dx_B}{dr}=0.
\end{aligned}
\tag{7}
$$

From (3a) and (3b), (7) is rewritten as

$$(r+q_LG'(1-\bar{q}-x_B)x_B)\frac{d\bar{q}}{dr}-q_LG'(1-\bar{q}-x_B)\bar{q}\frac{dx_B}{dr}=0.$$

Then, the optimal royalty rate is

$$\tilde{r}^{l1}=-q_LG'(1-\bar{q}-x_B)x_B-q_LG'(1-\bar{q}-x_B)\bar{q}\frac{\sigma_B}{\theta_B}.$$

The first order condition for Firm C, (3b), with $x_C=\bar{q}$ and $r=\tilde{r}^{l1}$ is rewritten as

$$
\begin{aligned}
&(q_H-q_L)G(1-\bar{q})+q_LG(1-\bar{q}-x_B)-[(q_H-q_L)G'(1-\bar{q})\\
&+q_LG'(1-\bar{q}-x_B)]\bar{q}-c+q_LG'(1-\bar{q}-x_B)x_B
\end{aligned}
\tag{8}
$$

$$+ q_L G'(1-\bar{q}-x_B)\bar{q}\frac{\sigma_B}{\theta_B} = 0.$$

(6) and (8) are the same. Therefore, two cases are equivalent.

2. φ^{e1} with $\bar{q} = x_A + x_C$ is

$$\begin{aligned}\varphi^{e1} =& [(q_H - q_L)G(1-\bar{q}) + q_L G(1-\bar{q}-x_B)]\bar{q} - c(x_A) - c(x_C) \\ & - (q_L G(1-\bar{q}-x_B-)x_B - c(x_B)).\end{aligned}$$

$\tilde{\varphi}^{l1}$ with $\bar{q} = x_C$ is written as

$$\begin{aligned}\tilde{\varphi}^{l1} =& [(q_H - q_L)G(1-\bar{q}) + q_L G(1-\bar{q}-x_B)]\bar{q} - c(\bar{q}) \\ & - (q_L G(1-\bar{q}-x_B-)x_B - c(x_B)) \\ =& \varphi^{e1} + c(x_A) + c(x_C) - c(x_A + x_C).\end{aligned}$$

If the cost function is strictly convex,

$$c(x_C) < \frac{x_C}{x_A + x_C}c(x_A + x_C) + \left(1 - \frac{x_C}{x_A + x_C}\right)c(0) = \frac{x_C}{x_A + x_C}c(x_A + x_C),$$

$$c(x_A) < \frac{x_A}{x_A + x_C}c(x_A + x_C) + \left(1 - \frac{x_A}{x_A + x_C}\right)c(0) = \frac{x_A}{x_A + x_C}c(x_A + x_C).$$

Then,

$$c(x_A) + c(x_C) < c(x_A + x_C).$$

This means that separation of production between two firms is more efficient than concentration to one firm.

> This property of the cost function is called strict *super-additivity*. The strict convexity of the cost function with zero fixed cost implies strict super-additivity.

Thus, when $x_A + x_C$ in the case of entry with a license and x_C in the case of license without entry are equal, φ^{e1} is larger than $\tilde{\varphi}^{l1}$, and the maximum value of φ^{e1} is larger than the maximum value of $\tilde{\varphi}^{l1}$. Hence, threat of entry by two-step auction is credible.

3. Similarly to the case of strictly convex cost function, if the cost function is strictly concave, we find

$$c(x_A) + c(x_C) > c(x_A + x_C).$$

This means that concentration of production to one firm is more efficient than separation between two firms.

> This property of the cost function is called strict *sub-additivity*. The strict concavity of the cost function with zero fixed cost implies strict sub-additivity.

Thus, when $x_A + x_C$ in the case of entry with a license and x_C in the case of license without entry are equal, $\tilde{\varphi}^{l1}$ is larger than φ^{e1}, and the maximum value of $\tilde{\varphi}^{l1}$ is larger than the maximum value of φ^{e1}. Hence, two-step auction is not credible. □

7. THE OPTIMAL STRATEGIES: AN EXAMPLE

Analyses of optimal strategies in general distribution and cost functions case seems to be complicated. We will consider an example. We assume that $\rho = F(\theta)$ has a uniform distribution. Then, $\rho = \theta$, $\theta = G(\rho) = \rho$, $F'(\theta) = G'(\rho) = 1$ and $F''(\theta) = G''(\rho) = 0$. We consider a case of convex cost function. The cost function of the firms is $c(x_i) = \frac{1}{2}q_L x_i^2$, $i = A, B, C$. Denote $q_H = tq_L$, $t > 1$. We present summaries of the calculation results.

