

Title	International trade and wage inequality in a dynamic model
Sub Title	
Author	Gupta, Manash Ranjan Dutta, Priya Brata
Publisher	Keio Economic Society, Keio University
Publication year	2014
Jtitle	Keio economic studies Vol.50, (2014.) ,p.29- 49
JaLC DOI	
Abstract	This paper extends the one commodity two sector dynamic model of Kiley (1999) introducing two different commodities to be produced in two sectors and introducing international knowledge spill over from the rest of the world to the home country and localized knowledge spillover from the more advanced modern sector to the less advanced traditional sector. We analyse the effect of opening of international trade on the skilled unskilled wage inequality in the long run equilibrium. We show that the relationship between the skilled-unskilled wage ratio and the skilled unskilled labour endowment ratio under autarky is ambiguous; and the nature of this relationship depends on the degree of consumer's indifference substitution between the two final goods. However, when international trade is opened, its effect on this skilled-unskilled wage ratio in the long run equilibrium depends not only on the degree of consumer's indifference substitution between the two final goods but also on the intensity of spillover effects as well as on the intercountry difference in factor endowments.
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-20140000-0029

慶應義塾大学学術情報リポジトリ(KOARA)に掲載されているコンテンツの著作権は、それぞれの著作者、学会または出版社/発行者に帰属し、その権利は著作権法によって保護されています。引用にあたっては、著作権法を遵守してご利用ください。

The copyrights of content available on the KeiO Associated Repository of Academic resources (KOARA) belong to the respective authors, academic societies, or publishers/issuers, and these rights are protected by the Japanese Copyright Act. When quoting the content, please follow the Japanese copyright act.

INTERNATIONAL TRADE AND WAGE INEQUALITY IN A DYNAMIC MODEL

Manash Ranjan GUPTA

*Economic Research Unit, Indian Statistical Institute,
Kolkata, India*

and

Priya Brata DUTTA

*Economics and Politics Department, Visva-Bharati University,
Bhirkhum, India*

First version received March 2014; final version accepted September 2014

Abstract: This paper extends the one commodity two sector dynamic model of Kiley (1999) introducing two different commodities to be produced in two sectors and introducing international knowledge spill over from the rest of the world to the home country and localized knowledge spillover from the more advanced modern sector to the less advanced traditional sector. We analyse the effect of opening of international trade on the skilled unskilled wage inequality in the long run equilibrium. We show that the relationship between the skilled-unskilled wage ratio and the skilled unskilled labour endowment ratio under autarky is ambiguous; and the nature of this relationship depends on the degree of consumer's indifference substitution between the two final goods. However, when international trade is opened, its effect on this skilled-unskilled wage ratio in the long run equilibrium depends not only on the degree of consumer's indifference substitution between the two final goods but also on the intensity of spillover effects as well as on the intercountry difference in factor endowments.

Key words: Wage Inequality, Skilled labour, Unskilled labour, International knowledge spillovers, Dynamic Model.

JEL Classification Number: F13, F43, J31, O30, O41.

Acknowledgments. We are thankful to the referee of this journal for his interesting and helpful comments on an earlier version of this paper. The present version is revised following his comments. Remaining errors are ours.

Communicating author, email address: priyabratadutta@gmail.com (P.B.Dutta.)

Copyright©2014, by the Keio Economic Society

1. INTRODUCTION

Economic growth experienced by different countries in the world over the last two decades can be attributed to trade liberalization or globalization. The conventional belief is that globalization leads to an improvement in welfare both from the aggregative and distributive perspectives. However, with regard to its distributive effect, various empirical works point out that income inequality has grown in various countries in the form of a decline in income and employment of unskilled labour compared to those of skilled labour. This growing income inequality is experienced in US during 1960s¹ and in European countries between 1978 and 1988.² We find similar picture in many developing countries too. Wage inequality has increased in many Latin American and South Asian countries in the mid 1980s.³ However, the experience of East Asian countries between 1960s and 1970s goes in favour of the conventional theory that a greater openness to international trade tends to narrow the skilled unskilled wage gap.⁴ Different studies point out different reasons for this increase in income inequality. Trade liberalization and technological progress are considered to be the main two controversial reasons of this phenomenon.⁵ However, there are other causes of this growing wage inequality, for example, international outsourcing,⁶ increase in the price of skill intensive good,⁷ entry of overpopulated low income countries like Bangladesh, China, India, Indonesia, and Pakistan in the global market⁸ etc.

There exists a lot of theoretical works dealing with the issue of this growing wage inequality. Some of them adopt the framework of static competitive general equilibrium models⁹ of small open economies in which there exist two different types of labour-skilled and unskilled. The ratio of wage rate of the skilled worker to that of the unskilled worker is taken as a measure of wage inequality in these models. There are

¹ See, for example, Bound and Johnson (1992), Autor et al. (1998) Leamer (2000), Acemoglu (2002a) etc. According to Juhn et al. (1993) wage differentials by education actually increased during the 1960's and then declined significantly over the 1970s.

² See, for example, Lawrence (1994), Katz et al. (1992) etc.

³ See, for example, Wood (1997), Dev (2000), Borjas and Ramey (1993), Banga (2005), Beyer et al. (1999) etc.

⁴ See, for example, Wood (1997).

⁵ According to Wood (1998), Beyer et al. (1999), Green et.al. (2001), Behrman et al. (2000), Isgut (2001) etc. trade liberalization is to blame for this growing wage inequality. However, Wood (1997, 1998), Dev (2000) and Göğ and Strobl(2002) are of the view that technological progress worsens wage inequality through an increase in the relative demand for skilled labour. Esquivel and Lo'pez (2003) shows that technological change aggravates but trade liberalization lowers wage inequality in Mexico.

⁶ See Feenstra and Hanson (1997) in this context.

⁷ See Harrison and Hanson (1999), Hanson and Harrison (1999) and Beyer, Rojas, Vergara (1999) in this context.

⁸ See Wood (1997) in this context.

⁹ See, for example, Beladiet. al. (2008), Chaudhuri and Yabuuchi (2007, 2008), Chaudhuri (2004, 2008), Marjit and Kar (2005), Yabuuchi and Chaudhuri (2007), Marjit and Acharyya (2003), Marjit (2003), Xu (2003), Marjitet.al. (2004), Marjit and Acharyya (2006), Kar and Beladi (2004), Zhu and Trefler (2005) etc. in this context

some other static models¹⁰ dealing with these problems using product variety structure with monopolistic competition.

Some other authors adopt dynamic models to analyse this problem. They also consider two types of labour skilled and unskilled; and then explain skilled unskilled wage inequality in the long run equilibrium of their models. Among those works, Galor and Moav (2000), Aghion et al. (1999) and Aghion (2002)¹¹ deal with these problems in developed countries. Ripoll (2005) explains wage inequality on the basis of differences in trade policies; and, Kiley (1999), Acemoglu (2002a, 2002b) explains wage inequality with the help of endogenous adoption of skill-biased or unskilled biased technologies. Here also the wage ratio stands for the degree of inequality.

