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KEIO ECONOMIC STUDIES **51**, 1–17 (2015)

MULTIPLIER THEORY AND PUBLIC GOODS: A MACROECONOMICS OF THE MIXED ECONOMIES

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Abstract: In this paper we reconsider and extend the economic theory of deep depression, initiated by Keynes (1936), taking explicit account of the concept of public goods largely blurred in his general theory and after. To contrive the model of secular stagnation where the interest rate falls to zero and monetary policy is restricted by the zero-bound, we concentrate on "real economy" model featuring the theory of public finance. In so doing, we synthesize the traditional theory of public finance and the Keynesian theory of fiscal policy; and cast new light to their welfare significance. In stead of the Samuelsonian neo-classical synthesis, we propose the neo-Keynesian (or the mixed economy) synthesis that requires optimization of public good provision as well as full employment of labor.

Key words: multiplier, public goods, secular stagnation, public finance, neo-Keynesian synthesis
JEL Classification Number: E12, E22, E24, H30, H40

1. INTRODUCTION

Since the collapse of the Bubble in early 1990, the Japan's government expenditure continued to expand for more than 20 years, but failed to improve the business condition leaving the national rate of unemployment at high levels. In the meantime, the government deficit increased and consequently the balance of the government bond accumulated enormously. This experience cast doubt to the effectiveness of fiscal policy[1]intended to increase the aggregate income and employment in Japan. The Keynesian multiplier theory originated in the midst of the great depression of 1930s has served as

Acknowledgments. The draft of this chapter was first presented at the workshop presided by Ryuzo Sato on November 29, 2014. I benefited from instructive comments by Sato on the contributions by Richard Musgrave regarding the concept of public goods, together with constructive discussions from other attendants of the workshop. I am indebted to helpful comments by Masatoshi Tsumagari and an anonymous referee of *Keio Economic Studies*.

[1] Fiscal policy is here defined as a macroeconomic policy designed to stabilize national income and aggregate employment by controlling the government revenue and expenditure. It should be distinguished from public finance in the narrow sense limited to the provision of public goods.

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the cornerstone of the fiscal policy in the standard textbook for a long time. It emphasized multiplier effects of government deficit expenditure on the national income and employment symbolizing the Keynesian revolution in the history of macroeconomic policy. The recent experience of the stagnation of Japan and other countries, however, revealed that the multiplier effect was not as large as believed in the past. Moreover, frequent use of fiscal policy undermined the government fiscal discipline giving rise to inefficient resource allocation in Japan as well as in many other countries.

The multiplier theory typically abstracts from the coexistence of public and private goods regarding the aggregate output as a composite product. Keynes considered public works as an important element of fiscal policy, but somehow glossed over the distinction of private and public goods when he talked of the national product as if it were one good tradable in the market. At the time of the General Theory, the concept of public goods introduced by Musgrave (1959) and Samuelson (1954, 1955) was not known yet. In their interpretation, public goods differ from private goods in that they are provided by the dictation of the government while private goods are provided through demand and supply in the market. This distinction is not really understood even now, after more than half a century.

Another problem with the multiplier theory is its disregard of welfare economics. It tends to undervalue the welfare effect of public goods, while overestimating the income and employment effects arising from public works. In fact, the use of unemployed workers for the production of public goods may yield an increase in social welfare surpassing its direct income and employment effects. In terms of the simple Keynesian theory, however, an expansion of the government deficit expenditure brings about a far more increase in welfare through its multiplier effects than its direct meager effects though the accompanying provision of public goods. A well-known illustration given by Keynes himself is the employment of workers for burying old bottles in disused coal mines and digging them up again. It would increase the real income of the community through its multiplier effect even though there is no sense in such a project in terms of social welfare.

One of the most controversial problems with the multiplier theory was whether the public expenditure financed by the government deficit would really create much more national income than its face value. In order to finance the project, the government must issue the corresponding value of bonds, which it has to repay later by taxation in the future. The real burden of the project to taxpayers is, therefore, equivalent, regardless of whether it is financed by deficit or by taxation. This recognition due to Ricardo and Barro, is now well known as the equivalence theorem.[2] The simple multiplier theory ignores the message of this theorem completely. In fact, many public work projects put into practice in Japan by deficit financing during the "lost two decades" since the collapse of the bubble failed to revive the slumped economy. Moreover, they created a number of public facilities such as dams and buildings detrimental to environments and

[2] Ricardo's idea is recorded in his "Essays in the Funding System," in McCulloch (1888). Barro (1974, 1979) formalized his idea clearly in the words of modern economics.

social welfare. During the decades, the government deficit increased and its resulting debt continued to increase at unprecedented pace. In the 21st century, the sovereign risks attributable to government debt financing began to threaten business prospects of many countries all over the world.

In what follows, we reconsider the effects of fiscal policy using a simple real model of macroeconomies. In the absence of innovations and new frontiers, the effective demand of the economy may become insufficient to ensure the full employment even when the interest rates decline to zero like the "liquidity trap" envisioned by Hicks (1937).[3]

In Section 2, we consider the simple real economy without public goods, in which only private goods are traded in the market. Money and government are assumed away. This basic model is useful for the purpose of illuminating the deep depression economy envisioned by Hicks (1937) as "liquidity trap" where monetary policy becomes ineffective and fiscal policy does not exist. Krugman (1998) reintroduced the concept of Hicksian liquidity trap within the framework of IS-LM equilibrium. The fundamental cause of such a depression is deficiency of effective private demand. In order to remedy such a situation, it is necessary to introduce government fiscal policy conjugating private and public goods.