License to one firm without entry not using two-step auction case: The optimal royalty rate and the total payoff of Firm A are

$$\tilde{r}^{l1} = -\frac{q_L}{3} < 0,$$

$$\tilde{r}^{l1}x_C + \tilde{L}^{l1} = \frac{q_L(9t^2 - 6t - 2)}{12(3t+1)}.$$

Licenses to two firms without entry not using two-step auction case: The optimal royalty rate and the total payoff of Firm A are

$$\tilde{r}^{l2} = \frac{q_L t^2}{4t+1} > 0,$$

$$\tilde{r}^{l2}(x_B + x_C) + \tilde{L}^{l2} = \frac{q_L(108t^4 + 36t^3 - 45t^2 - 28t - 4)}{12(3t+1)^2(4t+1)}.$$

Entry without license case: The profit of Firm A is

$$\pi_A^{e0} = \frac{q_L(2t-1)^2(2t+1)}{2(4t+1)^2}.$$

Entry with license to one firm case: The optimal royalty rate and the total payoff of Firm A are

$$\tilde{r}^{e1} = \frac{q_L(t+1)^2(9t^2 - 12t - 1)}{2(3t+1)(3t^2+12t+1)},$$

$$\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1} = \frac{q_L(9t^4 + 30t^3 - 8t^2 - 14t - 1)}{4(3t+1)(3t^2+12t+1)}.$$

If $1 < t < \frac{\sqrt{5}+2}{3}$, we have $\tilde{r}^{e1} < 0$.

Entry with licenses to two firms case: The optimal royalty rate and the total payoff of Firm A are

$$\tilde{r}^{e2} = \frac{2q_L t^2(t+1)^2}{(2t+1)(2t^2+6t+1)} > 0,$$

$$\begin{aligned}&\pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} \\ &= \frac{q_L}{4(2t+1)(3t+1)^2(2t^2+6t+1)(3t^2+12t+1)^2}(324t^{10} + 3672t^9 + 14904t^8 \\ &+ 25368t^7 + 14805t^6 - 3318t^5 - 7781t^4 - 3660t^3 - 777t^2 - 78t - 3).\end{aligned}$$

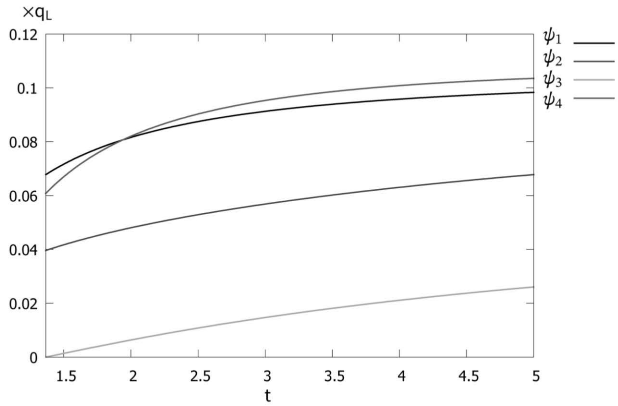

Figure 1. Optimal strategy for Firm A when $t > \frac{\sqrt{3}+1}{2}$

License to one firm without entry using two-step auction case: The optimal royalty rate and the total payoff of Firm A are

$$\hat{r}^{l1} = -\frac{q_L(3t-1)}{3(6t+1)} < 0,$$

$$\hat{r}^{l1}x_C + \hat{L}^{l1} = \frac{q_L}{24(3t+1)^2(6t+1)(3t^2+12t+1)^2}(2916t^8 + 22842t^7 + 44307t^6 - 7452t^5 - 23031t^4 - 8838t^3 - 1371t^2 - 120t - 5).$$

Licenses to two firms without entry using two-step auction case: The optimal royalty rate and the total payoff of Firm A are

$$\hat{r}^{l2} = \frac{q_L t^2}{4t+1} > 0,$$

$$\hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} = \frac{q_L}{4(3t+1)^2(4t+1)(3t^2+12t+1)^2}(324t^8 + 2700t^7 + 6345t^6 + 2454t^5 - 1761t^4 - 1728t^3 - 521t^2 - 66t - 3).$$

Comparing $\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1}$ and $\tilde{r}^{l1}x_C + \tilde{L}^{l1}$,

$$\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1} - (\tilde{r}^{l1}x_C + \tilde{L}^{l1}) = \frac{q_L(3t-1)(15t+1)}{12(3t+1)(3t^2+12t+1)} > 0.$$

Therefore, threat of entry by two-step auction is credible. About this example we get the following results.

