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RELATIVE PROFIT MAXIMIZATION AND THE CHOICE OF STRATEGIC VARIABLES IN DUOPOLY

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Abstract: We study implications of the choice of strategic variables, price or quantity, by firms in a duopoly with differentiated goods in which each firm maximizes its relative profit. We consider general demand and cost functions, and show that the choice of strategic variables is irrelevant in the sense that the conditions of relative profit maximization for the firms are the same in all situations, and so any combination of strategy choice by the firms constitutes a sub-game perfect equilibrium in a two stage game such that in the first stage the firms choose their strategic variables and in the second stage they determine the values of their strategic variables. We define the relative profit of a firm as the ratio of its profit over the total profit. But, even if we define the relative profit of a firm as the difference between the profits of firms, we can show the same result.

Key words: relative profit maximization, choice of strategic variables, duopoly.
JEL Classification Number: D43, L13.

1. INTRODUCTION

We study implications of the choice of strategic variables, price or quantity, by firms in a duopoly with differentiated goods in which each firm maximizes its relative profit. We consider general demand and cost functions, and show that the choice of strategic variables is irrelevant in the sense that the conditions of relative profit maximization for the firms are the same in all situations, and so any combination of strategy choice by the firms constitutes a sub-game perfect equilibrium in a two stage game such that

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in the first stage the firms choose their strategic variables and in the second stage they determine the values of their strategic variables. We define the relative profit of a firm as the ratio of its profit over the total profit (ratio case). But, even if we define the relative profit of a firm as the difference between the profits of firms (difference case), we can show the same result.

In another paper, Tanaka (2013), we have shown a similar result in the difference case with a simple model in which demand functions are symmetric and linear, firms have the same cost functions and their marginal costs are constant. This paper extends this result to a case of general demand and cost functions, and we consider the ratio case instead of the difference case.[1]

In recent years, maximizing relative profit instead of absolute profit has aroused the interest of economists. Please see Gibbons and Murphy (1990), Lu (2011), Matsumura, Matsushima and Cato (2013), Miller and Pazgal (2001), Vega-Redondo (1997), Schaffer (1989) and Satoh and Tanaka (2013).[2]

In Vega-Redondo (1997), it is argued that, in a homogeneous good case, if firms maximize their relative profits, a competitive equilibrium can be induced. But in the case of differentiated goods, the result under relative profit maximization is different from the competitive result.

Miller and Pazgal (2001) has shown the equivalence of price strategy and quantity strategy in a delegation game when owners of firms control managers of firms seek to maximize an appropriate combination of absolute and relative profits. But in their analyses owners of firms themselves still seek to maximize absolute profits of their firms. On the other hand, in this paper we do not consider a delegation problem, and we assume that firms, or owners of firms, seek to maximize their relative profits.

We believe that seeking relative profit or utility is based on human nature. Even if a person earns a large salary and if their brother/sister or close friend earns more, then they are not sufficiently happy and may be disappointed. In contrast, even if a person is very poor and if their neighbor has even less, then they may be consoled by that fact. Our interpretation is related to the classical relative income hypothesis of consumption theory by Duesenberry (1949), which states that the satisfaction (or utility) an individual derives from a given consumption level depends on its relative magnitude in the society, for example, relative to the average consumption, rather than its absolute level. In addition, firms not only seek to improve their own performance but also to outperform rival firms in the industry. The TV audience-ratings race and market share competition by breweries, automobile manufacturers, convenience store chains, and mobile-phone carriers (especially in Japan) are examples of such firm behavior.

[1] In Tanaka (2014) we analyzed a Stackelberg model of duopoly under relative profit maximization, and have shown that the equilibrium output and price of the good of the leader and those of the follower are equal, that is, the role of leader or follower is irrelevant to the equilibrium.

[2] In Matsumura, Matsushima and Cato (2013) and Satoh and Tanaka (2013) it is assumed that each firm maximizes the weighted sum of its absolute and relative profits. In such a case, however, the equivalence of quantity-quantity competition, price-quantity competition, quantity-price competition and price-price competition does not hold. It holds under pure relative profit maximization.