In Section 3, we modify the preceding setup and assume that the government intervenes in the economy providing public goods and transfer payment by means of taxation. It abides by balanced budget in the sense that its payment for public goods and transfer is fully financed by taxation. The amount of public goods is determined by the government democratically or dictatorially, while the amount of private goods (including labor services required for production of public goods) is determined by the adjustment of income in the market. It will be shown that the national disposable income expressed in terms of private goods becomes constant independently of the level of the government taxation. One unit of government expenditure generates one unit of private goods and associated labor services meaning that the multiplier of government expenditure is exactly unity.

In Section 4, we develop a two-good model in which public as well as private goods are considered explicitly and distinctly. When the economy is underemployed, an increase in the government expenditure increases employment and the output of the public good, but does not affect the national disposable income. If the public good is defined to be useful by a given social utility function, however, it clearly increases social welfare. Furthermore, if the public good is so designed to stimulate the demand for private goods, the fiscal multiplier will be strengthened. Once full employment is realized, a further increase in government expenditure, i.e., a further increase in the provision of public goods gives rise to "crowding out," or a corresponding decrease in the supply of private goods. This means that the opportunity cost of public goods is positive rather than zero.

In Section 5, we explore the effects of macroeconomic policy under full employment.

[3] Ohyama (1987, 2004, 2007) developed a version of the *IS–LM* model with a microeconomic foundation to characterize the "liquidity trap" as its special case.

In this phase, an increase in public expenditure may not be justified in view of the given social utility function even though the public good is useful on its own. In other words, it may be desirable to decrease government expenditure. The optimal provision of the public good is illustrated by looking at how the social indifference curve intersects with the production frontier of public and private goods. The purpose of public finance in this phase is to optimize the supply of the public good rather than just to preserve full employment. At a first glance, this idea may seem to resemble the neoclassical synthesis introduced by Samuelson (1954). But as we argue in Section 5, the government must realize full employment and optimize the supply of the public good at the same time. Thus it should be named the neo-Keynesian (or mixed system) synthesis of employment and public finance.

2. THE PRINCIPLE OF EFFECTIVE DEMAND

Let us begin by considering a simplest model in which there is only one good. It is supposed to be a private good used for production, investment as well as for consumption and tradable in the market. For simplicity, it may be named "rice," the most popular Japanese staple. A unit of rice is supposed be produced using a units of homogeneous labor. The representative worker is bestowed a given amount of leisure and is supposed to work for h hours per day for the wage rate w^C determined by labor contract on the basis of the social convention. Figure 1 depicts the representative worker's indifference curves between wage rate and leisure and the determination of contract wage on the vertical line showing the given labor hour. Contract wage is supposed to be negotiated at a value between reservation wage, $\underline{w}$, which the worker requires at least and unity which the employers can pay at most, or $\underline{w} \leq w^C \leq 1$. See McDonald and Solow (1981) and Ohyama (1987) for models of wage bargaining under unemployment.

The total number of workers in the economy is denoted $\bar{N}$. Let us denote by N^S the number of workers who are willing to be employed, or the supply of workers and by N^D the number of workers demanded by the economy. When contract wage satisfies the above inequality, the economy is in the state of full employment, or $N^S = \bar{N}$. Figure 2 shows how the supply of workers may not be equilibrated to the demand for workers, or $\bar{N} \geq N^D$. We assume here that the demand of workers falls short of the supply of workers giving rise to the emergence of involuntary unemployment. Such a situation occurs when the demand for national product at full employment is insufficient to absorb the supply of national product at full employment as in the case of the Great Depression in the US and the "Lost Two Decades" after the Bubble in Japan.

What then are the determinants of social demand for national product? In the present setup abstracting from the government, social private demand for national product consists of private consumption C and investment demand I. Private consumption demand depends on contract wage rate w^C, real rate of interest r, national income Y and capital stock K, which may be written as an aggregate consumption function

