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ON AN ENDOGENOUS LEADER–FOLLOWER RELATIONSHIP AND NETWORK COMPATIBILITY†

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Abstract: We consider an endogenous leader–follower relationship in a network goods market, such as those found in the information and communication technology industries, where we observe network externalities and product compatibility (hereafter, network compatibility effects). Using the framework of an endogenous timing game, we examine how network compatibility effects affect strategic relationships between firms and the leader–follower relationship. In particular, we demonstrate that if there are sufficient asymmetric network compatibility effects between the firms, there is a unique subgame perfect Nash equilibrium in the endogenous timing game, where the firm providing the product with a large (small) network compatibility effect becomes a leader (follower) in the case of quantity competition. However, in the cases of price competition and quantity competition with consumers' *ex ante* expectations for network size, the reverse result arises.

Key words: Simultaneous-move game, sequential-move game, endogenous timing game, network externality, product compatibility, product substitutability.

JEL Classification Number: D21, D43, D62, L15.

1. INTRODUCTION

Since the early 21st century, along with progress in information and communication technologies, we have witnessed the proliferation of products and services exhibiting network externalities in network industries (e.g., smartphones, application software, and Internet services).[1] In addition, compatibility (or connectivity) between products and services with different brand names is important for both providers and (potential) users. In particular, compatibility is likely to enhance the utility of users because the interaction with other products and services improves performance.

As an example, we consider the VHS–Betamax war of the early 1980s and the competition between Blu-ray and HD DVD at the beginning of the 2000s. These products

†Toshimitsu (2015) provides the basis for the earliest version of this paper.

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were for the most part incompatible; therefore, the firms producing them competed on quantities to increase their market share. Importantly, this type of competition highlights the struggle to become the market leader and with it the *de facto* standard. We also observe that mobile phone companies and Internet service providers commit to their price contract strategies and consumers subscribe to them at the listed prices. This implies that the firms compete on prices in the network market where their products and services are fully compatible with each other. In this case, we may interpret the battle between firms as an effort to secure price leadership.

Furthermore, let us consider application software markets (e.g., word processing software, spreadsheets, and database management systems). Suppose that *product 1* (*2*) is a perfectly compatible (incompatible) product, i.e., a user of *software 1* can use data files made by *software 2* as well as *software 1*, whereas a user of software 2 can only use data files made by *software 2*. This implies that *software 1* is the *de facto* standard in the software market. Thus, we can imagine that *software firm 2* providing an incompatible product needs to compete at a lower price, whereas *software firm 1* providing a perfectly compatible product competes at a higher price.

Taking instant messaging software like AOL Instant Messenger, Yahoo! Messenger, and others as examples, Belleflamme and Peitz (2010, p. 546) argue that "(1) there seems to be early-mover advantage related to the launch of network goods; (2) entrants tend to favor compatibility, while incumbent firms tend to prefer incompatibility." Accordingly, we pose the following research questions. What firms, i.e., providers of (in)compatible products, take the first- or second-mover advantage? For firms facing both network externalities and compatibility, choosing to be either a leader or a follower is a critical decision. In other words, is it advantageous to be a first-mover among competing firms in network markets?

In the field of industrial organization, many studies have considered the choice of firm roles in market or timing decisions with respect to strategic variables, including price, quantity, and other firm activities. With the first- and second-mover advantage, when comparing the Stackelberg and Cournot–Nash equilibria, we find that firms prefer to be leaders (followers) if the strategic relationships between them indicate that they are substitutes (complements) with respect to the relevant strategic variables. Equivalently, the same is true if we find negative (positive) slopes of the reaction functions in the relevant strategic variable space.[2]

For endogenous leadership to hold in a duopolistic game, there are requirements for certain asymmetric characteristics between the firms themselves, the attributes of their products, and their strategic variables. For example, extending the strategic taxonomy of

[1] Birke (2009) surveys the empirical literature on network effects.

[2] For example, Gal-Or (1985) demonstrates that firms are willing (unwilling) to commit first when the reaction functions are downward (upward) sloping. In this case, the firms have a first (second)-mover advantage. Dowrick (1986) considers the conditions whereby firms agree upon the choice of role of leader and follower in the Stackelberg duopoly model and demonstrates that each firm prefers to be a leader when the slope of the reaction functions is downward. In contrast, each firm prefers that the other firm will be the leader when the slope of the reaction functions is upward.

Fudenberg and Tirole (1984), Tombak (2006) examines a "strategic asymmetry" two-stage game where one firm (the other firm) regards its rival's second-stage strategic variable as a strategic complement (substitute).

In this paper, with respect to our research questions posed above, we consider how network externalities and product compatibility (hereafter, network compatibility effects) affect the timing decision of a firm to commit to its production or price. By focusing on the network compatibility effects, we demonstrate that a leader–follower relationship endogenously develops among competing firms in a horizontally differentiated product market, as based on the extended game with observable delay developed by Hamilton and Slutsky (1990) and Amir and Grilo (1999).

Using the framework in Hamilton and Slutsky (1990), Tremblay et al. (2013) develop a model in a related paper in which both the timing of play and the strategic choice variables, i.e., quantity and price, are endogenous. In their model, a firm choosing quantity (price) is a leader (follower). Similarly, Lambertini and Tampieri (2012) demonstrate that a firm producing a low (high)-quality product is a leader (follower) in a vertically differentiated quantity-setting duopoly model (see also Lambertini and Tedeschi, 2007). As an example, they detail the introduction of solid state (transistor) circuitry in the replacement of vacuum tube designs.

As discussed, with respect to endogenous timing decisions, the asymmetric strategic space (i.e., price vs. quantity) in Tremblay et al. (2013) and the asymmetric product quality (i.e., low vs. high quality) in Lambertini and Tampieri (2012) are important considerations. In this paper, we show that sufficient asymmetric network compatibility effects between the firms are important in constructing an endogenous leader–follower relationship. That is, we demonstrate that the firm providing the product with a larger (smaller) network compatibility effect emerges as a leader (follower) in the case of quantity competition. However, we also reconsider this result from the viewpoint of the mode of competition and the formation of consumer expectations for network size.

2. THE MODEL

2.1. *Quantity competition, consumer expectations, and network compatibility effect*

We consider duopolistic quantity competition in a market of horizontally differentiated products with network externalities. Based on the framework in Economides (1996), we assume a linear inverse demand function for *product i* as follows:

$$p_i = A - q_i - \gamma q_j + f(S_i^e), \quad i, j = 1, 2,\ i \neq j\,, \tag{1}$$

where A is the intrinsic market size, $q_i(q_j)$ is the output level of *firm* i (j), and $\gamma \in (0, 1)$ represents the level of product substitutability and implies a horizontal difference between the products (e.g., brand names). If $\gamma \to 1(0)$, *product i* becomes perfectly substitutable (independent). The network externality function is given by $f(S_i^e)$, where S_i^e is the expected network size of *firm i*'s product. We also assume a linear network externality function, $f(S_i^e) = aS_i^e$, where $a \in (0, 1)$ denotes the degree of network externality. Furthermore, using the formulation of Shy (2001, p. 62), the expected network size of *firm i* is given by:

$$S_i^e = q_i^e + \alpha_i q_j^e, \quad i, j = 1, 2, \ i \neq j \,, \tag{2}$$

where $\alpha_i \in [0, 1]\,, \quad i = 1, 2$, is the degree of *product i*'s compatibility with *product j*.

