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# MONOPOLISTIC CHOICE OF PRODUCT SPECIFICATIONS WHEN HIGHER END PRODUCT SPECIFICATIONS PROVIDE IMPERFECT SIGNALS OF THE PERFORMANCE OF THE LOW END PRODUCT

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*Abstract*: In this paper we introduce the possibility that customers can only observe technical specifications and are unable to identify the exact performance level of a product a priori. We explore the possibility that firms making technically sophisticated high end products induce the belief among customers that the low end products they produce are likely to perform better because of their higher technical skill. We show that in the presence of this quality spillover effect, the standard result of monopolistic quality discrimination breaks down. It is possible that in equilibrium, customers are provided quality levels above or below the socially optimal level, irrespective of the group they belong to.

**Key words:** technical specification and performance, quality spillovers, product differentiation, monopolistic quality discrimination.

**JEL Classification Number:** D21, L12, L15.

## 1. INTRODUCTION

Consider the case of a firm that offers different varieties of the same basic product to customers. The varieties differ in the levels of sophistication of their technical features. In such cases, it is fairly commonplace for customers to refer to different varieties as

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high end and low end products, where high end products are the more technically sophisticated products that are expected to perform better than the less technically sophisticated low end products. Ostensibly, firms resort to such business strategies because they face a heterogeneous customer base with variations in individual incomes and valuation of quality of performance. Thus, loosely speaking, less sophisticated products are designed for customers who have low incomes and/or are unwilling to pay much for superior levels of performance; and sophisticated products are designed for customers who have higher incomes and/or attach greater value to product performance. The firm is thus faced with the following decision problem: what is the optimal menu of products (and corresponding prices) that it should offer to its customers? The decision problem becomes more complex once one recognizes that a firm may have a fairly good idea about the distribution of the types of customers it faces, but may not have a foolproof way of identifying a customer's type *a priori*. In that case, a mistake in quality selection and pricing by the firm can result in customers who value quality more opting to purchase products designed for customers who have a lower valuation of quality. The net result from the firms' perspective is that it would earn a lower level of profit than it could have because it failed to induce people who are willing to pay more for higher quality of performance actually pay for higher performance. At the heart of the literature on monopolistic quality discrimination lie two questions: what is the menu of "quality" and corresponding prices that the firm should select to induce buyers of each type to buy the product that was designed for them, and also maximize its total profit? And how does this menu differ from the one that a completely informed social planner would offer if he were seeking to maximize social welfare?

In the standard model of quality discrimination by a monopolistic seller who faces a heterogeneous set of customers, the problem is characterized by the rent extraction-efficiency tradeoff. In order to save the high type consumers' information rent, the monopolist comes to terms with the efficiency loss in a trade with low type consumers. In the standard model, no distinction, however, is made between the level of technical sophistication of a product and its performance. The two are lumped together under a "catch-all" term, quality. In practice, buyers observe technical specifications and may use other information—brand names, feedback from previous users, advertisement, market shares, etc.—to arrive at a judgement about the kind of performance that can be expected from the product. This, in turn, helps to determine a buyer's willingness to pay for a unit of the product. In this paper we introduce the possibility that customers can only observe technical specifications and are unable to identify the exact performance level of the product *a priori*. In that sense, the goods in our model are experience goods: a customer may form *a priori* beliefs about the quality of performance of the good based on a study of its technical specification, hearsay evidence, etc., but is able to form a clear idea of performance only after consumption. Advertisements for cars, electronic equipments like computers, audio and video systems provide technical details that are often largely meaningless for the average customer. In particular, we assume that buyers can form an idea (more specifically, a probability distribution) of performance levels based on the quality specification such that a "higher" quality specification generates a better distribution of performance in the First-Order Stochastic Dominance (FSD) sense. This implies that a customer with a given valuation of quality is willing to pay more for a better quality specification[1]. This per se does not affect the basic result of the theory of monopolistic quality discrimination in any meaningful way. The result breaks down once we introduce some spillover effects as in Kim and Kim (1996) who introduced technical spillover effects into the model by assuming that the cost of production of the low end quality falls as the quality of the high end product increases.