$$C = c(w^C, r, Y, K) \tag{1}$$

assuming

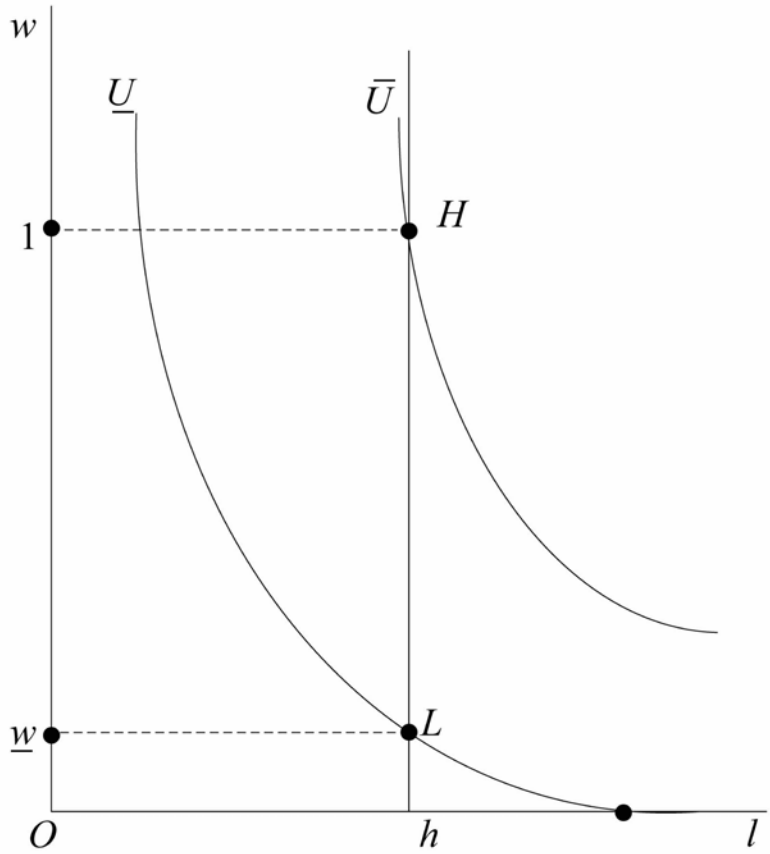

Figure 1. Standard Labor Hour and Reservation Wage.

$$c_w = \frac{\partial c}{\partial w^C} \geq 0\,, \quad 1 > c_Y \geq 0\,, \quad c_r = \frac{\partial c}{\partial r} \leq 0\,, \quad c_K = \frac{\partial c}{\partial K} \geq 0\,.$$

For simplicity, investment demand I is supposed to be given exogenously depending on the firm's long-term expectation among other factors.

The market equilibrium condition under unemployment is

$$Y = c(w^C, Y, K) + I \tag{2}$$

where Y works as an adjustment factor to equilibrate the demand and supply of private goods given w^C, K and I. Figure 3 shows the equilibrium national income Y_E at the intersectionof E aggregate demand curve Dd and 45° line, while Figure 4 illustrates the determination of national disposable income. Figure 5 shows the equilibrium F_B or E_{UG} supported by the government expenditure G_B.[4] In the absence of government intervention, market mechanism fails to realize full employment in this situation. The

[4] Keynes (1936), p. 25. "The point of intersection of the aggregate demand function and the aggregate supply function will be called *the effective demand.*

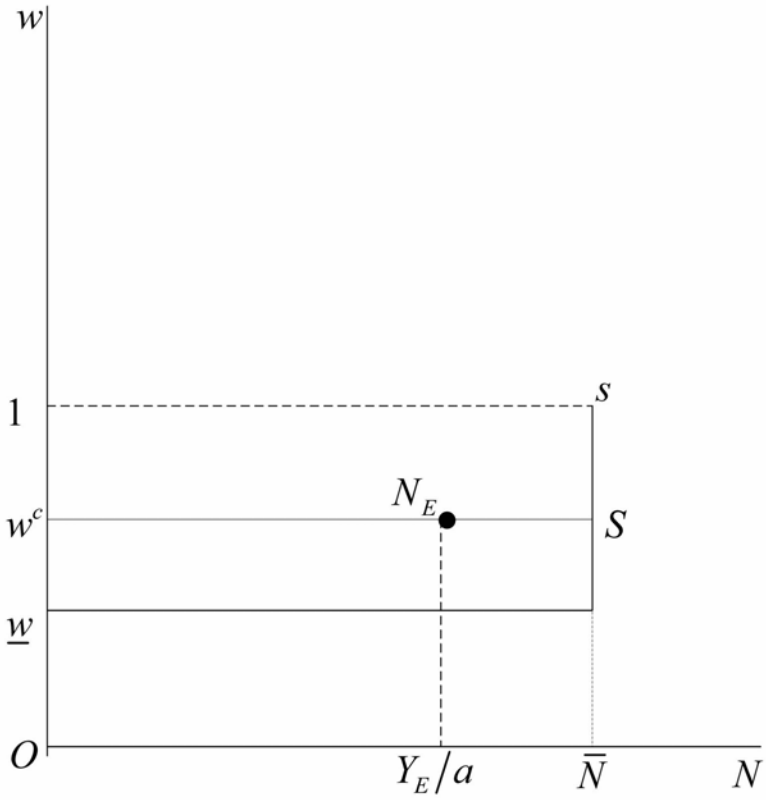

Figure 2. Supply Curve of Labor.

market equilibrium income falls short of full employment income, Y_F, by "deflationary gap" FG in case of Figure 3, or by $G_A G_B$ in case of Figure 5 implying the existence of corresponding unemployment. Reverting to Figure 2, we observe that the deflationary gap coincides with unemployment equal to segment $N_E S$. Figure 3 is essentially identical to the familiar illustration of "Keynesian Cross," whereas Figure 5 is invoked to take full advantage of the newly introduced general equilibrium model covering private and public goods.[5]

[5] The private goods used in the production of public goods may be subject to decreasing returns to proportion, given the stock of private capital. In such a case, the production frontier of private and public goods becomes convex to the origin and that the aggregate supply function depends positively on the relative price of public goods.

This modification is not necessary, however, for our conclusions but is agreeable to the Keynesian concepts of aggregate supply functions.