Here, and related to the concept of fulfilled expectation equilibria, we assume that consumers make expectations for network size after the firms' output decisions.[3] This implies that the firms can commit to the output level, so that consumers believe the output levels and then form expectations for the network size, i.e., $q_i^e = q_i\,, \quad i = 1, 2$ (i.e., the case of consumers' *ex post* expectations). Thus, it holds that $S_i^e = S_i = q_i + \alpha_i q_j$, where S_i is the actual network size of *firm i*'s product.[4]

Based on equations (1) and (2), the inverse demand function for *firm i* is represented by:[5]

$$p_i = A - (1-a)q_i - (\gamma - a\alpha_i)q_j\,, \quad i, j = 1, 2, \ i \neq j \,. \tag{3}$$

As for equation (3), we assume that the own-price effect exceeds the cross-price effect, i.e., $\left|\frac{dp_i}{dq_i}\right| > \left|\frac{dp_i}{dq_j}\right|, \quad i, j = 1, 2, \ i \neq j$. In this case, it follows that $1 - a > |\gamma - a\alpha_i|\,, \quad i = 1, 2$. Hereafter, $a\alpha_i, i = 1, 2$, denotes a network compatibility effect of *firm i*, which is attributed to the nature of product complementarity (connectivity). Furthermore, in light of equation (3), this effect implies the ability to absorb demand spillovers from the other firm.

To simplify, we assume that production costs are zero, because we readily observe low and even negligible marginal running costs in Internet businesses. The profit function is expressed as:

$$\pi_i = p_i q_i = \left\{A - (1-a)q_i - (\gamma - a\alpha_i)q_j\right\} q_i\,, \quad i = 1, 2\,. \tag{4}$$

2.2. Cournot–Nash equilibrium in a simultaneous-move game

We derive Cournot–Nash equilibrium in the case of consumers' *ex post* expectations for network size. In view of equation (4), given the output level of *firm j*, *firm i* decides its output level, incorporating the network compatibility effect. The first-order condition (FOC) for profit maximization is given by:

$$\frac{\partial \pi_i}{\partial q_i} = A - 2(1-a)q_i - (\gamma - a\alpha_i)q_j = 0, \quad i, j = 1, 2, \ i \neq j \,. \tag{5}$$

Thus, we have the reaction function for *firm i* as follows:

$$q_i = \frac{A}{2(1-a)} - \frac{\gamma - a\alpha_i}{2(1-a)} q_j, \quad i, j = 1, 2, \ i \neq j \,. \tag{6}$$

[3] Katz and Shapiro (1985) and Economides (1996) assume that consumers make their expectations for network size before firms' output decisions and thus the firms cannot affect the network size (i.e., the case of consumers' *ex ante* expectations). Section 3.2 examines that case.

[4] Strictly speaking, we consider subgame perfect Nash equilibria in which consumers observe output levels (capacities) before making actual consumption decisions. Given consumers have to make their choice given the choices of all other consumers in the Nash equilibrium, each consumer's beliefs about the behavior of other consumers are confirmed.

[5] Assuming a homogeneous product market, i.e., $\gamma = 1$, and symmetric compatibilities, i.e., $\alpha_i = \alpha, i = 1, 2$, Ji and Daitoh (2008) derive the following inverse demand function: $p_i = A - (1-a)q_i - (1 - a\alpha)q_j\,, \quad i, j = 1, 2, \ i \neq j$.

Given equation (6), it follows that $\frac{\partial q_i}{\partial q_j} < (>)0 \Leftrightarrow \gamma > (<)a\alpha_i, i, j = 1, 2,\ i \neq j$. This implies that a strategic substitutionary (complementarity) relationship arises between the firms if the degree of the network compatibility effect is smaller (larger) than that of product substitutability. That is, although the intrinsic nature of the products is substitutionary, if the degree of the network compatibility effect is sufficiently large, the nature of the products can change to be complementary.

Using equation (5), we rewrite the profit function as follows: $\pi_i = p_i q_i = (1-a)(q_i)^2, i = 1, 2$. Thus, we obtain the external effect of an increase in the output level of *firm* j on the profit of *firm* i:

$$\frac{\partial \pi_i}{\partial q_j} = 2(1-a)q_i \frac{\partial q_i}{\partial q_j} < (>)0 \Leftrightarrow \gamma > (<)a\alpha_i, i, j = 1, 2,\ i \neq j\,. \tag{7}$$

For our analysis, we make the following assumptions:

ASSUMPTIONS.

(i) *Asymmetric compatibility*: $1 \geq \alpha_1 > \alpha_2 \geq 0$.
(ii) *Strong network externality*: $a > \gamma$.

Based on equation (5), we derive the following Cournot–Nash equilibrium in the case of consumers' *ex post* expectations:

$$q_i^N = \frac{A\{2(1-a) - (\gamma - a\alpha_i)\}}{D}, \quad i = 1, 2\,, \tag{8}$$

where $D \equiv 4(1-a)^2 - (\gamma - a\alpha_1)(\gamma - a\alpha_2) > 0$ and $2(1-a) - (\gamma - a\alpha_i) > 0, i = 1, 2$. Superscript N denotes the Cournot–Nash equilibrium in the case of consumers' *ex post* expectations.

Considering equations (6) and (7), here, we focus on the case that sufficient asymmetric network compatibility effects between the firms arise, i.e., $a\alpha_1 > \gamma > a\alpha_2$. In this case, the strategic complementarity (substitutionary) relationship for *firm 1* (*2*) holds and thus the reaction curve of *firm 1* (*2*) is upward (downward) sloping, as characterized by a game with strategic heterogeneity (see Monaco and Sabarwal, 2016).

2.3. *Stackelberg equilibrium in a sequential-move game and comparison*

We consider a Stackelberg game. Without loss of generality, suppose that *firm j* (i) is a leader (follower). That is, *firm j* commits to its output level, and after observing this, *firm i* decides the output level. Thus, as for *firm i* (a follower), we obtain equations (5) and (6).

Now, the profit function of *firm j* as a leader can be expressed as:

$$\pi_j = \left\{A - (1-a)q_j - (\gamma - a\alpha_j)q_i[\bullet]\right\} q_j, i, j = 1, 2,\ i \neq j\,,$$

where $q_i[\bullet]$ is given by equation (6). The FOC is given by:

$$\frac{\partial \pi_j}{\partial q_j} = A - 2(1-a)q_j - (\gamma - a\alpha_j)q_i[\bullet] + \frac{(\gamma - a\alpha_i)(\gamma - a\alpha_j)}{2(1-a)} q_j = 0\,,$$
$$i, j = 1, 2,\ i \neq j\,.$$

Thus, we derive the following Stackelberg equilibrium in the case of quantity competition with consumers' *ex post* expectations:

$$q_j^L = \frac{A\left\{2(1-a) - (\gamma - a\alpha_j)\right\}}{D - (\gamma - a\alpha_1)(\gamma - a\alpha_2)}\,, \tag{9}$$

$$q_i^F = \frac{A\left\{D - 2(1-a)(\gamma - a\alpha_i)\right\}}{2(1-a)\{D - (\gamma - a\alpha_1)(\gamma - a\alpha_2)\}}\,, \tag{10}$$

where $i, j = 1, 2,\ i \neq j$. Superscript $L(F)$ denotes a leader (follower) in the Stackelberg equilibrium. We can similarly obtain the outcomes in the case of opposing roles, i.e., *firm j* (i) is a follower (leader).