In the next section we mention some related literature. In Section 3 we present the model of this paper. In Section 4 we investigate the relationship between inverse and ordinary demand functions. In Section 5 we analyze the choice of strategic variables in a duopoly under relative profit maximization. In Section 6 we mention the relationship between the difference case and the ratio case.

2. RELATION TO THE PREVIOUS STUDIES

There are many studies about the choice of strategic variables in a duopoly under absolute profit maximization. The most famous paper is Singh and Vives (1984). They showed that in a differentiated duopoly if the goods are substitutes (complements), it is a dominant strategy for each firm to choose the quantity (price) as a strategic variable. Using a geometric analysis of a duopoly Cheng (1985) showed that if the goods are substitutes (complements), Cournot equilibrium prices (quantities) are higher than Bertrand equilibrium prices (quantities), and a quantity (price) strategy dominates a price (quantity) strategy. Tasnádi (2006) formulated a model in which firms endogenously choose strategic variables in an oligopoly where they produce homogeneous goods under capacity constraints, and showed that every firm chooses the quantity in the equilibrium. Reisinger and Ressner (2009) analyzed a duopoly model with stochastic demand in which firms first commit to a strategic variable and compete afterwards, and showed that firms set prices if uncertainty is high compared to the degree of substitutability and quantities if the reverse holds true. Matsumura and Ogawa (2012) analyzed the endogenous choice of a price or a quantity contract in a mixed duopoly with a public firm, and showed that choosing the price contract is a dominant strategy for both firms, whether the goods are substitutes or complements. Tanaka (2001a) analyzed an oligopoly with differentiated goods, and showed that quantity strategy is the best response for each firm when all other firms choose a price strategy; thus the Bertrand equilibrium does not constitute a sub-game perfect equilibrium in a two-stage game such that in the first stage firms choose strategic variables and in the second stage they determine the levels of their strategic variables. A quantity strategy is also the best response for each firm when all other firms choose a quantity strategy; therefore the Cournot equilibrium constitutes a sub-game perfect equilibrium. Tanaka (2001b) analyzed a duopoly in which the products of the firms are vertically differentiated, that is, there are a high quality firm and a low quality firm, and showed that if the goods are substitutes, a quantity strategy dominates a price strategy for both firms. These studies are conducted under the assumption of absolute profit maximization.

Klemperer and Meyer (1986) presented a model where price-quantity choice does not matter. This result is the same as ours. However, they analyzed a one-stage game in which firms simultaneously determine strategic variables and the levels of their strategic variables. On the other hand, other works mentioned above analyzed a two-stage game in which firms determine their strategic variables in the first stage, and determine the levels of their strategic variables in the second stage. The model of this paper is also a two-stage game.

3. THE MODEL

There are two firms, A and B. They produce differentiated (substitutable or complementary) goods. The outputs of Firm A and B are denoted by x_A and x_B, the prices of the goods of Firm A and B are p_A and p_B. The inverse demand functions of the goods produced by the firms are

$$p_A = p_A(x_A, x_B)\,, \ \text{and} \ p_B = p_B(x_A, x_B)\,. \tag{1}$$

From these inverse demand functions the ordinary demand functions are derived as follows.

$$x_A = x_A(p_A, p_B)\,, \ \text{and} \ x_B = x_B(p_A, p_B)\,.$$

We have

$$\frac{\partial p_A}{\partial x_A} < 0\,, \ \text{and} \ \frac{\partial p_B}{\partial x_B} < 0\,.$$

We assume that the effect of a change in the output of a good on its price is larger than the effect on the price of another good. Then,

$$\left|\frac{\partial p_A}{\partial x_A}\right| > \left|\frac{\partial p_B}{\partial x_A}\right|,\ \left|\frac{\partial p_A}{\partial x_A}\right| > \left|\frac{\partial p_A}{\partial x_B}\right|,\ \left|\frac{\partial p_B}{\partial x_B}\right| > \left|\frac{\partial p_A}{\partial x_B}\right|,\ \left|\frac{\partial p_B}{\partial x_B}\right| > \left|\frac{\partial p_B}{\partial x_A}\right|,$$

and, we have

$$\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A} > 0\,.$$

4. RELATIONS BETWEEN INVERSE DEMAND FUNCTIONS AND ORDINARY DEMAND FUNCTIONS

Let us investigate the relations between the inverse demand functions and the ordinary demand functions.