In this paper we explore the possibility that firms making technically sophisticated high end products induce the belief among customers that the low end products they produce are likely to perform better because of their higher technical skill. This positive externality is thus unidirectional. Automobile manufacturers, high fashion apparel and accessory manufacturers, etc. produce low end products that are often valued more by customers than comparable products made by companies that do not make the high end products. While part of the reason could lie in brand snobbery, a likely alternative explanation could be that customers expect firms, that have the technical skill to make high quality products, are also likely to make good quality low end products. In that sense, brand names are bearers of information as in Wernerfelt (1988), Tadelis (1999) and Cabral (2000). Our problem is different from the ones addressed in these papers in that we explore the effects of quality spillovers on quality choices made by a seller under second degree quality discrimination. In Tadelis (1999), each product is sold under a different name and these names can be traded. The model looks at equilibrium in the market for names. Wernerfelt (1988) and Cabral (2000) focus on the firm's decision to brand stretch, given fixed qualities. In Cabral (2000), firms introduce new products whose qualities are known to them but are unknown to buyers *a priori*. *Ex post*, buyers can update their beliefs about the quality of the product based on their experience about the performance of the product. A firm that introduces new good using its established brand name faces the possibility that its reputation may suffer if the buyers' experience is not good. In our analysis brand stretching is definitional: a firm that produces high end products knows that its high end quality specification affects customers' valuation of its lower end products. The question is: will a firm facing two types of customers provide a quality level above the socially optimal level to its high valuation customers in equilibrium? We show in this paper that in the presence of a quality spillover effect, the standard result of monopolistic quality discrimination breaks down. The rent extraction-efficiency tradeoff is affected by the one-sided externality discussed above, and so the optimal levels of product qualities solving the monopolist's screening problem differ from the standard case. Among a host of possibilities, it is possible that in equilibrium, customers are provided quality levels above or below the socially optimal level irrespective of whether they belong to high end group or the low end group.

The paper is organized as follows: In section 2 we present the formal model and derive the principal result. Section 3 offers some concluding remarks.

[1] Our formal specification draws heavily on Tadelis (1999) and Cabral (2000).

## 2. THE MODEL

Consider a monopolist producing a vertically differentiated good. In the standard formulation of the model, there are (say) two types of customers indexed by $i = 1, 2$ and there are $n_i$ customers of type $i$, where $n_i > 0$. Each customer buys at most one unit of the good. The monopolist chooses characteristics $(q_1, q_2)$ from the interval $[\underline{q}, \overline{q}]$ and a corresponding set of prices $(p_1, p_2)$ for the product. Potential customers observe the monopolist's choice of $(q_1, q_2)$ and $(p_1, p_2)$ before deciding the type of product they wish to purchase. What this presumes is that the announced quality specification provides an exact (degenerate distribution) signal to the buyer of the nature of performance that he can expect from the product. In other words, the goods in the standard model are not experience goods. The critical feature of our model is that our goods are experience goods: buyers can observe quality specifications *a priori*, but these specifications provide a noisy signal of the quality of performance that they can expect when they use it.

Let $r \in [\underline{r}, \overline{r}]$ designate the actual performance of the product. This is not directly observable before the product is used. However, customers can form beliefs about the likelihood of different performance levels based on the announced quality specification. Such beliefs are captured by a cumulative distribution function, say $F(r|q_j, q_k)$, $q_j$, $q_k = 1, 2, q_j \neq q_k$, where the first element $q_j$ designates the quality purchased and the second element $q_k$ is the other available quality. The standard monopolistic quality discrimination model then reduces to the special case where there is a monotonically increasing function $x : [\underline{q}, \overline{q}] \rightarrow [\underline{r}, \overline{r}]$ such that $F(r|q_j, q_k) = 0$ if $r < x(q_j)$ and equals 1 otherwise. Notice that $x(\cdot)$ does not depend upon $q_k$.

The utility that a type-i customer derives from the consumption of the good is a function of the performance level of the product and a type specific taste parameter $\alpha_i$:

$$u^i = u(r, \alpha_i), \quad \text{with } \partial u(r, \alpha_i)/\partial r > 0 \quad \text{and} \quad \partial^2 u(r, \alpha_i)/\partial r^2 < 0, \quad i = 1, 2 \quad (1)$$

Hence, the willingness of customer i to pay for a product of quality specification $q_j$ when $q_k$ is the other quality that is available, is

$$V_j^i(q_j, q_k) = \int_{\underline{r}}^{\overline{r}} u(r, \alpha_i) dF(r|q_j, q_k), \quad i = 1, 2, \quad j, k = 1, 2, \quad j \neq k \quad (2)$$

$$= u(\overline{r}, \alpha_i) F(\overline{r}|q_j, q_k) - u(\underline{r}, \alpha_i) F(\underline{r}|q_j, q_k) - \int_{\underline{r}}^{\overline{r}} F(r|q_j, q_k) du(r, \alpha_i)^2,$$

$$i = 1, 2, \quad j, k = 1, 2, \quad j \neq k$$

Since $u(r, \alpha_i)$is differentiable in r and since $F(\overline{r}|q_j, q_k) = 1$ and $F(\underline{r}|q_j, q_k) = 0$,

[2] See T.M. Apostol (1974), p. 144.

$$V_j^i(q_j, q_k) = u(\overline{r}, \alpha_i) - \int_{\underline{r}}^{\overline{r}} (\partial u(r, \alpha_i)/\partial r) F(r|q_j, q_k) dr \tag{2'}$$

We assume that the total utility as well as the marginal utility is higher for the type-2 customers than for the type-1 ones:

$$u(r, \alpha_2) > u(r, \alpha_1) \quad \forall r \in [\underline{r}, \overline{r}] \tag{3}$$

$$u_r(r, \alpha_2) > u_r(r, \alpha_1) \quad \forall r \in [\underline{r}, \overline{r}] \tag{4}$$

