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COMMERCIAL FISHING WITH PREDATOR-PREY INTERACTION REVISITED

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Abstract: The non-extinct long-run equilibrium in commercial fishing with predator-prey interaction and without entry of new fleet is locally stable. No limit cycle arises for the movement of fish stocks of both species. Dynamic properties of the long-run equilibrium in which at least one species of fish becomes extinct are also analyzed.

Key words: commercial fishing, predator-prey interaction, equilibrium, limit cycle.
JEL Classification Number: Q22, Q57.

1. INTRODUCTION

Commercial fishing of a single or multiple species have been much analyzed by economists and bioeconomists since the appearances of the seminal paper by Smith (1969) and of the comprehensive book on mathematical bioeconomics by Clark (1976). The recent works on single species fishery include Ruseski (1998), Szidarovszky and Okuguchi (1998), Szidarovszky, Ilieva and Okuguchi (2002), Okuguchi (2003), Szidarovszky, Okuguchi and Kopel (2005), and Erjaee and Okuguchi (2006), among others. Chaudhuri (1986), Mesterton-Gibbons (1996), Kar and Chaudhuri (2003), Das, Mukherjee and Chaudhuri (2009), and Kar and Chakraborty (2010) are among the important papers on multi-species fishery. An excellent exposition of the original Lotka-Volterra differential equation for predator-prey interaction between two species of fish is Gandolfo (1997). In our earlier paper(Okuguchi, 1984), we have formulated a commercial fishing model with predator-prey interaction and with free entry of fleet, and proved the local stability of the unique non-extinct steady state equilibrium fish stocks. We have also conducted a comparative static analysis of the non-extinct steady state equilibrium in relationship to exogenous changes in the values of the market and biological parameters. We have not, however, examined whether there exists a limit cycle for the model. In this paper we will simplify our earlier model by assuming away entry of new fleet and prove, on the basis of a mathematical theorem of Bendixon-du Lac,

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that no limit cycle occurs for the movement of stocks of prey and predator. We will also give a dynamic analysis of the long-run equilibrium in which at least fish of one species becomes extinct. The non-existence of limit cycle in our model is in sharp contrast to the result of Gandolfo (1997) for the original Lotka-Volterra equation which does not take into consideration of commercial fishing. We note that since multiple equilibrium steady states exist in our model, we can not discuss the global stability of any equilibrium state using the well-known theorem of Olech on the global stability of a stationary point for a two variable system of differential equations.

2. MODEL AND ANALYSIS

Let the first and second species of fish be predator and prey, respectively, and let the stocks (biomass) of the two species in the absence of fishing change according to the following system of differential equations[1].

$$\begin{aligned} \frac{dx_1}{dt} &= r_1x_1\Big(1-\frac{x_1}{K_1}\Big)-\theta_1x_1x_2 \equiv f^{(1)}(x_1,x_2)\,, \\ \frac{dx_2}{dt} &= r_2x_2\Big(1-\frac{x_2}{K_2}\Big)-\theta_2x_1x_2 \equiv f^{(2)}(x_1,x_2)\,, \end{aligned} \tag{1}$$

where t is continuous time; r_i is the intrinsic growth rate,K_i is the carrying capacity, and x_i is the stock (biomass), all of the i-th species, $\theta_1 < 0$ (predator) and $\theta_2 > 0$ (prey)[2]. Let

$$C_i(h_i, x_i) \equiv \frac{\gamma_i h_i^2}{2x_i}, \quad i = 1, 2 \tag{2}$$

be the cost function for selectively harvesting the $i-$th species in the amount h_i, where γ_i is a positive constant. The cost function of this type was first used by Leung and Wang (1976), and it is derived on the basis of fish harvesting function which depends on fishing efforts and fish stock, as shown by Szidarovszky and Okuguchi (1998). By definition, the profit function for harvesting the i-th species is

$$\pi_i \equiv p_i h_i - \frac{\gamma_i h_i^2}{2x_i}, \quad i = 1, 2\,, \tag{3}$$

where p_i is the price of harvested fish of the i-th species. The prey may have no economic value.In this case only preditor is harvested.

Taking into account maximization of (3) with respect to h_i, we get the following system of differential equations for the changes of fish stocks in the presence of commercial fishing.

$$\begin{aligned} \frac{dx_i}{dt} &= g^{(i)}(x_i, x_j) \equiv f^{(i)}(x_i, x_j) - \frac{p_i x_i}{\gamma_i} \\ &= x_i(-a_i x_i - \theta_i x_j + c_i), \quad i \neq j, \;\; i, j = 1, 2\,, \end{aligned} \tag{4}$$

[1] There are alternative models for predator-prey interaction between two species of fish (see Okuguchi, 1984, and Kar and Chakraborty, 2010) or two species of wild animals (see Kawata, 2007).