First consider a case where the strategic variables of the firms are the prices. Differentiating (1) with respect to p_A given p_B yields

$$1 = \frac{\partial p_A}{\partial x_A}\frac{\partial x_A}{\partial p_A} + \frac{\partial p_A}{\partial x_B}\frac{\partial x_B}{\partial p_A}\,, \ \text{and} \ 0 = \frac{\partial p_B}{\partial x_A}\frac{\partial x_A}{\partial p_A} + \frac{\partial p_B}{\partial x_B}\frac{\partial x_B}{\partial p_A}\,.$$

From these equations we get

$$\frac{\partial x_A}{\partial p_A} = \frac{\frac{\partial p_B}{\partial x_B}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}\,, \ \text{and} \ \frac{\partial x_B}{\partial p_A} = -\frac{\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}\,. \tag{2}$$

Similarly, differentiating (1) with respect to p_B given p_A, we obtain

$$\frac{\partial x_B}{\partial p_B} = \frac{\frac{\partial p_A}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}\,, \ \text{and} \ \frac{\partial x_A}{\partial p_B} = -\frac{\frac{\partial p_A}{\partial x_B}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}\,. \tag{3}$$

We have

$$\frac{\partial x_A}{\partial p_A} < 0\,, \ \text{and} \ \frac{\partial x_B}{\partial p_B} < 0\,.$$

If the goods of Firm A and Firm B are substitutes, then

$$\frac{\partial p_A}{\partial x_B} < 0\,,\quad \frac{\partial p_B}{\partial x_A} < 0\,,\quad \frac{\partial x_B}{\partial p_A} > 0,\quad \text{and}\quad \frac{\partial x_A}{\partial p_B} > 0\,.$$

On the other hand, if they are complements, then

$$\frac{\partial p_A}{\partial x_B} > 0\,,\quad \frac{\partial p_B}{\partial x_A} > 0\,,\quad \frac{\partial x_B}{\partial p_A} < 0\,,\quad \text{and}\quad \frac{\partial x_A}{\partial p_B} < 0\,.$$

Also we have

$$\frac{\partial x_A}{\partial p_A}\frac{\partial x_B}{\partial p_B} - \frac{\partial x_A}{\partial p_B}\frac{\partial x_B}{\partial p_A} > 0\,.$$

Next consider a case where the strategic variable of one firm is the price and that of the other firm is the quantity (output). Assume that Firm A determines the price of its good and Firm B determines its output. The inverse demand function for Firm B is written as follows.

$$p_B = p_B(x_A(p_A, p_B), x_B)\,. \tag{4}$$

Differentiating (4) with respect to p_A given x_B, we get

$$\frac{\partial p_B}{\partial p_A} = \frac{\partial p_B}{\partial x_A}\frac{\partial x_A}{\partial p_A} + \frac{\partial p_B}{\partial x_A}\frac{\partial x_A}{\partial p_B}\frac{\partial p_B}{\partial p_A} = \frac{1}{1-\frac{\partial p_B}{\partial x_A}\frac{\partial x_A}{\partial p_B}}\frac{\partial p_B}{\partial x_A}\frac{\partial x_A}{\partial p_A}\,.$$

From (3) we have

$$\frac{1}{1-\frac{\partial p_B}{\partial x_A}\frac{\partial x_A}{\partial p_B}} = \frac{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B}}\,.$$

Thus, using (2),

$$\frac{\partial p_B}{\partial p_A} = \frac{\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}}\,. \tag{5}$$

On the other hand, the ordinary demand function of the good of Firm A is written as

$$x_A = x_A(p_A, p_B(x_A, x_B))\,. \tag{6}$$

Differentiating (6) with respect to x_B given p_A, and using (3) we get

$$\frac{\partial x_A}{\partial x_B} = \frac{\partial x_A}{\partial p_B}\frac{\partial p_B}{\partial x_B} + \frac{\partial x_A}{\partial p_B}\frac{\partial p_B}{\partial x_A}\frac{\partial x_A}{\partial x_B} = \frac{1}{1-\frac{\partial p_B}{\partial x_A}\frac{\partial x_A}{\partial p_B}}\frac{\partial x_A}{\partial p_B}\frac{\partial p_B}{\partial x_B} = -\frac{\frac{\partial p_A}{\partial x_B}}{\frac{\partial p_A}{\partial x_A}}\,. \tag{7}$$