Based on equations (8), (9), and (10), comparing the output levels in the Cournot–Nash and the Stackelberg equilibria, we derive the following relationships:

$$q_j^L > (<) q_j^N \Leftrightarrow (\gamma - a\alpha_i)(\gamma - a\alpha_j) > (<) 0\,, \tag{11}$$

$$q_j^F > (<) q_j^N \Leftrightarrow \gamma - a\alpha_i < (>) 0\,, \tag{12}$$

$$q_j^L > (<) q_j^F \Leftrightarrow (\gamma - a\alpha_i)(\gamma - a\alpha_j) > (<) 0\,, \tag{13}$$

where $i, j = 1, 2,\ i \neq j$. Given Assumptions (i) and (ii), and, using equations (11), (12), and (13), we directly derive the following outcomes.

LEMMA 1.

(i) *If* $1 > a\alpha_1 > a\alpha_2 > \gamma$, *then* $q_i^L > q_i^F > q_i^N, i = 1, 2$.
(ii) *If* $\gamma > a\alpha_1 > a\alpha_2 \geq 0$, *then* $q_i^L > q_i^N > q_i^F, i = 1, 2$.
(iii) *If* $1 > a\alpha_1 > \gamma > a\alpha_2 \geq 0$, *then* $q_1^N > q_1^F > q_1^L$, *and* $q_2^F > q_2^N > q_2^L$.

In Lemma 1 (i), because the degree of network compatibility effects of both firms is larger than that of the product substitutability, it appears that the firms compete in a complementary product market. Conversely, in Lemma 1 (ii), there is a standard Cournot competition in a substitutionary product market. In particular, Lemma 1 (iii) implies that the firms providing different properties of the products compete on quantities. That is, the product with a large (small) network compatibility effect corresponds to a complementary (substitutionary) product.

Furthermore, based on Lemma 1, we derive the following relationships regarding the profits in the Cournot–Nash equilibrium and the Stackelberg equilibria.

LEMMA 2.

(i) *If* $1 > a\alpha_1 > a\alpha_2 > \gamma$, *then* $\pi_i^L > \pi_i^N, \pi_i^F > \pi_i^N, i = 1, 2$.
(ii) *If* $\gamma > a\alpha_1 > a\alpha_2 \geq 0$, *then* $\pi_i^L > \pi_i^N > \pi_i^F, i = 1, 2$.
(iii) *If* $1 > a\alpha_1 > \gamma > a\alpha_2 \geq 0$, *then* $\pi_1^L > \pi_1^N > \pi_1^F, \pi_2^L > \pi_2^N$, *and* $\pi_2^F > \pi_2^N$.

Proof. See Appendix 1.

Given Lemma 2 (ii) and (iii), we note that taking the first move (i.e., choosing leadership) is a dominating strategy if the rival firm is a strategic substitute, because the rival firm provides the product with a smaller network compatibility effect under quantity competition with consumers' *ex post* expectations.

Table 1. The payoff matrix in the endogenous timing game

Firm 1 \ *Firm 2*	*L (Leader)*	*F (Follower)*
L (Leader)	π_1^N, π_2^N	π_1^L, π_2^F
F (Follower)	π_1^F, π_2^L	π_1^N, π_2^N

Note: We omit superscript Bin the case of price competition in Section 3.1. Furthermore, the profits in the case of quantity competition with consumers' *ex ante* expectations are expressed as: $\pi_i^{FEC} = \pi_i^N, \quad \pi_i^l = \pi_i^L, \quad \pi_i^f = \pi_i^F, \quad i = 1, 2.$

2.4. *Subgame perfect Nash equilibrium in the endogenous timing game*

Applying the extended game with observable delay developed by Hamilton and Slutsky (1990), we demonstrate an endogenous leader–follower relationship. That is, by introducing the stage of timing decision (i.e., choosing either a leader or a follower) before market competition, we derive a subgame perfect Nash equilibrium (SPNE) in an endogenous timing game (see Table 1).[6]

In Lemma 2 (i), if the degree of network compatibility effects of both firms is larger than that of the product substitutability, both firms prefer being a follower to a leader and to playing a simultaneous-move game. In this case, because the reaction curves of both firms are upward sloping, they have a second-mover advantage. Thus, considering Theorem V (A. ii) in Hamilton and Slutsky (1990) and Lemma 1 in Yang et al. (2009), because *firm i* (j) chooses to be a leader (follower), $i, j = 1, 2, \; i \neq j$, there are multiple SPNE in the extended game with observable delay.

In Lemma 2 (ii), if the degree of network compatibility effects of both firms is smaller than that of the product substitutability, both firms prefer being a leader to a follower and to playing a simultaneous-move game. In this case, because the reaction curves of both firms are downward sloping, they have a first-mover advantage. That is, choosing leadership is a dominating strategy for both firms. Thus, considering Theorem V (A. i) in Hamilton and Slutsky (1990) and Lemma 2 in Yang et al. (2009), there is a unique SPNE in the extended game with observable delay.

In Lemma 2 (iii), i.e., $a\alpha_1 > \gamma > a\alpha_2$,there are sufficient asymmetric network compatibility effects between the firms. That is, if the degree of a network compatibility effect for *firm 1* (*2*) is larger (smaller) than that of product substitutability, *firm 1* prefers being a leader to a follower and to playing a simultaneous-move game, whereas *firm 2* prefers being the follower to a leader and to playing a simultaneous-move game. In this case, choosing leadership is a dominating strategy for *firm 1* because a strategic substitute for *firm 2* holds. In other words, the reaction curve of *firm 2* is downward sloping. Thus, considering Theorem V (B) in Hamilton and Slutsky (1990), there is such a unique SPNE in the extended game with observable delay that *firm 1* (*2*) is a leader (follower). See S_1 in Figure 1. Therefore, we derive the following proposition.

[6] See also Figure 1 in Amir and Grilo (1999, p. 5)

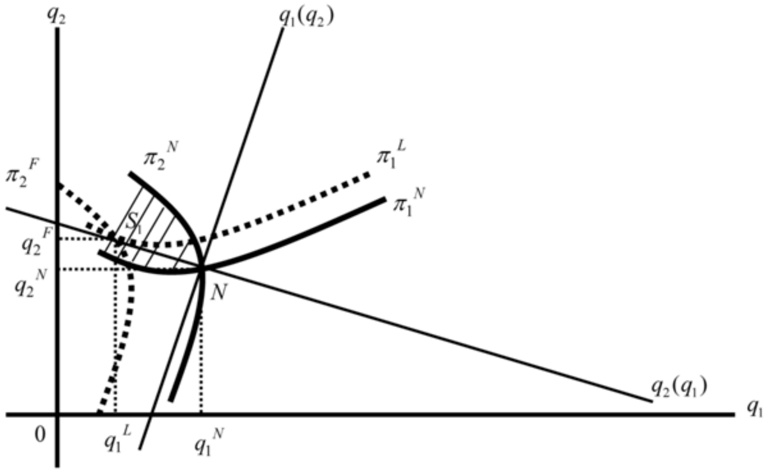

Figure 1. Quantity competition with asymmetric network compatibility effects: $a\alpha_1 > \gamma > a\alpha_2$.