Kiley (1999), Acemoglu (2002a, 2002b), Fang et al. (2008) develop two sector dynamic models with a single final commodity. So they can not analyse the effects of trade on wage inequality; and our present work makes an attempt to analyse this problem. Acemoglu (2003) shows that the increase in the supply of skilled labour raises the market size of skill-complementary inputs and this, in turn, raises demand for skilled labour leading to a rise in the wage rate of skilled labour. As U.S. economy trades with LDC's, the level of demand and thus prices of the skill intensive products increase. Demand for skill complementary inputs are increased in U.S. leading to an increase in the relative demand for skilled labour and to a consequent increase in the skilled-unskilled wage inequality in U.S. The LDC'S which are skill abundant adopt the skilled complementary technologies of U.S. and hence face an increase in skilled-unskilled wage inequality. However, skill-scarce LDCs experience a decline in skilled-unskilled wage inequality. However, Acemoglu (2003) does not describe the effect of international and inter-sectoral knowledge spillover on skilled-unskilled wage inequality.

The present model is an extension of Kiley (1999) with two final commodities and with international trade and international and inter-sectoral technology spillover. In Kiley (1999), the cost of developing a new specific intermediate good depends on the number of varieties of those specific intermediate goods available and on the level of existing research but not on the inter-sectoral knowledge spillover effect. The question of international knowledge spillover does not arise in that model because Kiley (1999) considers a closed economy. However, in the present model, this cost also depends on the intensity of international as well as inter-sectoral knowledge spillover effect. The motivation of the present research comes from the existence of several empirical studies which support the incidence of this knowledge spillover. This empirical literature includes works of Bransetter (2001), Coe and Helpman (1995), Coe et al. (1997), Lichtenberg and Potterie (1998), Griliches (1992), Keller (2002) etc. They show that foreign R&D has beneficial effects on domestic productivity and these positive effects are stronger with the degree of openness. According to Keller (2002), the contribution of R&D in the industry itself is about 50%, and in other domestic industries is about 30%. The remaining 20% is contributed to foreign industries.

¹⁰ See for example Anwar and Rice (2009), Anwar (2009, 2006), Glazer and Ranjan (2003) etc.

¹¹ The paper describes wage inequality on the basis of Schumpeterian growth theory.

In our model, the cost of developing new intermediate goods is reduced due to international and inter-sectoral knowledge spillover. We have two final goods in this model which are not perfect substitutes. We derive many interesting results. The skilled-unskilled wage ratio is affected by the intensity of international knowledge spillover directly as well as through inter country difference in relative factor endowments. In the long run equilibrium of a closed economy, the relationship between the skilled-unskilled wage ratio and the skilled unskilled labour endowment ratio is ambiguous; and the nature of this relationship depends on the degree of consumer's indifference substitution between the two final goods. A direct relationship may be obtained only if these two goods are highly substitutes. In the one commodity model of Kiley (1999), we always obtain such a positive relationship. However, when international trade is opened, we always find a positive relationship between the wage ratio and the domestic factor endowment ratio in the long run equilibrium of a small open economy. The effect of the opening of trade on the long run equilibrium level of skilled unskilled wage inequality depends not only on the inter-country difference of factor endowment ratios but also on the intensity of spillover effects. If factor endowment ratios in both the countries are equal and if the foreign country has a larger endowment of each of the two factors, then the globalization policy makes the home country face a rise in the degree of skilled-unskilled wage inequality in the presence of spillover effects.

This paper is organized as follows. Section 2 describes the model and section 3 analyses properties of it's the balanced growth equilibrium. The equilibrium under autarky is described in subsection 3.1 and the effect of opening of trade is analysed in sub section 3.2. Concluding remarks are made in section 4.

2. THE MODEL

There are two countries in the world; and each of them has two factors of production—unskilled labour and skilled labour. The unskilled labour is used to produce a traditional final good, Y^U ; and an advanced final good, Y^S , is produced by skilled labour. These two goods are not perfect substitutes to consumers. In Kiley (1999), both the sectors produce the same commodity. Some intermediate goods complement skilled labour and other intermediate goods complement unskilled labour. Intermediate goods complementing skilled labour are denoted by X ; and those complementing unskilled labour are denoted by Z . New intermediate goods are developed by the R&D sector in each of the two countries. Production technologies in the two final good sectors are identical in both the countries. However, these two countries may differ in terms of their factor endowments. H (H^*) and L (L^*) stand for skilled labour endowment and unskilled labour endowment of the home (foreign) country respectively.

The home country is a small open economy and thus is a price taker in the world market. However, the foreign country, which is basically the rest of the world, behaves like a closed economy. It means that foreign country is also an open economy but the market share of the foreign country is so large that the volume of trade of the home country is negligible in the determination of the world prices of commodities. Then

the commodity prices are determined in the world market as if there is not any home country in the picture.

Due to international knowledge spillover from the advanced R&D sector in the rest of the world to that in the home country, the advanced R&D sector in the home country enjoys a cost advantage in the production of new intermediate goods used for the production of the advanced final good. Also there exists localized knowledge spillover from the advanced R&D sector to the traditional R&D sector in the home country; and this gives a cost advantage to the traditional R&D sector of the home country. However, there is neither any international knowledge spillover nor any inter-sectoral knowledge spillover in the model of Kiley(1999). Markets for final goods and primary factors in both the countries are perfectly competitive. Intermediate goods are rented; and every intermediate good producer is a monopolist in the rental market. Factors are internationally immobile; and consumers in both the countries have identical tastes.

2.1.1. Final goods

The description of two final goods producing sectors is identical to that found in Kiley (1999). The production functions of two final goods produced by firm i at time t are specified as follows.

$$Y_{it}^U = (L_{it})^{(1-a)} \int_0^{M(t)} (Z_{itj})^a dj; \quad (1)$$

and

$$Y_{it}^S = (H_{it})^{(1-a)} \int_0^{N(t)} (X_{itj})^a dj, \quad \text{with } 0 < a < 1. \quad (2)$$

Here, Z_{itj} and X_{itj} are the quantities of j th variety of intermediate good and L_{it} and H_{it} are the quantities of unskilled labour and skilled labour used by i th competitive firm in the production of traditional final good and advanced final good respectively at the time point t . Numbers of varieties of intermediate goods that complement skilled labour and unskilled labour at time point t are denoted by $N(t)$ and $M(t)$ respectively.

The instantaneous profit functions of the i th competitive firm in sectors producing Y^U and Y^S are given by

$$\Pi_i^U = (L_{it})^{(1-a)} \int_0^{M(t)} (Z_{itj})^a dj - \int_0^{M(t)} P_{jt}^U Z_{itj} dj - W_t^U L_{it}; \quad (3)$$

and,

$$\Pi_i^S = P_F (H_{it})^{(1-a)} \int_0^{N(t)} (X_{itj})^a dj - \int_0^{N(t)} P_{jt}^S X_{itj} dj - W_t^S H_{it}. \quad (4)$$

Π_i^U and Π_i^S stand for profit of the i th firm in the U sector and in the S sector respectively in terms of its own sectors product. Π_i^U is maximized with respect to L_{it} and Z_{itj} ; and Π_i^S is maximized with respect to H_{it} and X_{itj} given the input prices. Here, P_{jt}^U and P_{jt}^S are rental prices of the j th intermediate good used as input in the traditional final good sector and in the advanced final good sector, respectively; and these are expressed in terms of the final product of the corresponding sector. P_F is the relative price of the advanced final good in terms of the traditional final good. W_t^U and W_t^S are wage rates

of unskilled labour and skilled labour, respectively, at time point t expressed in terms of the product of the corresponding sector. First order conditions of profit maximization in both the sectors to be valid at each t are given by followings.

