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UNIONISED LABOUR MARKET, EFFICIENCY WAGE AND ENDOGENOUS GROWTH

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Abstract: In this paper, we analyse the effect of unionisation in the labour market on the growth rate of the economy in the presence of 'Efficiency Wage Hypothesis'. We use both 'Efficient Bargaining' model and 'Right to Manage' model to solve the negotiation problem. Unionisation raises the negotiated wage rate as well as the effort (efficiency) level of the worker. In the case of 'Efficient Bargaining' model, unionisation in the labour market in general lowers the number of workers if the labour union is not highly employment oriented. However, irrespective of labour union's orientation, it raises the effort (efficiency) level per worker. As a result, if the labour union is not highly wage oriented, effective employment measured in efficiency unit is increased; and this leads to a rise in the growth rate of the economy. However, in the 'Right to Manage' model of bargaining, unionisation in the labour market raises worker's effort level but lowers the number of workers irrespective of the orientation of the labour union; and raises effective employment and balanced growth rate if the exogenously given income tax rate is lower than a critical value.

Key words: Labour union, efficiency wage hypothesis, endogenous growth, 'efficient bargaining' model, 'right to manage' model.

JEL Classification Number: J51, O41, J31.

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1. INTRODUCTION

In recent years, many European countries are suffering from high unemployment rate as well as from low economic growth rate. The school of thought in favour of free market, blames the prominent presence of labour unions in European countries for this situation; and it also opines in favour of reducing labour market frictions and raising workers' efficiency to cope up with this problem. However, a number of studies have estimated positive union productivity differentials for unionized firms using industry or firm level data.[1] Moreover, Asteriou and Monastiriotis (2001) finds a positive economy-wide effect of unionism on productivity and productivity growth in 18 OECD countries utilizing newly developed econometric methods for the estimation of dynamic panel models. These empirical results opposing the general view against trade unionism makes it very important to theoretically analyse the effect of unionisation in the labour market on the rate of economic growth.

There exists a set of theoretical literature[2] dealing with the effect of unionisation on the long run growth rate of the economy. However, these works do not focus on the role of 'Efficiency Wage Hypothesis'.[3,4] Since unionisation raises the wage rate, and according to 'Efficiency Wage Hypothesis', effort (efficiency) level per worker should rise[5]; and this may produce an overall positive effect on the production level. The empirical literature also confirms the strong existence of 'Efficiency Wage Hypothesis'. Peach and Stanley (2009) makes an empirical analysis on efficiency wage and labour productivity. The survey paper of Cooper and Kagel (2009) provides vast empirical evidence for efficiency wages in the literature of experimental economics, such as, Fehr et al. (1993), Cooper and Lightle (2013), Gneezy (2004), Falk et al. (2008) etc. Several other experimental studies such as Fehr et al. (1998), Charness (1996, 2000), Fehr and Falk (1999), Gächter and Falk (2002), Falk et al. (1999), Hannan et al. (2002), Brandts and Charness (2004) and Fehr et al. (2004) report that the mean effort is, in general, positively related to the offered wage; and this is consistent with the interpretation that workers, on average, work more when higher wages are offered. Rotemberg (2006) is also an important survey paper to be mentioned in this context. So it has become very important to analyse the effect of unionisation in the labour market in the presence of this efficiency wage hypothesis. Many theoretical works consider the role of efficiency wage on union firm bargaining; and the set includes Garino and Martin (2000), Marti (1997),

[1] Some examples are Brown and Medoff (1978), Clark (1980) and Gregg et al. (1993).

[2] See, for example, Bräuninger (2000), Lingens (2003a, 2003b), Irmen and Wigger (2002/2003), Sorensen (1997), Palokangas (1996, 2004), Chang et al. (2007), Adjemian et al. (2010), Lai and Wang (2010), Ramos-Parreño and Sánchez-Losada (2002) etc.

[3] See, for example, Solow (1979), Yellen (1984), Stiglitz (1976), Shapiro and Stiglitz (1984), Akerlof (1982, 1984), Akerlof and Yellen (1986) etc. for a discussion on efficiency wage hypothesis.

[4] An earlier version of Palokangas (2004), i.e., Palokangas (2003) incorporates 'Efficiency Wage Hypothesis' in his model, but does not emphasis on its role while determining the effect of unionisation. In fact, in a footnote in that paper, the author states, "*However, the results in this paper hold even if the effort per worker is wholly inflexible... ...*". The published version of the paper, i.e., Palokangas (2004) does not incorporate the 'Efficiency Wage Hypothesis'.

[5] See sections 9.2 and 9.3 of Romer (2006).

Mauleon and Vannetelbosch (2003) and Pereau and Sanz (2006). Meeusen et al. (2011) and Lindbeck and Snower (1991) also analyse the implication of efficiency wage theory in labour economics. However, none of them analyses the effect of unionisation on economic growth.