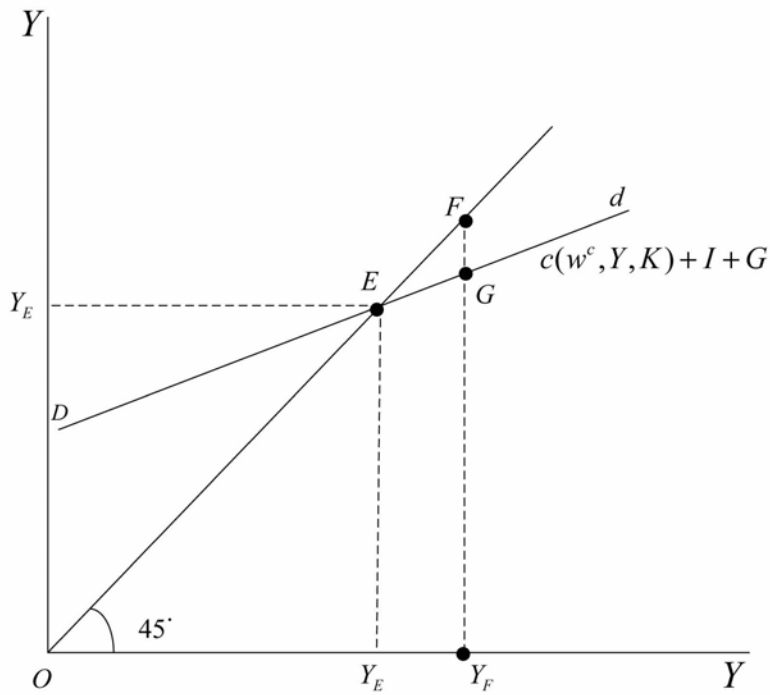

Figure 3. The Principle of Effective Demand.

3. GOVERNMENT AND FISCAL POLICY (TWO GOOD GENERAL EQUILIBRIUM)

Let us now introduce government and consider the multiplier effects of changes in government expenditure and other exogenous variables on national income and related endogenous variables. Suppose that proportion α of national tax revenue T is to be spent on government expenditure G on public goods and proportion $(1-\alpha)$ on transfer payments R:

$$T = G + R \tag{3}$$

$$G = \alpha T \tag{4}$$

$$R = (1-\alpha)T \tag{5}$$

$$Z = Y - T + R = Y - \alpha T \tag{6}$$

With the introduction of government, private consumption function is modified as

$$C = c(w^c, Y - \alpha T, K) \tag{7}$$

Given the reservation wage w^C, tax T, private investment I and consumption function, the new market equilibrium condition is written

$$Y - \alpha T = c(w^c, Y - \alpha T, K) + I \tag{8}$$

determines equilibrium income Y_E and disposable income

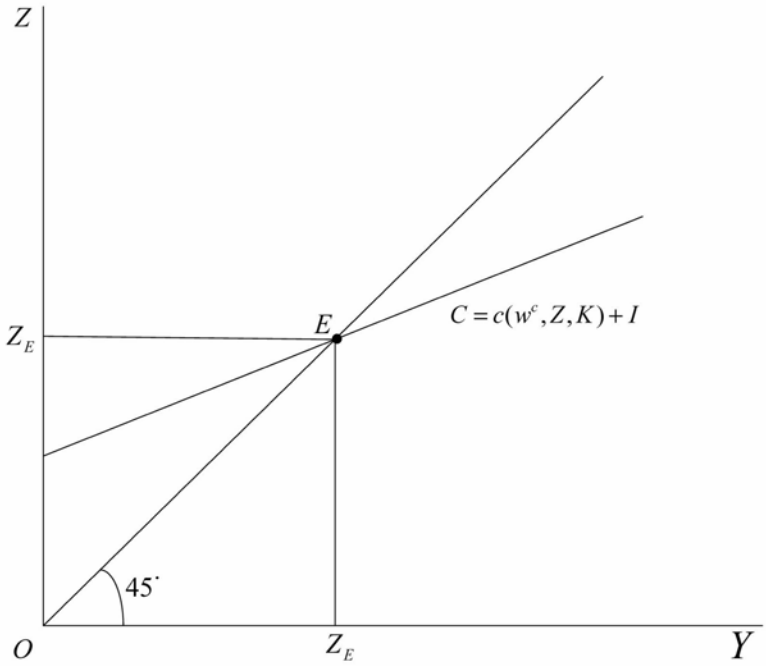

Figure 4. The Determination of National Disposable Income.

$$Z(= Y_E - \alpha T)\,.$$

The effect of an increase in autonomous investment on equilibrium income and disposable income is shown by

$$\frac{\partial Y_E}{\partial I} = \frac{\partial Z_E}{\partial I} = \frac{1}{1 - c_Z}\,. \tag{9}$$

Marginal propensity to spend out of disposable income, c_Z, is assumed to positive and smaller than 1, which implies that the investment multiplier is positive and could be much more than 1. This conclusion is nothing more than the familiar investment multiplier. The effect of a rise in contract wage, w^C, is not quite clear but often supposed to be positive as workers often spend more than capitalists on private goods.

How about the effect of government expenditure? First suppose that the government spends its tax revenue on useless goods. This is exactly the case of Keynesian parable of burying old bottles in disused coal mines and digging them up again. The effect of an increase in government expenditure under balanced budget is

$$\frac{\partial Y_E}{\partial T} = \alpha\,. \tag{10}$$

To be sure, the balanced budget multiplier is equal to unity, or

$$\frac{\partial Y_E}{\partial G} = 1\,. \tag{11}$$

Needless to say, the multiplier effect on the equilibrium disposable income becomes

$$\frac{\partial Z_E}{\partial G}\left(=\frac{\partial Z_E}{\partial T}\right)=0\,. \tag{12}$$

PROPOSITION 1 (balanced budget multiplier). *Suppose that unemployment prevails under balanced budget. Given the proportion of government transfer payment at* $(1-\alpha)$ *of tax revenue, a unit increase of tax increases equilibrium national income by* α *but leaves equilibrium disposable income unaffected.*

Standard textbook of macroeconomics tells us that the balanced budget multiplier is just unity. Here, we say that if the proportion of government transfer is set at $1-\alpha$ of tax revenue, tax based multiplier becomes equal to α or smaller than 1. If $\alpha = 1$, the standard balanced budget multiplier of unity obtains. The larger the value of α, the smaller becomes the multiplier.