S_1 is a Stackelberg equilibrium where *firm 1* (*2*) is a leader (follower). The shaded area represents the Pareto-superior set.

PROPOSITION 1. *There is a SPNE where the firm providing the product with a larger (smaller) network compatibility effect than product substitutability is a leader (follower) under quantity competition with consumers' ex post expectations.*

If the degree of a network compatibility effect is larger (smaller) than that of product substitutability, i.e., $a\alpha_1 > \gamma > a\alpha_2$, an increase in the output level of the rival firm increases (decreases) the profit of the firm. In this case, it follows that $q_1^N > q_1^L$ and $q_2^F > q_2^N$. Thus, *firm 1* providing the product with a larger network compatibility effect (i.e., the fully compatible product) has a strong incentive to take the lead by committing to the lower output level because an increase in the output level of *firm 2* increases its market price and thus its profit. That is, the large network compatibility effect implies the high ability to absorb demand spillovers from the rival firm. On the other hand, f*irm 2* providing the product with a smaller network compatibility effect (the incompatible product) prefers being a follower to a leader and increases its output level because the lower output level of *firm 1* increases its profit.

Furthermore, the Stackelberg equilibrium is Pareto improving for both firms, when compared with the Cournot–Nash equilibrium. This implies that if there are sufficient asymmetric network compatibility effects between the firms, semicollusion arises in the market whereby the firm providing the product with a larger network compatibility effect commits to the lower output level in advance. The other firm providing the product with a smaller network compatibility effect then sets the higher output level.

3. DISCUSSION

3.1. Price competition

To reconsider the result in the case of quantity competition, i.e., Proposition 1, we examine the case of price competition. If products are substitutes, it is usual that strategic substitutes (complements) hold in the case of quantity (price) competition.

Taking equation (3), we derive the following direct demand function of *firm i.*

$$q_i = \frac{\{(1-a)-(\gamma - a\alpha_i)\}A - (1-a)p_i + (\gamma - a\alpha_i)p_j}{(1-a)^2 - (\gamma - a\alpha_1)(\gamma - a\alpha_2)}, \quad i, j = 1, 2,\ i \neq j\,. \tag{14}$$

Based on equation (14), using the FOC, we derive the reaction function for *firm i* as follows:

$$p_i = \frac{\{(1-a)-(\gamma - a\alpha_i)\}A}{2(1-a)} + \frac{\gamma - a\alpha_i}{2(1-a)}p_j, \quad i, j = 1, 2,\ i \neq j\,. \tag{15}$$

In this case, the strategic relationships depend on the degree of a network compatibility effect and product substitutability. Based on equation (15), we obtain the following Bertrand–Nash equilibrium:

$$p_i^{BN} = \frac{A\left\{2(1-a)^2 - (1-a)(\gamma - a\alpha_i) - (\gamma - a\alpha_1)(\gamma - a\alpha_2)\right\}}{D}, \quad i = 1, 2\,, \tag{16}$$

where $2(1-a)^2 - (1-a)(\gamma - a\alpha_i) - (\gamma - a\alpha_1)(\gamma - a\alpha_2) > 0 i = 1, 2$. Superscript *BN* denotes the Bertrand–Nash equilibrium.

Next, assuming that *firm j* (i) is a leader (follower), we can derive the Stackelberg equilibrium in a sequential-move game as follows:

$$p_j^{BL} = \frac{A\left\{2(1-a)^2 - (1-a)(\gamma - a\alpha_j) - (\gamma - a\alpha_1)(\gamma - a\alpha_2)\right\}}{D - (\gamma - a\alpha_1)(\gamma - a\alpha_2)}\,, \tag{17}$$

$$p_i^{BF} = \frac{A\left[(1-a)H - (\gamma - a\alpha_i)G\right]}{2(1-a)\{D - (\gamma - a\alpha_1)(\gamma - a\alpha_2)\}}\,, \tag{18}$$

where $H \equiv D - 2(\gamma - a\alpha_1)(\gamma - a\alpha_2) > 0$ and $G \equiv D - 2(1-a)^2 > 0$. Because $H - G = 2\left\{(1-a)^2 - (\gamma - a\alpha_1)(\gamma - a\alpha_2)\right\} > 0$ and $1 - a > |\gamma - a\alpha_i|$ hold, it follows that $p_i^{BF} > 0, \quad i = 1, 2$. Superscript *BL* (*BF*) denotes a leader (follower) in the Stackelberg equilibrium in price competition.

Following the same procedure as in the case of quantity competition, we have the results as Lemma 3 and 4 in Appendix 2. In particular, we look at Lemma 4 (iii), i.e., $a\alpha_1 > \gamma > a\alpha_2$,where there is sufficient asymmetry between the firms as for network compatibility effects. In this case, the reaction curve of *firm 1* (2) is downward (upward) sloping and the external effect on the profit of *firm 1* (2) is negative (positive). In this case, choosing leadership is a dominating strategy for *firm 2*, because a strategic substitute for *firm 1* arises. Thus, in view of Lemma 2 (ii) and (iii) and Lemma 4 (i) and (iii), we can state that choosing leadership is a dominating strategy for the firm if the strategic relationship for the rival firm is substitutionary, irrespective of the mode of

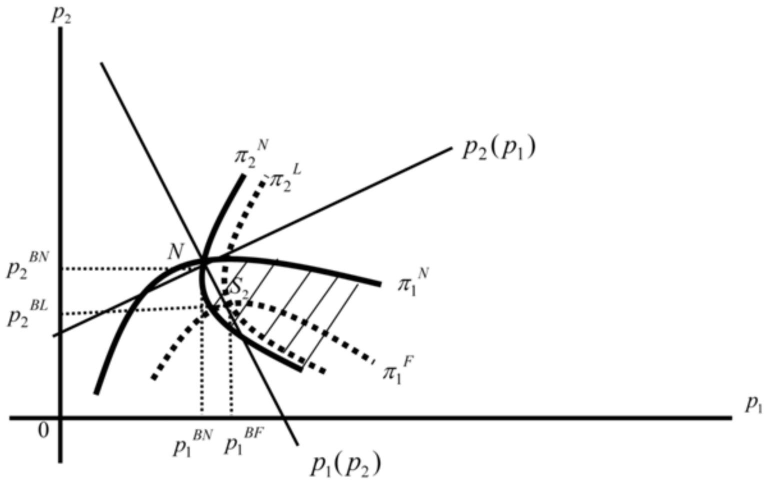

Figure 2. Price competition with asymmetric network compatibility effects: $a\alpha_1 > \gamma > a\alpha_2$.

S_2i s a Stackelberg equilibrium where *firm 2* (*1*) is a leader (follower). The shaded area represents the Pareto-superior set.

competition.

Based on Table 1, and considering Theorem V (B) in Hamilton and Slutsky (1990), the SPNE in endogenous timing of the extended game with observable delay is unique. See S_2 in Figure 2. Therefore, we derive the following proposition.