It is easy to see that equation (2) and inequality (3) imply $V_1^2 > V_1^1$ and $V_2^2 > V_2^1$. Consider now a customer of type $i$ who is considering purchasing the good with quality specification $q_j$. If the value of $q_j$ increases, i.e. the quality specification of the good under consideration improves, then this generates the belief in the customer's mind that the probability of getting a better level of performance from the product is higher than before. A change in the quality specification of the other (quality) good available, i.e. a change in the value of $q_k$, has asymmetric effects. In the presence of the one-sided spillover effect, mentioned earlier, the probability distribution of performance at the lower end "improves" as the quality specification at the higher end increases, while the probability distribution of performance levels at the higher end remains unaffected by changes in the lower end quality specification. If the good being considered by the customer is the one with the lower quality specification then an improvement in the quality specification of the higher end product has a "positive" impact on the customer's beliefs about the performance he can expect from the product he is looking at. On the other hand, if the good under consideration is at the higher end of the quality spectrum being offered, a change in the quality specification of the low end product has no impact on the customer's beliefs about the performance that he can expect from the high end product. To capture this intuition we assume that in each of the first two cases, the lower quality specification generates a "better" distribution of performance in the First-Order Stochastic Dominance (FSD) sense. In the last case, a change in $q_k$ does not have any effect on the probability distribution of performances.

The Distribution $F(x, y)$ first-order stochastically dominates the distribution $F(x, y'), y \neq y'$, if, for every non-decreasing function u: $R \to R$ we have

$$\int_x u(x) dF(x, y) \geq \int_x u(x) dF(x, y') \tag{5}$$

A well-known result is that

$$\int_x u(x) dF(x, y) \geq \int_x u(x) dF(x, y') \Leftrightarrow F(x, y) \leq F(x, y') \quad \text{for every } x \,.$$

Thus, in formal language, we assume[3]:

[3] It is to be noted that $F(r|q_j, q_k)$ is defined as the cumulative distribution of performance where $q_j$ is the quality purchased and $q_k$ is the other alternative quality.

(I) for all $q_k, q_j, q'_j \in [\underline{q}, \overline{q}]$, and $q_j > q'_j$, $F(r|q_j, q_k) \leq F(r|q'_j, q_k)$ for every $r \in [\underline{r}, \overline{r}]$ with the strict inequality holding at some $r$;

(II) for all $q_k, q_j, q'_j \in [\underline{q}, \overline{q}], q'_j > q_j > q_k$, $F(r|q_k, q'_j) \leq F(r|q_k, q_j)$ for every $r \in [\underline{r}, \overline{r}]$ with the strict inequality holding at some $r$; and

(III) for all $q_k, q_j, q'_j \in [\underline{q}, \overline{q}], q_k > q'_j > q_j$, $F(r|q_k, q_j) = F(r|q_k, q_{j'})$ for every $r \in [\underline{r}, \overline{r}]$.

If $F(\cdot)$ is differentiable with respect to $q_j, q_k$ then assumptions (I′), (II′) and (III′) stated below are equivalent to assumptions (I), (II) and (III) respectively:

for any $q_j, q_k \in [\underline{q}, \overline{q}]$,

(I′) $\partial F(r|q_j, q_k)/\partial q_j \leq 0$ for every $r \in [\underline{r}, \overline{r}]$ with a strict inequality holding at some $r$;

(II′) if $q_j > q_k$, $\partial F(r|q_k, q_j)/\partial q_j \leq 0$ for every r $\in [\underline{r}, \overline{r}]$ with a strict inequality holding at some $r$; and

(III′) if $q_j \leq q_k$, $\partial F(r|q_j, q_k)/\partial q_j = 0$ for every r $\in [\underline{r}, \overline{r}]$.

PROPOSITION 1.

(i) $\forall q_j, q_k \in [\underline{q}, \overline{q}]$, $\partial u(r, \alpha_i)/\partial r > 0$ *and (I′) imply* $\partial V^i_j(q_j, q_k)/\partial q_j \geq 0$;

(ii) $\forall q_j, q_k \in [\underline{q}, \overline{q}]$ *such that* $q_j > q_k$, $\partial u(r, \alpha_i)/\partial r > 0$ *and* (II′) *imply* $\partial V^i_j(q_j, q_k)/\partial q_j \geq 0$;

(iii) $\forall q_j, q_k \in [\underline{q}, \overline{q}]$ *such that* $q_j < q_k$, $\partial u(r, \alpha_i)/\partial r > 0$ *and* (III′) *imply* $\partial V^i_j(q_j, q_k)/\partial q_j = 0$.