The present paper attempts to develop a model to analyse the effect of unionisation in the labour market on economic growth in the presence of 'Efficiency Wage Hypothesis'. The model developed here is an AK model with a unionised labour market and with an unemployment benefit scheme. It is an extension of the model of Chang et al. (2007) with 'Efficiency Wage Hypothesis'. In this model, we use two alternative versions of bargaining models — the 'Efficient Bargaining' model of McDonald and Solow (1981) and the 'Right to Manage' model of Nickell and Andrews (1983).

We combine the 'Efficiency Wage Hypothesis' and union firm wage bargaining theories in a unified model because they may be either mutually reinforcing[6] or conflicting.[7] The reinforcing effect takes place because 'Efficiency Wage Hypothesis' makes it easier for the union to raise wage in a bargaining environment. The adverse effect of rent sharing is reduced because higher labour productivity is associated with higher wage. In contrast, the conflicting effect indicates that the greater is the labour union's bargaining strength, the less incentives for firms to drive up wages due to efficiency—wage consideration. Our analysis in this paper provides support to the reinforcing effect hypothesis.

We derive interesting results from this model. In the 'Efficient Bargaining' model, unionisation in the labour market in general lowers the number of workers if the labour union is not highly employment oriented. However, irrespective of labour union's orientation, it raises the effort (efficiency) level per worker. As a result, if the labour union is not highly wage oriented, effective employment measured in efficiency unit is increased; and this leads to a rise in the growth rate of the economy. However, in the 'Right to Manage' model, unionisation raises worker's effort level but lowers the number of workers irrespective of the orientation of the labour union; and raises effective employment and balanced growth rate if the rate of income tax used to finance unemployment benefit expenditure is very lower than a critical value.

The paper is organised as follows. In section 2, we describe the basic model and analyse the effect of unionisation with 'Efficient Bargaining'. In section 3, we do the same with a 'Right to Manage' model. Section 4 concludes the paper.

2. THE BASIC MODEL WITH 'EFFICIENT BARGAINING'

The representative competitive firm produces the final good, Y, using private capital, K, and effective labour, eL; and its production function[8] is given by

$$Y = AK^{\alpha}(eL)^{\beta}\bar{K}^{1-\alpha}\text{satisfying } \alpha, \beta, \alpha+\beta \in (0,1)\,. \tag{1}$$

[6] See, for example, Summers (1988), Garino and Martin (2000), Meeusen et al. (2011).

[7] See, for example, Lindbeck and Snower (1991).

[8] This production function is identical to that in Chang et al. (2007) except for the fact that Chang et al. (2007) does not consider effort of workers, e.

Here $A > 0$ is a time independent technology parameter. $\bar{K}$ represents the average amount of capital stock of all firms available in the economy; and $0 < \alpha < 1$ ensures that external effect of capital is positive. Here L denotes the number of workers and e represents the efficiency (effort) level of the representative worker.[9] Existence of decreasing returns to private inputs in the production technology leads to a positive super normal profit in the competitive equilibrium; and this acts as the rent in the bargaining process to be negotiated between the employers' association and the labour union. Following Chang et al. (2007), we assume that a fixed quantity of land is necessary for a firm to operate; and thus the number of firms is fixed even in the presence of positive profit.[10]

We introduce the 'Efficiency Wage Hypothesis'[11] which states that the efficiency level of a worker, e, varies positively with the premium of wage over his alternative reservation income. For simplicity, we assume that the wage income and the reservation income, which is equal to unemployment benefit per unemployed worker, b, in this model, are taxed at equal rates. So the worker's effort function is given by [12]

$$e = h\left(\frac{[1-\tau]w}{[1-\tau]b}\right)^{\delta} = h\left(\frac{w}{b}\right)^{\delta} . \tag{2}$$

Here τ is the rate of income tax; and h is a positive parameter representing worker's effort level when $\delta = 0$. Here δ represents the elasticity of effort with respect to the relative wage rate; and it is assumed to satisfy $0 < \delta < 1$. Chang et al. (2007) does not consider 'Efficiency Wage Hypothesis'. In Chang et al. (2007), $e \equiv 1$, i.e., $\delta = 0$ and $h = 1$.

The firm maximises profit, π, defined as

$$\pi = Y - wL - rK . \tag{3}$$

Here w and r represent the wage rate and the rental rate on capital respectively.

Capital market is perfectly competitive. The equilibrium value of the rental rate on capital is determined by the supply-demand equality in the capital market. The demand function for capital is derived from firms' profit maximisation exercise; and it is given by

$$r = \alpha A K^{\alpha-1}(eL)^{\beta}\bar{K}^{1-\alpha} = \frac{\alpha Y}{K} . \tag{4}$$

The government finances the unemployment benefit scheme by imposing an exogenously given rate of income tax, τ; and balances its budget at each point of time. The budget balancing equation is given by

$$\tau Y + \tau b\,(1-L) = b\,(1-L) . \tag{5}$$

[9] We assume that all workers have identical effort levels.