PROPOSITION 2. *There is a SPNE where the firm providing the product with a smaller (larger) network compatibility effect than product substitutability is a leader (follower) under price competition with consumers' ex post expectations.*

The result in Proposition 2 lies counter to that in Proposition 1. The firm providing the product with a smaller (larger) network compatibility effect than product substitutability is a Stackelberg price leader (follower). That is, if the network compatibility effect is smaller (larger) than a certain level of product substitutability, an increase (decrease) in the price of the rival firm increases the profit of the firm. In this case, in providing the product with a smaller network compatibility effect, *firm 2* has an incentive to take leadership and to commit to the lower price, i.e., $p_2^{BN} > p_2^{BL}$.Conversely, in providing the product with a larger network compatibility effect, *firm 1* prefers being a follower to a leader because the lower price of the rival firm increases its profit, i.e., $\pi_1^{BF} > \pi_1^{BN}$.

Furthermore, without network externalities, the follower's price is less than the leader's price. This outcome is similar to that in a standard Stackelberg price competition. However, in the case of $a\alpha_1 > \gamma > a\alpha_2$, it holds that $p_1^{BF} > p_2^{BL}$. That is, the follower can set a higher price than the leader's price because the follower's product is sufficiently compatible with the leader's product. As a result, it may be usual for the price of a fully compatible product to be higher than that of an incompatible product.

3.2. Consumers' ex ante expectations

Here, we examine the case where consumers make expectations before the output decisions, i.e., the case of quantity competition with consumers' *ex ante* expectations, which is similar to the two-firm case of Economides (1996). This analysis demonstrates whether the different formation of consumer expectations for network size affects the equilibria in market competition, and thus the SPNE in the endogenous timing game.

Using equation (1), the profit function can be represented by:

$$\pi_i = \left\{A - q_i - \gamma q_j + f(S_i^e)\right\} q_i, \quad i, j = 1, 2,\ i \neq j\,.$$

We first derive Cournot–Nash equilibrium in the case of consumers' *ex ante* expectations for network size. Given the expected network size and the other firm's output, the FOC for profit maximization is given by:

$$\frac{\partial \pi_i}{\partial q_i} = A - 2q_i - \gamma q_j + f(S_i^e) = 0, \quad i, j = 1, 2,\ i \neq j\,, \tag{19}$$

and we thus obtain the following reaction function:

$$q_i = \frac{A + f(S_i^e) - \gamma q_j}{2}, \quad i, j = 1, 2,\ i \neq j\,. \tag{20}$$

Following the same procedure as in Katz and Shapiro (1985) and Economides (1996), we derive the fulfilled expectation Cournot (*FEC*) equilibrium. That is, it holds that $q_i^e = q_i$ and $q_j^e = q_j$, and in the *FEC* equilibrium, we have $S_i^e = q_i + \alpha_i q_j, i, j = 1, 2,\ i \neq j$. Thus, we obtain:

$$q_i^{FEC} = \frac{\{(2-a) - (\gamma - a\alpha_i)\}\, A}{(2-a)^2 - (\gamma - a\alpha_1)(\gamma - a\alpha_2)}, \quad i = 1, 2\,. \tag{21}$$

In view of equations (8) and (21), we obtain $q_i^{FEC} < q_i^N$, $i = 1, 2$. That is, the output level (and thus consumer surplus) in the case of consumers' *ex ante* expectations is lower than in the case of consumers' *ex post* expectations. This implies that an *ex ante* rational expectation for network size is not beneficial for consumers.

Second, we consider a Stackelberg game with two stages where *firm i* (j) is a follower (leader). We derive a Stackelberg equilibrium using backward induction. In the second stage, given the output level of *firm* j and the expected network size, *firm* i decides the output level to maximize its profit. Thus, we have equation (19).

In the first stage, taking the reaction function given by equation (20) and the given network sizes of both products, i.e., S_i^e and S_j^e, *firm* j decides the output level to maximize its profit, i.e., $\pi_j = \left\{A - q_j - \gamma q_i + f(S_j^e)\right\} q_j$, where $q_i = \frac{A + f(S_i^e)}{2} - \frac{\gamma}{2} q_j, \quad i, j = 1, 2,\ i \neq j$. In this case, the FOC is given by:

$$\frac{\partial \pi_j}{\partial q_j} = A - 2q_j - \gamma q_i + \frac{\gamma^2}{2} q_j + f(S_j^e) = 0\,, \tag{22}$$

In the Stackelberg equilibrium under the fulfilled expectations, we have equations (20), (22), $q_i^e = q_i$, and $q_j^e = q_j$, $i, j = 1, 2,\ i \neq j$. Thus, we derive the following two equations:

$$A-(2-a)q_i-(\gamma-a\alpha_i)q_j=0\,, \tag{23}$$

$$A-(\gamma-a\alpha_j)q_i-(2-a-\frac{\gamma^2}{2})q_j=0, \quad i,j=1,2,\ i\neq j\,. \tag{24}$$

Therefore, we obtain the fulfilled expectation Stackelberg equilibrium as follows:

$$q_i^f=\frac{\left\{(2-a)-\frac{\gamma^2}{2}-(\gamma-a\alpha_i)\right\}A}{(2-a)^2-\frac{(2-a)\gamma^2}{2}-(\gamma-a\alpha_1)(\gamma-a\alpha_2)}\,, \tag{25}$$

$$q_j^l=\frac{\left\{(2-a)-(\gamma-a\alpha_j)\right\}A}{(2-a)^2-\frac{(2-a)\gamma^2}{2}-(\gamma-a\alpha_1)(\gamma-a\alpha_2)}, \quad i,j=1,2,\ i\neq j\,. \tag{26}$$

Using equations (21), (25), and (26), we obtain the following results:

$$q_j^l>q_j^f\,, q_j^l>q_j^{FEC} \text{ and } q_i^f>(<)q_i^{FEC}\Leftrightarrow a\alpha_i>(<)\gamma, \quad i,j=1,2,\ i\neq j\,. \tag{27}$$

Here, and to consider the implications of the different formation of consumer expectations for network size, we address the behavior of a leader in the Stackelberg game. In view of equations (6) and (20), when deciding the output levels, and whether firms can incorporate the network compatibility effects or not, it is important to understand the different formation of consumer expectations. That is, in the case of consumers' *ex ante* expectations, because the firms cannot affect the network size, as in equation (19), strategic substitutes always arise under quantity competition. Therefore, a leader in a Stackelberg game is going to increase the output level higher than in the Cournot–Nash game.

Conversely, in the case of consumers' *ex post* expectations, because firms can affect the network size, as in equation (6), the strategic relationships depend on the degree of network compatibility effects. This, in turn, affects the firm's output decision. That is, if strategic complements arise, even though under quantity competition, a leader in a Stackelberg game is going to decrease the output level lower than in the Cournot–Nash game.

With respect to the profits, we obtain the following results.

LEMMA 5.

(i) $\pi_i^l>\pi_i^{FEC}, i=1,2.$

(ii) $\pi_i^f>(<)\pi_i^{FEC}\Leftrightarrow a\alpha_i>(<)\gamma, i=1,2.$

Proof. See Appendix 3.

In Lemma 5, suppose $a\alpha_i<\gamma, \quad i=1,2$. In this case, choosing a leader is a dominating strategy for both firms. Thus, there is a unique SPNE in the extended game with observable delay. As a result, the Cournot–Nash equilibrium arises in market competition. Conversely, suppose $a\alpha_i>\gamma, \quad i=1,2$. In this case, both firms prefer being a follower to a leader and to playing a simultaneous-move game. Both firms then have a second-mover advantage. Thus, there are two SPNE in the extended game with observable delay.

