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RELATIVE PROFIT MAXIMIZATION AND IRRELEVANCE OF LEADERSHIP IN STACKELBERG MODEL

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Abstract: We study the Stackelberg equilibrium in a symmetric duopoly with differentiated goods in which each firm maximizes its relative profit that is the difference between its profit and the profit of the rival firm. We show that the equilibrium output and price of the good of the leader and those of the follower are equal, that is, the role of leader or follower is irrelevant to the equilibrium, and the equilibrium outputs and prices do not change between the case where the firms are quantity setting firms and the case where the firms are price setting firms. We assume that demand functions are linear and symmetric, the marginal costs of the firms are common and constant, and the fixed costs are zero.

Key words: differentiated duopoly, relative profit maximization, Stackelberg model, irrelevance of leadership.

JEL Classification Number: D43, L13.

1. INTRODUCTION

We study the Stackelberg equilibrium in a symmetric duopoly with differentiated goods in which each firm maximizes its relative profit that is the difference between its profit and the profit of the rival firm. We show that the equilibrium output and price of the good of the leader and those of the follower are equal, that is, the role of leader or follower is irrelevant to the equilibrium, and the equilibrium outputs and prices do not change between the case where the firms are quantity setting firms and the case where the firms are price setting firms. We assume that demand functions are linear and symmetric, the marginal costs of the firms are common and constant, and the fixed costs are zero.

In recent years, maximizing relative profit instead of absolute profit has aroused the interest of economists. From an evolutionary perspective, Schaffer (1989) demonstrates with a Darwinian model of economic natural selection that if firms have market power, profit-maximizers are not necessarily the best survivors. According to Schaffer (1989),

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a unilateral deviation from Cournot equilibrium decreases the profit of the deviator, but decreases the other firm's profit even more. On the condition of being better than other competitors, firms that deviate from Cournot equilibrium achieve higher payoffs than the payoffs they receive under Cournot equilibrium. In Vega-Redondo (1997), it is argued that, under a general equilibrium framework, if firms maximize relative profit, a Walrasian equilibrium can be induced.

On the other hand, Lundgren (1996) shows that by making managerial compensation depend on relative profits rather than absolute profits, the incentives for oligopoly collusion can be eliminated. Kockesen et al. (2000) have shown that under some conditions a firm which strives to maximize relative profit will outperform a firm which maximizes absolute profit. Bolton and Ockenfels (2000) conducted an analysis considering an individual utility function that brings about a feeling of compassion toward an individual with a relatively lower material payoff and simultaneously brings about envy of other individuals with a higher material payoff.

As demonstrated by Matsumura, Matsushima and Cato (2013) evaluations of managers' performances are often based on their relative performance. Outperforming managers often obtain good positions in the management job markets. And the spiteful behavior as well as reciprocal behavior or altruistic behavior is closely related to the objective functions based on relative performance. The use of relative performance evaluation has been empirically supported by Gibbons and Murphy (1990) [1].

In another paper Tanaka (2013a) we have shown that in a duopoly with differentiated goods under linear demand functions when firms maximize relative profits, a Cournot equilibrium and a Bertrand equilibrium coincide. In Tanaka (2013b) we studied the choice of strategic variables by firms in a two stage game of duopoly with linear demand functions such that in the first stage the firms choose their strategic variables and in the second stage they determine the values of their strategic variables, and we have shown that when the firms maximize their relative profits, the choice of strategic variables is irrelevant to the outcome of the game in the sense that the equilibrium outputs, prices and profits of the firms are the same in all situations, and so any combination of strategy choice by the firms constitutes a sub-game perfect equilibrium in the two stage game.

The result of this paper is an extension of theses results.

2. THE MODEL

There are two firms, A and B. They produce differentiated substitutable goods. Denote the output of Firm A and B, respectively, by x_A and x_B, the prices of the goods of Firm A and B, respectively, by p_A and p_B. The marginal costs of the firms are common, and equal $c > 0$. There is no fixed cost.

The inverse demand functions of the goods produced by the firms are

$$p_A = a - x_A - bx_B , \tag{1}$$

and

[1] For these arguments about relative profit maximization we refer to Lu (2011).

$$p_B = a - x_B - bx_A\,, \tag{2}$$

where $a > c$ and $0 < b < 1$. x_A represents the demand for the good of Firm A, and x_B represents the demand for the good of Firm B. The prices of the goods are determined so that demand of consumers for each firm's good and supply of each firm are equilibrated.