$$P_{jt}^U = (L_{it})^{(1-a)} a (Z_{itj})^{(a-1)} ; \quad (5)$$

$$P_{jt}^S = P_F (H_{it})^{(1-a)} a (X_{itj})^{(a-1)} ; \quad (6)$$

$$W_t^S = (1-a) P_F N(t) H_{it}^{-a} (X_{itj})^a ; \quad (7)$$

and,

$$W_t^U = (1-a) M(t) L_{it}^{-a} (Z_{itj})^a . \quad (8)$$

2.1.2. Intermediate goods

Intermediate goods are nontraded and are durable in nature without having any depreciation; and these are rented to final good sectors. One unit of intermediate good of either type is required to produce one unit of the corresponding final good. The intermediate good can be used from the period in which it is developed. Then the discounted present value of profit of the j th intermediate good producer, who supplies it to the final good sectors, over the infinite time horizon are given by followings.

$$V_j^S(t) = \int_t^\infty e^{-r(\tau-t)} \left(\frac{P_{j\tau}^S}{P_F} - r \right) X_{j\tau} d\tau ; \quad (9)$$

and,

$$V_j^U(t) = \int_t^\infty e^{-r(\tau-t)} (P_{j\tau}^U - r) Z_{j\tau} d\tau . \quad (10)$$

Here, $V_j^S(t)$ and $V_j^U(t)$ are the discounted present values of profits earned from renting; and r is the constant real interest rate that plays the role of marginal cost of renting as well as of the rate of discounting future return.

Now, from equations (5) and (6), we derive demand functions for j th intermediate good of the i th firm in the two final goods sectors as follows.

$$Z_{itj} = L_{it} \left(\frac{a}{P_{jt}^U} \right)^{\left(\frac{1}{1-a} \right)} ; \quad (11)$$

and,

$$X_{itj} = H_{it} \left(\frac{a P_F}{P_{jt}^S} \right)^{\left(\frac{1}{1-a} \right)} . \quad (12)$$

The producer cum rentier of each of these intermediate goods is a monopolist maximizing its corresponding rental income subject to the demand constraint. Both the demand functions shown by equations (11) and (12) have constant price elasticities of demand denoted by $\left(\frac{1}{1-a} \right)$; and these imply that monopoly profit maximizing prices of intermediate goods are also constant and given as follows.

$$\frac{P_{j\tau}^S}{P_F} = P_{jt}^U = \frac{r}{a} . \quad (13)$$

So these monopoly prices vary neither across varieties nor over time. Now, using the assumption that labour endowments are fixed and time independent and also using equations (11), (12) and (13), we obtain equilibrium quantities of intermediate goods given by

$$X = H \left(\frac{a^2}{r} \right)^{\left(\frac{1}{1-a} \right)} \quad (14)$$

and

$$Z = L \left(\frac{a^2}{r} \right)^{\left(\frac{1}{1-a} \right)} \quad (15)$$

So aggregate uses of intermediate goods are linear in terms of specific labour endowments in each of the two sectors.

Using equations (1), (2), (14) and (15) we obtain aggregate output of two final good producing sectors as follows.

$$Y_t^U = M(t) L \left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} ; \quad (16)$$

and

$$Y_t^S = N(t) H \left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} . \quad (17)$$

So level of outputs of final goods are linear in terms of number of varieties of specific intermediate goods; and this ensures that the rate of growth of final output in a particular sector is equal to the rate of expansion of the number of varieties of intermediate goods specific to that sector, given the factor endowment.

Now, using equations (9), (10), (13), (14) and (15) we obtain infinite time horizon discounted present values of profit of intermediate good producers for advanced and traditional final good sectors as follows:

$$V^S(t) = \left(\frac{1-a}{a} \right) X ; \quad (18)$$

and

$$V^U(t) = \left(\frac{1-a}{a} \right) Z . \quad (19)$$

2.1.3. R&D sector

Expansions of the number of skill augmenting or unskill augmenting intermediate goods cause growth in output of the corresponding final good sector. However, this expansion process is costly. New intermediate goods are developed through R&D activities. We follow Kiley (1999) to assume that the cost of development of new specific intermediate goods varies positively with the number of varieties of specific intermediate goods and inversely with the level of existing research ($R(t)$). It is also assumed that existing research level, $R(t)$, is same in both the countries. However, there exists positive inter-sectoral spillover effect from the skill augmenting R&D activities to

the unskill augmenting R&D activities in the home country because unskilled workers learn how to improve their efficiency while working with skilled workers. Also there exists positive international spillover effect from skill augmenting R&D activity in the foreign country to that in the home country because knowledge capital whose accumulation generates skill is always internationally mobile. However, there is no international spillover effect between unskilled labour augmenting activities of two countries because they are not at all connected to each others. Kiley (1999) does not consider any spillover effect. Costs of developing skill augmenting new intermediate goods and unskill augmenting new intermediate goods are denoted by Γ^S and Γ^U , respectively; and these cost functions are given as follows

$$\Gamma^S = \frac{\delta_1 \left[\frac{N(t)}{R(t)} \right]^{K_1}}{\left[\frac{N^*(t)}{R(t)} \right]^{K_2}}, \quad \text{with } K_1 > 1, \quad \delta_1 > 0 \quad \text{and} \quad K_2 > 0; \quad (20)$$

and,

$$\Gamma^U = \frac{\delta_2 \left[\frac{M(t)}{R(t)} \right]^{K_1 q}}{\left[\frac{N(t)}{R(t)} \right]^{K_3}} \quad \text{with } K_1 q > 1, \delta_2 > q \geq 1, \quad K_3 > 0 \quad \text{and} \quad K_1 q > K_3. \quad (21)$$

These cost functions are increasing and convex in terms of $N(t)$ and $M(t)$ respectively because $K_1 > 1$ and $K_1 q > 1$. However, the presence of knowledge spillover always produces a downward effect on these cost functions because $K_2 > 0$ and $K_3 > 0$. Here δ_1 and δ_2 are the reciprocals of the productivity parameters in the two R&D sectors.

$K_2 > 0$ in equation (20) implies that the positive international knowledge spillover is allowed from the advanced R & D sector of the foreign country to that in the home country. Here $N^*(t)$ is the number of varieties of advanced intermediate goods in the foreign country and the numerical value of K_2 denotes the magnitude of international knowledge spillover efficiency.

$K_3 > 0$ in equation (21) implies that the positive inter-sectoral knowledge spillover takes place from the advanced R & D sector to the traditional R & D sector in the home country; and the numerical value of K_3 denotes the magnitude of inter-sectoral knowledge spillover efficiency. This is less than the own technical efficiency parameter of the traditional R&D sector, denoted by $K_1 q$. We go back to Kiley (1999) when $K_2 = K_3 = 0$ and $q = 1$. It should also be noted that results of this model may be changed substantially if restrictions imposed on these parameters are altered. We follow Kiley (1999) to assume $K_1 > 1$. However, what River-Batiz and Romer (1991) and Wang et al. (2009) assume is equivalent to assuming $K_1 < 0$.

Markets for R & D designs are perfectly competitive. In competitive equilibrium of the R & D sector, we also have

$$\Gamma^S = V^S; \quad (22)$$

and,

$$\Gamma^U = V^U. \quad (23)$$

If the value of the firm producing the intermediate good is greater than cost of developing the R & D design entry would occur until the cost equals the value. So equations (4.22) and (4.23) are the conditions of no entry and no exit in the advanced and traditional R & D sector respectively.

We follow Kiley (1999) to assume that the level of existing research, denoted by $R(t)$, grows over time at a constant rate.

So, we have

$$\dot{R} = gR \quad (24)$$

where g is the exogenous growth rate.