Furthermore, suppose that there are sufficient asymmetric network compatibility effects between the firms, i.e., $a\alpha_1 > \gamma > a\alpha_2$. In this case, it holds that $\pi_1^f > \pi_1^{FEC}$ and $\pi_2^f < \pi_2^{FEC}$. This implies that *firm 1* prefers being a follower to playing a simultaneous-move game, whereas *firm 2* prefers playing a simultaneous-move game to being a follower. Thus, choosing leadership is a dominating strategy for *firm 2* because a strategic substitute for *firm 2* arises. As a result, there is such a unique SPNE in the extended game with observable delay that *firm 2* (*1*) takes the role of leader (follower). Let us summarize the result as follows:

PROPOSITION 3. *If consumers make expectations for network size before the firms' decision in quantity competition, there is a SPNE where the firm providing the product with a smaller (larger) network compatibility effect than product substitutability is a leader (follower).*

In the case of consumers' *ex ante* expectations, the firms cannot incorporate the network compatibility effects when deciding the output level. Accordingly, it is the dominating strategy for the firm with a smaller network compatibility effect to commit to the higher output level in advance, i.e., $q_2^l > q_2^{FEC} (> q_2^f)$. This behavior is identical to that of a Stackelberg leader in the familiar quantity competition case. On the other hand, the firm with a larger network compatibility effect is going to set a lower output level. However, because the larger network compatibility effect implies the greater ability to absorb demand spillovers from *firm 2*, it holds in equilibrium that $(q_1^l >) q_1^f > q_1^{FEC}$. As a result, the profit of *firm 1* increases more than in the Cournot–Nash equilibrium.

3.3. *Implications of endogenous leadership in the presence of network compatibility effects*

Assuming that the leader (follower) is an incumbent (entrant), we should first interpret the two points presented by Belleflamme and Peitz (2010, p. 546), as discussed in the introduction. That is, (1) there is a first-mover advantage related to the launch of network goods; and (2) a follower tends to favor compatibility, while a leader tends to prefer incompatibility. Because competing firms do not perfectly control (or conjecture) consumer expectations for network size at the start point in new network products and services markets, e.g., Betamax vs. VHS in video recorder markets, Apple vs. Microsoft in personal computer markets, and i-OS vs. Android operating systems, they cannot affect the network size of their products. This situation may correspond to the case of quantity competition with consumers' *ex ante* expectations in Section 3.2. In this case, we demonstrate that a firm providing an incompatible product is a leader while a firm providing a compatible product is a follower (see Proposition 3).

Conversely, we envisage that firms compete for new customers because each firm already has an *installed base* of customers from past competition or competition in the markets for other areas (countries). In this case, competing firms can control and affect the network size. This corresponds to the case of consumers' *ex post* expectations (Section 2 and 3.1). For example, we observe more severe price cutting competition between Internet service providers and mobile phone companies providing perfectly compatible services. As in Section 3.1, this case is one in which the firms choose leadership in price competition and thus a Bertrand–Nash equilibrium arises in a simultaneous-move game. In other words, there may not be endogenous leadership in the case of price competition with compatible products. However, as in Proposition 2, for example, even with price competition, we can appreciate that Apple, in providing an incompatible product, may be a leader (first mover), while Google, in providing a compatible product, is a follower (second mover).

Finally, we consider the case of an automated teller machine (ATM) network, comprising both ATMs and bank cards, such that ATMs and bank cards are complementary products. However, ATMs are substitutes for one another, as are different bank cards.[7] Suppose there is competition between the ATMs network services of a regional small-sized bank and a mega bank in a single local area. One customer, who is a member of the regional bank, uses the ATM terminals of both banks with no fees, whereas another customer, who is a member of the mega bank, cannot use an ATM terminal of the regional bank without additional fees. Therefore, if we assume that the regional bank provides compatible services of the ATM network while the mega bank provides incompatible services, we may say that the regional bank is a leader of ATM networks in the local area while the mega bank is a follower (see Proposition 1).

4. CONCLUSION

In this paper, we consider the formation of an endogenous leadership in a network products and services market. We show that the properties of a firm-specific product, i.e., product substitutability and network compatibility, determine the strategic relationships and the external effects on profits. Furthermore, the formation of consumer expectations before (after) the output decisions, i.e., *ex ante* (*ex post*) expectations for network size, and the mode of competition, i.e., quantity or price, affect the endogenous leader–follower relationships.

In the case of consumer expectations following the output decisions, we demonstrate that given sufficient asymmetric network compatibility effects between products, the firm providing the product with a larger (smaller) network compatibility effect than a certain level of product substitutability is the Stackelberg leader under quantity (price) competition. Furthermore, if we suppose that a firm providing a fully compatible product is a small firm, whereas a firm providing an incompatible product is a large firm, we can say that in the markets displaying strong network externalities, to increase profit, the small firm should commit to its production in quantity competition but delay setting its price in price competition.

However, in the case of quantity competition with consumer expectations preceding the output decisions, as pointed out in Belleflamme and Peitz (2010, p. 546), the firm (incumbent) providing an incompatible product is a leader, whereas the firm (entrant) providing a fully compatible product is a follower.

We appreciate that our model depends on specific assumptions, e.g., linearity of the

[7] ATM network competition may not completely correspond to the case of quantity competition.

functions. However, by focusing on the properties of the products associated with network externalities and compatibility, we have illustrated precisely which effects of these properties determine the endogenous distribution of roles. Furthermore, the assumption of a strong network externality is very strong. Otherwise, strategic substitutes (complements) always arise under quantity (price) competition, and our main results do not hold.

Although we assume exogenously determined product compatibility, we should consider the level of compatibility as a firm's strategic variable. Thus, we should examine Stackelberg leadership in the context of endogenous product compatibility choice (see Maggi, 1996). For similar reasons, although we assume that asymmetric network compatibility is also exogenously given, we should consider how asymmetric network compatibility arises between firms.

In relation to these points, if we assume that an "open resource strategy" corresponds to the choice of perfectly compatible network goods and services and a "closed (control) strategy" corresponds to the choice of incompatible network goods and services, we can consider endogenous leadership in the context of an "open resource strategy" and a "closed (control) strategy." That is, we may say that in the Internet business, Apple, in choosing a closed (control) strategy, becomes a leader (first mover), and Google, in choosing an open resource strategy becomes, a follower (second mover).

Appendix

Appendix 1. Proof of Lemma 2[8]

Taking as given the levels of output in the Cournot–Nash and Stackelberg equilibria, we compare the profits. We define the profit of *firm i* in the Cournot–Nash equilibrium as follows:

$$\pi_i^N \equiv \pi_i\left(q_i^N, q_j^N\right), \quad i, j = 1, 2, \quad i \neq j\,. \tag{A.1}$$

Furthermore, if *firm i* is a leader, its profit can be represented by: $\pi_i^L \equiv \pi_i\left(q_i^L, q_j(q_i^L)\right)$. Similarly, if *firm i* is a follower, its profit is $\pi_i^F \equiv \pi_i\left(q_i(q_j^L), q_j^L\right), i, j = 1, 2, \quad i \neq j$.

Using equations (6), (7), and Lemma 1, with respect to the following three cases, we compare the profit as a leader (or a follower) in the Stackelberg equilibrium and that in the Cournot–Nash equilibrium.

Case (i) $1 > a\alpha_1 > a\alpha_2 > \gamma$.