Differentiating (4) with respect to x_B given p_A, and using (7) we obtain

$$\frac{dp_B}{dx_B} = \frac{\partial p_B}{\partial x_B} + \frac{\partial p_B}{\partial x_A}\frac{\partial x_A}{\partial x_B} = \frac{\partial p_B}{\partial x_B} - \frac{\frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}}\,. \tag{8}$$

Differentiating (6) with respect to p_A given x_B, and using (2), (3) and (5) we obtain

$$\begin{aligned}\frac{dx_A}{dp_A} &= \frac{\partial x_A}{\partial p_A} + \frac{\partial x_A}{\partial p_B}\frac{\partial p_B}{\partial p_A}\\ &= \frac{\frac{\partial p_B}{\partial x_B}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}} - \left(\frac{\frac{\partial p_A}{\partial x_B}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}\right)\frac{\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}} = \frac{1}{\frac{\partial p_A}{\partial x_A}}\,.\end{aligned} \tag{9}$$

Similarly in the case where Firm A's strategic variable is the quantity and Firm B's strategic variable is the price,

$$\frac{\partial p_A}{\partial p_B} = \frac{\frac{\partial p_A}{\partial x_B}}{\frac{\partial p_B}{\partial x_B}}, \quad \frac{\partial x_B}{\partial x_A} = -\frac{\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_B}{\partial x_B}}, \quad \frac{dp_A}{dx_A} = \frac{\partial p_A}{\partial x_A} - \frac{\frac{\partial p_B}{\partial x_A}\frac{\partial p_A}{\partial x_B}}{\frac{\partial p_B}{\partial x_B}} \text{ and } \frac{dx_B}{dp_B} = \frac{1}{\frac{\partial p_B}{\partial x_B}}.$$

5. CHOICE OF STRATEGIC VARIABLES UNDER RELATIVE PROFIT MAXIMIZATION

In this section we analyze the choice of strategic variables by the firms. We define the relative profit of each firm as the ratio of its absolute profit over the total absolute profit of two firms. However, when we define the relative profits as the difference between the profits of the firms, we can show the same result.

5.1. Quantity-quantity competition

Let denote the absolute profits of Firm A and B, respectively, by π_A and π_B. Then,

$$\pi_A = p_A(x_A, x_B)x_A - c_A(x_A),$$

and

$$\pi_B = p_B(x_A, x_B)x_B - c_B(x_B).$$

The relative profits of Firm A and Firm B are denoted by Φ_A and Φ_B. They are written as follows,

$$\Phi_A = \frac{\pi_A}{\pi_A + \pi_B}, \text{ and } \Phi_B = \frac{\pi_B}{\pi_A + \pi_B}.$$

Firm A determines x_A so as to maximize Φ_A. The condition for maximization of Φ_A is written as follows.

$$\frac{\partial \pi_A}{\partial x_A}(\pi_A + \pi_B) - \pi_A\left(\frac{\partial \pi_A}{\partial x_A} + \frac{\partial \pi_B}{\partial x_A}\right) = 0.$$

This is rewritten as

$$\frac{\partial \pi_A}{\partial x_A}\pi_B - \pi_A\frac{\partial \pi_B}{\partial x_A} = 0.$$

Similarly the condition for maximization of Φ_B is

$$\frac{\partial \pi_B}{\partial x_B}\pi_A - \pi_B\frac{\partial \pi_A}{\partial x_B} = 0.$$

Then, we have

$$\left(p_A - c_A' + x_A\frac{\partial p_A}{\partial x_A}\right)\pi_B - x_B\frac{\partial p_B}{\partial x_A}\pi_A = 0, \tag{10}$$

and

$$\left(p_B - c_B' + x_B\frac{\partial p_B}{\partial x_B}\right)\pi_A - x_A\frac{\partial p_A}{\partial x_B}\pi_B = 0. \tag{11}$$