The ordinary demand functions for the goods of the firms are obtained from these inverse demand functions as follows,

$$x_A = \frac{1}{1-b^2}[(1-b)a - p_A + bp_B]\,, \tag{3}$$

and

$$x_B = \frac{1}{1-b^2}[(1-b)a - p_B + bp_A]\,. \tag{4}$$

We consider a model of Stackelberg competition. Firm A is a leader and Firm B is a follower. We analyze two cases. One is a case where the firms, the leader and the follower, are quantity setting (Cournot type) firms. And in the other case the firms are price setting (Bertrand type) firms.

3. SUMMARY OF ABSOLUTE PROFIT MAXIMIZATION CASE

In this subsection for reference we summary the results of the case of absolute profit maximization.

First assume that Firm A and B are quantity setting firms. Firm B determines its output given the output of Firm A. Then, we get the reaction function of Firm B as follows.

$$x_B = \frac{1}{2}(a - bx_A - c)\,. \tag{5}$$

The leader, Firm A, determines its output given the reaction function of Firm B in (5). The equilibrium outputs of Firm A and B are, respectively,

$$x_A^Q = \frac{(2-b)(a-c)}{2(2-b^2)}\,,$$

and

$$x_B^Q = \frac{(4-b^2-2b)(a-c)}{4(2-b^2)}\,.$$

The equilibrium prices of the goods of Firm A and B are, respectively

$$p_A^Q = \frac{(4+b^3-2b^2-2b)a + (4-b^3-2b^2+2b)c}{4(2-b^2)}\,,$$

and

$$p_B^Q = \frac{(4-b^2-2b)a + (4-3b^2+2b)c}{4(2-b^2))}\,.$$

Comparing x_A^Q with x_B^Q,

$$x_A^Q - x_B^Q = \frac{b^2(a-c)}{4(2-b^2)} > 0\,.$$

Thus, the equilibrium output of the leader is larger than that of the follower.

Next, assume that Firm A and B are price setting firms, Firm B determines the price of its good given the price of the good of Firm A. Then, we get the reaction function of Firm B as follows,

$$p_B = \frac{1}{2}[(1-b)a + bp_A + c]\,. \tag{6}$$

The leader, Firm A, determines the price of its good given the reaction function of Firm B in (6). The equilibrium prices of the goods of Firm A and B are, respectively,

$$p_A^P = \frac{(1-b)(2+b)a + (2+b-b^2)c}{2(2-b^2)}\,,$$

and

$$p_B^P = \frac{(1-b)(4-b^2+2b)a + (4-b^2+2b-b^3)c}{4(2-b^2)}\,.$$

The equilibrium outputs of Firm A and B are, respectively

$$x_A^P = \frac{(2+b)(a-c)}{4(1+b))}\,,$$

and

$$x_B^P = \frac{(4-b^2+2b)(a-c)}{4(1+b)(2-b^2)}\,.$$

Comparing x_A^P with x_B^P,

$$x_A^P - x_B^P = \frac{-b^2(a-c)}{4(2-b^2)} < 0\,.$$

Thus, the equilibrium output of the leader is smaller than that of the follower.

4. RELATIVE PROFIT MAXIMIZATION

In this section we consider a case of relative profit maximization. The relative profit of Firm A (or B) is the difference between its profit and the profit of Firm B (or A). Denote the relative profit of Firm A by Π_A and that of Firm B by Π_B.

4.1. Quantity Setting Competition

Assume that the firms, Firm A and B, are quantity setting firms. The relative profit of Firm B is written as, using the inverse demand function,

$$\Pi_B = (a - x_B - bx_A)x_B - cx_B - (a - x_A - bx_B)x_A + cx_A\,. \tag{7}$$

Firm B determines its output *given the output of Firm A* so as to maximize Π_B. The condition for relative profit maximization of Firm B is

$$a - 2x_B - bx_A - c + bx_A = 0\,. \tag{8}$$

Then, the output of Firm B is obtained as follows,

$$\tilde{x}_B = \frac{a-c}{2}\,. \tag{9}$$

This is the reaction function of Firm B, but it does not depend on x_A, and it is the equilibrium output of Firm B. The leader, Firm A, determines its output given the reaction function of Firm B in (9) so as to maximize its relative profit. The relative profit of Firm A is

$$\Pi_A = \left[a - x_A - \frac{b}{2}(a-c)\right]x_A - cx_A - \left[a - \frac{1}{2}(a-c) - bx_A\right] \times \frac{1}{2}(a-c) + c \times \frac{1}{2}(a-c)\,. \quad (10)$$