[2] According to Kihara and Shimada (1988), Pacific halibut, walleye pollock, yellow Irish lord, thorny sculpin, Greenland turbot, etc. are predators of Pacific cod.

where $a_i \equiv \frac{r_i}{K_i} > 0$, $c_i \equiv r_i - \frac{p_i}{\gamma_i}$, $i = 1, 2$. The sign of c_i is indeterminate. We therefore assume it to be positive. The steady state equilibrium fish stocks x_1^* and x_2^* satisfy

$$g^{(i)}(x_i^*, x_2^*) = 0, \quad i = 1, 2. \tag{5}$$

It is clear that there are four steady states. Our main interest in this paper, however, is the one in which both species of fish exist. Defining $\Delta \equiv a_1 a_2 - \theta_1\theta_2 > 0$ and solving

$$a_i x_i + \theta_i x_j = c_i, \quad i \neq j, \ i, j = 1, 2 \tag{6}$$

we have

$$x_i^* = \frac{c_i a_j - \theta_i c_j}{\Delta}, \quad i \neq j, \ i, j = 1, 2. \tag{7}$$

Clearly, $x_1^* > 0$, but the sign of x_2^* is indeterminate. We must therefore assume that

$$a_1 c_2 > \theta_2 c_1 \tag{8}$$

to ensure the prey to be non-extinct in the steady state equilibrium.

We are now in a position to analyze the dynamic property of predator –prey in the neighborhood of the non-extinct steady state. Letting $X_i \equiv x_i - x_i^*$ and taking into account the linear approximation of $g^{(i)}(x_1, x_2)$ in the neighborhood of the equilibrium, we have

$$\frac{dX_i}{dt} = -a_i x_i^* X_i - \theta_i x_i^* X_j, \quad i \neq j, \ i, j = 1, 2. \tag{9}$$

The characteristic equation for the coefficient matrix of the right hand side of (9) is

$$\begin{vmatrix} \lambda + a_1 x_1^* & \theta_1 x_i^* \\ \theta_2 x_2^* & \lambda + a_2 x_2^* \end{vmatrix} = 0. \tag{10}$$

This leads to

$$\lambda_1 + \lambda_2 = -(a_1 x_1^* + a_2 x_2^*) < 0, \quad \lambda_1\lambda_2 = x_1^* x_2^* \Delta > 0, \tag{11}$$

showing that both characteristic roots are negative. This proves the local stability of the non-extinct equilibrium in our predator-prey model of commercial fishing.

Our next and main task is to prove that our fishing model does not yield any limit cycle. Before tackling this problem we need to cite the following theorem (see Clark, 1976).

Bendixon-du Lac theorem: Let

$$\begin{aligned} \frac{dx_1}{dt} &= x_1(a_{11}x_1 + a_{12}x_2 + b_1) \equiv f_1(x_1, x_2) \\ \frac{dx_2}{dt} &= x_2(a_{21}x_1 + a_{22}x_2 + b_2) \equiv f_2(x_1, x_2) \end{aligned} \tag{12}$$

be a two variable system of differential equations, and assume that

$$A \equiv \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0,$$

$$B \equiv a_{11}b_2(a_{12} - a_{22}) + a_{22}b_1(a_{21} - a_{11}) \neq 0.$$

Define further that

$$\sigma_1 \equiv \frac{a_{22}(a_{21}-a_{11})}{A}, \quad \sigma_2 \equiv \frac{a_{11}(a_{12}-a_{22})}{A},$$
$$g(x_1, x_2) \equiv x_1^{\sigma_1 - 1} x_2^{\sigma_2 - 1}.$$

Then, (12) does not yield any limit cycle in the first quadrant if

$$\frac{\partial}{\partial x_1}(g f_1) + \frac{\partial}{\partial x_2}(g f_2)$$

does not change sign in the first quadrant of the phase plane.

We calculate A and B for our dynamic system (4) as follows.

$$A = \Delta = \begin{vmatrix} -a_1 & -\theta_1 \\ -\theta_2 & -a_2 \end{vmatrix} > 0,$$
$$B = -a_1 c_2(-\theta_1 + a_2) - a_2 c_1(-\theta_2 + a_1)$$
$$< (\theta_1\theta_2 - a_1 a_2)c_1 + (c_1\theta_2 - a_1 c_2)a_2 < 0,$$

where we have taken into account (8) and $\theta_1 < 0$. Now let

$$\sigma_1 \equiv \frac{-a_2(-\theta_2 + a_1)}{\Delta}, \quad \sigma_2 \equiv \frac{-a_1(-\theta_1 + a_2)}{\Delta},$$
$$g(x_1, x_2) \equiv x_1^{\sigma_1 - 1} x_2^{\sigma_2 - 1}.$$