5.2. Price-quantity competition

Assume that Firm A chooses the price of its good as a strategic variable, and Firm B chooses the output as a strategic variable. In this case the absolute profits of Firm A and B are written as follows,

$$\pi_A = p_A x_A(p_A, p_B(x_A, x_B)) - c_A(x_A(p_A, p_B(x_A, x_B)) ,$$

and

$$\pi_B = p_B(x_A(p_A, p_B(x_A, x_B)), x_B)x_B - c_B(x_B) .$$

The relative profit of Firm A and that of Firm B, which are denoted by Φ_A and Φ_B, are also written as follows,

$$\Phi_A = \frac{\pi_A}{\pi_A + \pi_B}, \text{ and } \Phi_B = \frac{\pi_B}{\pi_A + \pi_B} .$$

Firm A determines p_A so as to maximize Φ_A given x_B. The condition for maximization of Φ_A is written as follows.

$$\frac{\partial \pi_A}{\partial p_A}(\pi_A + \pi_B) - \pi_A\left(\frac{\partial \pi_A}{\partial p_A} + \frac{\partial \pi_B}{\partial p_A}\right) = 0 .$$

This is rewritten as

$$\frac{\partial \pi_A}{\partial p_A}\pi_B - \pi_A\frac{\partial \pi_B}{\partial p_A} = 0 .$$

Firm B determines x_B so as to maximize Φ_B given p_A. The condition for maximization of Φ_B is

$$\frac{\partial \pi_B}{\partial x_B}\pi_A - \pi_B\frac{\partial \pi_A}{\partial x_B} = 0 .$$

Then, we have

$$\left[x_A + (p_A - c'_A)\frac{dx_A}{dp_A}\right]\pi_B - \frac{\partial p_B}{\partial p_A}x_B\pi_A = 0 , \tag{12}$$

and

$$\left(p_B + x_B\frac{dp_B}{dx_B} - c'_B\right)\pi_A - (p_A - c'_A)\frac{\partial x_A}{\partial x_B}\pi_B = 0 . \tag{13}$$

From (5) and (9), we rewrite (12) as follows.

$$\left[x_A + (p_A - c'_A)\frac{1}{\frac{\partial p_A}{\partial x_A}}\right]\pi_B - \frac{\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}}x_B\pi_A = 0 .$$

Thus, the following equation is derived.

$$\left(\frac{\partial p_A}{\partial x_A}x_A + p_A - c'_A\right)\pi_B - \frac{\partial p_B}{\partial x_A}x_B\pi_A = 0 . \tag{14}$$

This is the same as (10) which is the condition of relative profit maximization for Firm A in the case of quantity-quantity competition.

From (7) and (8), we rewrite (13) as follows.

$$\left[p_B + \left(\frac{\partial p_B}{\partial x_B} - \frac{\frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}}\right)x_B - c'_B\right]\pi_A + (p_A - c'_A)\frac{\frac{\partial p_A}{\partial x_B}}{\frac{\partial p_A}{\partial x_A}}\pi_B = 0 . \tag{15}$$

From (14)

$$p_A - c'_A = \frac{\pi_A}{\pi_B}\frac{\partial p_B}{\partial x_A}x_B - \frac{\partial p_A}{\partial x_A}x_A\,.$$

Substituting this into (15),

$$\left[p_B + \left(\frac{\partial p_B}{\partial x_B} - \frac{\frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}}\right)x_B - c'_B\right]\pi_A + \left(\frac{\pi_A}{\pi_B}\frac{\partial p_B}{\partial x_A}x_B - \frac{\partial p_A}{\partial x_A}x_A\right)\frac{\frac{\partial p_A}{\partial x_B}}{\frac{\partial p_A}{\partial x_A}}\pi_B = 0\,.$$

Then, the following equation is derived.

$$\left(p_B - c'_B + \frac{\partial p_B}{\partial x_A}x_B\right)\pi_A - \frac{\partial p_A}{\partial x_B}x_A\pi_B = 0\,. \tag{16}$$

This is the same as (11) which is the condition of relative profit maximization for Firm B in the case of quantity-quantity competition.

These results mean that quantity-quantity competition and price-quantity competition are equivalent.

5.3. *Quantity-price competition*

Assume that Firm B chooses the price of its good as a strategic variable, and Firm A chooses the output as a strategic variable. In this case Firm B determines the price of its good given the output of Firm A, and Firm A determines its output given the price of the good of Firm B. Interchanging A with B, by the same methods as those in the previous subsection we can show that quantity-quantity competition and quantity-price competition are equivalent.