These conclusions are the results of our implicit assumption that the net effect of transfer payments on aggregate demand is zero. In fact, however, transfer payments are often related to social security benefits, which distributes income from rich to poor people thereby increasing aggregate consumption expenditure. This is one of the most controversial problems regarding the simultaneous reform of tax and social security systems recently discussed in Japan. In this paper we assume that consumers are homogeneous and therefore transfer payments tend to decrease government expenditure on private goods used in the production of public goods.

It should also be noted that a tax-financed government expenditure does not affect the disposable national income. This is because an increase in T brings about an equal increase in Y_E/α under balanced budget. The increase in the government expenditure financed by tax increases government expenditure for public goods and therefore labor employment in the production of public goods but it does not affect the income and employment in the private sector. Caesar's money used for the provision of the public goods is returned to Caesar through taxation, as it were.

While the aggregate employment increases, per capita income of employed workers decreases in the face of constant aggregate income. Thus, the increase in the government expenditure amounts to work (income) sharing among potential workers.

PROPOSITION 2 (work and income-sharing effect of government expenditure). *An increase in government expenditure on public goods increases aggregate employment and realizes work and income sharing among workers.*

The government is supposed to provide public good and public investment in addition to transfer payment. Here, public good is defined to be flow good, distinguished from public capital stock (or social common capital in the word of Uzawa (1974)).[6] Here, public good is specified as a labor service used in the production of public good in collaboration with public capital. Public investment is investment in public capital stock like roads, harbors, parks, embankments, national defense forces etc. that are publicly

[6] Also see Musgrave (1959), pp. 13–14. The merit wants considered there are related to flow public goods.

owned. We will consider the economic significance of public investment and public capital stock in more detail in the next section.

For simplicity, 1 unit of public good is supposed to be produced using a unit of private good. The total number of workers N employed in national production is the sum of workers employed in the government and private sectors:

$$N = X + aG\,, \tag{13}$$

where X denotes the output of the private good. Note that G is equal to the amount of labor employed in the production of the public good. Figure 5 shows the equilibrium of the mixed economy where the government undertakes to provide the public good and the private sector supplies private goods through market. The vertical axis measures the government G expenditure and the horizontal axis that of the private good. The equilibrium employment N_E contains unemployed workers when

$$N_E = X_E + aG < \bar{N}. \tag{14}$$

The curve Tt depicts a straight line production frontier between G and X on the simplifying assumption that the labor coefficient a is given and fixed. The curves Ss are the social indifference curves between G and X on the assumption that the marginal utility of the public goods decreases given the aggregate consumption of the private goods.

The curve S_U is the social indifference curve where the government expenditure on public goods is given at G_A and the consumption of the private goods Z_O is correspondingly determined on S_U. As pointed out above, an increase in the government expenditure increases the aggregate income and employment in the same proportion, but what can we say about its welfare effects? An increase in the aggregate income and employment would increase the expected utility of the potential workers by increasing the probability of their employment. Moreover, the associated increase in the provision of public goods would increase the welfare (social utility) by itself. To make this point clear, let us define the social utility function (the graphical representation of which is the social indifference curve) as

$$W = u(X, G)\,, \quad u_X > 0\,, \quad u_G > 0\,. \tag{15}$$

PROPOSITION 3 (expansion path in the presence of unemployment). *The expansion of tax-financed government expenditure shifts the equilibrium point under unemployment to increase social welfare along the expansion path* $\overline{E_A E_B}$.

According to the old Keynesian view, a bond-financed government expenditure brings about a multiplied increase in national income, whereas a tax-financed increase in government expenditure results in an equal increase in national income. As already argued, this view is untenable in that it overlooks the future tax burden of the expenditure increase. Furthermore, it focuses only on the income effects of the expenditure increase neglecting its welfare effects. In light of (15), an increase in tax-financed government expenditure creates extra income and employment without incurring opportunity costs under unemployment. It is bound to increase social welfare.

There are some public goods that complement and promote private consumption and

investment. For example, the provision of care and nurse services by the government may increase consumption by creating leisure time for housekeepers. The maintenance of public roads and port facilities may be important for business opportunities and investments in general. Under these circumstances, consumption function may be rewritten as a function of government expenditure as well as on real interest rate r, national income and on capital stock:

$$C = c(w^C, r, Y - \alpha T, K, \alpha T) \tag{16}$$

and investment function may be similarly modified as

$$I = i(r, \alpha T, K) \tag{17}$$

where $c_r < 0 \quad i_r < 0$.

PROPOSITION 4 (government expenditure complementing private expenditure). *The balanced budget multiplier of a government expenditure that complements private expenditure has a multiplier greater than α under unemployment.*[7]

After all the prevalence of unemployment stems from deficiency of effective demand, the result of unwillingness on the part of people to consume or invest. It may reflect the shortage of public goods in the broad sense including those that complement private expenditure.