[10] Number of firms is normalized to unity. The equilibrium in the product market is always a short run competitive equilibrium with positive profit. Lai and Wang (2010) and Chang et al. (2007) also assume that union — firm bargaining takes place in such competitive production sector with decreasing returns.

[11] See footnote 5.

[12] Danthine and Kurmann (2006) has also used almost similar functional form.

Here $(1 - L)$ is the unemployment level; and this equation solves for b in terms of L and τ. Here unemployment benefits are taxed at equal rate. Empirically, these may be taxable or may be exempted from tax.[13] Major results obtained in this paper remain unchanged if unemployment benefits are assumed to be exempted from tax or are taxed at different rates. We consider taxation on unemployment benefits at the same rate for the sake of analytical simplicity. Even if the tax rates are different but exogenous, $b(1 - L)$ would be proportional to Y.

The labour union in this model derives utility from the hike in the after tax wage rate over the after tax unemployment benefit rate[14] as well as from the size of the membership. All employed workers are assumed to be members of the union as it is a closed shop labour union. The utility function of the labour union is given by

$$u_T = ([1-\tau]w - [1-\tau]b)^m L^n = [1-\tau]^m (w-b)^m L^n \,. \tag{6}$$

Here u_T stands for the level of utility of the labour union. m and n represent elasticities of labour union's utility with respect to wage premium and with respect to number of members respectively. The labour union is said to be 'wage oriented' ('employment oriented') ('neutral') if $m > (<)(=)\, n$. Chang et al. (2007) contains a brief discussion about these parameters.

We now consider the 'Efficient Bargaining' model where the wage rate and the number of employed workers are determined jointly by the labour union and the firm at the firm level. They maximise the generalised Nash product function given by

$$\psi = (u_T - \overline{u_T})^\theta (\pi - \bar{\pi})^{(1-\theta)} \quad \text{satisfying} \quad 0 < \theta < 1 \,. \tag{7}$$

Here $\overline{u_T}$ and $\bar{\pi}$ stand for the reservation utility level of the labour union and the reservation profit level of the firm respectively. Bargaining disagreement discontinues production process and this implies $\overline{u_T} = \bar{\pi} = 0$. The relative bargaining power of the labour union is represented by θ. Unionisation is defined as an exogenous increase in the relative bargaining power of the labour union, i.e. in the value of θ.

Finally, using equations (3), (6) and (7), we obtain

$$\psi = \{(1-\tau)^m (w-b)^m L^n\}^\theta \{Y - wL - rK\}^{(1-\theta)} \,. \tag{8}$$

Here ψ is to be maximised with respect to w and L. Using equations (1), (2), (4) and (5), and two first order conditions of optimisation, we solve for optimal w and L.[15] These are given by

$$L^* = \frac{(1-\tau)\Theta_2\Theta_4}{(1-\tau)\Theta_2\Theta_4 + \tau\Theta_1\Theta_3} \,; \tag{9}$$

and

[13] We are indebted to a referee of this journal who points out this aspect.

[14] Irmen and Wigger (2003), Lingens (2003a) and Lai and Wang (2010) assume that the difference between the bargained wage rate and the competitive wage rate is an argument in the labour union's utility function. Contrary to this, Adjemian et al. (2010) and Chang et al. (2007) consider the difference between the bargained wage rate and the unemployment benefit; and we follow them.

[15] Derivation of optimal w and L is provided in Appendix A.

$$w^* = \frac{AK^{\alpha}h^{\beta}\bar{K}^{1-\alpha}(1-\tau)^{\beta-1}}{([1-\tau]\Theta_2\Theta_4+\tau\Theta_1\Theta_3)^{\beta-1}}\left(\frac{\Theta_2^{\beta}\Theta_3^{\beta\delta}}{\Theta_1{\Theta_4}^{1-\beta(1-\delta)}}\right). \tag{10}$$

Here,

$$\Theta_1 = (1-\theta+\theta n) > 0\,, \tag{11}$$

$$\Theta_2 = [\theta n(1-\alpha)+\beta(1-\theta)] > 0\,, \tag{12}$$

$$\Theta_3 = [\theta n(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)] > 0 \tag{13}$$

and

$$\Theta_4 = [\theta(n-m)\,(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]\,. \tag{14}$$

We assume Θ_4 to be positive to ensure $0 < L^* < 1$. This assumption implies that the elasticity of union's utility with respect to relative wage cannot be far greater than the corresponding elasticity with respect to the size of membership. If the union is neutral or employment oriented, i.e., $m \leq n$, then Θ_4 is always positive. From equation (9), we obtain

$$\frac{\partial L^*}{\partial \tau} = -\frac{\Theta_2\Theta_4\Theta_1\Theta_3}{[(1-\tau)\Theta_2\Theta_4+\tau\Theta_1\Theta_3]^2} < 0\,. \tag{9.a}$$

Equation (9.a) shows that the number of employed workers varies inversely with the rate of income tax used to finance unemployment benefit. This is so because a rise in the tax rate raises unemployment benefit per worker; and this lowers union's utility from the wage hike. As a result, the wage rate is increased and the employment level is reduced.