2.1.4. Consumers equilibrium

The representative consumer consumes each of the two final goods; and her problem is to maximize the discounted present value of instantaneous utility over the infinite time horizon. It is given by

$$\int_0^\infty e^{-\gamma t} [\beta C_U^{-\rho} + (1 - \beta) C_S^{-\rho}]^{-\frac{1}{\rho}} dt, \quad \text{with } -1 < \rho < \infty \quad \text{and with } \rho \neq 0.$$

This is maximized subject to the intertemporal budget constraint given by

$$\int_0^\infty e^{-rt} [C_U + P_F C_S] dt = \int_0^\infty e^{-rt} [W_L L + W_H H] dt.$$

Here $W_L L$ and $W_H H$ can be added because equation (13) shows that prices of intermediates are same in two sectors. γ is the constant consumption rate of discount; C_U and C_S are the consumption levels of two goods of the representative consumer; ρ is the substitution parameter of the two goods in the utility function; r is the interest rate; and $(W_L L + W_H H)$ is the total income of the representative consumer.¹²

Now, solving the consumer's utility maximization problem, we obtain

$$P_F = \frac{(1 - \beta) C_U^{(\rho+1)}}{\beta C_S^{(\rho+1)}}. \quad (25)$$

Here the R.H.S. of equation (25) represents the marginal rate of indifferent substitution between two final goods.

3. BALANCED GROWTH EQUILIBRIUM

Along a balanced growth path, levels of output of the two sectors (Y^U and Y^S), consumption levels of the two goods (C_U and C_S), the stock of existing research level (R), the number of varieties of skill and unskilled complements (M and N) and the wage rates of two types of labour (W_U and W_S) grow at same rate g .¹³

¹² If we consider two different representative consumers- one for skilled workers and the other for unskilled, then also we should have equation (25) for the equilibrium of each of them provided that their preferences are identical.

¹³ This rate of growth is obtained from equation (24).

Now, we want to examine the effect of the opening of trade on skilled unskilled relative wage in the home country in the balanced growth equilibrium. So we derive the skilled-unskilled wage ratio in the balanced growth equilibrium in the following two cases: (i) the home country is closed to international trade and (ii) the home country is a small open economy whose relative product price P_F is equal to that obtained in the competitive equilibrium in the rest of the world. Kiley (1999) can not analyse the effect of the opening of trade on skill-unskilled wage ratio with the one final good model. We now use superscripts *tr* and *au* to denote the variables of the home country under free trade and under autarky respectively.

3.1. Wage inequality under autarky

Under autarky, there is no effect of international knowledge spillover. So $K_2 = 0$. Supply equals to demand in the competitive equilibrium for each of two final goods market in the home country. Hence we have following two equations

$$Y^S = C_S + \frac{d \left(\int_0^{N(t)} X_{itj} dj \right)}{dt}; \quad (26)$$

and,

$$Y^U = C_U + \frac{d \left(\int_0^{M(t)} Z_{itj} dj \right)}{dt}. \quad (27)$$

Equations (26) and (27) show that total supply of each of the two products is equal to total consumption demand plus total investment demand for that product. Here intermediate goods are modeled as durables as in Romer (1990). So only the newly invented intermediate goods use resources.

Using $K_2 = 0$ and also using equations (14), (15), (18), (19), (22) and (23), we obtain the following equation¹⁴

$$\left(\frac{N}{M} \right)^{au} = A \left(\frac{H}{L} \right)^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)} \left(\frac{1}{L} \right)^{\left[\frac{K_1(1-q) - K_3}{K_1^2 q} \right]}; \quad (28)$$

where

$$A = \left[\frac{1}{\delta_1} \right]^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)} [\delta_2]^{\frac{1}{K_1 q}} \left[\left(\frac{1-a}{a} \right) \left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} \right]^{\left[\frac{K_1(q-1) - K_3}{K_1^2 q} \right]}.$$

Then, using equations (14), (15), (16), (17), (25), (26), (27) and (28), we obtain the competitive equilibrium relative price of the advanced final good under autarky as follows.¹⁵

¹⁴ Derivation of equation (28) in detail is obtained in Appendix (A).

¹⁵ Derivation of equation (29) in detail is obtained in Appendix (B).

$$\begin{aligned}
P_F^{au} &= \frac{(1-\beta)}{\beta} \left(\frac{ML}{NH} \right)^{(\rho+1)} \\
&= \frac{(1-\beta)}{\beta} \left[A^{-1} \left(\frac{L}{H} \right)^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right) + 1} (L)^{\left[\frac{K_1(1-q) - K_3}{K_1^2 q} \right]} \right]^{(\rho+1)}. \quad (29)
\end{aligned}$$

Using equations, (7), (8), (14) and (15), we have¹⁶

$$\begin{aligned}
\left(\frac{W^S}{W^U} \right)^{au} &= P_F^{au} \left(\frac{N}{M} \right)^{au} \\
&= (A)^{-\rho} \frac{(1-\beta)}{\beta} \left(\frac{L}{H} \right)^{\rho \left(\frac{K_1 q - K_3}{K_1^2 q} \right) + \rho + 1} (L)^{\rho \left[\frac{K_1(1-q) - K_3}{K_1^2 q} \right]}. \quad (30)
\end{aligned}$$

When the two goods are perfectly substitutes, $\rho = -1$; and hence, in this case, $\left(\frac{W^S}{W^U} \right)$ varies directly with H if $K_1 q > K_3$. $\left\{ \rho \left(\frac{K_1 q - K_3}{K_1^2 q} \right) + \rho + 1 \right\} < 0$ is valid even if ρ is very close to minus unity. So, we can establish the following proposition.

PROPOSITION 1. *Skilled-unskilled wage ratio varies positively with the stock of domestic skilled labour in the long run equilibrium of the closed economy only if two final goods are highly substitute.*

We now provide intuitive explanations behind this proposition. The effect on skilled-unskilled wage ratio can be divided into two parts: (i) Price effect that comes from the change in the relative price and (ii) skill discrepancy effect resulting from the change in the ratio of number of intermediate goods.

An increase in the domestic skilled labour endowment lowers the relative price of the advanced final good, and the magnitude of this price effect depends on the degree of indifferent substitution between two final goods. On the other hand, the ratio of number of intermediate goods moves in favour of the advanced good sector in this case following the skill discrepancy effect. So the two effects are opposite to each others but the skill discrepancy effect does not depend on the degree of indifferent substitution between two final goods. Therefore, if the degree of indifference substitution is sufficiently large, the skill discrepancy effect dominates the price effect. Price effect is nil when two goods are perfect substitutes. Thus we obtain a positive relationship between the skilled-unskilled wage ratio and the size of the domestic skilled labour endowment.

If the consumer's utility function is Cobb-Douglas, i.e., if $\rho = 0$, then from equation (28), (29) and (30), we obtain

$$\left(\frac{W^S}{W^U} \right)^{au} = \frac{(1-\beta)}{\beta} \left(\frac{L}{H} \right); \quad (30A)$$

and this equation (30A) gives an inverse relationship between $\left(\frac{W^S}{W^U} \right)^{au}$ and $\left(\frac{H}{L} \right)$ along

¹⁶ Derivation of equation (30) in detail is obtained in Appendix (C).

a rectangular hyperbola. This is important because Kiley (1999) who develops an one final good model always obtains a positive relationship between $\left(\frac{W^S}{W^U}\right)^{au}$ and $\left(\frac{H}{L}\right)$. Also the expression of the relative wage is independent of productivity parameters in the R&D sector and of technology parameters in the final good sector in this special case with $\rho = 0$; but this is not true in Kiley (1999). However, equation (30A) is identical to the corresponding equation¹⁷ obtained in Wang, et al. (2009) who also solves a similar problem with $\rho = 0$.