4. FISCAL POLICY UNDER FULL EMPLOYMENT

In the presence of unemployment, it is possible to carry out the production of the public good by using labor from the pool of unemployed workers without reducing the output of private goods, i.e., incurring no opportunity costs. Once full employment is achieved and maintained, however, this is no longer the case. In Figure 5, indifference curve S_B passing though E_B cut production frontier Tt from down left to right above This means that the supply of the public good is insufficient and there are rooms for improving social welfare by increasing its supply. The socially optimal equilibrium is shown by E^* where indifference curve S^* touches Tt. Under full employment, however, an increase in the supply of the public good is achievable only if the supply of the private good is correspondingly decreased along Tt. Throughout the process, labor moves from the private sector to the public sector and the equilibrium point from E_B to F^* and the social welfare increases along the expansion path (or adjustment path) $E_B F^*$. Figure 6 depicts the case where the output of the public good at the socially optimal point F^* is smaller than that of E_B. It is desirable to decrease the production of the public good and increase that of the private good along the expansion path (or adjustment path) $E_B F^*$.

[7] Aside from the demand increasing effects from the introduction of public goods considered here, it is also worth taking note of the supply increasing effects from the public goods. In the present model, however, the supply shocks of this kind do not affect the short –run adjustment process under unemployment. We shall explore the long-run effects of public investment (increase in the stock of public capital) later in the next section.

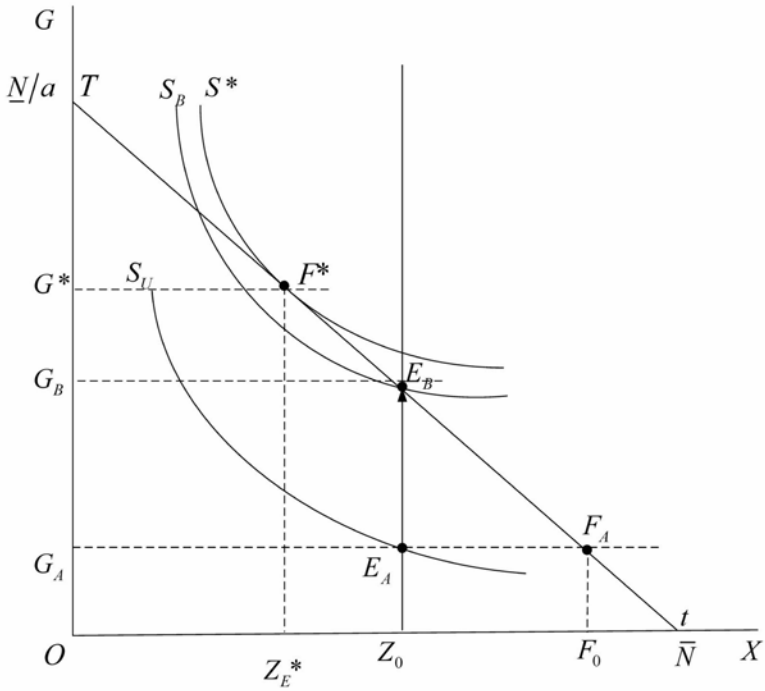

Figure 5. An Expansion Path of the Mixed Economy Equilibrium.

PROPOSITION 5 (Expansion Path under Full Employment). *When the government expenditure that ensures the full employment differs from its socially optimal level, a further adjustment has to be pursued along the expansion path (or adjustment path) $E_B F^*$ of the production frontier. To be more precise, if $G^* > G_B$, the government must increase the its expenditure and the supply of public goods. In contrast, if $G^* < G_B$, the government must decrease its expenditure and the supply of public goods. The social welfare increases in the process of the adjustment process along $E_B F^*$.*

5. NEO-KEYNESIAN SYNTHESIS

Early in 1950's, Samuelson put forward the idea that once the government succeeds in maintaining full employment, labor embodied in the employed workers will be optimally allocated through market mechanism, and named it the "neo-classical synthesis".[8] If full employment is maintained by the government's fiscal policy, the resource allocation of private goods will be achieved by market mechanism optimally to maximize social welfare. Within the simple model devoid of public goods, the fundamental theorem of welfare economics is established that general equilibrium becomes Pareto optimal under the condition of perfect competition and in the absence of externalities.

[8] See Samuelson (1955), p. 212.