1. If $t > \frac{\sqrt{3}+1}{2}$ (≈ 1.366), licenses to two firms without entry strategy is optimal for Firm A. Please see Figure 1. In this figure

$$\psi_1 = \hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} - (\tilde{r}^{l1}x_C + \tilde{L}^{l1}),$$
$$\psi_2 = \hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} - (\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1}),$$

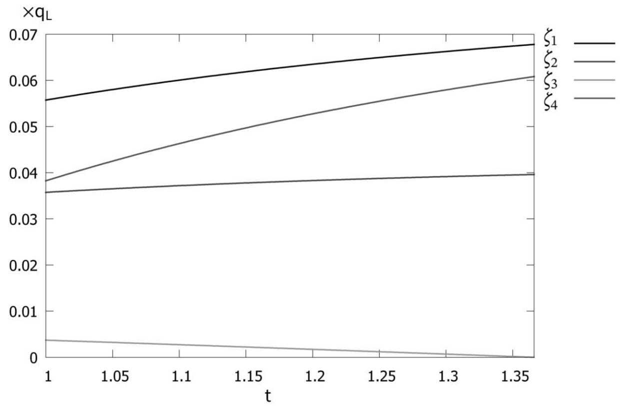

Figure 2. Optimal strategy for Firm A when $1 < t < \frac{\sqrt{3}+1}{2}$

$$\psi_3 = \hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} - (\pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2}),$$
$$\psi_4 = \hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} - \pi_A^{e0}.$$

2. If $1 < t < \frac{\sqrt{3}+1}{2}$, entry with licenses to two firms strategy is optimal for Firm A. Please see Figure 2. In this figure

$$\zeta_1 = \pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} - (\tilde{r}^{l1}x_C + \tilde{L}^{l1}),$$
$$\zeta_2 = \pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} - (\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1}),$$
$$\zeta_3 = \pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} - (\hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2}),$$
$$\zeta_4 = \pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} - \pi_A^{e0}.$$

8. CONCLUDING REMARK

We have analyzed the choice of options for the innovating firm under oligopoly with vertical product differentiation to enter the market with or without licensing its technology for producing a higher quality good to the incumbent firm, or to license its technology without entry using a combination of a royalty per output and a fixed license fee. We have shown that the results depend on the form of the cost function. Analyses of optimal strategies in general distribution and cost functions case seems to be complicated. It is the theme of the future research.

APPENDICES

A. DETAILED ANALYSIS OF DEMAND FUNCTIONS

If a consumer with taste parameter ξ buys one unit of a good of quality k at price p, his utility is equal to $y - p + \xi k$. Let ξ_0 be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the high-quality good. Then,

$$\xi_0 = \frac{p_H}{q_H}.$$

Let ξ_L be the value of ξ for which the corresponding consumer is indifferent between buying nothing and buying the low-quality good. Then,

$$\xi_L = \frac{p_L}{q_L}.$$

Let ξ_H be the value of ξ for which the corresponding consumer is indifferent between buying the low-quality good and buying the high-quality good. Then

$$\xi_H = \frac{p_H - p_L}{q_H - q_L}.$$

We find

$$\xi_0 = \frac{(q_H - q_L)\xi_H + q_L\xi_L}{q_H}.$$

Therefore, $\xi_L \geq \xi_0 \geq \xi_H$ or $\xi_H > \xi_0 > \xi_L$.

For $\xi > (<)\xi_L$,

$$y - p_L + \xi q_L > (<) y.$$

For $\xi > (<)\xi_0$,

$$y - p_H + \xi q_H > (<) y.$$

For $\xi > (<)\xi_H$,

$$y - p_H + \xi q_H > (<) y - p_L + \xi q_L.$$

A.1. License to one firm without entry case

In this case Firm C produces the high-quality good, and Firm B produces the low-quality good. Let x_H be the demand for the high-quality good and x_L be the demand for the low-quality good. Then, we get