Actually, the model of Kiley (1999) is a specialcase of the present model with $\rho = -1$, $K_3 = 0$ and $q = 1$. In this special case, from equation (28), (29) and (30), we obtain

$$\left(\frac{W^S}{W^U}\right)^{au} = \frac{(1-\beta)}{\beta} \left[\frac{\delta_2}{\delta_1}\right]^{\frac{1}{K_1}} \left(\frac{H}{L}\right)^{\frac{1}{K_1}}; \quad (30B)$$

and this equation with $\beta = \frac{1}{2}$ is identical to the expression of skilled-unskilled wage ratio derived in Kiley (1999)¹⁸ when $\beta = \frac{1}{2}$. This equation (30B) clearly shows a positive relationship between $\left(\frac{W^S}{W^U}\right)^{au}$ and $\left(\frac{H}{L}\right)$. Here $\rho = -1$ implies that the two final goods are perfect substitutes; and this is analytically equivalent to Kiley (1999) assumption that both the sectors produce the same good.

3.2. Wage inequality under trade

When the home country is open to international trade, we have $K_2 > 0$ because a positive international knowledge spillover effect is present. Then, adopting a similar process as used in the previous sub-section and using equations (14), (15), (18), (19), (22) and (23), we obtain the following equation¹⁹

$$\left(\frac{N}{M}\right)^{tr} = \left(\frac{N}{M}\right)^{au} B; \quad (31)$$

where,

$$B = \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) \left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} (H^*) \right]^{\left[\frac{K_2 \{K_1 q - K_3\}}{K_1^3 q} \right]}.$$

Here $B = 1$ if $K_2 = 0$; and equations (28) and (31) are identical in this case. Equations to be satisfied in the free trade equilibrium of the foreign country (rest of the world), which is assumed to be a closed economy, are as follows

$$Y^{S*} = C_S^* + \frac{d \int_0^{N^*(t)} X_{itj}^* dj}{dt}; \quad (26F)$$

¹⁷ See equation (18) in Wang, et al. (2009).

¹⁸ The expression of wage inequality is derived in Kiley (1999) using equations (11), (12) and (18c) in his model; and is shown as follows.

$\frac{W^S}{W^U} = \left[\frac{\delta_2}{\delta_1}\right]^{\frac{1}{K_1}} \left(\frac{H}{L}\right)^{\frac{1}{K_1}}.$

¹⁹ Detailed derivation of equation (31) is given in Appendix (D).

$$Y^{U*} = C_U^* + \frac{d \int_0^{M^*(t)} Z_{itj}^* dj}{dt}. \quad (27F)$$

and,

$$\left(\frac{N}{M}\right)^* = A \left(\frac{H^*}{L^*}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left(\frac{1}{L^*}\right)^{\left[\frac{K_1(1-q) - K_3}{K_1^2 q}\right]}. \quad (28F)$$

So, using equations (14), (15), (16), (17), (25), (26F) and (27F), we obtain competitive equilibrium value of relative price of the advanced good in the rest of world as follows.²⁰

$$\begin{aligned} P_F^* &= P_F^{tr} = \frac{(1-\beta)}{\beta} \left(\frac{M^* L^*}{N^* H^*}\right)^{(\rho+1)} \\ &= \frac{(1-\beta)}{\beta} \left[A^{-1} \left(\frac{L^*}{H^*}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)+1} (L)^{\left[\frac{K_1(1-q) - K_3}{K_1^2 q}\right]} \right]^{(\rho+1)}. \end{aligned} \quad (29F)$$

The small open home country is a taker of this relative price. So, under freetrade, equation (30) showing skilled-unskilled relative wage of the home country is modified as follows.

$$\left(\frac{W^S}{W^U}\right)^{tr} = P_F^{tr} \left(\frac{N}{M}\right)^{tr} = P_F^{tr} \left(\frac{N}{M}\right)^{au} B. \quad (30F)$$

So in the case of free trade, there are three effects: the price effect, the skill discrepancy effect and the international spillover effect. Price effect and international spillover effect play important role in this context. Relative price is determined in the foreign country. Opening of trade lowers the relative price in the home country; and thus raises the skilled-unskilled wage ratio there. When the endowment of foreign skilled labour is larger than that of domestic skilled labour, we find the international spillover effect to operate. This further raises the relative demand for skilled labour in the home country; and so skilled-unskilled wage ratio goes up there. Skill discrepancy effect works in the opposite direction but is dominated by the price effect and international spillover effect.

Then, using equations (29F) and (30F), we obtain the following equation

$$\left(\frac{W^S}{W^U}\right)^{tr} = \frac{(1-\beta)}{\beta} \left(\frac{L^*}{H^*}\right)^{(\rho+1)} \frac{\left(\frac{N}{M}\right)^{tr}}{\left(\frac{N^*}{M^*}\right)^{(\rho+1)}}. \quad (32)$$

We can now establish the following proposition.

PROPOSITION 2. *If $\rho = 0$, then in the long run equilibrium, the skilled-unskilled wage ratio of the small open home country may vary positively (inversely) with the skilled unskilled labour endowment ratio in the home (foreign) country.*²¹

²⁰ Derivation of equation (29F) in detail is obtained in Appendix (E).

²¹ This proposition may not be always valid because skilled-unskilled wage ratio also depends on H^* and $\frac{L^*}{L}$ separately in equation (32). We are thankful to the referee of this journal for pointing out this problem.

Here equation (32) shows that the relative wage in the home country is affected by factor endowments of the foreign country. This is so because the home country is a taker of the relative product price determined in the competitive international market and because there is an international knowledge spillover effect. Also both the parameters, K_2 and K_3 , enter into the R.H.S. expression of equation (32). So not only the magnitude of localized knowledge spillover but also the magnitude of international knowledge spillover affects the skilled-unskilled relative wage of the small open home country. However, in Wang et al. (2009), this relative wage is only subject to the localized knowledge spillover effect and not to the globalized knowledge spillover effect. This is so because, in Wang et al. (2009), the effect of international knowledge spillover comes only through the difference in relative factor endowments between the home country and the rest of the world. However, in our model, it comes directly as well as indirectly through the endowment difference. However, if $\rho + 1 = 0$ and $K_2 = 0$, then equation (32) is reduced to the following.

$$\left(\frac{W^S}{W^U}\right)^{tr} = \frac{(1-\beta)A}{\beta} \left(\frac{H}{L}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left(\frac{1}{L}\right)^{\left[\frac{K_1(1-q) - K_3}{K_1^2 q}\right]}. \quad (32B)$$

This equation (32B) implies that, when the two final goods are perfect substitutes and when there is no international knowledge spillover effect, then the relative wage of the small open home country is independent of factor endowments of the foreign country. If, further, we assume that $K_3 = 0$ and $q = 1$, then equation (32B) is reduced to equation (30B). So, in this very special case where we get back the model of Kiley (1999), there is no effect of international trade on wage inequality in the home country. If two commodities are perfect substitutes, then a two commodity system works like an one commodity system because the consumer, in equilibrium, consumes one of the two commodities. However, there is the effect of trade on wage inequality when $\rho + 1 \neq 0$.