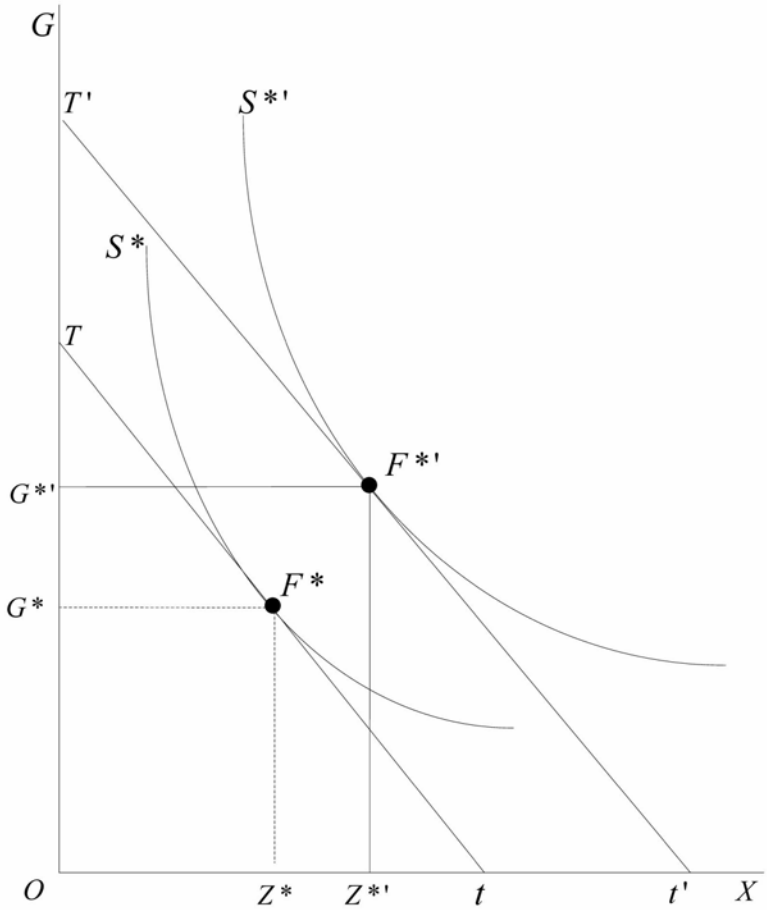

Figure 6. Productivity Improvement in Private Good Production.

In the present model the shortage of social public goods in the short run and social capital stock in the long run may depress consumption and investment bringing about short run unemployment and long run stagnation as mentioned above. Thus the government provision of social public goods in the short run and the social capital stock in the long run may exert positive externalities in the sense that it promotes private consumption and investment easing short run unemployment and long run stagnation. Figures 5 and 6 illustrate how the optimal allocation of national resources may be achieved by the government intervention along with market mechanism starting from the unemployment equilibrium E_A through full employment equilibrium E_B culminating in the optimal equilibrium E^*.

PROPOSITION 6 (Neo-Keynesian or mixed-economy synthesis). *Suppose that given*

resource endowment, production technology and consumers' preference, static stationary state prevails where net investment and saving become zero and real interest rate stabilizes at the natural level. Also suppose that perfect competition prevails in the market for private goods with no technological externalities. The government is assumed to anticipate the short-run as well as the long-run effects of public investment with perfect foresight. The government should then be able to use expenditure policy appropriately to increase employment in the short-run and maintain full employment and achieve the optimal provision of public goods in the long-run. This design of economic policy may be named the neo-Keynesian (or the mixed-economy) synthesis.[9]

So far we have abstracted from the dynamic effects of investment on production technology and formation of resources. It is the essence of investment either private or public, however, that it improves the quality of technology and resources. We have to reconsider the present model taking explicit account of these dynamic effects of investment. Let us divide government expenditure G into government consumption G_C and government investment G_I. The former is supposed to produce public services and the latter to improve the quality and efficiency of public capital stock K_G. Similarly, private investment I is supposed to improve the quality and efficiency of private capital stock K. Considering these relationship, we may rewrite the employment constraint as

$$N(K, K_G) = X + a(K_G)G \tag{14'}$$

where $a(K_G)$ is supposed to be a decreasing function of K_G since government investment certainly increases future government capital stock thereby improving labor productivity in the production of public goods (or lowering labor coefficient a) and $N(K, K_G)$ shows the national supply of labor in efficiency units as an increasing function of K, K_G since private and government capital stock serve as the basis of labor supply in efficiency units.

Figure 6 illustrates the effects of public and private investment on the position and shape of production frontier. The present frontier is shown by Tt. An increase in private and public investment improving labor productivity in projects such as environment, roads, harbor facilities and education may increase the supply of labor measured in efficiency units bringing about a parallel upward shift of production frontier expected in future to $T't'$. Given the social indifference curves as before, the present and future optimal equilibria are shown by F^*, $F^{*\prime}$ or the points of contact with $T't'$. Barring inferior goods, social consumption of the present and future goods will increase in that event. Similarly, Figure 7 depicts how the production frontier shifts as a result of an increase in public investment on account of its expansive effects on future public capital stock. It will decrease the labor coefficient a of the public good and expand the production frontier asymmetrically from Tt to $T't$. The relative cost of future public good will decrease and the optimal production point will change from F^* to $F^{*\prime}$. As a result, the future output of the public good will normally increase, but the future output of the

[9] For simplicity, we have assumed away the externalities of any kind. It would be easy to extend this proposition introducing the government tax-cum subsidy scheme to internalize externalities.