Now, from equations (1), (2), (5), (9) and (10), we obtain the effort level per worker as given by[16]

$$e^* = h\left(\frac{\Theta_3}{\Theta_4}\right)^{\delta}. \tag{15}$$

From equations (9) and (15), we obtain effective level of employment i.e., the level of employment in efficiency unit. It is given by

$$e^*L^* = h\frac{(1-\tau)\Theta_2\Theta_4^{1-\delta}\Theta_3^{\delta}}{(1-\tau)\Theta_2\Theta_4+\tau\Theta_1\Theta_3}\,. \tag{16}$$

The representative household derives instantaneous utility only from consumption of the final good.[17] She maximises her discounted present value of instantaneous utility over the infinite time horizon subject to the intertemporal budget constraint. The household's problem is given by the following.

$$Max\int_0^{\infty}\frac{c^{1-\sigma}-1}{1-\sigma}\exp(-\rho t)dt \tag{17}$$

subject to, $\quad \dot{K} = (1-\tau)\,[wL+rK+\pi]+(1-\tau)\,b\,(1-L)-c \qquad (18)$

[16] Derivation is provided in Appendix B.

[17] For simplicity, we do not incorporate the disutility stemming from work effort, e, into the representative household's utility function.

and $\quad K(0) = K_0 \quad (K_0 \text{ is historically given})\,.$

Here c denotes the consumption level of the representative household; and σ and ρ are the two parameters representing the elasticity of marginal utility of consumption and the rate of discount respectively. It is assumed that the rate of unemployment is same for all households. The representative household saves and invests the rest of his income left after consumption and there is no depreciation of private capital.

Solving this dynamic optimisation problem, we obtain the growth rate of consumption as given by

$$g = \frac{\dot{c}}{c} = \frac{(1-\tau)\alpha A K^{\alpha-1}(eL)^{\beta}\bar{K}^{1-\alpha} - \rho}{\sigma}\,. \tag{19}$$

We assume a symmetric equilibrium where $\bar{K} = K$, i.e., all firms have equal amount of capital; and hence, from equation (19), we obtain the growth rate of consumption given by

$$g = \frac{\dot{c}}{c} = \frac{(1-\tau)\alpha A(eL)^{\beta} - \rho}{\sigma}\,. \tag{20}$$

The economy is always in the steady state equilibrium; and so g is always time-independent. It does not have transitional dynamic properties because this is an AK model. In equilibrium, all variables like number of workers, L, income tax rate, τ, rental rate on capital, r, effort level of worker, e, and effective employment, eL, are time-independent. Capital stock, K, final output, Y, negotiated wage rate, w^*, firm's profit, π, and unemployment benefit, b, grow at equal rates in the steady-state equilibrium.

2.1. *Effect of unionisation:*

From equations (9), (11), (12), (13) and (14), we obtain

$$\frac{\partial L^*}{\partial \theta} = \frac{\tau(1-\tau)(1-\alpha-\beta)\left[(n-m)\left\{\beta(1-\delta)\Theta_1\Theta_2 + n\theta(1-\alpha-\beta)\Theta_3\right\} - \beta^2\delta n(1-\delta)\Theta_1^2\right]}{\left[(1-\tau)\Theta_2\Theta_4 + \tau\Theta_1\Theta_3\right]^2}\,. \tag{21}$$

Equation (21) shows that the effect of unionisation on the employment of workers consists of two components. First component is union's orientation effect on employment. It is ambiguous in sign and depends on the nature of orientation of the labour union. Second component is the substitution effect on employment. An increase in worker's efficiency lowers the employer's demand for workers. So the second component is always negative. The net effect depends on the relative strength of these two effects. We find that employment orientation property of the labour union is necessary but not sufficient to establish a positive relationship between unionisation and the number of workers. When the labour union is wage oriented or even neutral, unionisation must reduce the number of workers. In Chang et al. (2007), the effect of unionisation on employment consists only of orientation effect, i.e., it depends only on the nature of orientation of the labour union.

Now, from equations (13), (14) and (15), we obtain

$$\frac{\partial e^*}{\partial \theta} = \delta h \left(\frac{\Theta_3}{\Theta_4}\right)^{\delta-1} \frac{m\,(1-\alpha-\beta)\,\beta(1-\delta)}{\Theta_4^2} > 0\,. \tag{22}$$

Equation (22) shows that the efficiency level of the representative worker varies positively with the degree of unionisation in the labour market. Negotiated wage rate is increased with the increase in the relative bargaining power of the labour union; and this induces the worker to put greater effort. This positive relationship between unionisation and effort level is valid only in the presence of 'Efficiency Wage Hypothesis'.