Then,

$$\frac{\partial}{\partial x_1}\left\{g(x_1, x_2) g^{(1)}(x_1, x_2)\right\} + \frac{\partial}{\partial x_2}\left\{g(x_1, x_2) g^{(2)}(x_1, x_2)\right\}$$
$$= g(x_1, x_2)\{\sigma_1(-a_1 x_1 - \theta_1 x_2 + c_1) - a_1 x_1 + \sigma_2(-\theta_2 x_1 - a_2 x_2 + c_2) - a_2 x_2\}$$
$$= \frac{g(x_1, x_2)\{(a_2\theta_2 - a_1 a_2)c_1 + (a_1\theta_1 - a_1 a_2)c_2\}}{\Delta}.$$

The expression in the brace of the numerator of the above expression, therefore, the last expression above does not change its sign. Hence, (4) satisfies all conditions for the Bendixon-du Lac theorem for non-existence of limit cycle. This completes the proof of the non-existence of limit cycle in our commercial fishing model with predator-prey interaction.This result is in sharp contrast to the one of Gandolfo (1997) of the existence of limit cycle in the original Lotka-Volterra prey-preditor interaction model without fishing.

3. EXTINCT EQUILIBRIA

The differential equation system (4) has three steady state equilibria in which at least one species become extinct. We now discuss the dynamics for these equilibria.

Case 1. $x_1^* = x_2^* = 0$. In this case the characteristic equation for the coefficient matrix for the Taylor expansion of (4) in the neighborhood of the equilibrium has positive roots $\lambda_1 = c_1, \lambda_2 = c_2$, showing that $(0, 0)$is an unstable node.

Case 2. $x_1^* = 0, x_2^* = \frac{c_2}{a_2}$. In this case $\lambda_1 = -\frac{c_2\theta_1}{a_2} + c_1 > 0, \lambda_2 = -c_2 < 0$. This shows that $(0, \frac{c_2}{a_2})$ is a saddle point.

Case 3. In this case $\lambda_1 = -c_1 < 0, \lambda_2 = -\frac{c_1\theta_2}{a_1} + c_2 > 0$ and the equilibrium

$(\frac{c_1}{a_1}, 0)$ is a saddle point. Note that λ_2 becomes positive by virtue of the assumption (8) introduced above.

Incorporating these information we can now depict a phase diagram for (4) as below, where $E_0 = (0, 0)$, $E_1 = \left(\frac{c_1}{a_1}, 0\right)$, $E_2 = \left(0, \frac{c_2}{a_2}\right)$, $E^* = \left(x_1^*, x_2^*\right)$.

4. CONCLUSION

In this paper we have proved that under the crucial assumption (8), limit cycle does not arise in the commercial fishing model which takes into account predator-prey interaction but assumes away entry of new fleet. We have also proved that the unique non-extinct steady state equilibrium is locally stable and characterized the properties of the equilibria which cause extinction of at least one species of fish. Finally, we have given a phase diagram for the solution of (4). The arrows in the diagram shows that the non-extinct steady state equilibrium is likely to be reached in the long run if the initial fish stocks of both species are positive, and that no limit cycle arises.

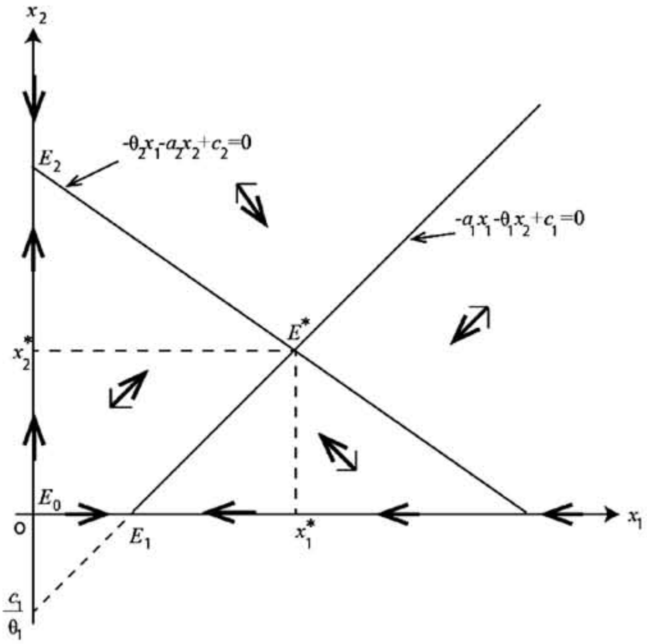

Figure 1. Phase diagram: No limit cycle arises and E^* is reachable from any non-extinct initial state of both species.

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