*Proof.* (i) Applying Leibniz Rule to differentiate (2′) with respect to $q_j$

$$\partial V^i_j/\partial q_j \quad = 0 - \int_{\underline{r}}^{\overline{r}} (\partial u(r, \alpha_i)/\partial r)(\partial F(r|q_j, q_k)/\partial q_j)dr \quad i = 1, 2$$

Since, marginal utility $\partial u(r, \alpha_i)/\partial r > 0$ it follows from assumption (I′) that

$$\partial V^i_j(q_j, q_k)/\partial q_j \geq 0\,.$$

(ii) Since, marginal utility $\partial u(r, \alpha_i)/\partial r > 0$ it follows from assumption (II′) that

$$\partial V^i_j(q_j, q_k)/\partial q_j \geq 0\,.$$

(iii) Since, marginal utility $\partial u(r, \alpha_i)/\partial r > 0$ it follows from assumption (III′) that

$$\partial V^i_j(q_j, q_k)/\partial q_j = 0\,.$$

□

In the special case where $q_1 < q_2$, Proposition 1 says that:

$$\partial V^i_1(q_1, q_2)/\partial q_1 > 0 \qquad i = 1, 2 \tag{6a}$$

$$\partial V^i_1(q_1, q_2)/\partial q_2 > 0 \qquad i = 1, 2 \tag{6b}$$

$$\partial V^i_2(q_2, q_1)/\partial q_1 = 0 \qquad i = 1, 2 \tag{6c}$$

$$\partial V^i_2(q_2, q_1)/\partial q_2 > 0 \qquad i = 1, 2 \tag{6d}$$

PROPOSITION 2.

$$q_1 < q_2 \textit{ and } (\mathrm{III}') \textit{ imply } \partial^2 V_2^i(q_2, q_1)/\partial q_1 \partial q_2 = 0, \quad i = 1, 2.$$

*Proof.* $\partial^2 V_2^i(q_2, q_1)/\partial q_1 \partial q_2 = \partial(\partial V_2^i(q_2, q_1)/\partial q_2)/\partial q_1 = \partial(\partial V_2^i(q_2, q_1)/\partial q_1)/\partial q_2 = 0$ (using (III′). □

Finally, assume that the total cost of production, $C(q)$, and the marginal cost vary positively with the quality:

$$dC(q)/dq > 0 \quad \text{and} \quad d^2C(q)/dq^2 > 0. \tag{7}$$

### 2.1. *The Social Planner's Solution*

To establish the benchmark qualities against which we shall measure the distortion that the profit-maximizing monopolist introduces in the quality spectrum, we first look at the quality choices of the Social Planner who chooses quality levels to maximize social welfare. In accordance with analytical custom, we assume that the social planner, unlike the monopolist, is fully informed about each potential customer's preference pattern.

Let W be the social welfare level. Hence the problem of the social planner is:

$$\underset{(q_1, q_2)}{\text{Max}} \; W = n_1[V_1^1(q_1, q_2) - C(q_1)] + n_2[V_2^2(q_2, q_1) - C(q_2)]$$

Let $(q_1^*, q_2^*)$ be the unique interior extremum of the welfare-maximization problem, that is, $q_1^* \in [\underline{q}, \overline{q}]$ and $q_2^* \in [\underline{q}, \overline{q}]$. Then $(q_1^*, q_2^*)$ satisfy the following first-order conditions:

$$\partial W/\partial q_1 = n_1[\partial V_1^1(q_1, q_2)/\partial q_1 - dC(q_1)/dq_1] = 0 \tag{8}$$

$$\partial W/\partial q_2 = n_1[\partial V_1^1(q_1, q_2)/\partial q_2] + n_2[\partial V_2^2(q_2, q_1)/\partial q_2 - dC(q_2)/dq_2] = 0 \tag{9}$$

Let $q_1 = h(q_2)$ be the solution to (8) and $q_1 = g(q_2)$ be the solution to (9).

Assume further that $q_1^* < q_2^*$. Let the locus of $(q_1, q_2)$ satisfying equations (8) and (9) be denoted by $M_1$ and $M_2$ respectively. The slopes of $M_1$ and $M_2$ are given by:

$$\begin{aligned}(dq_2/dq_1)_{M1} &= -(\partial^2 W/\partial q_1^2)/(\partial^2 W/\partial q_1 \partial q_2) \qquad (10)\\ &= -[\partial^2 V_1^1(q_1, q_2)/\partial q_1^2 - d^2C(q_1)/dq_1^2]/\partial^2 V_1^1(q_1, q_2)/\partial q_1 \partial q_2\\ (dq_2/dq_1)_{M2} &= -(\partial^2 W/\partial q_1 \partial q_2)/(\partial^2 W/\partial q_2^2)\\ &= -n_1[\partial^2 V_1^1(q_1, q_2)/\partial q_1 \partial q_2]/A \qquad (11)\end{aligned}$$

where $A = n_1 \partial^2 V_1^1(q_1, q_2)/\partial q_2^2 + n_2 \partial^2 V_2^2(q_2, q_1)/\partial q_2^2 - n_2 d^2C(q_2)/dq_2^2$

Second-order conditions require that the principal minors of the relevant Hessian determinant evaluated at $(q_1^*, q_2^*)$ alternate in sign:

$$i.e., \quad \partial^2 W/\partial q_1^2 < 0, \partial^2 W/\partial q_2^2 < 0 \tag{12}$$

$$\text{and} \quad \begin{vmatrix} \partial^2 W/\partial q_1^2 & \partial^2 W/\partial q_2 \partial q_1 \\ \partial^2 W/\partial q_1 \partial q_2 & \partial^2 W/\partial q_2^2 \end{vmatrix} > 0 \tag{13}$$

The following Lemma describes the slopes of $M_1$ and $M_2$.