Using equations (30) and (30F), we have

$$\begin{aligned} \Delta &= \left(\frac{W^S}{W^U}\right)^{tr} - \left(\frac{W^S}{W^U}\right)^{au} = (P_F^{tr} B - P_F^{au}) \left(\frac{N}{M}\right)^{au} \\ &= \frac{(1-\beta)L}{\beta H} \left[B \left(\frac{L^*}{H^*}\right)^{1+\frac{(K_1 q - K_3)}{K_1^2 q}} \left(\frac{H}{L}\right)^{1+\frac{(K_1 q - K_3)}{K_1^2 q}} \left(\frac{L^*}{L}\right)^{\frac{K_1(1-q) + K_3}{K_1^2 q}} - 1 \right]. \end{aligned} \quad (33)$$

Here Δ is a measure of the change in wage inequality in the home country caused by the opening of trade when $\rho = 0$. Equation (33) shows that the effect of trade on skilled-unskilled wage ratio depends on the inter-country differences in the levels of factor endowments and on the intensity of localized and international spillover effects. If factor endowment ratios are same in both the countries, i.e., $\frac{H}{L} = \frac{H^*}{L^*}$, then, with a high value of H^* and with $L^* > L$, we find that $\Delta > 0$ when at least one of the parameters $K_1(1-q)$, K_2 and K_3 takes a positive value. Hence we can establish the following proposition.

PROPOSITION 3. *If factor endowment ratios in both the countries are equal and if the foreign country has a larger endowment of each of the two factors, then opening of trade raises skilled-unskilled wage ratio in the home country in the presence of international and/or intra-sectoral spillover effect.*

Proposition 3 implies that, if the home country is very small compared to the rest of the world then opening of trade worsens its skilled-unskilled wage inequality problem. We now turn to provide intuitive explanations behind this proposition. In the case of free trade, there are three effects: the price effect, the skill discrepancy effect and the international spillover effect. Price effect and international spillover effect play important role in this context. Relative price is determined in the foreign country. Opening of trade lowers the relative price in the home country; and thus raises the skilled-unskilled wage ratio there. When the endowment of foreign skilled labour is larger than that of domestic skilled labour, we find the international spillover effect to operate. This further raises the relative demand for skilled labour in the home country; and so skilled-unskilled wage ratio goes up there. Skill discrepancy effect works in the opposite direction but is dominated by price effect and international spillover effect.

Now, equation (33) also shows that, if there is no international or intra-sectoral spillover i.e. $K_2=K_3=0$ and $q=1$, then $\Delta=0$. In this case, we go back to Kiley (1999) model and here skilled-unskilled wage inequality remains unchanged even after the opening of trade. So trade alone can not affect the skilled-unskilled wage inequality in this model. Trade can aggravate this problem only in the presence of spillover effect. In Acemoglu (2003), trade does not cause international technology spillover. In Wang et al. (2009), trade leads to international technology spillover but the effect of trade on the change in wage inequality is independent of the magnitude of the international technology spillover effect parameter.

4. CONCLUSION

The present paper develops a dynamic model following the line of Kiley (1999) to explain the skilled unskilled wage inequality in the long run equilibrium. The special property of this extended model is that it analyses the effect of international trade and international knowledge spill over from the rest of the world to the home country and localized knowledge spillover from the more advanced sector to the less advanced sector. We consider two final commodities with imperfect substitution between them to analyse the effect of trade, while the model of Kiley (1999) adopts an one commodity framework. The cost of developing a new intermediate good does not depend on any kind of knowledge spillover in Kiley (1999) model but does depend on those in our model.

We derive many interesting results from this model. In the long run equilibrium of a closed economy, the relationship between the skilled-unskilled wage ratio and the skilled unskilled labour endowment ratio is ambiguous; and the nature of this relationship depends on the degree of consumer's indifference substitution between the two final goods. A direct relationship in that case may be obtained only if these two goods

are highly substitutes. In the one commodity model of Kiley (1999), we always obtain such a positive relationship. However, when international trade is opened, the relationship between the wage ratio and the domestic factor endowment ratio is always direct in nature in the long run equilibrium of the small open economy. The effect of the opening of trade on the long run equilibrium skilled-unskilled wage ratio depends not only on the inter country difference of factor endowment ratios but also on the intensity of spillover effect. If factor endowment ratios in both the countries are equal and if the foreign country has a larger endowment of each of the two factors, then the globalization policy makes the home country face a rise in the degree of skilled-unskilled wage inequality in the presence of spillover effects.

However, our model fails to consider many important aspects of reality. We assume that only final goods are traded but intermediate goods can not be traded. The level of existing research is assumed to grow over time at an exogenously given rate. So along the balanced growth path, all other endogenous variable of the model grow at the same exogenous rate. So the long run equilibrium growth rate in this model is exogenous. We assume the foreign country (rest of the world) to be a closed economy. Identical production functions are assumed to exist in both the skilled labour using sector and the unskilled labour using sector. Also, imperfection of final commodity markets and international factor motilities are not considered. The assumption of representative household consisting of skilled labour as well as of unskilled labour is also a restrictive one. The possibility of unemployment is also ruled out. We plan to do further research in future removing these problems.

REFERENCES

- Acemoglu, D. (2002a). Technical Change, Inequality, and the Labor Market. *Journal of Economic Literature* 40), 7–72.
- Acemoglu, D. (2002b). Directed Technical Change. *Review of Economic Studies* (69), 781–809.
- Acemoglu, D. (2003). Patterns of Skill Premia. *The Review of Economic Studies* 70 (2), 199–230.
- Aghion, P. (2002). Schumpeterian Growth Theory and the Dynamics of Income Inequality. *Econometrica* 70 (3), 855–882.
- Aghion, P., Caroli, E., & Penalosa, C. G. (1999). Inequality and Economic Growth: The Perspective of the New Growth Theories. *Journal of Economic Literature* 37 (4), 1615–1660.
- Anwar, S. (2006). Factor mobility and Wage Inequality in the Presence of Specialisation-Based External Economics. *Economics Letters* 93(1), 88–93.
- Anwar, S. (2009). Wage inequality, welfare and downsizing. *Economics Letters* 103, 75–77.
- Anwar, S., & Rice, J. (2009). Labour Mobility and Wage Inequality in The Presence of Endogenous Foreign Investment. *Economic Modelling* 26 (6), 1135–1139.
- Autor, D. H., Katz, L. F., & Krueger, A. B. (1988). Computing Inequality: Have Computers Change Labor Market. *The Quarterly Journal of Economics* 113 (4), 1169–1213.
- Banga, R. (2005). Liberalisation and Wage Inequality in India. ICRIER Working Paper 156 March.
- Behrman, J., Birdsall, N., & Szekely, M. (2000). Economic Reform and Wage Differentials in Latin America. BID Working Paper 435 October.
- Beladi, H., Chaudhuri, S., & Yabuuchi, S. (2008). Can International Factor Mobility Reduce Wage Inequality in a Dual Economy? *Review of International Economics* 16(5), 893–903.
- Beyer, H., Rojas, P., & Vergara, R. (1999). Trade Liberalization and Wage Inequality. *Journal of Development Economics* 59, 103–123.
- Borjas, G., & Ramey, V. (1993). Foreign Competition, Market Power and Wage Inequality: Theory and