Suppose *firm i* is a leader. In this case, because it holds that $\frac{dq_j}{dq_i} > 0$ and $\frac{\partial \pi_i}{\partial q_j} > 0$, $i, j = 1, 2,\ i \neq j$, we derive:

$$\left.\frac{d\pi_i}{dq_i}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial \pi_i}{\partial q_i}\right|_{(q_i^N, q_j^N)} + \left.\frac{\partial \pi_i}{\partial q_j}\frac{dq_j}{dq_i}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial \pi_i}{\partial q_j}\frac{dq_j}{dq_i}\right|_{(q_i^N, q_j^N)} > 0\,. \tag{A.2}$$

Thus, we have $\pi_i^L > \pi_i^N$, because $q_i^L > q_i^N, i, j = 1, 2,\ i \neq j$.

[8] With respect to the proof of Lemma 2, we follow the procedure in Jinji (2004).

Next, suppose *firm i* is a follower. In this case, from Lemma 1 (i), it follows that $q_j^L > q_j^N$, $j = 1, 2$. Based on Lemma 1 (i), we similarly derive:

$$\left.\frac{d\pi_i}{dq_j}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_i}{\partial q_j}\right|_{(q_i^N, q_j^N)} + \left.\frac{\partial\pi_i}{\partial q_i}\frac{dq_i}{dq_j}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_i}{\partial q_j}\right|_{(q_i^N, q_j^N)} > 0\,. \tag{A.3}$$

Thus, it holds that $\pi_i^F > \pi_i^N, \quad i = 1, 2$. Therefore, it holds that $\pi_i^L > \pi_i^N, \quad \pi_i^F > \pi_i^N$, $i = 1, 2$.

Case (ii) $\gamma > a\alpha_1 > a\alpha_2 \geq 0$.

Suppose *firm i* is a leader. In this case, because it holds that $\frac{dq_j}{dq_i} < 0$ and $\frac{\partial\pi_i}{\partial q_j} < 0$, $i, j = 1, 2,\ i \neq j$,we derive:

$$\left.\frac{d\pi_i}{dq_i}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_i}{\partial q_i}\right|_{(q_i^N, q_j^N)} + \left.\frac{\partial\pi_i}{\partial q_j}\frac{dq_j}{dq_i}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_i}{\partial q_j}\frac{dq_j}{dq_i}\right|_{(q_i^N, q_j^N)} > 0\,. \tag{A.4}$$

In this case, from Lemma 1 (ii), it holds that $q_i^L > q_i^N$, $i = 1, 2$. Thus, we have $\pi_i^L > \pi_i^N$, $i = 1, 2$.

Next, suppose *firm i* is a follower. In this case, from Lemma 1 (ii), it follows that $q_j^L > q_j^N, \quad j = 1, 2$. Based on Lemma 1 (ii), we derive:

$$\left.\frac{d\pi_i}{dq_j}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_i}{\partial q_j}\right|_{(q_i^N, q_j^N)} + \left.\frac{\partial\pi_i}{\partial q_i}\frac{dq_i}{dq_j}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_i}{\partial q_j}\right|_{(q_i^N, q_j^N)} < 0\,. \tag{A.5}$$

Thus, it holds that $\pi_i^N > \pi_i^F, \quad i = 1, 2$. Therefore, it follows that $\pi_i^L > \pi_i^N > \pi_i^F,\ i = 1, 2$.

Case (iii) $1 > a\alpha_1 > \gamma > a\alpha_2 \geq 0$.

First, suppose *firm 1* (*2*) is a leader (follower). For *firm 1*, we obtain:

$$\left.\frac{d\pi_1}{dq_1}\right|_{(q_1^N, q_2^N)} = \left.\frac{\partial\pi_1}{\partial q_1}\right|_{(q_1^N, q_2^N)} + \left.\frac{\partial\pi_1}{\partial q_2}\frac{dq_2}{dq_1}\right|_{(q_1^N, q_2^N)} = \left.\frac{\partial\pi_1}{\partial q_2}\frac{dq_2}{dq_1}\right|_{(q_1^N, q_2^N)} < 0\,, \tag{A.6}$$

because $\frac{\partial\pi_1}{\partial q_2} > 0$ and $\frac{dq_2}{dq_1} < 0$. Based on Lemma 1 (iii), because it holds that $q_1^N > q_1^L$, we have $\pi_1^L > \pi_1^N$.

Furthermore, for *firm 2*, we derive:

$$\left.\frac{d\pi_2}{dq_1}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_2}{\partial q_1}\right|_{(q_i^N, q_j^N)} + \left.\frac{\partial\pi_2}{\partial q_2}\frac{dq_2}{dq_1}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_2}{\partial q_1}\right|_{(q_i^N, q_j^N)} < 0\,. \tag{A.7}$$

Because, based on Lemma 1 (iii), it holds that $q_1^N > q_1^L$, we have $\pi_2^F > \pi_2^N$.

Second, suppose *firm 2* (*1*) is a leader (follower). In this case, for *firm 2*, we obtain:

$$\left.\frac{d\pi_2}{dq_2}\right|_{(q_1^N, q_2^N)} = \left.\frac{\partial\pi_2}{\partial q_2}\right|_{(q_1^N, q_2^N)} + \left.\frac{\partial\pi_2}{\partial q_1}\frac{dq_1}{dq_2}\right|_{(q_1^N, q_2^N)} = \left.\frac{\partial\pi_2}{\partial q_1}\frac{dq_1}{dq_2}\right|_{(q_1^N, q_2^N)} < 0\,. \tag{A.8}$$

because $\frac{\partial\pi_2}{\partial q_1} < 0$ and $\frac{dq_1}{dq_2} > 0$. Based on Lemma 1 (iii), it holds that $q_2^N > q_2^L$. Thus, we have $\pi_2^L > \pi_2^N$.

Furthermore, for *firm 1*, we derive:

$$\left.\frac{d\pi_1}{dq_2}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_1}{\partial q_2}\right|_{(q_i^N, q_j^N)} + \left.\frac{\partial\pi_1}{\partial q_1}\frac{dq_1}{dq_2}\right|_{(q_i^N, q_j^N)} = \left.\frac{\partial\pi_1}{\partial q_2}\right|_{(q_i^N, q_j^N)} > 0\,. \tag{A.9}$$

Because it holds that $q_2^N > q_2^L$, we have $\pi_1^N > \pi_1^F$. Therefore, it holds that $\pi_1^L > \pi_1^N > \pi_1^F, \quad \pi_2^L > \pi_2^N$, and $\pi_2^F > \pi_2^N$.

APPENDIX 2. THE CASE OF PRICE COMPETITION

Using equations (15), (16), and (17), we derive the following relationships:

$$p_j^{BL} > (<) p_j^{BN} \Leftrightarrow (\gamma - a\alpha_i)(\gamma - a\alpha_j) > (<) 0\,, \tag{A.10}$$

$$p_j^{BF} > (<) p_j^{BN} \Leftrightarrow \gamma - a\alpha_i > (<) 0\,, \tag{A.11}$$

$$p_j^{BL} > (<) p_j^{BF} \Leftrightarrow (\gamma - a\alpha_i)(\gamma - a\alpha_j) > (<) 0\,, \tag{A.12}$$

where $i, j = 1, 2,\ i \neq j$. Thus, using equations (A.10), (A.11), and (A.12), we obtain the following results.

LEMMA 3.