LEMMA 1. *Slopes of $M_1$ and $M_2$ depend on the sign of $\partial^2 W/\partial q_1 \partial q_2$. However, $M_1$ is "steeper" than $M_2$ irrespective of the sign of $\partial^2 W/\partial q_1 \partial q_2$.*

*Proof.* It follows immediately from equations (10), (11) and condition (12) that if $\partial^2 W/\partial q_1 \partial q_2$ is positive then $M_1$ and $M_2$ are both positively sloped; and they are both negatively sloped when $\partial^2 W/\partial q_1 \partial q_2$ is negative.

Again equations (10) and (11) imply that

$$
\begin{aligned}
&(dq_2/dq_1)_{M1} - (dq_2/dq_1)_{M2} \\
&= -[(\partial^2 W/\partial q_1^2)/(\partial^2 W/\partial q_1 \partial q_2) - (\partial^2 W/\partial q_1 \partial q_2)/(\partial^2 W/\partial q_2^2)] \\
&= -[((\partial^2 W/\partial q_1^2)(\partial^2 W/\partial q_2^2) - (\partial^2 W/\partial q_1 \partial q_2)^2)/((\partial^2 W/\partial q_1 \partial q_2)(\partial^2 W/\partial q_2^2))] \\
&= 1/(\partial^2 W/\partial q_1 \partial q_2)[-((\partial^2 W/\partial q_1^2)(\partial^2 W/\partial q_2^2) - (\partial^2 W/\partial q_1 \partial q_2)^2)/\partial^2 W/\partial q_2^2]
\end{aligned}
$$

The second order conditions for maximum (12) and (13) imply that the bracketed term is positive. Hence,

$$\text{if } \partial^2 W/\partial q_1 \partial q_2 > 0 \text{ then at } (q_1^*, q_2^*), (dq_2/dq_1)_{M1} > (dq_2/dq_1)_{M2} > 0 \text{ and}$$

$$\text{if } \partial^2 W/\partial q_1 \partial q_2 < 0 \text{ then at } (q_1^*, q_2^*), (dq_2/dq_1)_{M1} < (dq_2/dq_1)_{M2} < 0$$

i.e., $M_1$ is "steeper" than $M_2$ in both the cases. □

From Proposition 2 it follows that $\partial^2 W/\partial q_1 \partial q_2 = n_1 \partial^2 V_1^1(q_1, q_2)/\partial q_1 \partial q_2$. This can be either positive or negative. Now $\partial^2 V_1^1(q_1, q_2)/\partial q_1 \partial q_2 = \partial/\partial q_2(\partial V_1^1(q_1, q_2)/\partial q_1)$ shows how quality of high end product has an effect on the consumers' marginal willingness to pay for the low end product. If the effect is positive (negative), i.e., $\partial^2 V_1^1(q_1, q_2)/\partial q_1 \partial q_2 > 0$ ($< 0$) then the consumers' marginal willingness to pay for the low end product increases (decreases) as quality at the high end improves (deteriorates).

Figures 1.a and 1.b show the optimum choice of quality of the Social Planner in the two cases.

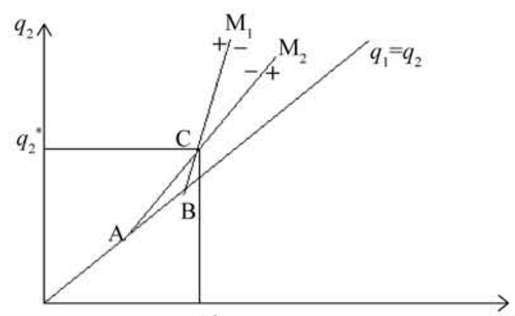


Figure 1a. The Social Planner's optimum choice of quality (point C) when $\partial^2 V_1^1(q_1, q_2)/\partial q_1 \partial q_2 > 0$.

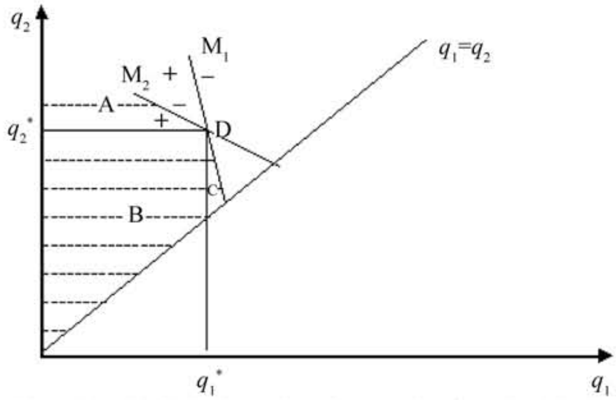


Figure 1b. The Social Planner's optimum choice of quality (point D) when $\partial^2 V_1^1(q_1, q_2)/\partial q_1 \partial q_2 < 0$.