1. When $p_H \geq q_H$ ($\xi_0 \geq 1$) and $p_L \geq q_L$ ($\xi_L \geq 1$), we have $x_H = 0$ and $x_L = 0$.
2. When $p_H < q_H$ ($\xi_0 < 1$) and $p_L \geq \frac{p_H}{q_H} q_L$ ($\xi_L \geq \xi_0 \geq \xi_H$), we have $x_H = 1 - F(\xi_0)$ and $x_L = 0$.
3. When $p_L < q_L$ ($\xi_L < 1$), $p_H > \frac{p_L}{q_L} q_H$ ($\xi_H > \xi_0 > \xi_L$) and $p_H - p_L \geq q_H - q_L$ ($\xi_H \geq 1$), we have $x_H = 0$ and $x_L = 1 - F(\xi_L)$.
4. When $p_L < q_L$ ($\xi_L < 1$), $p_H > \frac{q_H}{q_L} p_L$ ($\xi_H > \xi_0 > \xi_L$) and $p_H - p_L < q_H - q_L$ ($\xi_H < 1$), we have $x_L = F(\xi_H) - F(\xi_L)$ and $x_H = 1 - F(\xi_H)$.

From this demand function we obtain the inverse demand function as follows.

1. When $x_H > 0$ and $x_L > 0$, we have $p_H = (q_H - q_L)G(1 - x_H) + q_L G(1 - x_H - x_L)$ and $p_L = q_L G(1 - x_H - x_L)$.
2. When $x_H > 0$ and $x_L = 0$, we have $p_H = q_H G(1 - x_H)$ and $p_L = q_L G(1 - x_H)$.
3. When $x_H = 0$ and $x_L > 0$, we have $p_H = q_H - q_L + q_L G(1 - x_L)$ and $p_L = q_L G(1 - x_L)$.
4. When $x_H = 0$ and $x_L = 0$, we have $p_H = q_H$ and $p_L = q_L$.

This is a continuously differentiable inverse demand function with the domain $0 \leq x_H \leq 1$ and $0 \leq x_L \leq 1$. We have $x_H = x_C$ and $x_L = x_B$.

A.2. Entry with license to one firm case

In this case Firms A and C produce the high-quality good, and Firm B produces the low-quality good. The inverse demand function is the same as that in Case A.1 with $x_H = x_A + x_C$ and $x_L = x_B$.

A.3. Entry without license case

In this case Firm A produces the high-quality good, and Firms B and C produce the low-quality good. The inverse demand function is the same as that in Case A.1 with $x_H = x_A$ and $x_L = x_B + x_C$.

A.4. Licenses to two firms without entry case

In this case Firms B and C produce the high-quality good. Let x_H be the demand for the high-quality good. Then, we get

1. When $p_H \geq q_H$ ($\xi_0 \geq 1$), we have $x_H = 0$.
2. When $p_H < q_H$ ($\xi_0 < 1$), we have $x_H = 1 - F(\xi_0)$.

Then, the inverse demand function is described as follows.

1. When $x_H > 0$, we have $p_H = q_H G(1 - x_H)$.
2. When $x_H = 0$, we have $p_H = q_H$.

This is a continuously differentiable inverse demand function with the domain $0 \leq x_H \leq 1$. We have $x_H = x_B + x_C$.

A.5. Licenses to two firms with entry case

In this case all firms produce the high-quality good. Let $x_H = x_A + x_B + x_C$. The inverse demand function is the same as that in Case A.4.

B. DETAILS ABOUT $\frac{dx_a}{dr}$, $\frac{dx_b}{dr}$ AND $\frac{dx_c}{dr}$ IN SECTION 4.2.

Differentiating (1a), (1b) and (1c) with respect to r, we obtain

$$\frac{dx_A}{dr} = \frac{\sigma_{AB}\sigma_B - \sigma_{AC}\theta_B}{\Delta'}, \quad \frac{dx_B}{dr} = \frac{\sigma_{AC}\sigma_B - \sigma_B\theta_A}{\Delta'}, \quad \frac{dx_C}{dr} = \frac{\theta_A\theta_B - \sigma_{AB}\sigma_B}{\Delta'},$$

where

$$\theta_A = \frac{\partial^2 \pi_A}{\partial x_A^2} = -2[(q_H - q_L)G'(1 - x_A - x_C) + q_L G'(1 - x_A - x_B - x_C)] + [(q_H - q_L)G''(1 - x_A - x_C) + q_L G''(1 - x_A - x_B - x_C)]x_A - c''(x_A),$$

$$\theta_B = \frac{\partial^2 \pi_B}{\partial x_B^2} = -q_L[2G'(1 - x_A - x_B - x_C) - G''(1 - x_A - x_B - x_C)x_B] - c''(x_B),$$