The condition for relative profit maximization of Firm A is

$$a - 2x_A - \frac{b}{2}(a-c) - c + \frac{b}{2}(a-c) = 0\,. \quad (11)$$

The equilibrium output of Firm A is obtained as follows,

$$\tilde{x}_A = \frac{a-c}{2}\,.$$

The equilibrium prices of the goods of Firm A and B are, respectively

$$\tilde{p}_A = \frac{(1-b)a + (1+b)c}{2}\,,$$

and

$$\tilde{p}_B = \frac{(1-b)a + (1+b)c}{2}\,.$$

We find $\tilde{x}_A = \tilde{x}_B$ and $\tilde{p}_A = \tilde{p}_B$. Therefore, whether a firm is the leader or the follower does not affect the equilibrium outputs and prices at the Stackelberg equilibrium under relative profit maximization when the firms are quantity setting firms.

4.2. Price Setting Competition

Assume that the firms, Firm A and B, are price setting firms. The relative profit of Firm B is written as, using the ordinary demand function,

$$\Pi_B = \frac{1}{1-b^2}[(1-b)a - p_B + bp_A](p_B - c) - \frac{1}{1-b^2}[(1-b)a - p_A + bp_B](p_A - c)\,. \quad (12)$$

Firm B determines the price of its good *given the price of the good of Firm A* so as to maximize Π_B. The condition for relative profit maximization of Firm B is

$$(1-b)a - 2p_B + bp_A + c - b(p_A - c) = 0\,. \quad (13)$$

The price of the good of Firm B is written as follows,

$$\hat{p}_B = \frac{(1-b)a + (1+b)c}{2}\,. \quad (14)$$

This is the reaction function of Firm B in this case, but it does not depend on p_A, and it is the equilibrium price of the good of Firm B. The leader, Firm A, determines the price of its good given the reaction function of Firm B in (14). The relative profit of Firm A is

$$\begin{aligned}\Pi_A = & \frac{1}{1-b^2}\left\{(1-b)a - p_A + b\frac{(1-b)a+(1+b)c}{2}\right\}(p_A - c) \\ & - \frac{1}{1-b^2}\left\{(1-b)a - \frac{(1-b)a+(1+b)c}{2} + bp_A\right\} \\ & \times \left[\frac{(1-b)a+(1+b)c}{2} - c\right]. \end{aligned} \tag{15}$$

The condition for relative profit maximization of Firm A is

$$(1-b)a - 2p_A + b\frac{(1-b)a+(1+b)c}{2} + c - b\frac{(1-b)a+(1+b)c}{2} + bc = 0\,. \tag{16}$$

The equilibrium price of the good of Firm A is obtained as follows,

$$\hat{p}_A = \frac{(1-b)a+(1+b)c}{2}\,.$$

The equilibrium outputs of Firm A and B are, respectively

$$\hat{x}_A = \frac{a-c}{2}\,,$$

and

$$\hat{x}_B = \frac{a-c}{2}\,.$$

We find $\hat{x}_A = \hat{x}_B$ and $\hat{p}_A = \hat{p}_B$. Therefore, whether a firm is the leader or the follower does not affect the equilibrium outputs and prices at the Stackelberg equilibrium under relative profit maximization when the firms are quantity setting firms.

Further we find $\tilde{x}_A = \hat{x}_A$, $\tilde{x}_B = \hat{x}_B$, $\tilde{p}_A = \hat{p}_A$ and $\tilde{p}_B = \hat{p}_B$. Thus, the Stackelberg equilibrium when the firms are quantity setting firms and that when the firms are price setting firms coincide.

5. CONCLUDING REMARKS

From the results of the previous sections we get the following conclusion.

> At the Stackelberg equilibrium in a duopoly with differentiated goods in which each firm maximizes its relative profit that is the difference between its profit and the profit of the rival firm, the output and price of the good of the leader and those of the follower are equal, that is, the role of leader or follower is irrelevant to the equilibrium, and the equilibrium outputs and prices do not change between the case where the firms are quantity setting firms and the case where the firms are price setting firms. We assumed that demand functions are linear and symmetric, the marginal costs of the firms are common and constant, and the fixed costs are zero.

Relative profit maximization is another model of imperfect competition in addition to Cournot and Bertrand models. Under relative profit maximization distinction of Cournot, Bertrand and Stackelberg is meaningless. In monopoly and perfect competitive economy relative profit maximization coincides with absolute profit maximization.

Assuming that firms seek to maximize some weighted average of absolute and relative profits may be more realistic. In this paper, however, we have presented striking results under the assumption of genuine relative profit maximization.

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