### 2.2. *Quality Choice under Asymmetric Information: The Monopolist's Solution*

Unlike the Social planner, the monopolist suffers from an informational deficiency: he is unable to identify the type of each customer *a priori*. Under this condition, the monopolist's problem is to select a profit maximizing pair of customer type specific contracts $(p_1, q_1)$ and $(p_2, q_2)$ such that for each type of customer there is at least one acceptable contract (individual rationality); and no customer is better off accepting a contract designed for a customer whose type is different from his own (incentive compatibility). Formally, the monopolist's problem is:

$$\underset{(p_i, q_i), \quad i = 1, 2}{\text{Max}} \quad \Pi = \sum_{i=1}^{2} [p_i - C(q_i)] n_i$$

subject to:

(i) $p_1 \leq V_1^1(q_1, q_2)$

(ii) $p_2 \leq V_2^2(q_2, q_1)$

(iii) $V_1^1(q_1, q_2) - p_1 \geq V_2^1(q_2, q_1) - p_2$

(iv) $V_2^2(q_2, q_1) - p_2 \geq V_1^2(q_1, q_2) - p_1$

Conditions (i) and (ii) are the individual rationality constraints for the first and second customer types respectively; and conditions (iii) and (iv) are their respective incentive compatibility constraints. It is possible, of course, that no such separating menu of contracts exists and the profit maximizing strategy is to offer the same contract for all types of customers (pooling contract). A second possibility is that it is best for the monopolist to serve only one type of customer (partial market coverage). In what follows we assume that a separating menu of contracts exists. It is fairly straightforward to show that only constraints (i) and (iv) bind in equilibrium[4] and so after some straightforward substitutions the monopolist's problem reduces to:

$$\underset{(q_1,q_2)}{\text{Max}} \quad [V_1^1(q_1,q_2) - C(q_1)]n_1 + [V_2^2(q_2,q_1) + V_1^1(q_1,q_2) - V_1^2(q_1,q_2) - C(q_2)]n_2$$

The first-order conditions for an interior maximum are:

$$n_1[\partial V_1^1(q_1,q_2)/\partial q_1 - dC(q_1)/dq_1] + n_2[\partial V_1^1(q_1,q_2)/\partial q_1 - \partial V_1^2(q_1,q_2)/\partial q_1] = 0 \tag{14}$$

and

$$n_1\partial V_1^1(q_1,q_2)/\partial q_2 + n_2[\partial V_2^2(q_2,q_1)/\partial q_2 - dC(q_2)/dq_2] + n_2[\partial V_1^1(q_1,q_2)/\partial q_2 - \partial V_1^2(q_1,q_2)] = 0 \tag{15}$$

The second-order conditions are assumed to hold. Let $(q_1^m, q_2^m)$ be the unique profit-maximizing quality levels of the monopolist.

We are now in a position to answer the fundamental question that we ask in this paper: if improvements in the higher end quality specification have a positive effect on customers' beliefs about the performance that they can expect from products at the lower end of the quality spectrum, how do the quality levels chosen by the monopolist differ from those of the social planner? Figures 1.a and 1.b provide an intuitive idea of the answer to this question. Consider first the case where $\partial^2 W/\partial q_1\partial q_2 > 0$ [Figure 1.a]. Since $(q_1^*, q_2^*)$ is by assumption a unique extremum, $\partial^2 W/\partial q_1^2 < 0$ for all $(q_1, q_2)$ in the interior of $[\underline{q}, \overline{q}] \times [\underline{q}, \overline{q}]$. This means that to the left of $M_1$, $\partial W/\partial q_1 > 0$ and to the right of $M_1$, $\partial W/\partial q_1 < 0$. Again, since by assumption $\partial^2 W/\partial q_1\partial q_2 = \partial/\partial q_1(\partial W/\partial q_2) > 0$, $\partial W/\partial q_2 > 0$ to the right of $M_2$ and $\partial W/\partial q_2 < 0$ to the left of $M_2$. Now, using equations (8) and (14):

$$\partial W(q_1^m, q_2^m)/\partial q_1 = -n_2[\partial V_1^1(q_1^m, q_2^m)/\partial q_1 - \partial V_1^2(q_1^m, q_2^m)/\partial q_1] > 0$$

Similarly, using equations (9) and (15):

$$\partial W(q_1^m, q_2^m)/\partial q_2 = -n_2[\partial V_1^1(q_1^m, q_2^m)/\partial q_2 - \partial V_1^2(q_1^m, q_2^m)/\partial q_2] > 0$$

It follows immediately then that $(q_1^m, q_2^m)$ must lie within the region 'ABC' of Figure 1.a that lies to the south west of $(q_1^*, q_2^*)$. In other words, $q_1^m < q_1^*$ and $q_2^m < q_2^*$. However, if $\partial^2 W/\partial q_1\partial q_2 < 0$, $\partial W/\partial q_2 > 0$ to the left of $M_2$ and $\partial W/\partial q_2 < 0$ to the right of $M_2$. Other arguments remaining same, Figure 1.b shows that $(q_1^m, q_2^m)$ may lie in any of the shaded regions 'A', 'B' or 'C'. In region 'A', $q_1^m < q_1^*$ and $q_2^m > q_2^*$, in region 'B', $q_1^m < q_1^*$ and $q_2^m < q_2^*$, in region 'C', $q_1^m > q_1^*$ and $q_2^m < q_2^*$.