Again, from equations (11), (12), (13), (14) and (16), we obtain

$$\begin{aligned}\frac{\partial e^* L^*}{\partial \theta} &= \frac{(n-m)\,(1-\tau)\,(1-\alpha-\beta)\,\tau h \Theta_3^{\delta+2}}{\left[(1-\tau)\,\Theta_2\Theta_4 + \tau\Theta_1\Theta_3\right]^2 \Theta_4^{\delta}} \\ &+ \frac{h(1-\tau)\beta\delta(1-\delta)(1-\alpha-\beta)m\left\{(1-\tau)\Theta_2^2\Theta_4 + \tau\Theta_1\Theta_3\theta n(1-\alpha-\beta)\right\}}{\Theta_3^{1-\delta}\Theta_4^{\delta}\left[(1-\tau)\Theta_2\Theta_4 + \tau\Theta_1\Theta_3\right]^2}\,.\end{aligned} \tag{23}$$

Equation (23) shows that unionisation affects effective employment through two channels—changing the number of workers and changing the effort level of the representative worker. The effect on the number of workers depends partially on the orientation of the labour union. However, the other effect is originated from the rise in the effort level of the worker; and hence this effect is always positive. So employment orientation property or neutrality property of the labour union is sufficient but not necessary to establish a positive relationship between effective employment and unionisation in the presence of 'Efficiency Wage Hypothesis'. This implies that, in the presence of 'Efficiency Wage Hypothesis', unionisation may raise effective employment through a rise in the efficiency level even if the number of workers is reduced. However, in the absence of this hypothesis, i.e., when $\delta = 0$, unionisation does not raise workers' effort level; and its effect on employment (number of workers) depends solely on the orientation of the labour union.

Now, equation (20) shows that the balanced growth rate, g, varies positively with the level of effective employment. So the effect of unionisation on the growth rate is qualitatively similar to that on effective employment. From equation (20) we obtain

$$\frac{\partial g}{\partial \theta} = \left(\frac{(1-\tau)\alpha A\beta(e^*L^*)^{\beta-1}}{\sigma}\right)\frac{\partial e^* L^*}{\partial \theta}\,. \tag{24}$$

Sign of $\frac{\partial g}{\partial \theta}$ depends on the sign of $\frac{\partial e^* L^*}{\partial \theta}$. In Chang et al. (2007), the nature of the growth effect of unionisation depends totally on the nature of orientation of the labour union because $e \equiv 1$ there. However, our model incorporates 'Efficiency Wage Hypothesis'; and so the effect on effective employment is crucial rather than the effect on the employment of workers.

Chang et al. (2007) shows that the employment effect and economic growth effect of unionisation are nil when labour union is neutral. However, in our model, each of them is positive in the presence of 'Efficiency Wage Hypothesis' even in this case.

So we can establish the following proposition.

PROPOSITION 1. *In case of the 'Efficient Bargaining' model, unionisation in the labour market in general lowers the number of workers if the labour union is not highly employment oriented. However, irrespective of labour union's orientation, it raises the effort (efficiency) level per worker. As a result, if the labour union is not highly wage oriented, effective employment measured in efficiency unit is increased; and this leads to a rise in the growth rate of the economy.*

3. THE 'RIGHT TO MANAGE' MODEL

In this section, we use the 'Right to Manage' model of bargaining where the two parties bargain only over the wage rate. The firm unilaterally decides the level of employment from its labour demand function obtained from its profit maximising behaviour. The inverted labour demand function of the representative firm is given by

$$w = \left[\beta A K^{\alpha} \bar{K}^{1-\alpha} L^{\beta-1} h^{\beta} b^{-\beta\delta}\right]^{\frac{1}{1-\beta\delta}} . \tag{25}$$

So the firm and the labour union jointly maximise the 'generalised Nash product' function given by equation (8), with respect to w only, subject to the firm's labour demand function given by equation (25). Using the first order condition and equations (1), (2), (4), (5) and (25), optimum values of L and w are obtained as[18]

$$L^{**} = \frac{\beta(1-\tau)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}{\begin{array}{c}\tau\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}\\ +\beta(1-\tau)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}\end{array}} < 1\,; \tag{26}$$

and

$$w^{**} = AK^{\alpha}\bar{K}^{1-\alpha}h^{\beta}\beta^{1+\beta\delta}L^{**\beta-1-\beta\delta}\tau^{-\beta\delta}(1-L^{**})^{\beta\delta}(1-\tau)^{\beta\delta} . \tag{27}$$

We assume

$$\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\} > \theta m(1-\beta)(1-\alpha-\beta)$$

to ensure that $L^{**} > 0$.

From equations (1), (2), (5) and (27), we obtain the efficiency level of the representative worker as given by

$$e^{**} = h\left[\frac{\beta}{\tau}\frac{(1-L^{**})(1-\tau)}{L^{**}}\right]^{\delta} . \tag{28}$$

Using equations (26) and (28), we obtain

$$e^{**} = h\left[\frac{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}\right]^{\delta} . \tag{28.a}$$

Equation (28.a) shows that efficiency level of the worker is independent of the exogenously given tax rate, τ. This is so because, as tax rate rises, unemployment benefit per

[18] We assume that second order condition of maximisation is satisfied.

worker is also increased. However, wage rate also rises in the same proportion and so the ratio of wage rate to unemployment benefit remains unchanged. So the workers' efficiency level also remains unchanged.