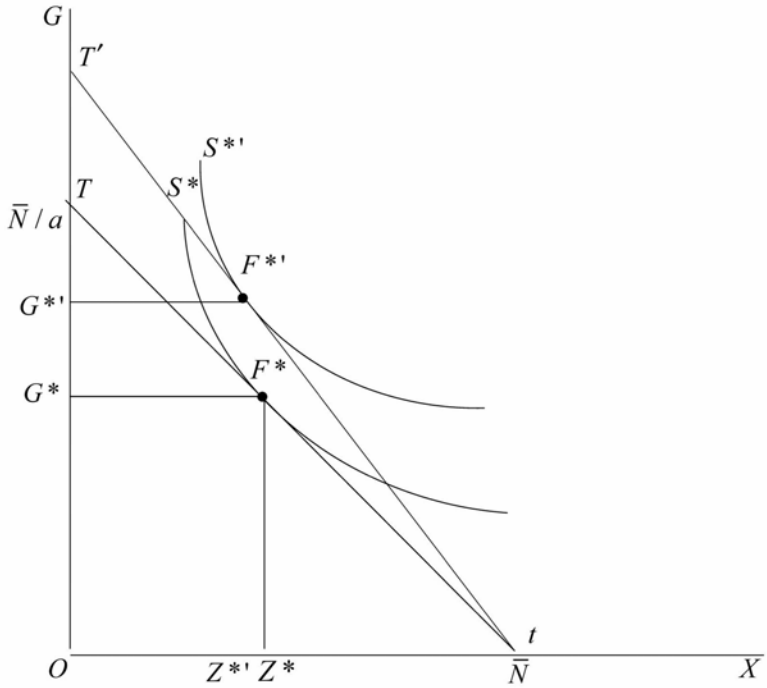

Figure 7. Productivity Improvement in Public Good Production.

private good may decrease.

PROPOSITION 7 (growth strategy). *Investment in general, both public and private, increases income and employment in the short run and work as the engine of economic growth by accumulating public and private capital stock in the long run.*[10]

Investment is capable of generating positive economic effects beyond consumption both in the short run and in the long run. As witnessed by the current difficulties in public finance in Japan and many other advanced countries, however, misguided public investment can lead to serious government indebtedness and tremendous misallocation of resources. Useless and untimely public investment destroy natural environment and educational facilities resulting in the loss of aggregate labor supply in efficiency units and contraction of production frontier. Therefore, it is vitally important to take into full account the welfare implications of public expenditure and investment in particular.

6. CONCLUDING REMARKS

In this paper we attempt to reconsider and extend the economic theory of deep depression initiated by Keynes (1936) taking explicit account of the concept of public

[10] The third arrow of "Abenomics" in the Japanese Economic policy calls for the growth strategy of useful public investment and deregulatory measures to improve the opportunities of private investment.

goods largely blurred in his general theory and after. To contrive the model of secular stagnation where the interest rate falls to zero and monetary policy is restricted by the zero-bound, we concentrate on real economy featuring on the theory of public finance. In so doing, we synthesize the traditional theory of public finance and the Keynesian fiscal theory and cast new light to their welfare significance.

(1) As the basic setting, we assume the Schumpeterian stationary state in which there are no market-originated private innovations and investment and the interest rate has fallen to zero. Using a simple macroscopic general equilibrium model we show that Keynesian involuntary unemployment equilibrium can arise there from deficient Keynesian effective demand. This underemployment equilibrium is realized through the adjustment of national income, not through the adjustment of interest rate nor through the adjustment of commodity prices. To reduce or remove unemployment in the equilibrium, it is necessary for the government to intervene by producing public goods or carrying out public investment. Conversely, the government intervention in this fashion is justified only under the present setup.

(2) In line with the Ricardo-Barro equivalence theorem, the government is supposed to abide by the balanced budget discipline. Even so, the government should be able to increase labor employment (private goods) in the government sector keeping the national disposable income at a constant level . This may be called the work (income) sharing effects of the government expenditure (Propositions 1 and 2).

(3) If the government expenditure is sufficiently large, it will realize full employment theoretically in no time. In reality, however, it may take some time to accomplish full employment. The process of adjustment toward full employment may be divided into discreet periods during which the government increases expenditure on goods and services continuously. It may be visualized as an expansion path of unemployment equilibrium as in $\overline{E_A E_B E^*}$ in Figure 5. If the public goods are useful, the national welfare will increase along the path (Proposition 3).

(4) The public good is supposed to be desirable in itself in the sense that it directly enhances social welfare, but it is often designed to increase the propensity to consume or invest thereby adding to effective demand. In such a case, the multiplier of the government expenditure will be greater than 1 even under balanced budget (Proposition 4).

(5) The production of the public good, whether it is useful or not, incurs opportunity cost except in the presence of involuntary unemployment. Starting from the initial unemployment equilibrium, an increase in the government expenditure will initially increase the production of the public good and employment without decreasing that of the private good, but as soon as full employment is maintained, there arises trade-off between the public good and the private good so that a further increase in the supply of the public good may not warrant improvement of national welfare. It may become necessary to decrease the supply of the public good on the way. The optimal supply of the public good must be determined in view of some welfare criterion and its opportunity cost in terms of the private good. Figure 6 illustrates this point using the social indifference curve and the production frontier (Proposition 5).

(6) Samuelson (1954) proposed the so-called neo-classical synthesis to the effect that

once full employment is established by the government, the market will accomplish the Pareto optimum of the rest of resource allocation. As shown above, however, the government's role goes beyond the achievement of full employment. It extends to the optimization of social welfare through appropriate provision of the public goods. The neo-Keynesian (or the mixed economy) synthesis that calls for optimization of public good provision as well as for full employment of labor is called for (Proposition 6).

(7) The foregoing analysis is applicable only to the "short run" setting where the production technology and labor resources of the economy and therefore the production frontier are given and fixed. Investment, public as well as private, is not only conducive to the short-run recovery but also to the "long-run" growth of the economy through its dynamic effects on the production frontier. In fact, it extends the future production frontier by means of the accumulation of public and private capital stock (Proposition 7). As argued by the advocateors of "Abenomics", public investment and active deregulation should serve as the growth strategy of the Japanese economy stagnant over 20 years since early 1990.

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