Before we formally state and prove this result, consider two numerical examples which corroborate this intuition about the relationship between the monopolist's optimal

[4] If (i) is satisfied, i.e., the net surplus of the low demand customers is positive then the high demand customers are automatically willing to purchase, i.e., (ii) is also satisfied. Moreover, the monopolist, who benefits from higher prices, is able to extract the entire surplus from the lower end. Hence, constraint (i) binds in equilibrium. The incentive compatibility constraint (iii) is not relevant as the monopolist is more interested to induce the high demand customers to reveal their true type.

quality levels and those of the social planner.

EXAMPLE 1. Let the performance level, $r$, takes up two distinct values—a high performance denoted by $r = 1$ and a low performance denoted by $r = 0$.

$$\begin{aligned} \text{Let} \quad P(1) &= 1 - [1/(q_1 + q_2)], q_1, q_2 \in [0, 1] \\ P(0) &= 1/(q_1 + q_2) \\ u(r, \alpha_i) &= \alpha_i \sqrt{r} \\ c(q_i) &= cq_i^2/2 \end{aligned}$$

In this case, $\partial^2 W/\partial q_1 \partial q_2 = n_1 \partial^2 V_1^1(q_1, q_2)/\partial q_1 \partial q_2 = -2n_1\alpha_1/(q_1 + q_2)^3 < 0$.

i) Let $n_1 = 100, n_2 = 20, \alpha_1 = 35, \alpha_2 = 40, c = .25$.

$$\begin{aligned} (q_1^*, q_2^*) &= (1.3528, 8.8204) \\ (q_1^m, q_2^m) &= (3.2612, 3.1965) \end{aligned}$$

i.e., the monopolist provides sub-optimal quality at high end but a quality higher than the optimal level at the lower end (region 'C' of Figure 1.b).

ii) Let $n_1 = 100, n_2 = 100, \alpha_1 = 35, \alpha_2 = 40, c = .25$.

$$\begin{aligned} (q_1^*, q_2^*) &= (2.0525, 6.2063) \\ (q_1^m, q_2^m) &= (1.8732, 6.1305) \end{aligned}$$

i.e., the monopolist provides sub-optimal quality at both ends (region 'B' of Figure 1.b).

iii) Let $n_1 = 30, n_2 = 20, \alpha_1 = 35, \alpha_2 = 50, c = .25$.

$$\begin{aligned} (q_1^*, q_2^*) &= (1.829, 6.92) \\ (q_1^m, q_2^m) &= (1.4794, 7.3754) \end{aligned}$$

Here the monopolist provides sub-optimal quality at lower end but a quality higher than the optimal level at the high end (region 'A' of Figure 1.b).

EXAMPLE 2. Let the performance level, r, take up two distinct values— a high performance denoted by $r = 1$ and a low performance denoted by $r = 0$.

$$\begin{aligned} \text{Let} \quad P(1) &= q_1 q_2, q_1, q_2 \in [0, 1] \\ P(0) &= 1 - q_1 q_2 \\ u(r, \alpha_i) &= \alpha_i \sqrt{r} \\ c(q_i) &= cq_i^2/2 \end{aligned}$$

In this case, $\partial^2 W/\partial q_1 \partial q_2 = n_1 \partial^2 V_1^1(q_1, q_2)/\partial q_1 \partial q_2 = n_1\alpha_1 > 0$.

Let $n_1 = 10, n_2 = 20, \alpha_1 = 3, \alpha_2 = 4, c = 5$.

$$\begin{aligned} (q_1^*, q_2^*) &= (.58537, .97561) \\ (q_1^m, q_2^m) &= (.16327, .81633) \end{aligned}$$

i.e., the monopolist provides sub-optimal quality at both ends.

Proposition 3 states and proves the relationship found graphically and numerically between the monopolist's optimal quality levels and those of the social planner.