The government's budget balance equation as well as the representative household's behaviour in this model is identical to that in the 'Efficient Bargaining' model. So equations and solutions derived here are same as those obtained in section 2 except that L^* is replaced by L^{**} and e^* is replaced by e^{**}.

Now, from equation (26), we have

$$\frac{\partial L^{**}}{\partial \theta} = \frac{-(1-\tau)\tau m\beta^2(1-\delta)(1-\alpha-\beta)(1-\beta)^2}{\left\{\begin{array}{c}\tau\,\{\theta n\,(1-\alpha-\beta)\,(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}\\ +\beta(1-\tau)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}\end{array}\right\}^2} < 0\,. \tag{29}$$

So, in this model, unionisation in the labour market lowers the number of workers to be employed irrespective of the orientation of the labour union.

Again, from equations (26) and (28.a), we obtain

$$\frac{\partial e^{**}}{\partial \theta} = \frac{\delta h\,\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}^{\delta-1}\,m\beta(1-\delta)(1-\alpha-\beta)(1-\beta)^2}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}^{\delta+1}} > 0\,; \tag{30}$$

and

$$\frac{\partial(e^{**}L^{**})}{\partial\theta} = \frac{h\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}^{\delta-1}m\beta^2(1-\delta)(1-\alpha-\beta)(1-\beta)^2(1-\tau)}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}^{\delta}} \cdot \frac{[\{\beta\delta(1-\tau)-\tau(1-\delta)\}\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}-\theta m(1-\beta)(1-\alpha-\beta)\beta\delta(1-\tau)]}{\left\{\begin{array}{c}\tau\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}\\ +\beta(1-\tau)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}\end{array}\right\}^2}\,. \tag{31}$$

Equations (29) and (31) clearly show that marginal effects of unionisation on number of workers and effective level of employment depend on the exogenously given tax rate. Equation (30) shows that, in the presence (absence) of 'Efficiency Wage Hypothesis', efficiency level of a worker goes up (does not change) with unionisation in the labour market. This result is similar to that obtained in the 'Efficient Bargaining' model. However, contrary to the 'Efficient Bargaining' model, equation (31) shows that the effect of unionisation on effective employment depends not on the orientation of the labour union but on the mathematical sign of $\{\beta\delta(1-\tau)-\tau(1-\delta)\}\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}-\theta m(1-\beta)(1-\alpha-\beta)\beta\delta(1-\tau)$. If this term is positive (negative), then the level of effective employment varies positively (inversely) with unionisation in the labour market. The RHS of equation (31) is positive if[19]

$$\tau < \bar{\tau} = \frac{\beta\delta\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}{\begin{array}{l}\beta\delta\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}\\ +(1-\delta)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}\end{array}}. \quad (32)$$

Here $1 > \bar{\tau} > 0$ due to the assumed parametric restriction[20] $\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}-\theta m(1-\beta)(1-\alpha-\beta) > 0$.

So, if the exogenously given tax rate is very low, then unionisation raises effective employment. The intuition behind this result is as follows. The relative change in the effective employment due to unionisation, i.e., $\frac{\frac{\partial(e^{**}L^{**})}{\partial\theta}}{(e^{**}L^{**})}$, is equal to the sum of relative change in the number of workers, $\frac{\frac{\partial L^{**}}{\partial\theta}}{L^{**}}$, and the relative change in the efficiency level, $\frac{\frac{\partial e^{**}}{\partial\theta}}{e^{**}}$. Now, equation (28.a) shows that the efficiency of a worker is independent of the exogenously given tax rate. So the relative change in the efficiency of the representative worker due to unionisation is also independent of the tax rate. However, the relative change in the number of workers due to unionisation varies inversely with the tax rate; and this can be shown using equations (26) and (29). From these two equations, we obtain

$$\frac{\partial\left(\frac{\frac{\partial L^{**}}{\partial\theta}}{L^{**}}\right)}{\partial\tau} = \frac{-m\beta^2(1-\delta)(1-\alpha-\beta)(1-\beta)^2}{\tau^2\left\{\begin{array}{l}\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}\\ +\beta(\frac{1}{\tau}-1)\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}\end{array}\right\}^2} < 0. \quad (33)$$

An increase in the labour union's bargaining power raises wage rate and thereby lowers the employment of workers. However, a rise in the tax rate raises unemployment benefit per worker. This raises the negotiated wage rate because unemployment benefit per worker is used as the reservation income in union's utility function. Hence a rise in the tax rate intensifies the negative effect of unionisation on the employment of workers. So, if the exogenously given rate of income tax used to finance the endogenously determined unemployment benefit is very low, then the effect of unionisation on the employment of workers would be weaker and thus unionisation may have a positive effect on effective employment, $e^{**}L^{**}$, in the presence of 'Efficiency Wage Hypothesis'. However, in the

[19] In this paper, the wage premium in the efficiency function given by equation (2) is specified as $\left(\frac{[1-\tau]w}{[1-\tau]b}\right)$. An alternative is $(1-\tau)(w-b)$, which is consistent with the specification in labour union's utility function given by equation (6). In this alternative specification, income tax will affect worker's effort and accordingly equations (15) and (28.a) will be changed. The condition given by equation (32) will also be changed accordingly. However, in this alternative specification, the effort level per worker, e, grows continuously in the steady state. So we did not choose that alternative specification.