5.4. *Price-price competition*

Assume that both firms choose the prices of their goods as their strategic variables. The relative profits of Firm A and B are

$$\Phi_A = \frac{\pi_A}{\pi_A+\pi_B} = \frac{p_A x_A(p_A,p_B) - c_A(x_A(p_A,p_B))}{p_A x_A(p_A,p_B) - c_A(x_A(p_A,p_B)) + p_B x_B(p_A,p_B) - c_B(x_B(p_A,p_B))}\,,$$

and

$$\Phi_B = \frac{\pi_B}{\pi_A+\pi_B} = \frac{p_B x_B(p_A,p_B) - c_B(x_B(p_A,p_B))}{p_B x_B(p_A,p_B) - c_B(x_B(p_A,p_B)) + p_A x_A(p_A,p_B) - c_A(x_B(p_A,p_B))}\,.$$

Firm A determines p_A so as to maximize Φ_A. The condition for maximization of Φ_A is written as follows.

$$\frac{\partial \pi_A}{\partial p_A}(\pi_A + \pi_B) - \pi_A\left(\frac{\partial \pi_A}{\partial p_A} + \frac{\partial \pi_B}{\partial p_A}\right) = 0\,.$$

This is rewritten as

$$\frac{\partial \pi_A}{\partial p_A}\pi_B - \pi_A\frac{\partial \pi_B}{\partial p_A} = 0\,.$$

Similarly the condition for maximization of Φ_B is

$$\frac{\partial \pi_B}{\partial p_B}\pi_A - \pi_B\frac{\partial \pi_A}{\partial p_B} = 0\,.$$

Then, we have

$$\left[x_A + (p_A - c'_A)\frac{\partial x_A}{\partial p_A}\right]\pi_B + (p_B - c'_B)\frac{\partial x_B}{\partial p_A}\pi_A = 0\,, \tag{17}$$

and

$$\left[x_B + (p_B - c'_B)\frac{\partial x_B}{\partial p_B}\right]\pi_A + (p_A - c'_A)\frac{\partial x_A}{\partial p_B}\pi_B = 0\,. \tag{18}$$

By some calculations (see Appendix) we get for Firm A

$$\left(x_A\frac{\partial p_A}{\partial x_A} + p_A - c'_A\right)\pi_B - x_B\frac{\partial p_B}{\partial x_A}\pi_A = 0\,.$$

This is the same as (10) which is the condition of relative profit maximization for Firm A in the case of quantity-quantity competition.

And for Firm B we get

$$\left(x_B\frac{\partial p_B}{\partial x_B} + p_B - c'_B\right)\pi_A - x_A\frac{\partial p_A}{\partial x_B}\pi_B = 0\,.$$

This is the same as (11) which is the condition of relative profit maximization for Firm B in the case of quantity-quantity competition.

We have shown the following result.

THEOREM 1. *In a duopoly with differentiated goods, whether the goods are substitutes or complements, quantity-quantity competition, price-quantity competition, quantity-price competition and price-price competition are all equivalent under relative profit maximization.*

Theorem 1 means that price and output as strategic variables in the first stage are indifferent for both firms. Thus we can conclude

THEOREM 2. *In a duopoly under relative profit maximization, whether the goods are substitutes or complements, any combination of the choice of strategic variables by two firms is a sub-game perfect equilibrium of the two stage game such that in the first stage the firms choose their strategic variables and in the second stage they determine the values of their strategic variables.*

6. NOTE ON THE DIFFERENCE CASE

In the difference case, in which the relative profit of a firm is defined as the difference between the profits of firms, the conditions of relative profit maximization for Firm A and B, for example, in the case of quantity-quantity competition are

$$\left(p_A - c'_A + x_A\frac{\partial p_A}{\partial x_A}\right) - x_B\frac{\partial p_B}{\partial x_A} = 0\,,$$

and

$$\left(p_B - c'_B + x_B\frac{\partial p_B}{\partial x_B}\right) - x_A\frac{\partial p_A}{\partial x_B} = 0\,.$$

Comparing them with (10) and (11), if $\pi_A = \pi_B$, the difference case and the ratio case are equivalent. But , if $\pi_A \neq \pi_B$, they are not equivalent. In the difference case as well as the ratio case, however, we can show the irrelevance of the choice of strategic variables.