- Evidence. NBER Working Paper 4556. NBER, Cambridge, Massachusetts, December.
- Bound, J., & Johnson, G. (1992). Changes in the Structure of Wages in the 1980s: An Evaluation of Alternative Explanations. *American Economic Review* 82, 371–392.
- Branstetter, L. G. (2001). Are Knowledge Spillovers International or Intranational in Scope? Microeconomic Evidence From the U. S. and Japan. *Journal of International Economics* 53, 53–79.
- Chaudhuri, S. (2004). International Migration of Skilled and Unskilled Labour, Welfare and Skilled-Unskilled Wage Inequality: A simple Model. *Journal of Economic Integration* 19(4), 726–741.
- Chaudhuri, S. (2008). Wage Inequality in a Dual Economy and International Mobility of Factors: Do Factor Intensities Always Matter? *Economic Modelling* 25, 1155–1164.
- Chaudhuri, S., & Yabuuchi, S. (2007). Economic Liberalization and Wage Inequality in the Presence of Labour Market Imperfection. *International Review of Economics and Finance* 16, 592–603.
- Chaudhuri, S., & Yabuuchi, S. (2008). Foreign Capital and Skilled- Unskilled Wage Inequality in a Developing Economy With Non-traded Goods. In S. Marjit, & E. Yu, *Contemporary and Emerging Issues In Trade Theory and Policy* (pp. 225–250). UK, North America, Japan, Malaysia, China: Elsevier Science & Technology Books.
- Coe, D. C., & Helpman, E. (1995). International R & D Spillovers. *European Economic Review* 39, 859–887.
- Coe, D. T., Helpman, E., & Hoffmaister, A. W. (1997). North-South R & D Spillovers. *The Economic Journal* 107 (440), 134–149.
- Dev, M. (2000). Economic Liberalization and Employment in South Asia. *Economic and Political Weekly*, 40–51.
- Esquivel, G., & Lopez, J. (2003). Technology, Trade and Wage Inequality in Mexico Before and After Nafta. *Journal of Development Economics* 72, 543–565.
- Fang, C., Huang, L., & Wang, M. (2008). Technology spillover and wage inequality. *Economic Modelling* 25, 137–147.
- Feenstra, R. C., & Hanson, G. H. (1997). Foreign Direct Investment and Relative Wages: Evidence from Mexico's Maquiladoras. *Journal of International Economics* 42, 371–394.
- Galar, O., & Moav, O. (2000). Ability-Biased Technological Transition, Wage Inequality, and Economic Growth. *The Quarterly Journal of Economics* 115 (2), 469–497.
- Glazer, A., & Ranjan, P. (2003). Preference Heterogeneity, Wage Inequality, and Trade. *Journal of International Economics* 60, 455–469.
- Gorg, H., & Strobl, E. (2002). Relative Wages, Openness and Skill-Biased Technological Change. IZA Discussion Paper Series 596 October.
- Green, F., Dickerson, A., & Arbache, J. (2001). A Picture of Wage Inequality and the allocation of Labour Through a Period of Trade Liberalization: The Case of Brazil. *World Development* 29(11), 1923–1939.
- Griliches, Z. (1992). The Search for R&D Spillovers. *The Scandinavian Journal of Economics* 94, S29–S47.
- Hanson, G. H., & Harisson, A. (1999). Trade Liberalization and Wage Inequality in Mexico. *Industrial and Labour Economics Review* 52, 271–288.
- Harrison, A., & Hanson, G. (1999). Who Gains from Trade Reform? Some Remmaning Puzzles. *Journal of Development Economics* 59, 125–154.
- Isgut, A. (2001). What's Difference about Exporters? Evidence from Colombian Manufacturing. *Journal of Development Studies* 37 (5), 57–82.
- Jhun, C., Murphy, K. M., & Pierce, B. (1993). Wage Inequality and the Rise in Returns to Skill. *The Journal of Political Economy* 101 (3), 410–442.
- Kar, S., & Beladi, H. (2004). Skill Formation and International Migration: Welfare Perspective of Developing Countries. *Japan and the World Economy* 16, 33–54.
- Katz, L., Loveman, G. W., & Blanchflower, D. G. (1992). A comparison of Changes in the structure of Wages in Four OECD countries. NBER.
- Keller, W. (2002). Trade and The Transmission of Technology. *Journal of Economic Growth* 7, 5–24.
- Kiley, M. T. (1999). The Supply of Skilled Labor and Skilled-biased Technological Progress. *Economic Journal* (109), 708–724.
- Lawrence, R. Z. (1994). Trade, Multinationals and Labour. NBER Working Paper 4836.
- Leamer, E. (2000). What's the Use of Factor Contents? *Journal of International Economics* 50, 51–71.
- Lichtenberg, F. R., & Potterie, B. V. (1998). International R&D spillovers: A Comment. *European Economic*

- Review 42, 1483–1491.
- Marjit, S. (2003). Economic Reform and Informal Wage—A general Equilibrium Analysis. *Journal of Development Economics* 72, 371–378.
- Marjit, S., & Acharyya, R. (2003). International Trade, Wage Inequality and the Developing Economy—A General Equilibrium Approach. Germany: Springer-Verlag.
- Marjit, S., & Acharyya, R. (2006). Trade Liberalization, Skill-linked Intermediate Production and the Two-sided Wage Gap. *The Journal of Policy Reform* 9(3), 203–217.
- Marjit, S., & Kar, S. (2005). Emigration and Wage Inequality. *Economic Letters* 88, 141–145.
- Marjit, S., Beladi, H., & Chakrabarti, A. (2004). Trade and Wage Inequality In Developing Countries. *Economic Inquiry* 42 (2), 295–303.
- Ripoll, M. (2005). Trade Liberalization and The Skill Premium in Developing Countries. *Journal of Monetary Economics* 52, 601–619.
- Rivera-Batiz, L. A., & Romer, P. (1991). Economic Integration and Endogenous Growth. *The Quarterly Journal of Economics* 106 (2), 531–555.
- Romer, P. (1990). Endogenous Technological Change. *The Journal of Political Economy* 98, S71–S102.
- Wang, M., Fang, C., & Huang, L. (2009). International knowledge spillovers and wage inequality in developing countries. *Economic Modelling* 25(1), 1208–1214.
- Wood, A. (1998). Globalisation and the Rise in Labor Market Inequalities. *The Economic Journal* 108, 1463–1482.
- Wood, A. (1997). Openness and Wage Inequality in Developing Countries—The Latin American Challenge to East Asian Conventional Wisdom. *World Bank Research Observer*, 33–57.
- Xu, B. (2003). Trade liberalization, Wage Inequality, and Endogenously Determined Non-traded Goods. *Journal of International Economics* 60, 417–431.
- Yabuuchi, S., & Chadhuri, S. (2007). International Migration of Labour and Skilled-unskilled Wage Inequality in a Developing Economy. *Economic Modelling* 24 (1), 128–137.
- Zhu, S. C., & Treffer, D. (2005). Trade and Inequality in Developing Countries: A General Equilibrium Analysis. *Journal of International Economics* 65, 21–48.