PROPOSITION 3. *If* $\partial^2 W/\partial q_1 \partial q_2 > 0$ *then* $q_1^m < q_1^*$ *and* $q_2^m < q_2^*$

*Proof.* It follows from Lemma 1 that

$$\text{if } \partial^2 W/\partial q_1 \partial q_2 > 0 \text{ then at } (q_1^*, q_2^*),$$
$$(dq_2/dq_1)_{M1} > (dq_2/dq_1)_{M2} > 0$$

Since, there is a unique maximum it follows that

$$g(q_2) \gtrless h(q_2) \text{ according as } q_2 \gtrless q_2^*.$$

Along $q_1 = h(q_2)$, $\partial W(q_1, q_2)/\partial q_1 = 0$ and since $\partial^2 W/\partial q_1^2 < 0$, at any $(q_1, q_2)$ where $q_1 < h(q_2)$, $\partial W(q_1, q_2)/\partial q_1 > 0$.

Again, along $q_1 = g(q_2)$, $\partial W(q_1, q_2)/\partial q_2 = 0$ and since $\partial^2 W/\partial q_1 \partial q_2 > 0$, at any $(q_1, q_2)$ where $q_1 > g(q_2)$, $\partial W(q_1, q_2)/\partial q_2 > 0$.

Equations (8) and (14) imply $\partial W(q_1^m, q_2^m)/\partial q_1 > 0$.

Similarly, equations (9) and (15) imply $\partial W(q_1^m, q_2^m)/\partial q_2 > 0$.

Hence,

$$g(q_2^m) < q_1^m < h(q_2^m) \qquad \text{(I)}$$

This implies $q_2^m < q_2^*$.

Again, since

$$h'(q_2) > 0, \quad h(q_2^m) < h(q_2^*) = q_1^* \qquad \text{(II)}$$

Combining (I) and (II) we get $g(q_2^m) < q_1^m < h(q_2^m) < h(q_2^*) = q_1^*$,

Hence, the monopolist provides sub-optimal qualities at both ends. □

PROPOSITION 4. *Let* $\partial^2 W/\partial q_1 \partial q_2 < 0$

(i) *If* $q_2^m > q_2^*$ *then the upper limit of* $q_1$ *is given by* $q_1 = g(q_2^m)$

(ii) *If* $q_2^m < q_2^*$ *then the upper limit of* $q_1$ *is given by* $q_1 = h(q_2^m)$

*Proof.* It follows from Lemma 1 that if $\partial^2 W/\partial q_1 \partial q_2 < 0$ then at $(q_1^*, q_2^*)$,

$$(dq_2/dq_1)_{M1} < (dq_2/dq_1)_{M2} < 0$$

Since, there is a unique maximum it follows that

$$g(q_2) \gtrless h(q_2) \text{ according as } q_2 \gtrless q_2^*.$$

$$\text{(i) If } q_2^m > q_2^* \text{ then } g(q_2^m) < g(q_2^*) = q_1^* [\text{since } g' < 0] \qquad \text{(III)}$$

Along $q_1 = g(q_2)$, $\partial W(q_1, q_2)/\partial q_2 = 0$ and since $\partial^2 W/\partial q_1 \partial q_2 < 0$, at any $(q_1, q_2)$ where $q_1 < g(q_2)$, $\partial W(q_1, q_2)/\partial q_2 > 0$.

From equations (9) and (15) we have $\partial W(q_1^m, q_2^m)/\partial q_2 > 0$.

Hence, for

$$q_2 > q_2^*, q_1^m < g(q_2^m) \qquad \text{(IV)}$$

Combining (III) and (IV) we get $q_1^m < g(q_2^m) < g(q_2^*) = q_1^*$

$$\text{(ii) If } q_2^m < q_2^* \text{ then } h(q_2^m) > h(q_2^*) = q_1^* [\text{since } h' < 0] \qquad \text{(V)}$$

Along $q_1 = h(q_2)$, $\partial W(q_1, q_2)/\partial q_1 = 0$ and since $\partial^2 W/\partial q_1^2 < 0$, at any $(q_1, q_2)$ where $q_1 < h(q_2)$, $\partial W(q_1, q_2)/\partial q_1 > 0$.

Equations (8) and (14) give $\partial W(q_1^m, q_2^m)/\partial q_1 > 0$.

$$\text{Hence, for } q_2 < q_2^*, q_1^m < h(q_2^m) \tag{VI}$$

Combining (V) and (VI) we get $q_1^m < h(q_2^m) > h(q_2^*) = q_1^*$

Hence, the monopolist may provide a product with a quality specification level above or below the socially optimal level at both ends. □

It may be noted here that when enhanced level of quality is provided at the higher end, it may not be possible to provide the maximum level of quality, $\overline{q}$, at the higher end. Since we have assumed the cost function to be increasing and convex in nature with respect to the quality of the product, the cost may increase at a faster rate than the quality and hence may not be profitable for the firm to increase the quality of the product beyond a certain level.

## 3. CONCLUSION

In this paper we have studied the impact of quality spillover effects in a vertically differentiated goods model. This spillover effect is one-sided and we assume that changes in the quality specification at the higher end affect the customers' perceptions about the performance of the lower quality good. However, changes in the lower end quality specification of the product have no effect on the customers' perceptions about the performance of the higher quality good. Under this situation, we show that the standard result breaks down. Quality distortion may take place at either end, and it may take the form of either enhanced level of quality or sub-optimal level of quality.

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