[20] See the line after equation (27).

absence of 'Efficiency Wage Hypothesis', i.e., with $\delta = 0$, unionisation always lowers employment level.

The rate of growth in this model is identical to that given by equation (20) in the 'Efficient Bargaining' model except that L^* and e^* are replaced by L^{**} and e^{**}. So the effect of unionisation on the growth rate is qualitatively similar to its effect on effective employment. So we can conclude that unionisation raises the growth rate if the exogenously given income tax rate is quite low; and this growth effect is independent of the nature of orientation of the union. This result is different from that obtained in the case of 'Efficient Bargaining' model where the effect of unionisation on growth depends partly on the nature of orientation of the labour union.

Important results derived in this section are summarised in the following proposition.

PROPOSITION 2. *In the 'Right to Manage' model of bargaining, unionisation in the labour market raises the efficiency level of the representative worker but lowers the number of workers to be employed irrespective of the orientation of the labour union. However, it raises (lowers) effective employment and the balanced growth rate of the economy if the exogenously given income tax rate is lower (higher) than a critical value.*

4. CONCLUSION

This paper develops a model to investigate the effect of unionisation in the labour market on the long run growth rate of an economy in the presence of 'Efficiency Wage Hypothesis'. Here we use two alternative versions of bargaining models—the 'Efficient Bargaining' model of McDonald and Solow (1981) and the 'Right to Manage' model of Nickell and Andrews (1983). The existing literature that analyses the role of unionisation on economic growth does not consider 'Efficiency Wage Hypothesis'.

We derive different results from these two versions of bargaining models. In the 'Efficient Bargaining' model, unionisation in the labour market in general lowers the number of workers if the labour union is not highly employment oriented. However, irrespective of labour union's orientation, it raises the effort (efficiency) level per worker. As a result, if the labour union is not highly wage oriented, effective employment measured in efficiency unit is increased; and this leads to a rise in the growth rate of the economy. However, in the 'Right to Manage' model, unionisation raises the effort level of the worker but reduces the number of workers to be employed irrespective of the orientation of the labour union. This raises effective employment and balanced growth rate of the economy when the exogenously given income tax rate is lower than a critical value.

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APPENDIX

APPENDIX A

Derivation of optimal w and L:
From equations (1) and (8), we obtain two first order conditions given by

$$\frac{\theta m}{(w-b)}+\frac{(1-\theta)\left[\beta\delta\frac{Y}{w}-L\right]}{Y-wL-rK}=0\,. \tag{A.1}$$

$$\frac{\theta n}{L}+\frac{(1-\theta)\left[\beta\frac{Y}{L}-w\right]}{Y-wL-rK}=0\,. \tag{A.2}$$

From equations (A.2) and (4), we obtain

$$\frac{Y}{wL}=\frac{(1-\theta+\theta n)}{[\theta n\,(1-\alpha)+\beta(1-\theta)]}\,. \tag{A.3}$$

From equations (A.1), (4) and (5), we obtain

$$\frac{\theta m}{1-\left(\frac{\tau Y}{w[1-L](1-\tau)}\right)}=\frac{(1-\theta)\left[1-\beta\delta\frac{Y}{wL}\right]}{\left[(1-\alpha)\,\frac{Y}{wL}-1\right]}\,. \tag{A.4}$$

Incorporating equation (A.3) in equation (A.4), we obtain

$$\frac{\theta m}{1-\left(\frac{\tau L}{[1-L]}\frac{(1-\tau)^{-1}(1-\theta+\theta n)}{[\theta n(1-\alpha)+\beta(1-\theta)]}\right)}=\frac{(1-\theta)\left[1-\frac{\beta\delta(1-\theta+\theta n)}{[\theta n(1-\alpha)+\beta(1-\theta)]}\right]}{\left[\frac{(1-\alpha)(1-\theta+\theta n)}{[\theta n(1-\alpha)+\beta(1-\theta)]}-1\right]}\,. \tag{A.4a}$$

Solving equation (A.4a), we obtain the optimal value of L as given in equation (9) in the body of the paper.
Now, using equations (1) and (5), we obtain

$$Y=\left[AK^{\alpha}\bar{K}^{1-\alpha}h^{\beta}L^{\beta}w^{\beta\delta}\tau^{-\beta\delta}\,(1-L)^{\beta\delta}\right]^{\frac{1}{1+\beta\delta}}\,. \tag{A.5}$$