APPENDIX (A). Derivation of equation (28)

Under autarky $K_2 = 0$. So, using equations (18) and (22), we obtain

$$[N(t)/R(t)] = \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X \right]^{\frac{1}{K_1}}. \quad (\text{A.1})$$

Similarly, from equations (19) and (23), we have

$$[M(t)/R(t)] = \left[\frac{1}{\delta_2} \left(\frac{1-a}{a} \right) Z \right]^{\frac{1}{K_1 q}} \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X \right]^{\frac{K_3}{K_1^2 q}}. \quad (\text{A.2})$$

Finally, from equations (A.1) and (A.2), we obtain

$$\frac{N}{M} = \frac{\left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X \right]^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)}}{\left[\frac{1}{\delta_2} \left(\frac{1-a}{a} \right) Z \right]^{\frac{1}{K_1 q}}}. \quad (\text{A.3})$$

Then, using equations (14), (15) and (A.3), we obtain

$$\left(\frac{N}{M} \right)^{au} = A \left(\frac{H}{L} \right)^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)} \left(\frac{1}{L} \right)^{\left[\frac{K_1 (1-q) - K_3}{K_1^2 q} \right]}; \quad (\text{A.4})$$

where

$$A = \left[\frac{1}{\delta_1} \right]^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)} [\delta_2]^{\frac{1}{K_1 q}} \left[\left(\frac{1-a}{a} \right) \left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} \right]^{\left[\frac{K_1(q-1) - K_3}{K_1^2 q} \right]}$$

This equation (A.4) is same as equation (28) in the body of the paper.

APPENDIX (B). Derivation of equation (29)

Using equations (25), (26) and (27), we obtain

$$\begin{aligned} P_F^{au} &= \frac{(1-\beta)}{\beta} \left(\frac{Y^U - \frac{d \left(\int_0^{M(t)} Z_{itj} dj \right)}{dt}}{Y^S - \frac{d \left(\int_0^{N(t)} X_{itj} dj \right)}{dt}} \right)^{(\rho+1)}, \\ \Rightarrow P_F^{au} &= \frac{(1-\beta)}{\beta} \left(\frac{Y^U - M Z g}{Y^S - N X g} \right)^{(\rho+1)}. \end{aligned} \quad (\text{B.1})$$

Using equations (14), (15), (16), (17) and (B.1), we obtain

$$\begin{aligned} P_F^{au} &= \frac{(1-\beta)}{\beta} \left(\frac{ML \left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} - ML \left(\frac{a^2}{r} \right)^{\left(\frac{1}{1-a} \right)} g}{NH \left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} - NH \left(\frac{a^2}{r} \right)^{\left(\frac{1}{1-a} \right)} g} \right)^{(\rho+1)}, \\ \Rightarrow P_F^{au} &= \frac{(1-\beta)}{\beta} \left(\frac{ML}{NH} \right)^{(\rho+1)} \left(\frac{\left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} - \left(\frac{a^2}{r} \right)^{\left(\frac{1}{1-a} \right)} g}{\left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} - \left(\frac{a^2}{r} \right)^{\left(\frac{1}{1-a} \right)} g} \right)^{(\rho+1)} \\ \Rightarrow P_F^{au} &= \frac{(1-\beta)}{\beta} \left(\frac{ML}{NH} \right)^{(\rho+1)} = \frac{(1-\beta)}{\beta} \left[A^{-1} \left(\frac{L}{H} \right)^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)} (L)^{\left[\frac{K_1(1-q) - K_3}{K_1^2 q} \right]} \right]. \end{aligned} \quad (\text{B.2})$$

This equation (B.2) is same as equation (29) in the body of the paper.

APPENDIX (C). Derivation of equation (30)

Using equations (7) and (8), we have

$$\frac{W^S}{W^U} = P_F \left(\frac{N}{M} \right) \left(\frac{X}{Z} \right)^a \left(\frac{H}{L} \right)^{-a}. \quad (\text{C.1})$$

Using equations (14), (15) and (C.1), we obtain

$$\frac{W^S}{W^U} = P_F \left(\frac{N}{M} \right) \left(\frac{H \left(\frac{a^2}{r} \right)^{\left(\frac{1}{1-a} \right)}}{L \left(\frac{a^2}{r} \right)^{\left(\frac{1}{1-a} \right)}} \right)^a \left(\frac{H}{L} \right)^{-a},$$

$$\Rightarrow \frac{W^S}{W^U} = P_F \left(\frac{N}{M} \right). \quad (C.2)$$

Now, in autarky equation (C.2) becomes

$$\left(\frac{W^S}{W^U} \right)^{au} = P_F^{au} \left(\frac{N}{M} \right)^{au}. \quad (C.3)$$

This equation (C.3) is same as equation (30) in the body of the paper.

APPENDIX (D). Derivation of equation (31)

Undertrade, $K_2 \neq 0$. So, using equations (18) and (22), we obtain

$$[N(t)/R(t)] = \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X \right]^{\frac{1}{K_1}} \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X^* \right]^{\frac{K_2}{K_1^2}}. \quad (D.1)$$

Similarly, from equations (19) and (23), we have

$$[M(t)/R(t)] = \left[\frac{1}{\delta_2} \left(\frac{1-a}{a} \right) Z \right]^{\frac{1}{K_1^q}} \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X \right]^{\frac{K_3}{K_1^2 q}} \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X^* \right]^{\frac{K_3 K_2}{K_1^3 q}}. \quad (D.2)$$

Finally, from equations (D.1) and (D.2), we obtain

$$\frac{N}{M} = \frac{\left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X \right]^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)} \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X^* \right]^{K_2 \left(\frac{K_1 q - K_3}{K_1^3 q} \right)}}{\left[\frac{1}{\delta_2} \left(\frac{1-a}{a} \right) Z \right]^{\frac{1}{K_1^q}}}. \quad (D.3)$$

Then, using equations (14), (15), (A.4) and (D.3), we obtain

$$\left(\frac{N}{M} \right)^{tr} = \left(\frac{N}{M} \right)^{au} B; \quad (D.4)$$

where,

$$B = \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) \left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} (H^*) \right]^{\left[\frac{K_2 \{ K_1 q - K_3 \}}{K_1^3 q} \right]}. \quad (D.5)$$

This equation (D.4) is same as equation (31) in the body of the paper.

APPENDIX (E). Derivation of equation (29F)

Using equations (25), (26F) and (27F), we obtain

$$\begin{aligned} P_F^* &= \frac{(1-\beta)}{\beta} \left(\frac{Y^{U^*} - \frac{d \int_0^{M^*(t)} Z_{itj}^* dj}{dt}}{Y^{S^*} - \frac{d \int_0^{N^*(t)} X_{itj}^* dj}{dt}} \right)^{(\rho+1)}, \\ \Rightarrow P_F^* &= \frac{(1-\beta)}{\beta} \left(\frac{Y^{U^*} - M^* Z^* g^*}{Y^{S^*} - N^* X^* g^*} \right)^{(\rho+1)}. \end{aligned} \quad (E.1)$$

Using equations (14), (15), (16), (17) (modified under trade) and (E.1), we obtain

$$\begin{aligned}
 P_F^* &= \frac{(1-\beta)}{\beta} \left(\frac{M^* L^* \left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - M^* L^* \left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)} g}{N^* H^* \left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - N^* H^* \left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)} g} \right)^{(\rho+1)}, \\
 \Rightarrow P_F^* &= \frac{(1-\beta)}{\beta} \left(\frac{M^* L^*}{N^* H^*} \right)^{(\rho+1)} \left(\frac{\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - \left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)} g}{\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - \left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)} g} \right)^{(\rho+1)}, \\
 \Rightarrow P_F^* &= P_F^{tr} = \frac{(1-\beta)}{\beta} \left(\frac{M^* L^*}{N^* H^*} \right)^{(\rho+1)} \\
 &= \frac{(1-\beta)}{\beta} \left[A^{-1} \left(\frac{L^*}{H^*} \right)^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right) + 1} (L)^{\left[\frac{K_1(1-q) - K_3}{K_1^2 q} \right]} \right]^{(\rho+1)}. \quad (E.2)
 \end{aligned}$$

This equation (E.2) is same as equation (29F) in the body of the paper.