7. CONCLUSION

We have shown that in an asymmetric duopoly with differentiated goods under relative profit maximization the choice of strategic variables is irrelevant in the sense that the conditions of relative profit maximization for the firms are the same in all situations. This result can be extended to a symmetric oligopoly in which all firms have the same cost function and demand functions are symmetric. In an asymmetric oligopoly, however, it may fail. It is a future issue.

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APPENDIX: CALCULATION OF PRICE-PRICE COMPETITION

Substituting (2) into (17) yields

$$\left[x_A + (p_A - c'_A)\frac{\frac{\partial p_B}{\partial x_B}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}\right]\pi_B + (p_B - c'_B)\frac{\frac{\partial p_B}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}\pi_A = 0\,.$$

Similarly substituting (3) into (18) yields

$$\left[x_B + (p_B - c'_B)\frac{\frac{\partial p_A}{\partial x_A}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}\right]\pi_A + (p_A - c'_A)\frac{\frac{\partial p_A}{\partial x_B}}{\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}}\pi_B = 0\,.$$

Rearranging the terms of these equations,

$$x_A\left(\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}\right)\pi_B + (p_A - c'_A)\frac{\partial p_B}{\partial x_B}\pi_B + (p_B - c'_B)\frac{\partial p_B}{\partial x_A}\pi_A = 0\,, \tag{19}$$

and

$$x_B\left(\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}\right)\pi_A + (p_B - c'_B)\frac{\partial p_A}{\partial x_A}\pi_A + (p_A - c'_A)\frac{\partial p_A}{\partial x_B}\pi_B = 0\,. \tag{20}$$

From (20)

$$p_B - c'_B = -\frac{1}{\frac{\partial p_A}{\partial x_A}\pi_A}\left[(p_A - c'_A)\frac{\partial p_A}{\partial x_B}\pi_B + x_B\left(\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}\right)\pi_A\right].$$

Substituting this into (19), and multiplying $\frac{\partial p_A}{\partial x_A}$ to both sides, we obtain

$$\begin{aligned} &x_A\pi_B\frac{\partial p_A}{\partial x_A}\left(\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}\right) + (p_A - c'_A)\pi_B\frac{\partial p_B}{\partial x_B}\frac{\partial p_A}{\partial x_A} \\ &- (p_A - c'_A)\pi_B\frac{\partial p_B}{\partial x_A}\frac{\partial p_A}{\partial x_B} - x_B\pi_A\frac{\partial p_B}{\partial x_A}\left(\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}\right) = 0\,. \end{aligned}$$

Then, since $\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A} > 0$, we get

$$\left(x_A\frac{\partial p_A}{\partial x_A} + p_A - c'_A\right)\pi_B - x_B\frac{\partial p_B}{\partial x_A}\pi_A = 0\,.$$

Similarly from (19)

$$p_A - c'_A = -\frac{1}{\frac{\partial p_B}{\partial x_B}\pi_B}\left[(p_B - c'_B)\frac{\partial p_B}{\partial x_A}\pi_A + x_A\left(\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}\right)\pi_B\right].$$

Substituting this into (20), and multiplying $\frac{\partial p_B}{\partial x_B}$ to both sides, we obtain

$$\begin{aligned} &x_B\pi_A\frac{\partial p_B}{\partial x_B}\left(\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}\right) + (p_B - c'_B)\pi_A\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} \\ &- (p_B - c'_B)\pi_A\frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A} - x_A\pi_B\frac{\partial p_A}{\partial x_B}\left(\frac{\partial p_A}{\partial x_A}\frac{\partial p_B}{\partial x_B} - \frac{\partial p_A}{\partial x_B}\frac{\partial p_B}{\partial x_A}\right) = 0\,. \end{aligned}$$

Then, we get

$$\left(x_B\frac{\partial p_B}{\partial x_B} + p_B - c'_B\right)\pi_A - x_A\frac{\partial p_A}{\partial x_B}\pi_B = 0\,.$$