$$\theta_C = \frac{\partial^2 \pi_C}{\partial x_C^2} = -2[(q_H - q_L)G'(1 - x_A - x_C) + q_L G'(1 - x_A - x_B - x_C)] + [(q_H - q_L)G''(1 - x_A - x_C) + q_L G''(1 - x_A - x_B - x_C)]x_C - c''(x_C),$$

$$\sigma_{AB} = \frac{\partial^2 \pi_A}{\partial x_A x_B} = -q_L G'(1 - x_A - x_B - x_C) + q_L G''(1 - x_A - x_B - x_C)x_A,$$

$$\sigma_{AC} = \frac{\partial^2 \pi_A}{\partial x_A x_C} = -(q_H - q_L)G'(1 - x_A - x_C) + q_L G'(1 - x_A - x_B - x_C)$$
$$+ [(q_H - q_L)G''(1 - x_A - x_C) + q_L G''(1 - x_A - x_B - x_C)]x_A,$$

$$\sigma_B = \frac{\partial^2 \pi_B}{\partial x_B x_A} = \frac{\partial^2 \pi_B}{\partial x_B x_C} = -q_L G'(1 - x_A - x_B - x_C) + q_L G''(1 - x_A - x_B - x_C)x_B,$$

$$\sigma_{CA} = \frac{\partial^2 \pi_C}{\partial x_C x_A} = -(q_H - q_L)G'(1 - x_A - x_C) + q_L G'(1 - x_A - x_B - x_C)$$
$$+ [(q_H - q_L)G''(1 - x_A - x_C) + q_L G''(1 - x_A - x_B - x_C)]x_C,$$

$$\sigma_{CB} = \frac{\partial^2 \pi_C}{\partial x_C x_B} = -q_L G'(1 - x_A - x_B - x_C) + q_L G''(1 - x_A - x_B - x_C)x_C,$$

$$\Delta' = \theta_A\theta_B\theta_C - \theta_A\sigma_B\sigma_{CB} - \theta_B\sigma_{AC}\sigma_{CA} - \theta_C\sigma_{AB}\sigma_B + \sigma_{AC}\sigma_B\sigma_{CB} + \sigma_{AB}\sigma_B\sigma_{CA}.$$

By the second order conditions, $\theta_A < 0, \theta_B < 0, \theta_C < 0$. From the stability conditions for oligopoly (Seade (1980) and Dixit (1986)) $\Delta' < 0$. We assume that the absolute values of θ_A, θ_B and θ_C are larger than those of σ's, and the magnitudes of σ_{AB} and σ_{AC} are similar. Then, we have $\frac{dx_C}{dr} < 0$. If the goods are strategic substitutes, σ's are negative and $\frac{dx_A}{dr}$ and $\frac{dx_B}{dr}$ are positive. If the goods are strategic complements, σ's are positive and $\frac{dx_A}{dr}$ and $\frac{dx_B}{dr}$ are negative.

C. DETAILS ABOUT $\frac{dx_a}{dr}$, $\frac{dx_b}{dr}$ AND $\frac{dx_c}{dr}$ IN SECTION 4.3.

Differentiating (2a), (2b) and (2c) with respect to r, we obtain

$$\frac{dx_A}{dr} = \frac{\sigma_A(\sigma_B - \theta_B + \sigma_C - \theta_C)}{\Delta},$$
$$\frac{dx_B}{dr} = \frac{\theta_A\theta_C - \theta_A\sigma_B + \sigma_A\sigma_B - \sigma_A\sigma_C}{\Delta},$$
$$\frac{dx_C}{dr} = \frac{\theta_A\theta_B - \theta_A\sigma_C + \sigma_A\sigma_C - \sigma_A\sigma_B}{\Delta},$$

where

$$\theta_A = -2q_H G'(1 - x_A - x_B - x_C) + q_H G''(1 - x_A - x_B - x_C)x_A - c''(x_A),$$
$$\theta_B = -2q_H G'(1 - x_A - x_B - x_C) + q_H G''(1 - x_A - x_B - x_C)x_B - c''(x_B),$$
$$\theta_C = -2q_H G'(1 - x_A - x_B - x_C) + q_H G''(1 - x_A - x_B - x_C)x_C - c''(x_C),$$
$$\sigma_A = -q_H G'(1 - x_A - x_B - x_C) + q_H G''(1 - x_A - x_B - x_C)x_A,$$
$$\sigma_B = -q_H G'(1 - x_A - x_B - x_C) + q_H G''(1 - x_A - x_B - x_C)x_B,$$
$$\sigma_C = -q_H G'(1 - x_A - x_B - x_C) + q_H G''(1 - x_A - x_B - x_C)x_C,$$
$$\Delta = \theta_A\theta_B\theta_C - \theta_A\sigma_B\sigma_C - \theta_B\sigma_A\sigma_C - \theta_C\sigma_A\sigma_B + \sigma_A\sigma_B\sigma_C.$$