Using equations (A.3) and (A.5), we obtain

$$\frac{\left[AK^{\alpha}\bar{K}^{1-\alpha}h^{\beta}L^{\beta}w^{\beta\delta}\tau^{-\beta\delta}\,(1-L)^{\beta\delta}\right]^{\frac{1}{1+\beta\delta}}}{wL}=\frac{(1-\theta+\theta n)}{[\theta n\,(1-\alpha)+\beta(1-\theta)]}\,.$$

$$\Rightarrow w=\left[AK^{\alpha}\bar{K}^{1-\alpha}h^{\beta}L^{\beta-(1+\beta\delta)}\tau^{-\beta\delta}\,(1-L)^{\beta\delta}\right]\left\{\frac{[\theta n\,(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)}\right\}^{1+\beta\delta}\,. \tag{A.6}$$

Using equations (A.6) and (9), we obtain the optimal value of w as given in equation (10) in the body of the paper.

Second order conditions:
From equations (A.1) and (A.2), we obtain

$$\frac{\frac{\partial^2\psi}{\partial w^2}\psi-\left(\frac{\partial\psi}{\partial w}\right)^2}{\psi^2}$$

$$=-\frac{\theta m}{(w-b)^2}+\frac{(1-\theta)\left[\beta\delta\,(\beta\delta-1)\,\frac{Y}{w^2}\,(Y-wL-rK)-\left\{\beta\delta\frac{Y}{w}-L\right\}^2\right]}{(Y-wL-rK)^2}\,; \tag{A.7}$$

$$\frac{\frac{\partial^2\psi}{\partial L^2}\psi-\left(\frac{\partial\psi}{\partial L}\right)^2}{\psi^2}$$

$$= -\frac{\theta n}{L^2} + \frac{(1-\theta)\left[\beta(\beta-1)\frac{Y}{L^2}(Y-wL-rK) - \left\{\beta\frac{Y}{L} - w\right\}^2\right]}{(Y-wL-rK)^2}; \tag{A.8}$$

and

$$\frac{\frac{\partial^2\psi}{\partial L\partial w}\psi - \frac{\partial\psi}{\partial L}\frac{\partial\psi}{\partial w}}{\psi^2} = \frac{\left[(\beta^2\delta\frac{Y}{wL} - 1)(Y-wL-rK) - \left\{\beta\delta\frac{Y}{w} - L\right\}\left\{\beta\frac{Y}{L} - w\right\}\right]}{(1-\theta)^{-1}(Y-wL-rK)^2}. \tag{A.9}$$

Using equations (1), (4), (5), (9), (11), (12), (13), (14), (A.3), (A.7), (A.8), (A.9) and $\frac{\partial\psi}{\partial L} = \frac{\partial\psi}{\partial w} = 0$, we obtain respectively

$$\frac{\frac{\partial^2\psi}{\partial w^2}}{\psi} = -\frac{(1-\theta+\theta m)\Theta_3^2 + (1-\alpha-\beta)(1-\theta)\beta\delta\Theta_1(1-\beta\delta)\theta m}{w^2(1-\alpha-\beta)^2\theta m(1-\theta)} < 0; \tag{A.10}$$

$$\frac{\frac{\partial^2\psi}{\partial L^2}}{\psi} = -\frac{\theta n(1-\alpha-\beta)\Theta_1 + \Theta_1\beta(1-\theta)(1-\beta)}{(1-\alpha-\beta)(1-\theta)L^2} < 0; \tag{A.11}$$

and

$$\frac{\frac{\partial^2\psi}{\partial L\partial w}}{\psi} = \frac{[\theta n + \beta(1-\theta) - \Theta_1\Theta_2]}{(1-\alpha-\beta)(1-\theta)wL}. \tag{A.12}$$

Now using equations (A.10), (A.11) and (A.12), we have

$$\frac{\frac{\partial^2\psi}{\partial w^2}\frac{\partial^2\psi}{\partial L^2} - \left(\frac{\partial^2\psi}{\partial L\partial w}\right)^2}{\psi^2}$$
$$= \frac{\begin{array}{c}\left\{(1-\theta+\theta m)\Theta_3^2 + (1-\alpha-\beta)(1-\theta)\beta\delta\Theta_1(1-\beta\delta)\theta m\right\} \\ \{\theta n(1-\alpha-\beta)\Theta_1 + \Theta_1\beta(1-\theta)(1-\beta)\} \\ -\{\theta n + \beta(1-\theta) - \Theta_1\Theta_2\}^2(1-\alpha-\beta)\theta m\end{array}}{w^2(1-\alpha-\beta)^3\theta m(1-\theta)^2L^2}. \tag{A.13}$$

We assume that the R. H. S. of equation (A.13) is positive in order to satisfy the second order conditions.

APPENDIX B

Derivation of equation (15):
From equations (2) and (5), we obtain

$$e = h\left(\frac{w(1-L)}{\tau Y}\right)^{\delta}. \tag{B.1}$$

Using equations (A.3) and (9), we obtain equation (15) in the body of the paper.
Derivations of section 3 are similar to that of section 2.