(i) *If* $1 > a\alpha_1 > a\alpha_2 > \gamma$, $p_i^{BL} > p_i^{BN} > p_i^{BF}, i = 1, 2$.
(ii) *If* $\gamma > a\alpha_1 > a\alpha_2 \geq 0$, $p_i^{BL} > p_i^{BF} > p_i^{BN}, \quad i = 1, 2$.
(iii) *If* $1 > a\alpha_1 > \gamma > a\alpha_2 \geq 0$, $p_1^{BF} > p_1^{BN} > p_1^{BL}$, *and* $p_2^{BN} > p_2^{BF} > p_2^{BL}$.

Furthermore, based on Lemma 3, using the same procedure as in Appendix 1, and equations (16) and (17), we derive the following relationships regarding the profits in the Bertrand–Nash and the Stackelberg equilibria.

LEMMA 4.

(i) *If* $1 > a\alpha_1 > a\alpha_2 > \gamma, \pi_i^{BL} > \pi_i^{BN} > \pi_i^{BF}, i = 1, 2$.
(ii) *If* $\gamma > a\alpha_1 > a\alpha_2 \geq 0, \pi_i^{BL} > \pi_i^{BN}, \pi_i^{BF} > \pi_i^{BN}, i = 1, 2$.
(iii) *If* $1 > a\alpha_1 > \gamma > a\alpha_2 \geq 0, \pi_1^{BL} > \pi_1^{BN}, \pi_1^{BF} > \pi_1^{BN}$, *and* $\pi_2^{BL} > \pi_2^{BN} > \pi_2^{BF}$.

Lemma 4 (i) implies that strategic substitutes for both firms arise. In this case, choosing leadership is a dominating strategy for both firms. Thus, there is a unique SPNE in the endogenous timing game.

Conversely, in Lemma 4 (ii), the slope and the external effect on profit are positive for both firms. Both firms then prefer being a follower to a leader and to playing a simultaneous-move game. As per the usual case in price competition, both firms have a second-mover advantage. Thus, considering Theorem V (A. ii) in Hamilton and Slutsky (1990) and Lemma 1 in Yang et al. (2009), there are two SPNE in the extended game with observable delay.

APPENDIX 3. PROOF OF LEMMA 5

We first consider the relationship between the leader's profit in the Stackelberg equilibrium and that in the *FEC* equilibrium.

The profit of *firm j* in the *FEC* equilibrium is expressed as:

$$\pi_j^{FEC} = \pi_j\left[q_j^{FEC}, q_i^{FEC}\right] = \left\{A - (1-a)q_j^{FEC} - (\gamma - a\alpha_j)q_i^{FEC}\right\} q_j^{FEC}\,,$$
$$i, j = 1, 2,\ i \neq j\,.$$

Similarly, the profit of *firm* j being a leader in the Stackelberg equilibrium is also given by:

$$\pi_j^l = \pi_j\left[q_j^l, q_i^f\right] = \left\{A - (1-a)q_j^l - (\gamma - a\alpha_j)q_i^f\right\} q_j^l, \quad i, j = 1, 2,\ i \neq j\,.$$

We now deal with the following profit function of *firm j*:

$$\pi_j = \pi_j\left[q_j, q_i\right] = \left\{A - (1-a)q_j - (\gamma - a\alpha_j)q_i\right\} q_j, \quad i, j = 1, 2,\ i \neq j\,. \tag{A.13}$$

In this case, totally differentiating equation (A.13), we have:

$$\begin{aligned} d\pi_j &= \frac{\partial \pi_j}{\partial q_j} dq_j + \frac{\partial \pi_j}{\partial q_i} dq_i \\ &= \left\{A - 2(1-a)q_j - (\gamma - a\alpha_j)q_i\right\} dq_j - (\gamma - a\alpha_j)q_j dq_i\,. \end{aligned} \tag{A.14}$$

When evaluating equation (A.14) at the *FEC* equilibrium, it holds that:

$$A - (2-a)q_j - (\gamma - a\alpha_j)q_i = 0, \quad i, j = 1, 2,\ i \neq j\,. \tag{A.15}$$

Taking equations (A.14) and (A.15), we can derive as follows:

$$\begin{aligned} \left.\frac{d\pi_j}{dq_j}\right|_{(q_j^{FEC}, q_i^{FEC})} &= \left.\frac{\partial \pi_j}{\partial q_j}\right|_{(q_j^{FEC}, q_i^{FEC})} + \left.\frac{\partial \pi_j}{\partial q_i}\frac{dq_i}{dq_j}\right|_{(q_j^{FEC}, q_i^{FEC})} \\ &= \left\{a - (\gamma - a\alpha_j)\left.\frac{dq_i}{dq_j}\right|_{(q_j^{FEC}, q_i^{FEC})}\right\} q_j^{FEC}\,, \end{aligned} \tag{A.16}$$

where $\left.\frac{\partial \pi_j}{\partial q_j}\right|_{(q_j^{FEC}, q_i^{FEC})} = aq_j^{FEC} (> 0)$ is not zero.

Furthermore, in view of equation (A.15), we derive $\left.\frac{dq_i}{dq_j}\right|_{(q_j^{FEC}, q_i^{FEC})} = -\frac{\gamma - a\alpha_i}{2-a}$. Thus, as for the second part of the equation in (A.16), we obtain the following relationship:

$$G \equiv a + \frac{(\gamma - a\alpha_i)(\gamma - a\alpha_j)}{2-a} > (<) 0\,. \tag{A.17}$$

With respect to equation (A.17), if either $\gamma > a\alpha_i, a\alpha_j$ or $\gamma < a\alpha_i, a\alpha_j$, it holds that $\operatorname{sgn} G > 0$. Since $q_j^l > q_j^{FEC}$, i.e., $dq_j > 0$, it holds that $\left.\frac{d\pi_j}{dq_j}\right|_{(q_j^{FEC}, q_i^{FEC})} > 0$. This implies that $\pi_j^l > \pi_j^{FEC}, \quad j = 1, 2.$

If either $a\alpha_j > \gamma > a\alpha_i$ or $a\alpha_i > \gamma > a\alpha_j$, the sign of $\frac{(\gamma - a\alpha_i)(\gamma - a\alpha_j)}{2-a}$ is negative. Because $1 \geq \alpha_i + \alpha_j$, we derive the following equation:

$$\operatorname{sgn} G = \operatorname{sgn}\{(2-a)a + \gamma(\gamma - a)\} \ . \tag{A.18}$$

Given Assumption 1, i.e., $1 > a > \gamma$, because it holds that $a(2 - a - \gamma) + \gamma^2 > 0$, we have $\operatorname{sgn} G > 0$. Therefore, we derive *Lemma 5 (i)*: $\pi_j^l > \pi_j^{FEC}, \quad j = 1, 2$.

To sum up, in the case where consumers make expectations for network size before firms' output decisions, the profit of a leader in the Stackelberg equilibrium is larger than that in the *FEC* equilibrium, regardless of the degree of network compatibility effects.

Taking equations (20) and (24), we can directly obtain the following relationship:

$$\pi_i^f > (<)\pi_i^{FEC} \Leftrightarrow q_i^f > (<)q_i^{FEC} \Leftrightarrow a\alpha_i > (<)\gamma, \ i = 1, 2 \,.$$

Therefore, we derive *Lemma 5* (ii).

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