By the second order conditions, $\theta_A < 0, \theta_B < 0, \theta_C < 0$. From the stability conditions for oligopoly (Seade (1980) and Dixit (1986)) $\Delta < 0$. We assume that the absolute values of θ_A, θ_B and θ_C are larger than those of σ_A, σ_B and σ_C, and that the magnitudes of σ's are similar. We have $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$. If the goods are strategic substitutes, σ's are negative and $\frac{dx_A}{dr}$ is positive. If the goods are strategic complements, σ's are positive and $\frac{dx_A}{dr}$ is negative.

D. PROOF OF LEMMA 1.

We assume

$$G(1 - x_A - x_C) - G'(1 - x_A - x_C)x_A > 0, \tag{9}$$

The first order condition for Firm A, (1a), means

$$\begin{aligned}(q_H - q_L)[G(1 - x_A - x_C) - G'(1 - x_A - x_C)x_A] \\ + q_L[G(1 - x_A - x_B - x_C) - G'(1 - x_A - x_B - x_C)x_A] = c'(x_A) > 0.\end{aligned} \tag{10}$$

Thus, (9) will hold. For example, if ρ has a uniform distribution, from (10)

$$q_H(1 - 2x_A - x_C) = q_L x_B + c'(x_A) > 0.$$

Then, we get

$$G(1 - x_A - x_C) - G'(1 - x_A - x_C)x_A = 1 - 2x_A - x_C > 0.$$

Assume that $x_A = x_B$ and (1b) is satisfied. Then,

$$q_L G(1 - x_A - x_B - x_C) - q_L G'(1 - x_A - x_B - x_C)x_A = c'(x_A).$$

Substituting this into the left hand side of (1a), we get

$$\frac{\partial \pi_A}{\partial x_A} = (q_H - q_L)[G(1 - x_A - x_C) - G'(1 - x_A - x_C)x_A].$$

By (9) this is positive. Therefore, $x_A > x_B$. If (1a) is satisfied, $x_A = x_C$ and $r \leq 0$, then the left hand side of (1c) is nonnegative. Thus, when $r \leq 0$, we have $x_C \geq x_A$ and $x_C > x_B$.

We can show the following results.

1. If the goods are strategic complements, then the optimal royalty rate is positive.
2. If the goods are strategic substitutes, then the optimal royalty rate may be positive or negative.

Proof. 1. If the goods are strategic complements, $\frac{dx_A}{dr}$, $\frac{dx_B}{dr}$ and $\frac{dx_C}{dr}$ are all negative. Then,

$$\begin{aligned}\left.\frac{d\varphi^{e1}}{dr}\right|_{r\leq 0} &= r\frac{dx_C}{dr} - (q_H - q_L)G'(1 - x_A - x_C)\left(x_C\frac{dx_A}{dr} + x_A\frac{dx_C}{dr}\right) \\ &\quad - q_L G'(1 - x_A - x_B - x_C) \\ &\quad \times \left[(x_C - x_B)\frac{dx_A}{dr} + (x_A - x_B)\frac{dx_C}{dr} + (x_A + x_C)\frac{dx_B}{dr}\right] > 0,\end{aligned}$$

because $x_A - x_B > 0$, $x_C - x_B > 0$ and $x_A + x_C > 0$. Thus, the optimal royalty rate is positive.

2. If the goods are strategic substitutes, $\frac{dx_A}{dr} > 0$, $\frac{dx_B}{dr} > 0$ and $\frac{dx_C}{dr} < 0$. Then, since $x_C \frac{dx_A}{dr} > 0$, $x_A \frac{dx_C}{dr} < 0$, $(x_C - x_B)\frac{dx_A}{dr} > 0$, $(x_A - x_B)\frac{dx_C}{dr} < 0$ and $(x_A + x_C)\frac{dx_B}{dr} > 0$, the optimal royalty rate may be positive or negative.

□

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